Simulating intra-household interactions for in- and out-of-home activity scheduling

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Abstract

Various interactions, time arrangements, and constraints exist for individuals scheduling their day as a member of a household, which affect their in-home as well as out-of-home activity schedule. However, the existing activity-based models are mostly based on the individual decision-making process, which are limited in their demonstration of behaviour. We simulate multiple intra-household interaction dimensions within the same framework and capture the coordination of the activity scheduling decisions among all household members. Our approach adopts the Optimisation-based Activity Scheduling Integrating Simultaneous choice dimensions (OASIS) framework, which is at the level of isolated individuals and focuses on out-of-home activity schedules. We jointly simulate in- and out-of-home activities and incorporate interactions into the framework. Our framework contributes to the state-of-the-art in activity-based modelling by explicitly capturing multiple interactions within the same model, such as the allocation of the private vehicle to household members, dividing household maintenance responsibilities, escorting, joint activity participation, and sharing rides. We operationalise the model using time-use-survey data from the United Kingdom. The simulation results demonstrate the ability of the framework to capture complex intra-household interactions. We then demonstrate how these interactions can cause individuals to deviate from their schedules planned in isolation. This is a general framework applicable to different household compositions and available resources.

Keywords:Intra-household interactions; Group decision-making; Daily scheduling; Activity-based modelling.

1 Introduction

1.1 Motivation

Activity-based models (ABMs) portray how people plan their activities and travels over a period of time such as a day. This approach has been of special interest to transportation modellers. These models try to replicate the actual decisions of travellers with more behavioural realism compared to the traditional trip-based models. Understanding and predicting complex behaviour and interactions throughout the day is the key to better demand-side management and adapting infrastructure systems (e.g. transportation, energy) to deliver critical services that meet the needs of society.

Individuals do not plan their day in isolation from other members of the household. Various interactions, time arrangements, and constraints need to be taken into account, affecting in-home as well as out-of-home activity schedules. For example, individuals in a household synchronize their schedules to create time window overlaps for joint activities such as jointly participating in a recreational activity. The coordination of schedules is not only limited to out-of-home activities but also includes joint in-home activities such as having a family dinner at home. Household members coordinate their travels as well, in order to travel together to joint activities, escort children, or simply share rides. Therefore, policies directly affecting the activity and travel patterns of an individual, such as earlier school starting times, can affect the schedule of multiple household members.

The members of a household also share responsibilities and resources with each other to satisfy household needs. These allocations are done such that it is the most desirable for the entire household rather than a specific member under social, spatial, and resource constraints. For instance, an employed partner might take public transportation to commute to work in order to leave the car for their partner to do the shopping. In addition, an adult individual might cancel a leisure activity to escort their children to school. The escorting duty affects the schedule and travel patterns of the adult members as they should accommodate the pick-up and drop-off activities into their schedule. Therefore, considering the interpersonal dependencies in a household, the activity schedule should be addressed from a group decision-making point-of-view rather than isolated agents in order to reflect reality.

In spite of the interest in activity scheduling and the substantial development of ABMs within transport modellers, only a limited number of studies examine household decision-making perspectives and consider the effect of intra-household interactions in their ABMs (Lai et al. 2019, Gliebe & Koppelman 2005, 2002). Most activity-based studies examine intra-household dependencies at the top-level of activity generation, rather than activity scheduling and travel planning. Early activity-based studies address inter-household interactions implicitly by using household characteristics as explanatory variables for individual decisions (Ho & Mulley 2015). These models cannot explicitly evaluate the impact of intra-household interactions on the schedule of individuals, however. In addition, they are not capable of examining policies aimed at groups rather than individuals. Therefore, capturing the inter-personal effects of household members on their daily schedules needs explicit modelling of household interactions.

Studies that consider inter-household interactions more explicitly, only incorporate one or a few interaction dimensions, such as joint travels (Vovsha et al. 2003) or escorting children (Vovsha & Petersen 2005), rather than encompassing multiple interactions within the same model. The majority of the existing explicit models consider homogeneous intra-household interactions rather than heterogeneous interactions. However, different household members influence household decisions to different extents depending on their status and personal characteristics. In addition, current activity-based models only focus on out-of-home activities and do not contain any information on activities performed at home. These limitations in the current literature leave us a gap to contribute to the state of the art by jointly modelling the time-use in the home alongside activity participation outside the home while explicitly capturing the intra-household interactions. Addressing this gap enables us to have a better estimation of the activity and travel patterns of people and thus, their associated demands.

1.2 Contributions and scope

In this paper, we propose a framework to simulate the daily activity schedules of individuals in a household, explicitly accounting for multiple complex interactions among household members. Our scheduling model is based on a mixed-integer utility optimisation approach. We adopt the OASIS framework (Pougala et al. 2022*b*), which is at the level of isolated individuals and focuses on out-of-home activity schedules. We build on the base model to capture interactions among members of the same household and jointly simulate in- and out-of-home activities. The fundamental assumption of our framework is that individuals do not plan their day in isolation from other members of the household. Our scheduling model contributes to the state-of-the-art in activity-based modelling by explicitly modelling multiple interaction dimensions within the same framework. It is more behaviourally realistic than the traditional ABMs as it simulates the activity schedules of individuals from a group decision-making point-of-view rather than isolated agents.

Another major advantage of our framework is its high level of flexibility. The scheduling problem is formulated as an optimisation problem. The intra-household interactions are captured through constraints and the objective function. Therefore, any interpersonal and temporal dependencies can be comfortably incorporated by modifying the constraints and/or terms of the objective function. One other merit of our model is its simultaneous simulation of different daily scheduling choice dimensions such as activity participation, activity location, activity schedule, activity duration, activity participation mode (solo/joint), and transport mode. This feature which is derived from the base OASIS approach captures trade-offs between different choice dimensions. Furthermore, we jointly model time-use in-home alongside activity participation outside the home within the same framework. This allows capturing the trade-offs between in- and out-of-home activities.

The following example interactions are operationalised in our framework: (i) household private vehicle ownership, (ii) allocation of the private vehicle to household members, (iii) sharing household maintenance responsibilities, (iv) joint activity participation, (v) joint travel to joint activities, and (vi) escorting. Due to the flexible nature of the framework, other interaction dimensions can be arbitrarily added.

It should be noted that interpersonal interactions beyond the household are referred to as social interactions and are out of the scope of this paper. Moreover, we explicitly focus on short-term interactions in our framework. Therefore, we assume long-term household decisions such as the number of household cars, partnerships, children, and home and work locations are exogenous.

The remainder of this manuscript is structured as follows. Section 2 gives a brief review of the literature on interactions in activity-based models. In Section 3, a detailed explanation of the model as well as its key components and features is provided. An illustrative example is presented in Section 4 to showcase the capabilities of the proposed framework. The concluding remarks and opportunities for future research are discussed in Section 5.

2 Relevant literature

Daily scheduling of individuals has been of interest to transport modellers as the demand for travel is assumed to be driven by participation in activities which are distributed in space and time (Hilgert et al. 2017, Bhat et al. 2004, Bowman & Ben-Akiva 2001, Axhausen & Gärling 1992, Chapin 1974, Hagerstrand 1970). There are two major research streams within the scope of activity-based models among transport modellers: (i) empirical rule-based/computational process models (Arentze & Timmermans 2004, Ettema et al. 2000, Pendyala et al. 1998, Golledge et al. 1994), and (ii) econometric models (Palma et al. 2021, Nurul Habib 2018, Bhat 2005, Charypar & Nagel 2005, Bowman & Ben-Akiva 2001, Recker et al. 1986).

Most of the conventional activity-based models in transportation research are based on individual decision-making process where the individuals are treated as isolated agents whose choices are independent of other decision-makers (Habib & Hui 2017, Bhat 2005). However, ignoring the interdependence between household members causes a biased simulation of activity-travel schedules as the schedule of household members are mutually dependent. In spite of the recognition of the importance of incorporating group decision-making paradigm into household travel behaviour in 1980s (Jones et al. 1987), studies on group choice models are relatively new and thus, limited due to methodological difficulties and data availability (Timmermans & Zhang 2009).

The intra-household dependencies in activity-travel behaviour have mostly been explored at the top-level of activity generation and much less at the level of household activity scheduling process, which will output their activity and travel schedules (Bhat et al. 2013, Arentze & Timmermans 2009, Bradley & Vovsha 2005). Early-activity travel models have considered intra-household interactions implicitly. For example, in the household-level Multiple Discrete Continuous Extreme Value (MDCEV) model (Bernardo et al. 2015, Bhat et al. 2013), the household is treated as the decision-maker who allocates activity patterns to its members such that the utility of the total household is maximised. However, as the household is treated as the only decision-maker, there is only one time constraint for the entire household, which is not applicable when household members have different constraints. In the literature, there are also examples of implicit consideration of intra-household interactions by using household characteristics as explanatory variables for individual decisions (Srinivasan & Athuru 2005). This, however, does not ensure the consistency of the choices. Moreover, most of the studies that consider the interactions explicitly, assume the intra-household interplays to be homogeneous. Thus, they do not consider the heterogeneous and context-dependent influence of members on household decisions (Timmermans 2009).

Existing research address only one or few aspects of household interactions within their studies such as resource allocation and usage decisions (Petersen & Vovsha 2005, Miller & Roorda 2003, Arentze & Timmermans 2000), task allocation (Zhang et al. 2005, Srinivasan & Bhat 2005, Vovsha et al. 2004), joint activity participation (Srinivasan & Bhat 2006, Gliebe & Koppelman 2002, Arentze & Timmermans 2000), or travel arrangements (Gupta et al. 2014, Roorda et al. 2006, Vovsha et al. 2003). Key papers studying household interactions in activity-based models are selected from the literature and their findings are discussed in this section.

Arentze & Timmermans (2000) have developed a sequential rule-based model, which simulates the allocation of the car in auto-deficient households to its members for work tours. They take the activities of other household members into consideration but their model is not based on the group decision-making paradigm. Petersen & Vovsha (2005) have proposed a nested-logit model to simulate car allocation and car type. Firstly, the activities are generated. Then, the generated activities are scheduled and out-of-home activities are distributed by travel tours. Joint travel arrangements are considered to consolidate travel needs. Finally, cars are allocated to the tours. Authors suggest that feedback between different stages are required to maintain the model consistency, which can only be accommodated by rule-based algorithms for complex feedbacks such as rescheduling and joint tour formation. Therefore, a model that simulates the choice dimensions simultaneously would be a suitable solution. Vovsha et al. (2004) have suggested a two-step sequential discrete-choice framework applied in a microsimulation fashion, which generates the total daily frequency of maintenance tours and then allocates the maintenance tours to household members for implementation. The microsimulation technique allows for the explicit incorporation of interactions. However, due to the sequential nature of the approach, the trade-offs between choice facets are not considered. Although the breakdown into sequential choice dimensions is convenient, it is oversimplified.

Miller & Roorda (2003) have proposed a sequential rule-based microsimulation model called TASHA, which simulates activity schedules and travel patterns of all individuals in a household. TASHA is a successful first attempt to operationalise a model with a group decision-making paradigm. Bhat et al. (2004) have developed a Comprehensive Econometric Microsimulator for Daily Activity-Travel Patterns (CEMDAP). CEMDAP includes two components: a generation-allocation model system, which simulates the activity participation decisions of individuals, and a scheduling model. Later, Pinjari et al. (2008) incorporates the inter-dependencies between the activity-travel patterns of children and their parents into the CEMDAP framework. The modeling system processes the students and workers before the non-workers and takes a sequential approach for each individual: first the decisions about mandatory activities, then the household maintenance, and finally discretionary and flexible activities are determined. The activity-travel patterns of all individuals in a household are generated in an interleaved fashion. Although both studies provide valuable insights into the effect of interactions on individuals' schedules, their sequential structure does not represent the true nature of the scheduling process. As different choice dimensions are interconnected, the trade-offs between them should be captured simultaneously.

Meister et al. (2005) extend the Genetic Algorithm (GA) scheduler model developed by Charypar & Nagel (2005) to the household level. They accommodate interactions and synchronizations between the schedules of members of a household. This scheduling framework is a multi-agent micro-simulator that generates schedules based on an iterative probabilistic optimisation using a genetic algorithm. However, Charypar & Nagel (2005) argue that GA is not a suitable approach to accommodate multiple social links as it cannot handle many agents simultaneously. Arentze & Timmermans (2009) introduce a utility-based multi-day activity generation model that takes within-household interactions into account. This model should be used alongside a scheduling model to determine the sequence of activities and travel demands as it does not address the scheduling phase. Gupta & Vovsha (2013) propose a joint work activity scheduling model in a multiple-worker household. Their model features exact and fuzzy schedule synchronization mechanisms between workers in a household. The formulation of this work-tour framework can be applied as a part of an ABMs followed by a detailed individual scheduling model.

In order to find the gap and place our research in the literature, we have done a feature comparison between selected existing key approaches, presented in Table 1. The reviewed features are as follows:

1. Simultaneous simulation (SS): all the scheduling choice dimensions such as activity participation, scheduling, location, transport mode, and accompaniment are simulated jointly unlike sequential models. This enhance the behavioural realism of the model and enables capturing trade-offs and interactions.

2. Explicit in-home activities (EIH): activities performed at home are scheduled within the same framework as the out-of-home activities.

3. Resource allocation (RA): the availability and allocation of resources such as the private vehicle and bathrooms are considered within the framework and impacts the scheduling decisions.

4. Task allocation (TA): the allocation of tasks related to the whole household to its members is considered in the model.

5. Joint participation (JP): joint activity participation is captured consistently in the schedules of members of the household.

6. Travel arrangements (TRA): the model can effectively capture synchronized and linked travels such as ride-sharing and escorting.

7. Multi-day dynamics (MDD): the model can accommodate day-to-day correlations between the daily schedules of individuals such as habit formation.

Table 1: Features comparison of the current approaches in the literature

Models	**SS**	**EIH**	**RA**	**TA**	**JP**	**TRA**	**MDD**
Miller & Roorda (2003)			X		X	X	
Bhat et al. (2004), Pinjari et al. (2008)				X		X	
Charypar & Nagel (2005), Meister et al. (2005)	X		X	X	X		
Arentze & Timmermans (2009)				X	X		X
Gupta & Vovsha (2013)				X	X	X	

Ignoring intra-household interactions can cause an overestimation of policy effects and lead to inappropriate actions and investments. Therefore, capturing interpersonal dependencies between individuals belonging to the same household enhances the consistency of the predicted choices and behaviour. Although the aforementioned studies provide ample insights into intra-household interactions in travel demand modelling, there is a gap for an operational household-level joint in- and out-of-home activity-based model that explicitly incorporates multiple heterogeneous interactions within the same framework using a simultaneous approach.

3 Modelling framework

We propose a modelling framework to simulate the joint scheduling process of a household, comprising several household members (called agents) over a time period. Our framework jointly models time-use in the home alongside activity participation outside the home within the same scheduling model. The framework considers the household as a single decision-making unit while encompassing the activity scheduling behaviour of all agents through the utility that each agent derives from their schedule. Agents schedule their day to maximize the total combined utility of the household, gained from the completed activities of all household members over the fixed time budget and according to both the agents' and the household's needs, preferences, and constraints. Therefore, it accounts for both individuals' constraints and the constraints that appear due to interpersonal dependencies within household members. We assume that the decision of each household is independent from the decisions of other households. Thus, it is sufficient to describe the model for one household.

We treat activity scheduling as a utility-optimisation problem. This functionality adopts the utility maximisation approach of the OASIS model. We assume that the agents in the household are unselfish, meaning that they coordinate their schedules for the benefit of the entire household rather than each aiming to maximize their own utility independently. The objective function in the household scheduling problem is as follows:

$$\max \sum_{n=1}^{n=N_m} w_n \, U_n \tag{1}$$

where n presents an agent having decision-making capabilities in the household. N_m is the number of agents in the household. w_n is the agent priority parameter, which captures the heterogeneous influence of household members on household decisions by accounting for how much relative priority is placed on the utility of each individual. U_n is the utility that agent n gains from her/his schedule over the considered time period. U_n can be either positive, negative, or zero. The utility function U_n explicitly captures the features related to the behaviour of agents in the household towards activities including both agent-specific and household-specific activities that are not solely associated with the agent doing them such as groceries, their related trips, and their interactions with other agents in the household.

The remainder of this section is structured as follows. First, we define the household scheduling problem in Section 3.1. Next, we give a brief synopsis of the optimisation-based scheduling approach of the OASIS framework in Section 3.2. We then present the formulation of our utility-based household scheduling model in Section 3.3.

3.1 Definitions

Consider an individual having decision-making capabilities called an agent n. The agent is a member of a household of N_m agents living together, each trying to schedule their activities over a time budget T. Each agent n considers participating in activities distributed in space and time including activities done at home. Among the activities, there are also household maintenance duties that are for satisfying the needs of the entire household rather than solely the needs of the agent who implements them such as groceries shopping and cleaning. Each activity a_n in the considered activity set A^n, is an action taking place at location ℓ_{a_n} with a start time x_{a_n} and duration τ_{a_n}. The agents also decide whether to participate in the activity jointly with other agent(s) or alone, captured by a binary variable called activity participation mode p_{a_n}. If consecutive actions are at different locations, they would be followed by a trip with a transport mode m_{a_n}. We consider an activity a_n as the combination of an action performed at location ℓ_{a_n} with a start time x_{a_n}, duration τ_{a_n}, participation mode p_{a_n}, and where required their associated trips with transport mode m_{a_n}. An action that can be performed at multiple locations, have multiple participation modes, or can have different transport modes is modelled as multiple unique activities. Figure 1 illustrates an activity unit.

Each activity a_n is associated with the following attributes:

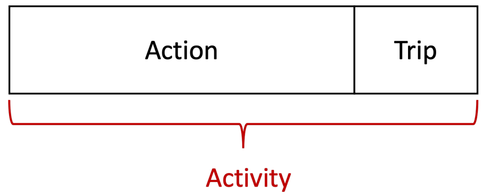


Figure 1: Definition of activity in our framework

- a minimum duration $\tau_{a_n}^{min}$,
- a time range indicating the desired duration of activity a_n: $[\tau_{a_n}^{*-}, \tau_{a_n}^{*+}]$ where $\tau_{a_n}^{min} \leq \tau_{a_n}^{*-} \leq \tau_{a_n}^{*+}$,
- a time interval indicating the preferred start time for activity a_n: $[x_{a_n}^{*-}, x_{a_n}^{*+}]$ where $x_{a_n}^{*-} \leq x_{a_n}^{*+}$,
- a time interval indicating the feasible time range during which activity a_n can take place: $[\gamma_{a_n}^{-}, \gamma_{a_n}^{+}]$ where $\gamma_{a_n}^{-} \leq \gamma_{a_n}^{+}$,
- a group G_{q_n} from the mutually-exclusive groups containing all possible combinations of locations, transport modes, and participation modes for each activity, and
- a scheduling flexibility k_{a_n}, which specifies how sensitive activity a_n is to schedule deviations from the preference.

Consider one of the household agents to be a child. Agents with restricted mobility such as children need to be escorted by an adult agent for their out-of-home activities. Chauffeuring children to school is an example of escorting activity. But, escorting is not limited to children or agents with restricted mobility. For instance, an adult agent may drive another adult member of the household to work in order to keep the car. Escorting can be done in two ways:

- *pick-up and drop-off*, where a core adult picks up/drops off the other agent from/to the activity location, and
- *escort and stay*, in which the adult accompanies the other agent throughout the entire tour (e.g, takes the agent to the activity location, stays throughout the activity, and takes her/him to the location of the next activity). In this case, sharing the same activity is not implied; instead, serving the other agent becomes a purposeful activity for the escorting agent.

Each household has limited resources. Consider a household owning N_r resources of the same kind. Cars are examples of household resources. Resources can be moving such as household private vehicles, or can be static such as the bathroom. Some activities might use a household resource r. A household resource has no independent decision-making capabilities and is purely used by and dependent on the decision-making agents. The schedule of the resources is constrained to that of the agents and has only associated constraints.

We summarise the model notations in Table 2, which presents a glossary of the terms used in the framework.

Table 2: Notations used in the framework (DV = Decision variable)

Notation	Name	Description	Type
n	Agent	An individual having decision making capabilities, determined by both preferences and constraints, $n \in \{1, 2, ...N_m\}$.	Input
r	Resource	A household resource used by the agents. Resources have no independent decision making capabilities and are purely dependent on the decision-making agents.	Input
N_m	Household size	Number of agents in the household.	Input
N_r	Number of household resources	The number of household resources of the same kind, which can be used by all its members upon availability.	Input
O_r	Resource occupancy	The number of agents using resource r at the same time.	Variable
C_r	Resource capacity	Maximum number of agents that can use resource r at the same time.	Input
T	Time budget	The time period over which the schedules are simulated.	Input
t	Time	The schedules are simulated over a time period T, with the start time at $t = 0$ until the end of the time horizon $t = T$.	Variable
A^n	Considered activity set	An activity set containing all activities a_n that agent n considers performing within her time budget T.	Input
E^r	Associated resource event set	An event set containing all possible events e_r that can be scheduled for resource r within the time budget T.	Input
a_n	Activity	Activity a_n that can be performed by agent n.	Input
e_r	Resource event	Event e_r that can be scheduled for resource r.	Input
ω_{a_n}	Activity participation	A binary variable equals to 1 if agent n participates in activity a_n, and 0 otherwise.	DV
ω_{e_r}	Event occurrence	A binary variable equals to 1 if event e_r is scheduled for resource r, and 0 otherwise.	DV
$z_{a_n b_n}$	Activity succession	A binary variable representing activity succession, which is 1 if agent n schedules activity b_n immediately after activity a_n, and 0 otherwise.	DV
$z_{e_r e'_r}$	Event succession	A binary variable representing resource event succession, which is 1 if event e'_r is scheduled immediately after event e_r for resource r, and 0 otherwise.	DV
ℓ_{a_n}	Activity location	Location for activity a_n.	Input
ℓ_{e_r}	Resource location	Resource location for event e_r.	Input
L_{a_n}	Activity location choice set	A discrete and finite location choice set containing all locations ℓ_{a_n} that agent n considers for activity a_n.	Input
M	Transport mode choice set	A discrete and finite list of considered transport modes.	Input

Continued on next page

Table 2 - Notations used in the framework (Continued)

Notation	Name	Description	Type
m_{a_n}	Transportation mode	The mode to travel from the location of the current activity, ℓ_{a_n}, to the location of the following activity, ℓ_{a+1_n}.	Input
$\rho(\ell_o, \ell_d, m)$	Travel time	The travel time between the locations ℓ_o and ℓ_d with mode m is characterized by $\rho(\ell_o, \ell_d, m)$.	Input
p_{a_n}	Activity participation mode	A binary variable, indicating engagement mode of activity a_n, which is 1 if performed jointly with other agent(s), and 0 if performed solo.	Input
τ_{a_n}	Activity duration	A positive continuous variable representing the duration of activity a_n.	DV
τ_{e_r}	Event duration	A positive continuous variable representing the duration of event e_r for resource r.	DV
$\tau^{\min}_{a_n}$	Minimum activity duration	Minimum duration of activity a_n.	Input
$[\tau^{*-}_{a_n}, \tau^{*+}_{a_n}]$	Desired activity duration range	A time range indicating the desired duration of activity a_n.	Input
x_{a_n}	Activity start time	A positive continuous variable representing the start time of activity a_n.	DV
x_{e_r}	Event start time	A positive continuous variable representing the start time of event e_r for resource r.	DV
$[x^{*-}_{a_n}, x^{*+}_{a_n}]$	Desired activity start time range	A time range indicating the desired start time of activity a_n.	Input
$[\gamma^{-}_{a_n}, \gamma^{+}_{a_n}]$	Feasible activity time range	A time range indicating the feasible time range during which activity a_n can take place.	Input
G_{q_n}	Activity group	Each activity a_n is associated with a group G_{q_n}, which contains all possible combinations of locations, transport modes, and participation modes of that activity.	Input
k_{a_n}	Activity scheduling flexibility	Specifies how sensitive activity a_n is to schedule deviations from the preference.	Input
w_n	Agent priority parameter	Relative weight capturing the priority that is placed on the schedule utility of each individual.	Input

3.2 Base optimisation framework: A brief synopsis of the OASIS framework

OASIS is a mixed integer optimisation scheduling framework based on random utility theory, considering multiple scheduling decisions simultaneously. It allows explicit capture of trade-offs between choices. The schedule of each agent is a sequence of activities over a time horizon T, resulting from the agent's choices such as activity participation, activity duration, activity timing, and transportation mode. The framework is defined under a set of constraints which determines the validity of the schedules at an individual level.

Each schedule for agent n is associated with a utility function U_n. U_n is made up of a generic utility (U^{gen}_n) linked to the entire schedule of the agent and utility components linked to the performed activities (U_{a_n}). U_{a_n} is specified as the sum of components capturing the agent's activity and travel behaviour (e.g. time sensitivity). The utility terms may also include a random error term, capturing the unobserved variables. The general form of U_n, the utility of the schedule for agent n, is defined as follows:

$$\begin{aligned} U_n &= U_n^{\text{gen}} + \sum_{a_n \in A^n} U_{a_n} \\ &= U_n^{\text{gen}} + \sum_{a_n \in A^n} \left(U_{a_n}^{\text{partic}} + U_{a_n}^{\text{start}} + U_{a_n}^{\text{duration}} + \sum_{b_n \in A^n} (U_{a_n,b_n}^{\text{travel}}) \right) \end{aligned} \tag{2}$$

where:

- U_n^{gen}**:** a generic utility capturing characteristics of the whole schedule not directly linked with any specific activity.
- $U_{a_n}^{\text{partic}}$**:** a utility term, which is purely associated with participation in activity a_n, irrespective of its timing and associated trips.
- $U_{a_n}^{\text{start}}$**:** a utility term which captures the perceived penalty of deviation in start time from the desired start time which can be either single values or time intervals.
- $U_{a_n}^{\text{duration}}$**:** a utility term which captures the perceived penalty of deviation in duration of activity a_n from the preference which can be either single values or time intervals.
- $U_{a_n,b_n}^{\text{travel}}$**:** a utility term associated with the trip from ℓ_{a_n} to ℓ_{b_n}. This utility term can include the penalty associated with travel time and other travel variables such as travel cost.

The optimisation framework is formulated as follows. Each agent n aims to maximise the utility gained from her/his schedule:

$$\max U_n \tag{3}$$

subject to a set of constraints:

$$\sum_{a_n} \sum_{b_n} (\omega_{a_n} \tau_{a_n} + z_{a_n b_n} \rho(\ell_{a_n}, \ell_{b_n}, m)) = T \tag{4}$$

$$\omega_{\text{dawn}_n} = \omega_{\text{dusk}_n} = 1 \tag{5}$$

$$\tau_{a_n} \geq \omega_{a_n} \tau_{a_n}^{min} \qquad \forall a_n \in A^n \tag{6}$$

$$\tau_{a_n} \leq \omega_{a_n} T \qquad \forall a_n \in A^n \tag{7}$$

$$z_{a_n b_n} + z_{b_n a_n} \leq 1 \qquad \forall a_n, b_n \in A^n, a_n \neq b_n \tag{8}$$

$$z_{a_n\ \text{dawn}_n} = z_{\text{dusk}_n\ b_n} = 0 \qquad \forall a_n, b_n \in A^n \tag{9}$$

$$\sum_{a_n} z_{a_n b_n} = \omega_{b_n} \qquad \forall b_n \in A^n, b_n \neq \text{dawn}_n \tag{10}$$

$$\sum_{b_n} z_{a_n b_n} = \omega_{a_n} \qquad \forall a_n \in A^n, a_n \neq \text{dusk}_n \tag{11}$$

$$(z_{a_n b_n} - 1)\,T \;\le\; x_{a_n} + \tau_{a_n} + z_{a_n b_n}\;\rho(\ell_{a_n}, \ell_{b_n}, m) - x_{b_n} \qquad \forall a_n, b_n \in A^n, a_n \neq b_n \quad (12)$$

$$(1 - z_{a_n b_n})\;T \ge x_{a_n} + \tau_{a_n} + z_{a_n b_n}\;\rho(\ell_{a_n}, \ell_{b_n}, m) - x_{b_n} \qquad \forall a_n, b_n \in A^n, a_n \neq b_n \quad (13)$$

$$\sum_{a_n \in G_{q_n}} \omega_{a_n} \le 1 \qquad \forall q_n = 1, ..., Q_n \quad (14)$$

$$\omega_{a_n} \ge \omega_{b_n} + z_{a_n b_n} - 1 \qquad \forall a_n \in A^n, \forall b_n \in A^n /\ G_{\text{home}} \quad (15)$$

$$\omega_{b_n} \ge \omega_{a_n} + z_{a_n b_n} - 1 \qquad \forall a_n \in A^n, \forall b_n \in A^n /\ G_{\text{home}} \quad (16)$$

$$x_{a_n} \ge \gamma_a^- \qquad \forall a_n \in A^n \quad (17)$$

$$x_{a_n} + \tau_{a_n} \le \gamma_a^+ \qquad \forall a_n \in A^n \quad (18)$$

Equation 4 defines the time budget constraint. Equation 5 is a boundary condition such that each schedule should start and end with dummy activities *dawn* and *dusk*, respectively. Equations 6 and 7 enforce activity duration consistencies with regard to a minimum duration and available time budget. Equations 8 - 11 are activity succession constraints such that each activity can be scheduled once (8), and each activity can have only one predecessor (with the exception of the first activity) and one successor (with the exception of the last activity). Equations 12 and 13 enforce time consistency between two consecutive activities such that each activity starts when the trip following the previous activity is finished. Equation 14 ensures that for each agent, at most one activity within a group of duplicate activities can be scheduled. Equations 15 and 16 enforce mode consistency between two different activities. Equations 17 and 18 ensure each activity is scheduled within its feasible time window.

The framework takes as input a set of considered out-of-home activities for an agent , as well as the agents' scheduling preferences and flexibility towards the activities. Due to the stochastic nature of the utility function presented in Equation 2, the model generates empirical instances of the distribution. A simulation technique is used to generate several draws from the distributions of the random terms, and then solve the optimisation problem explicitly for each realisation. The outcome of the model is a realisation from the distributions of valid schedules, presenting the schedules of the agents subject to their constraints and preferences.

For a comprehensive explanation of the base model, including a complete formal definition of its mathematical formulation and constraints, we direct the reader to the paper by Pougala et al. (2022*a*).

3.3 A utility-based household scheduling model

Intra-household interactions affect how members schedule their entire day, including both activities they do at home as well as those performed out of home. In our framework, we first ensure that the possible interaction aspects are captured in the utility function. We then specify the model constraints such that they allow the integration of in-home activities alongside activities outside the home in a single framework. Moreover, as the within-household interactions lead to additional and more complex constraints, we define household-level constraints to explicitly capture the interplays. Resource constraints, sharing household maintenance responsibilities, the joint participation of household members in activities, joint travels, escorting, and coordination of daily rhythms between household members are examples of intra-household interactions, which add to the complexity of the constraints in the household scheduling model.

The framework takes as input the household composition, scheduling preferences, activity flexibilities, household resources and their associated events sets, as well as, a considered activity set including their associated locations, transport modes, and participation modes for each agent in the household. They are utilized to define a distribution over possible schedules from which random realisations can be generated. The outcome of the model is a realisation from the distributions of valid schedules, presenting the schedules of the agents in the same household under both individual- and household-level constraints and preferences.

The rest of this chapter is laid out as follows. First, we outline the form of the utility function in our framework in Section 3.3.1. Section 3.3.2 then summarises the individual-level constraints, followed by Section 3.3.3 which specifies the household-level constraints. Section 3.3.4 then summarizes our framework.

3.3.1 Form of the utility function

The central element in the objective function of our framework is the utility function U_n, which captures the utility of the schedule for each agent n in the household. In order to consider interaction aspects within the utility function as well, we define the participation utility function $U_{a_n}^{\text{partic}}$ of Equation 2 as follows. It is notable that more complex forms of the utility function can be also utilized.

$$U_{a_n}^{\text{partic}} = U_{a_n}^{\text{location}} + U_{a_n}^{\text{joint}} + U_{a_n}^{\text{escort}} \tag{19}$$

where:

- $U_{a_n}^{\textbf{partic}}$**:** a utility term for agent n, which is purely associated with participation in activity a_n, irrespective of any schedule deviations and travel behaviour.
- $U_{a_n}^{\textbf{location}}$**:** a utility term, capturing the utility of different activity location choices. This term effectively tries to capture why people choose to leave home rather than participating in activities remotely. We define this term as being associated with the location of activity a_n.

$$U_{a_n}^{\text{location}} = \alpha_{\ell_{a_n}}^{\text{loc}} \ell_{a_n} \tag{20}$$

where $\alpha_{\ell_{a_n}}^{\text{loc}}$ is the location specific parameter for location ℓ_{a_n}, and ℓ_{a_n} is the location for activity a_n.

- $U_{a_n}^{\textbf{joint}}$**:** joint participation in activities is motivated by considerations such as (i) efficiency; which can be gained from time and/or money savings from substituting a single episode of joint activity for multiple individual activity episodes or might be lost in joint engagements due to coordination costs, (ii) altruism, which is a selfless regard in which an individual gains utility by benefiting someone other than oneself, and (iii) companionship. Depending on the type of activity and the household role, the significance of each motivating factor can vary.

The joint engagement is captured in the utility of the schedule of each agent with the term $U_{a_n}^{\text{joint}}$. We capture $U_{a_n}^{\text{joint}}$ as follows:

$$U_{a_n}^{\text{joint}} = \alpha_{a_n}^{\text{jnt}} \, p_{a_n} \tag{21}$$

where $\alpha_{a_n}^{\text{jnt}}$ is the parameter for activity a_n, capturing the (dis)utility of joint activity engagement, and p_{a_n} is the participation mode of activity a_n, which is 1 if the agent performs the activity jointly with other agent(s), and 0 otherwise.

- $U_{a_n}^{\textbf{escort}}$**:** escorting a household member is one of the aspects of intra-household interactions and has an important role in determining the activity schedules of its agents. The (dis)utility of doing an escorting task is considered with the utility term $U_{a_n}^{\text{escort}}$. This term can include variables such as time, additional distance travelled, and schedule adjustments to accommodate the escort activity. Here, we illustrate the framework with a specification involving activity time. By undertaking an escort, the accompanying agent would not be able to perform some activities that they could have done if they had not escorted the other member and thus, had saved that time for their personal activities. The extent of (dis)utility can also vary among agents with different employment status such as workers and non-workers. We formulate this utility term as follows:

$$U^{\text{escort}}_{a_n} = \theta^{\text{esc}}_s \lambda_{a_n} \tag{22}$$

where $\theta^{\text{esc}}_s \leq 0$ is a penalty parameter associated with escort duration for agents with employment status s, and λ_{a_n} is the escort indicator which is a binary variable indicating whether activity a_n is an escort (1), or not (0).

Looking from the group decision-making paradigm, in a household of N_m agents, agents are assumed to select their schedules such that the total household utility is maximized under both individual and household constraints. Therefore, agents in the household solve an optimisation problem with an objective function derived from Equations 1, 2, and 19 as follows:

$$\begin{aligned} &\max \sum_{n=1}^{n=N_m} \left(w_n \, (U^{\text{gen}}_n + \sum_{a_n \in A^n} U_{a_n}) \right) \\ &= \max \sum_{n=1}^{n=N_m} \left(w_n \, (U^{\text{gen}}_n + \sum_{a_n \in A^n} (U^{\text{partic}}_{a_n} + U^{\text{start}}_{a_n} + U^{\text{duration}}_{a_n} + \sum_{b_n \in A^n} U^{\text{travel}_m}_{a_n b_n})) \right) \end{aligned} \tag{23}$$

The problem is subject to a set of constraints, which account for the validity of schedules under both the individual-level and household-level restrictions and preferences.

3.3.2 Individual-level constraints

In this section, we present constraints determining the validity of the schedules at an individual-level, allowing the integration of in-home activities in the framework. The presented constraints are relevant to the schedules of all agents in the household.

- Boundary conditions: each schedule begins with the activity *Sleep_morn* and ends with the activity *Sleep_night*.

$$\sum_{a_n \in G_{\text{Sleep\_morn}_n}} \omega_{a_n} = 1 \qquad \forall n \in \{1, ..., N_m\} \tag{24}$$

$$\sum_{a_n \in G_{\text{Sleep\_night}_n}} \omega_{a_n} = 1 \qquad \forall n \in \{1, ..., N_m\} \tag{25}$$

- The first activity; *Sleep_morn*; cannot have any predecessors. Moreover, the last activity; *Sleep_night*; cannot have any successors.

$$\sum_{b_n \in G_{\text{Sleep\_morn}_n}} z_{a_n b_n} = 0 \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{26}$$

$$\sum_{b_n \in G_{\text{Sleep\_night}_n}} z_{b_n a_n} = 0 \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{27}$$

- Consistent transport modes in tours: in order to obtain consistent transport modes between consecutive activities needing travel, we constrain the choice of transport mode from ℓ_{a_n} to ℓ_{a+1_n} to the prior mode selections. A key principle in mode choice consistency is that if a private vehicle is chosen as the transport mode in a tour, it should be used throughout the entire tour until returned back home. Other transport modes are not subject to such constraint.

$$m_{a_n}^V \geq m_{b_n}^V + z_{a_n b_n} - 1 \qquad \forall a_n, b_n \in A^n, \ell_{b_n} \notin \{Home\}, \forall n \in \{1, ..., N_m\} \tag{28}$$

$$m_{b_n}^V \geq m_{a_n}^V + z_{a_n b_n} - 1 \qquad \forall a_n, b_n \in A^n, \ell_{b_n} \notin \{Home\}, \forall n \in \{1, ..., N_m\} \tag{29}$$

where $m_{a_n}^V$ is an indicator variable that is 1 if a private mode is chosen for activity a_n, and 0 otherwise.

- Consistent transport mode for each activity: for each activity in order to select a transport mode consistent with its travel needs, we constrain the mode choice for each activity a_n to its travel time to the following activity. For instance, agent n cannot choose any private, public, or active mode of transportation for activity a_n if its successive activity, b_n, is at the same location ($\ell_{a_n} = \ell_{b_n}$). Therefore, as there would be no trip, the associated transport mode should be *Null*. The model specifications for mode choice consistency for each activity are presented in Equations 30 and 31. $\rho^{\min}$ is the minimum travel time between the locations in the case study of interest.

$$\sum_{b_n \in A^n} \left(z_{a_n b_n}\ \rho(\ell_{a_n}, \ell_{b_n}, m_{a_n})\right) \geq \rho^{\min} * \omega_{a_n} \quad \forall a_n \in A^n, m_{a_n} \notin \{\textit{Null}\}, \forall n \in \{1, ..., N_m\} \tag{30}$$

$$\sum_{b_n \in A^n} \left(z_{a_n b_n}\ \rho(\ell_{a_n}, \ell_{b_n}, m_{a_n})\right) = 0 \qquad \forall a_n \in A^n, m_{a_n} \in \{\textit{Null}\}, \forall n \in \{1, ..., N_m\} \tag{31}$$

3.3.3 Household-level constraints

Constraints determining the validity of the schedules under inter-personal interactions between the agents in a household are presented in this section.

1. **Household private vehicle ownership:** household mobility tool ownership plays a critical role in transport mode choice. If the household doesn't own any private vehicles, no agent can choose a private vehicle as their transport mode.

$$\omega_{a_n} + m_{a_n}^V \leq N_V + 1 \qquad \forall a_n \in A^n, \forall n \in N_m \tag{32}$$

   where N_V is the number of household private vehicles and $m_{a_n}^V$ is an indicator variable that is 1 if a private mode is chosen for activity a_n, and 0 otherwise.

2. **Allocation of resources to household members**: Each household has limited resources. Thus, the resource availability and allocation is one of the pivotal aspects of intra-household interactions. Some resources move such as the private vehicle, and some do not such as the bathroom.

   In our framework, we consider an event schedule for the resources. Each resource has a capacity that limits the maximum number of agents that can use it at the same time. Moreover, the moving resources need a driver to move them. Therefore, their schedule is constrained to that of the agents in the household and additional physical constraints exist for the non-static resources. This is a general approach applicable to any household resource. It provides valuable information such as the resource location and occupancy (the number of agents using the resource) at each time step. The specifications for modeling resource constraints are as follows:

   - **Event schedule validity:** In order to obtain consistent event schedules, the following should be valid for each resource r.
     - Event succession constraint: two events can follow each other only once.

$$z_{e_r e'_r} + z_{e'_r e_r} \leq 1 \qquad \forall e_r, e'_r \in E^r, e_r \neq e'_r \tag{33}$$

     - Time consistency between two consecutive events:

$$\left(z_{e_r e'_r} - 1\right) T \leq x_{e_r} + \tau_{e_r} - x_{e'_r} \leq \left(1 - z_{e_r e'_r}\right) T \qquad \forall e_r, e'_r \in E^r \tag{34}$$

– Time budget constraint:

$$\sum_{e_r \in E^r} \frac{\tau_{e_r}}{O_{e_r}} = T \tag{35}$$

where O_{e_r} is occupancy of resource r at event e_r.

- **Capacity constraints:** a maximum number of agents can use a resource at the same same. Thus, at each resource event, the occupancy of each resource O_{e_r} should not exceed its capacity C_r.

$$O_{e_r} \leq C_r \qquad \forall e_r \in E^r \tag{36}$$

- **Physical constraints:** the moving resources, such as the private vehicle, need an agent to drive them. An event can be scheduled for a moving resource only if it is accompanied by an adult agent throughout the tour. Therefore, their event schedule should be consistent with the schedule of the adult agents in the household.

$$\omega_{e_r} = \omega_{a_n} \qquad \forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \forall n \in \{\text{Adults}\} \tag{37}$$

$$x_{e_r} = x_{a_n} + \tau_{a_n} \qquad \forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \ell_{a_n} \in \{\text{Home}\}, \forall n \in \{\text{Adults}\} \tag{38}$$

$$\tau_{e_r} = \sum_{b_n \in A^n} (z_{a_n b_n} \, \rho(\ell_{a_n}, \ell_{b_n}, \text{Driving})) \qquad \forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \ell_{a_n} \in \{\text{Home}\}, \forall n \in \{\text{Adults}\} \tag{39}$$

$$x_{e_r} = x_{a_n} \qquad \forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \ell_{a_n} \notin \{\text{Home}\}, \forall n \in \{\text{Adults}\} \tag{40}$$

$$\tau_{e_r} = \tau_{a_n} + \sum_{b_n \in A^n} (z_{a_n b_n} \, \rho(\ell_{a_n}, \ell_{b_n}, \text{Driving})) \qquad \forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \ell_{a_n} \notin \{\text{Home}\}, \forall n \in \{\text{Adults}\} \tag{41}$$

3. **Allocation of maintenance activities to household members:** Household maintenance activities are for satisfying the needs of the entire household rather than solely the needs of the agent who implements them. Therefore, the maintenance activities are associated with a significant degree of intra-household coordination, substitution, and allocation. Groceries shopping and cleaning are examples of household maintenance cores. These activities have different participation constraints; for example some might be mandatory and some not. These activities are allocated to an adult agent in the household for implementation and have an associated utility. The constraint for the maintenance activities for which the participation of exactly one adult agent suffices is as follows:

$$\sum_{n \in \text{Adults}} \omega_{\text{maintenance}_n} = 1 \tag{42}$$

4. **Joint activity participation and ride-share to joint activities:** one of the complex behavioural patterns in households is joint activity participation. Joint activity arrangements result from a collective decision process and require synchronisation among household members. Joint participation can be mainly observed in maintenance and leisure activities. For example, a family might schedule a joint weekend recreation activity with all the household members. In this study, we consider joint participation as an activity engagement that is fully joint in purpose, location, and time. The agents might travel together to

the location of the joint activity, as well. In this case, they would also need to coordinate their joint travel.

Here, we illustrate the model specifications, which accommodate the cases for non-joint tours, semi-joint tours, and fully-joint tours for shared activities. In the case of non-joint tours, the participants participate in the activity together, but do not share rides to or from the location of the joint activity. In the case of semi-joint tours, agents might travel together for either the inbound or outbound trips, but not both. In the case where all the participating agents travel together to the activity location, participate in the activity, and then share the ride to their next destination, the instance would be a fully-joint tour..

The model specifications for joint activity arrangements and ride-share to joint activities are presented as follows. We consider the joint activity participation as a constraint; if there is a joint activity, both members must participate, or the joint activity is canceled. Furthermore, the consistency of space and time for all participating members should be ensured.

$$\omega_{a_n} = \omega_{a_{n'}}, \quad \forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \forall n,\ n' \in \{a, ..., N_m\} \tag{43}$$

$$x_{a_n} = x_{a_{n'}}, \quad \forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \forall n,\ n' \in \{a, ..., N_m\} \tag{44}$$

$$\tau_{a_n} = \tau_{a_{n'}}, \quad \forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \forall n,\ n' \in \{a, ..., N_m\} \tag{45}$$

where p_{a_n} is the activity participation indicator. $p_{a_n} = 1$ indicates joint participation.

If a private transport mode is chosen for the joint activity, we should ensure that all the agents travelling together depart from the same location. To ensure this matter, we define linked activities; the coordination activity and the joint activity a_n, where the coordination activity must be scheduled immediately before the joint activity in order to enforce identical departing locations for all the accompanying agents.

$$\begin{aligned} &\omega_{\text{Coord}_n} = \omega_{\text{Coord}_{n'}} = \omega_{a_n} \\ &\qquad \forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \ell_{a_n} = \ell_{a_{n'}} \notin \{\text{Home}\}, \\ &\qquad\qquad m_{a_n} = m_{a_{n'}} = \text{Driving}, \forall n,\ n' \in \{a, ..., N_m\} \end{aligned} \tag{46}$$

$$\begin{aligned} &z_{\text{Coord},a_n} = z_{\text{Coord},a_{n'}} = \omega_{a_n} \\ &\qquad \forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \ell_{a_n} = \ell_{a_{n'}} \notin \{\text{Home}\}, \\ &\qquad\qquad m_{a_n} = m_{a_{n'}} = \text{Driving}, \forall n,\ n' \in \{a, ..., N_m\} \end{aligned} \tag{47}$$

$$\begin{aligned} &\tau_{\text{Coord}_n} = \tau_{\text{Coord}_{n'}} = \mu\, \omega_{a_n} \\ &\qquad \forall a_n \in A^n, \forall a_{n'} \in A^{n'}, p_{a_n} = p_{a_{n'}} = 1, \ell_{a_n} = \ell_{a_{n'}} \notin \{\text{Home}\}, \\ &\qquad\qquad m_{a_n} = m_{a_{n'}} = \text{Driving}, \forall n,\ n' \in \{a, ..., N_m\} \end{aligned} \tag{48}$$

Equations 46 to 48 ensure consistency of space and time for all participating members in shared rides; all the agents in the travel party should depart from the same location. The variable μ in Equation 48, is the duration needed for the agents to coordinate for the joint travel, i.e., 5 minutes.

5. **Escort:** one other important aspect of intra-household interactions, especially in households having agents with restricted mobility such as children, is escorting. Children strictly depend on adults for their mobility. This interaction between children and household heads involves travel arrangements, timing,

and spatial synchronization between members in which the chauffeur does not participate in the activity. Here, we consider escort as a trip chauffeured by one of the adults in the household with a private vehicle. Escorting by multiple household heads is not included in the presented specification, but can be adopted within the framework.

Escorting can be done either as a *pick-up and drop-off*, or as an *escort and stay*. In pick-up and drop-off, the core adult picks up/drops off the passenger from/to the activity location. In escort and stay, the adult accompanies the passenger throughout the entire tour (e.g, drives the agent to the activity location, stays throughout the activity, and drives the passenger to the location of the next activity). In this case, sharing the same activity is not implied; instead, serving the passenger becomes a purposeful activity for the escorting agent. We thus, add three new activity types to the activity choice set of the agent escorting accounting for escort and stay, escort and pick-up, and escort and drop-off. Each escort activity is associated with an indicator variable indicating its type, χ_{a_n}. χ_{a_n} is 0 for escort and stay, 1 for the pick-up, and 2 for the drop-off escort type. We then define a binary variable escort indicator, λ_{a_n}, for each activity a_n, which specifies whether activity a_n is/needs escort or not. λ_{a_n} is defined as follows:

- for agents needing escort: λ_{a_n} specifies whether agent n *needs* to be escorted for activity a_n (1), or not (0), and
- for agents providing escort: λ_{a_n} specifies whether activity a_n performed by agent n *is* an escort (1), or not (0).

The intra-household bundling constraints, characterizing either type of escorting, are as follows.

$$\sum_{n \in \text{Adults}} \omega_{a_n} = \omega_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1 \tag{49}$$

$$\sum_{n \in \text{Adults}} x_{a_n} = x_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1 \tag{50}$$

$$\sum_{n \in \text{Adults}} \tau_{a_n} = \tau_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 0 \tag{51}$$

$$\sum_{n \in \text{Adults}} \sum_{b_n \in A^n} (z_{b_n a_n}\ \ell_{b_n}) = \sum_{b_{\text{Passenger}} \in A^{\text{Passenger}}} \left(z_{b_{\text{Passenger}} a_{\text{Passenger}}}\ \ell_{b_{\text{Passenger}}}\right)$$
$$\forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 0 \tag{52}$$

$$\sum_{n \in \text{Adults}} \sum_{b_n \in A^n} (z_{a_n b_n}\ \ell_{b_n}) = \sum_{b_{\text{Passenger}} \in A^{\text{Passenger}}} \left(z_{a_{\text{Passenger}} b_{\text{Passenger}}}\ \ell_{b_{\text{Passenger}}}\right)$$
$$\forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 0 \tag{53}$$

$$\sum_{n \in \text{Adults}} \tau_{a_n} = \vartheta\ \omega_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 1 \tag{54}$$

$$\sum_{n \in \text{Adults}} \sum_{b_n \in A^n} (z_{a_n b_n}\ \ell_{b_n}) = \sum_{b_{\text{Passenger}} \in A^{\text{Passenger}}} \left(z_{a_{\text{Passenger}} b_{\text{Passenger}}}\ \ell_{b_{\text{Passenger}}}\right)$$
$$\forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 1 \tag{55}$$

$$\sum_{n \in \text{Adults}} \tau_{a_n} = \vartheta \ \omega_{a_{\text{Passenger}}} \qquad \forall a \ \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 2 \quad (56)$$

$$\sum_{n \in \text{Adults}} \sum_{b_n \in A^n} (z_{b_n a_n} \ \ell_{b_n}) = \sum_{b_{\text{Passenger}} \in A^{\text{Passenger}}} \left(z_{b_{\text{Passenger}} a_{\text{Passenger}}} \ \ell_{b_{\text{Passenger}}}\right)$$
$$\forall a \ \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 2 \quad (57)$$

where variable ϑ is the stop time duration needed to pick-up or drop-off the passenger. Equations 52, 53, 55, and57 ensure location consistency between the passenger and the adult escorting agent.

3.3.4 Summary

To summarize, in our household-level scheduling framework, the goal of the decision-making agents is to maximise the utility of the entire household (Equation 1), considering both the set of individual-level constraints (Equations 24-31), and the household-level constraints (Equations 32-57).

4 Empirical investigation

In this section, we show the capabilities of our proposed modeling framework using a case study. The objective is to show that the proposed framework works with multi-member households, accommodates complex interactions among household members, and generates feasible realisations of their daily schedules. We demonstrate how the interactions between the agents in a household can cause individuals to deviate from their schedules planned in isolation. For this purpose, first, a realisation of the daily schedules of the agents in the household is drawn from the simulation model calibrated for independent individuals without considering any inter-household interactions. We then operationalise the proposed framework for a household of 2 adult agents as well as a household of 3 which contains 2 adults and 1 child. The results and the schedule deviations are then discussed.

We rely on a real-world daily diary dataset in order to generate the inputs to illustrate the operationalised model. The data from the United Kingdom (UK) Time use survey (TUS) (Gershuny & Sullivan 2021) is used for this purpose. It includes information on respondents' socio-economic characteristics and those of their household, as well as detailed diary information on activity, location, and accompaniment. The $2016 - 2020$ survey contains 4360 time-use diaries from 2202 respondents. The data is collected in four waves among which the last three waves have been collected during the COVID-19 pandemic. To ensure sufficient diversity in schedules, we use only the data collected before the COVID-19 pandemic in 2016, which contains 1011 surveys from 659 respondents. We first describe the model inputs and calibrations for the case-study in Section 4.1. We then illustrate and discuss the model results in Section 4.2.

4.1 Model inputs and calibration

Two adult individuals are randomly chosen from the dataset and their reported activity schedules are used for activity choice set generation and scheduling preferences. Since the UK TUS does not contain any diaries of individuals under 18, a scheduling preference for a school student is synthesized.

In order to obtain the required inputs, the reported activity schedules in the dataset are used to derive the required inputs for the model such as activities choice set and scheduling preferences. As common in any conventional survey, not all the inputs are available in the dataset. For the missing inputs, we either obtain an estimator from the existing literature or use heuristics to estimate them from the data. Table 3 summarises the input data requirements of the operational model and the rigorous or heuristic solutions we apply in this case study.

Table 3: Model data requirements

Requirements	Acquired solution
Agent n	Selected adult individuals from the dataset, as well as, synthesized children.
Household	Different combinations of considered agents living together.
Household size N_m	Consider household of 2 and 3.
Resource r	We consider the household private vehicle as a resource.
Number of household private vehicles N_r	Set to 1.
Capacity of private vehicle C_r	Set to 4.
Set of considered activities A^n	Generated from the activity set of the actual schedule from the dataset via subset generation and randomly adding other historic activities from the dataset.
Resource event set E^r	Generated from the activity set of agents in addition to parking events for the private vehicle.
Set of considered activity locations L_{a_n}	Set of descriptive locations (home, work, school, other1, other2) from the dataset.
Set of considered transport modes M	Consider 2 transport modes accounting for private and public means (driving, public transport).
Travel times $\rho(\ell_o, \ell_d, Driving), \rho(\ell_o, \ell_d, PT)$	Consider the average travel time between each set of locations, by each transport mode from the dataset.
Desired start time $[x^{*-}_{a_n}, x^{*+}_{a_n}]$ and duration ranges $[\tau^{*-}_{a_n}, \tau^{*+}_{a_n}]$	Ranges are replaced by the the recorded values or their average values in the dataset. Lower and upper bounds are assumed identical.
Feasible time windows $[\gamma^-_{a_n}, \gamma^+_{a_n}]$	Obtain from analysis of start and end times for each activity, across the population in the dataset.
Flexibility profiles ka_n	Consider a discrete flexibility profile for each activity based on the literature (Pougala et al. 2022*a*).
Activity participation mode p_{a_n}	Consider 2 engagement modes; solo and joint.
Minimum activity duration $\tau^{min}_{a_n}$	Set to 10 minutes.
Time budget T	Set to 24 hours.
Agent priority parameter w_n	Set to 1 for all agents, meaning that no agent is prioritized over another.

We have also calibrated the model parameters in the operational model. The key specifications and assumptions of the model, as well as, its parameters are as follows:

1. The random error components in the utility terms of the model follow a standard normal distribution $\varepsilon \in \mathcal{N}(0, 1)$.
2. Each activity is associated with a level of scheduling flexibility, k_{a_n}. The activity flexibility levels, k_{a_n}, are defined using discrete indicators, describing three possible schedule deviation sensitivities; flexible (F), moderately flexible (MF), and not flexible (NF). Table 4 summarizes the flexibility assignments for each activity. The interpretation of these indicators and their associated penalty parameters for schedule deviations are identical to those in the case-study of Pougala et al. (2022*a*).
3. For the sake of simplicity, we assume that the model parameters are deterministic and homogeneous across the population. The values of the parameters have been obtained based on the existing literature. Table 5 summarizes the specifications of the model parameters.

Table 4: Flexibility profiles for activities in the UK TUS

Activity	Start flexibility	Duration flexibility
Sleep	Early:MF Late:MF	Short:MF Long:F
Work	Early:MF, Late:NF	Short:NF Long:NF
Education	Early:MF Late:NF	Short:NF Long:MF
Homecare	Early:MF Late:MF	Short:MF Long:MF
Personal care	Early:F Late:MF	Short:MF Long:F
Leisure	Early:F Late:F	Short:MF Long:F
Maintenance	Early:MF Late:MF	Short:MF Long:F

Table 5: Specifications of the model parameters

Parameter	Description	Value	Reference
θ^{tt}_{car}	Car travel time penalty parameter	-1	Pougala et al. (2022*a*)
θ^{tt}_{pt}	Public transport travel time penalty parameter	-0.4	Bierlaire (2018)
α^{jnt}	Joint participation reward parameter	0.1	Meister et al. (2005)
$\theta^{esc}_{a_n}$	Escort duration penalty parameter	-0.58	Vovsha & Petersen (2005)
$\alpha^{social}_{out\text{-}of\text{-}home}$	Social interactions reward parameter	0.3	

4.2 Results

We present three examples from the UK TUS:

(i) three independent individuals; two workers, identified as Sara and David, and a school student, Alice (Section 4.2.1): In the first example, we treat the considered agents as independent individuals.

(ii) a family of 2; a pair of adults with no child, Sara and David (Section 4.2.2): In the second example, we treat the considered adults as a cohabiting couple.

(iii) a family of 3; a pair of adults and a child; Sara, David, and Alice (Section 4.2.3): In the third example, we treat the considered individuals as a cohabiting couple with a child.

It is notable that agents in the aforementioned examples are progressions of the same individuals. For each example, the set of considered activities, preferences, locations, transport mode to the next activity, and activity participation mode (solo/joint) is presented in Table 6. Certain activities are duplicated to offer different location, transport mode, and participation mode choice options. In each example, for each agent, independent draws are generated from the distribution of error terms, and used to draw a realisation from the optimal schedules.

Table 6: Considered activities and preferences for each agent

Person	Schedule	Activity	Start time (hh:mm)	Duration (hh:mm)	Location	Mode	Participation mode
Sara	Independent indiv./ Fam. of 2/ Fam. of 3	Sleep_morn	00:00	6:30	Home	Null	Solo
		Sleep_morn	00:00	6:30	Home	PT	Solo
		Sleep_morn	00:00	6:30	Home	Driving	Solo
		Work	8:30	6:00	Work	PT	Solo
		Work	8:30	6:00	Work	Driving	Solo
		Work	8:30	6:00	Home	Null	Solo
		Work	8:30	6:00	Home	PT	Solo
		Work	8:30	6:00	Home	Driving	Solo
		Home care	14:30	7:40	Home	Null	Solo
		Home care	14:30	7:40	Home	PT	Solo
		Home care	14:30	7:40	Home	Driving	Solo
		Sleep_night	22:10	1:50	Home	Null	Solo
	Independent indiv.	Leisure	19:00	1:00	Other1	Driving	Solo
		Leisure	19:00	1:00	Other1	PT	Solo
	Fam. of 2/ Fam. of 3	Leisure	19:00	1:00	Other1	Driving	Joint
		Maintenance	14:40	1:10	Other2	PT	Solo
		Maintenance	14:40	1:10	Other2	Driving	Solo
	Fam. of 3	Escort_pick_up_leisure	-	-	Other1	Driving	Solo
		Escort_drop_off_leisure	-	-	Other1	Driving	Solo
		Escort_pick_up_education	-	-	School	Driving	Solo
		Escort_drop_off_education	-	-	School	Driving	Solo
David	Independent indiv./ Fam. of 2/ Fam. of 3	Sleep_morn	00:00	7:20	Home	Null	Solo
		Sleep_morn	00:00	7:20	Home	PT	Solo
		Sleep_morn	00:00	7:20	Home	Driving	Solo
		Personal care	7:20	0:30	Home	Null	Solo
		Personal care	7:20	0:30	Home	PT	Solo
		Personal care	7:20	0:30	Home	Driving	Solo
		Work	7:50	8:40	Work	PT	Solo
		Work	7:50	8:40	Work	Driving	Solo
		Work	7:50	8:40	Home	Null	Solo
		Work	7:50	8:40	Home	PT	Solo
		Work	7:50	8:40	Home	Driving	Solo
		Leisure	18:10	4:50	Home	Null	Solo
		Leisure	18:10	4:50	Home	PT	Solo
		Leisure	18:10	4:50	Home	Driving	Solo
		Leisure	18:10	4:50	Other1	PT	Solo
		Leisure	18:10	4:50	Other1	Driving	Solo

Continued on next page

Table 6: Considered activities and preferences for each agent (Continued)

		Sleep_night	22:00	2:00	Home	Null	Solo
	Fam. of 2/ Fam. of 3	Leisure	18:10	4:50	Other1	Driving	Joint
		Maintenance	14:40	1:10	Other2	PT	Solo
		Maintenance	14:40	1:10	Other2	Driving	Solo
	Fam. of 3	Escort_pick_up_leisure	-	-	Other1	Driving	Solo
		Escort_drop_off_leisure	-	-	Other1	Driving	Solo
		Escort_pick_up_education	-	-	School	Driving	Solo
		Escort_drop_off_education	-	-	School	Driving	Solo
Alice	Independent indiv./ Fam. of 3	Sleep_morn	00:00	7:00	Home	Null	Solo
		Sleep_morn	00:00	7:00	Home	Driving	Solo
		Personal care	7:00	1:00	Home	Null	Solo
		Personal care	7:00	1:00	Home	Driving	Solo
		Education	8:00	8:00	School	Driving	Solo
		Leisure	17:00	5:00	Home	Null	Solo
		Leisure	17:00	5:00	Home	Driving	Solo
		Leisure	17:00	5:00	Other1	Driving	Solo
		Sleep_night	22:00	2:00	Home	Null	Solo

4.2.1 Independent individuals

An example realisation of the generated schedules for Sara, David, and Alice, as independent individuals, are presented in Figures 2a, 2b, and 2c, respectively. For each agent, we have generated 1000 realisations of the schedules and arbitrarily selected one for illustration.

In the generated schedule for Sara, the flexible activity leisure is scheduled to be done at an out-of-home location and at a different time of day from her preferred timing. Although the out-of-home location choice has the disutility of travel time, it has a reward for social interactions, which outweighs the disutility of travelling and schedule deviations in the simulated activity sequence in this realisation.

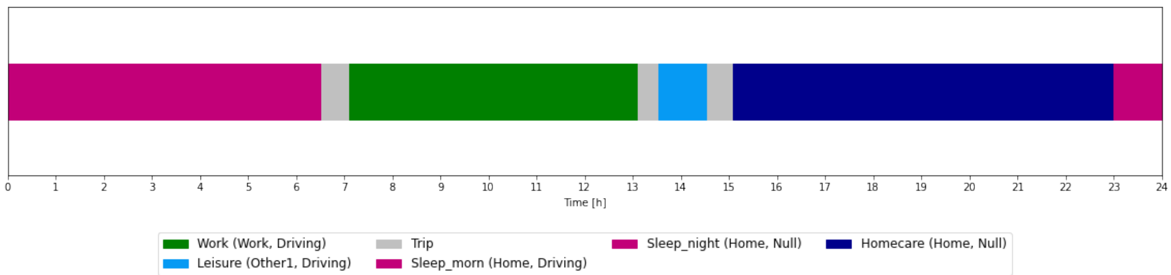
For David, the more constrained activity, work, does not diverge substantially from his preferred timing. Whereas, in the morning, the personal care activity is shortened to leave him time for travelling from home to the work location. The work and leisure location choices indicate the overall higher utility of out-of-home location choices in this realisation.

For Alice, the mandatory activity, education, is scheduled close to the desired timings. Leisure is scheduled for a longer duration than the preference and at an out-of-home location. The scheduling decisions have led to shorter personal care and sleep at night in order to leave her sufficient time for commutes.

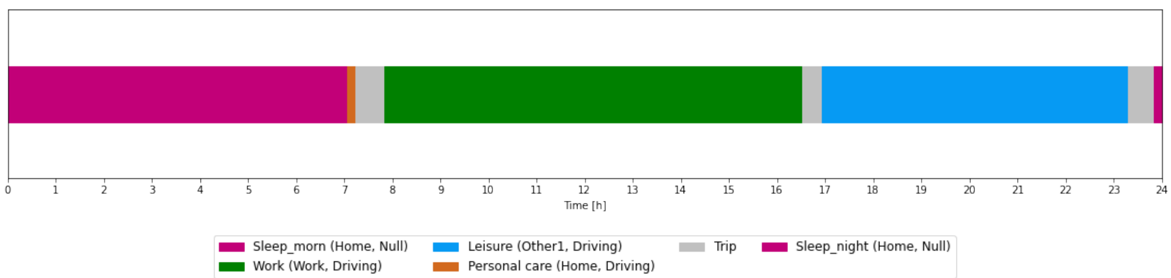
Overall, the results show that the less constrained activities such as personal care and leisure are more likely to be scheduled far from the preference or not scheduled at all, whereas the more constrained activities such as work and education do not diverge substantially from the preference. This example shows the trade-offs between different choice dimensions and the activities with conflicting timings.

4.2.2 Family of 2; 2 adults with no children

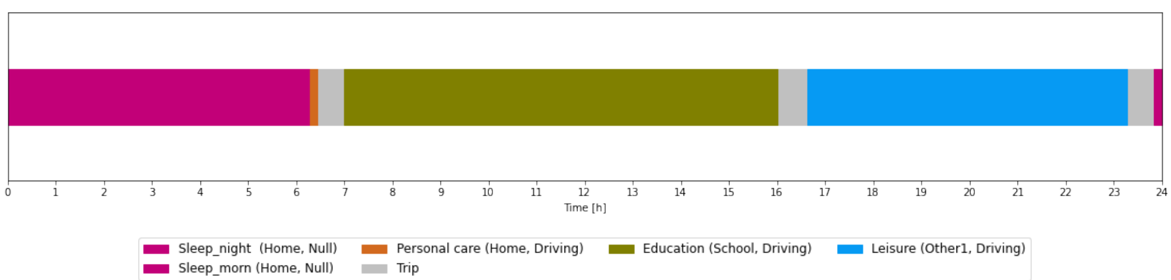
In this section, an example of a family of two with Sara and David as a cohabiting couple is illustrated. The household owns 1 car. 100 realisations of the schedules are generated for this example and one is arbitrarily



(a) Example generated schedule for Sara as an independent individual



(b) Example generated schedule for David as an independent individual



(c) Example generated schedule for Alice as an independent individual

Figure 2: Example generated schedules for Sara, David, and Alice as independent individuals

selected for illustration.

Figure 3 presents an arbitrarily selected realisation from the distribution of generated schedules. The outcomes of the model on car location sequence and occupancy are presented in Table 7. The column Parked_out indicator indicates whether the car is parked at an out-of-home location (1), or not (0). This example showcases the interactions within the household such as assigning daily household maintenance duties to the core adults, allocation of the car to household members, joint activity participation, and shared rides.

The selected realisation illustrates an example of joint activity participation. Sara and David jointly participate in a leisure activity at an out-of-home location in the evening and share a ride for the commute. The synchronization between the schedules of Sara and David for the joint activity engagement can be observed in Figure 3. Both Sara and David have deviated their schedule from their preference in order to create a time-window overlaps for the joint leisure activity. The simulated sequence for car location and occupancy, presented in Table 7, are also consistent with the schedules of the agents.

Moreover, the location and mode choices of the agents are compatible with the availability and allocation of the household car. In the generated schedule, Sara takes the car in the morning to travel to work and do the household maintenance (e.g., grocery shopping) on her way back from work to home. As the household owns only 1 car, the car would not be available to David. David has chosen home to work at home in order to save time on commuting.

The results show the capability of the modelling framework to simulate compatible schedules for the agents in multi-member households considering complex behaviours and interactions within members.

Table 7: Car location sequence and occupancy in the example of family of 2

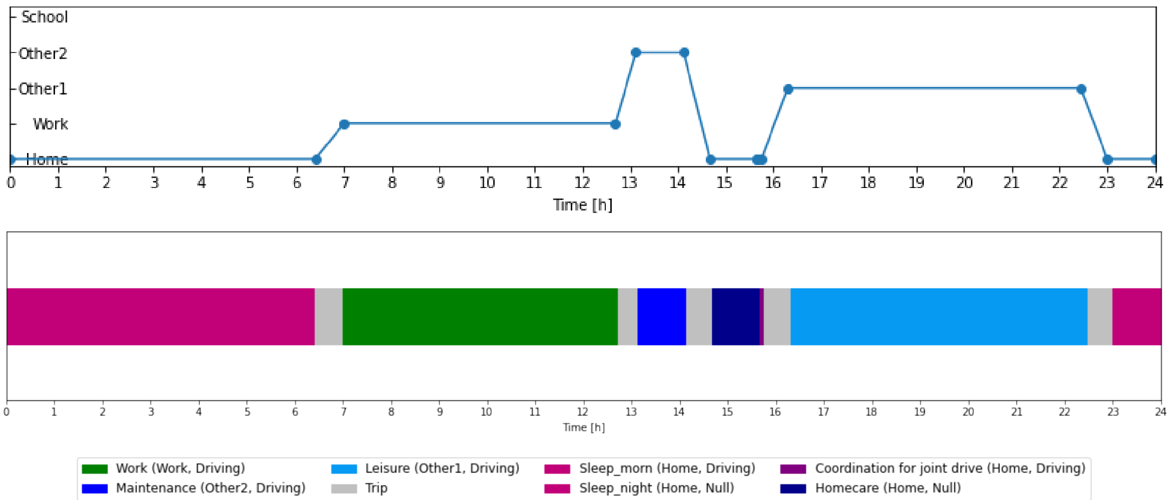
Location	Start time (hh:mm)	End time (hh:mm)	Duration (hh:mm)	Person using	Parked_out indicator	Car occupancy
Home	00:00	6:24	6:24	-	0	0
On the road	6:24	7:00	0:36	1	0	1
Work	7:00	12:41	5:41	1	1	0
On the road	12:41	13:07	0:26	1	0	1
Other2	13:07	14:07	1:00	1	1	0
On the road	14:07	14:40	0:33	1	0	1
Home	14:40	15:45	1:05	-	0	0
On the road	15:45	16:18	0:33	1&2	0	2
Other1	16:18	22:27	6:08	1&2	1	0
On the road	22:27	23:00	0:33	1&2	0	2
Home	23:00	24:00	1:00	-	0	0

4.2.3 Family of 3; 2 adults and 1 child

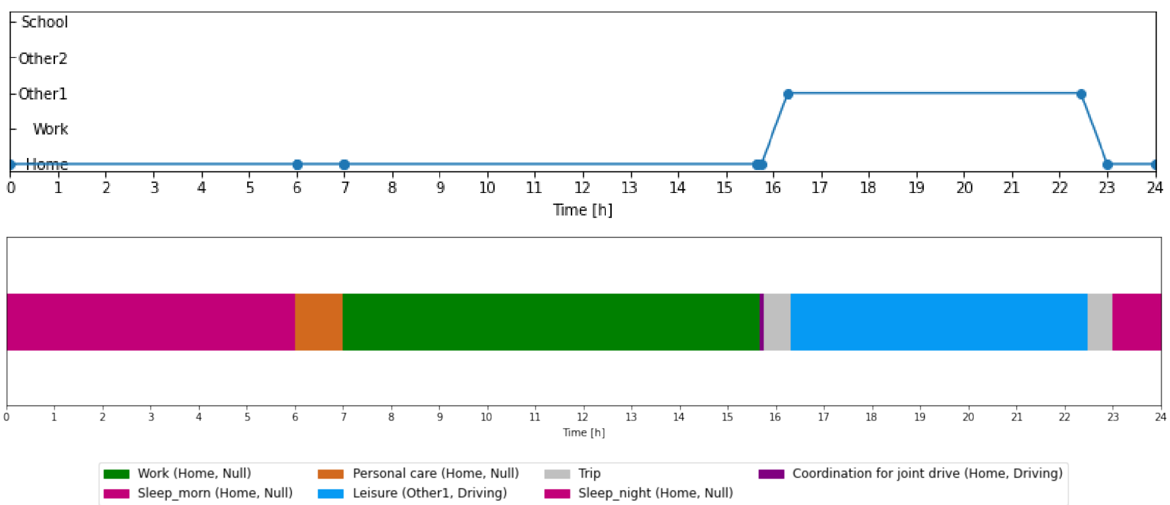
This section presents an example of a family of three, with Sara and David as a cohabiting couple and Alice being a school student. The household owns 1 car. Out of 100 simulation results for this example, one realisation is arbitrarily selected for the sake of illustration.

Figure 4 presents an arbitrarily selected realisation from the distribution of generated schedules. The outcomes of the model on car location sequence and occupancy are presented in Table 8. The column Parked_out indicator indicates whether the car is parked at an out-of-home location (1), or not (0).

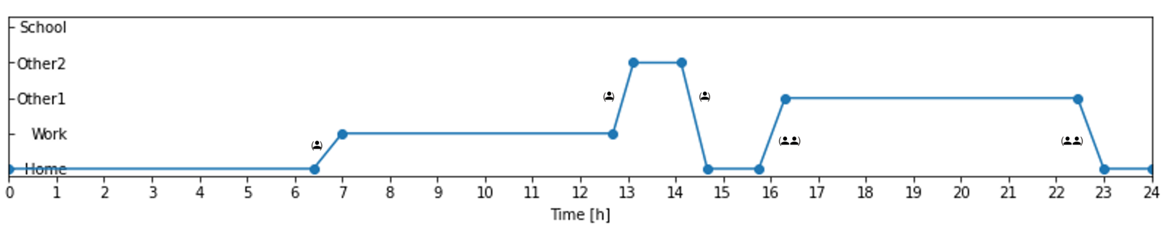
This example showcases a schedule realisation in which an adult should escort the children for their out-of-home activities. In the selected realisation, David drops-off and picks-up Alice by car on his home to work



(a) Generated schedule and location sequence for Sara



(b) Generated schedule and location sequence for David



(c) Generated location sequence for the car

Figure 3: Generated schedules and location sequences of Sara, David, and the car in the example of family of 2

tour. The synchronization between the schedules of David and Alice for the escort duty can be observed in Figure 4. The simulated sequence for car location and its occupancy, presented in Table 8, are consistent with the schedules of the agents.

Table 8: Car location sequence and occupancy in the example of family of 3

Location	Start time (hh:mm)	End time (hh:mm)	Duration (hh:mm)	Person using	Parked_out indicator	Car occupancy
Home	00:00	7:00	7:00	-	0	0
On the road	7:00	7:33	0:33	2&3	0	2
School	7:33	7:35	0:02	2	0	1
On the road	7:35	8:05	0:30	2	0	1
Work	8:05	16:45	8:40	2	1	0
On the road	16:45	17:11	0:26	2	0	1
School	17:11	17:13	0:02	2	1	1
On the road	17:13	17:46	0:33	2&3	0	2
Home	17:46	24:00	6:14	-	0	0

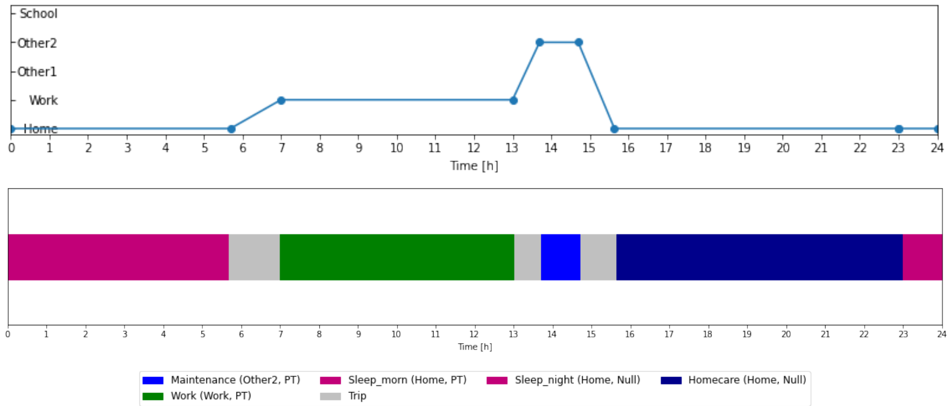
4.2.4 Distributions of schedules

For the considered examples, we aggregate the model outcomes generated from several iterations of the model and present the distribution of schedule frequencies over a day. We have run 1000 iterations of the model in the example of independent agents and 100 iterations in the multi-member examples. The schedule frequency for Sara, David, and Alice with model calibrations as independent agents and members of a multi-member family are presented in Figure 5, 6, and 7, respectively. The frequency distributions are stacked and the remaining grey area at each point in time presents trips.

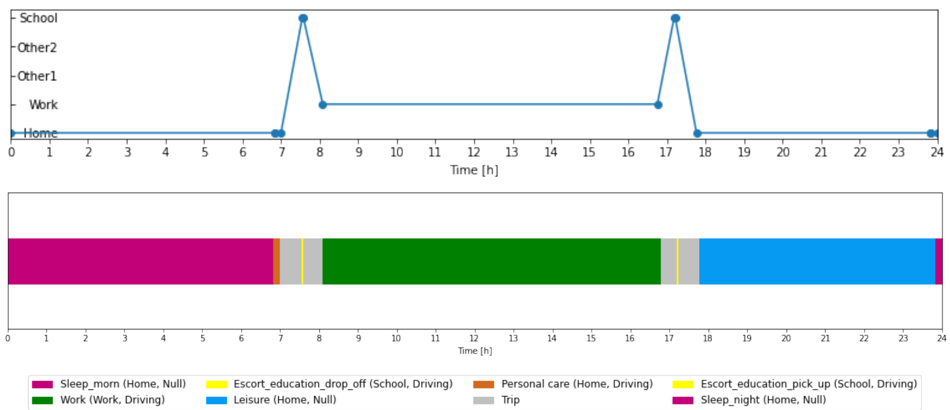
Looking at the progressions of the schedule frequency plots for each agent, we can observe that it is mostly the more flexible activities, which are affected due to intra-household coordinations. These are activities that are less penalised if deviated. For example, in Figures 5 for Sara, as the household size becomes larger, the leisure activity is shifted to be scheduled at later times and for shorter periods. It can be observed that as we move from the example of an independent agent towards the example of a family of 3, leisure activity covers a smaller time span in a day. This schedule deviation is caused by the addition of household duties (e.g. household maintenance and escort duties) to the agents' schedules when treated as a member of a multi-member household. Another influential factor for the observed trend is the household car availability limitation. The car availability limitation in an auto-deficient household restricts the mode choice of household members and thus, increases their usage of public transport. In general, public transport modes have longer travel times compared to driving. As the time budget is limited, the longer the commute times, the shorter the duration of the activities.

The activity location choices are also affected by intra-household interactions. For instance, as we can observe in Figure 7 for Alice, the location choice behaviour of the discretionary activity, leisure, is affected when the interactions are captured within the model. In the example of independent individuals, the leisure activity is scheduled at an out-of-home location in 94% of the generated schedules, whereas, leisure is always scheduled at home when the interaction with other household members is considered (Figure 8). As the children need to be escorted by an adult agent to out-of-home locations, an out-of-home activity location choice for Alice would require schedule synchronizations with the other adult agents. The schedule deviations and coordination costs for escorting causes the observed change in location choice behaviour.

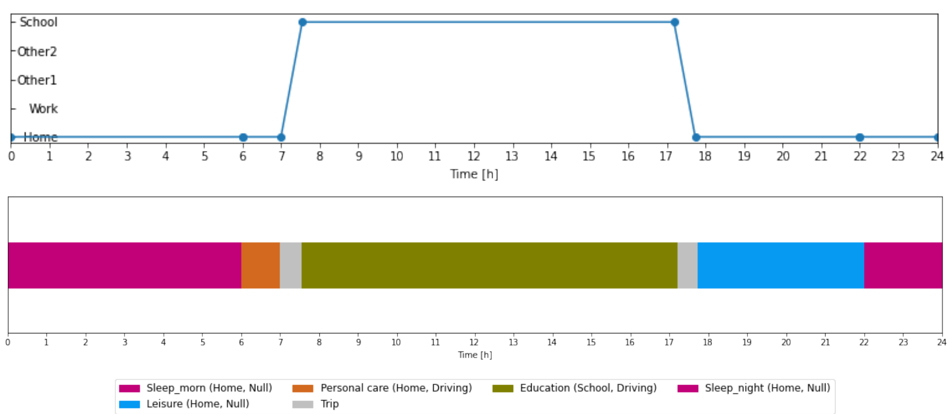
The results show that the simulation framework generates a reasonable distribution of schedules and can capture the change in scheduling behaviour of agents treated as an independent agent or as a member of the household.



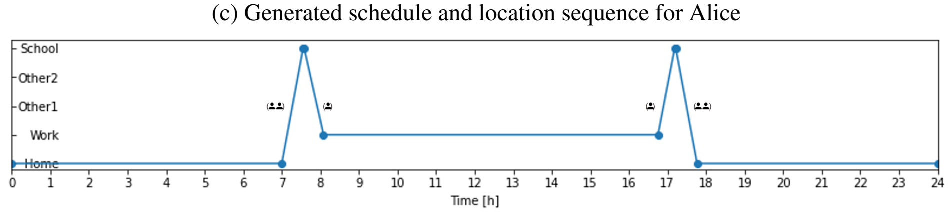
(a) Generated schedule and location sequence for Sara



(b) Generated schedule and location sequence for David

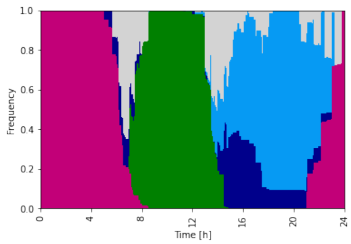


(c) Generated schedule and location sequence for Alice

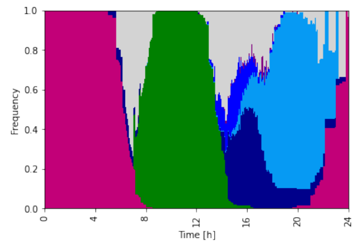


(d) Generated location sequence for the car

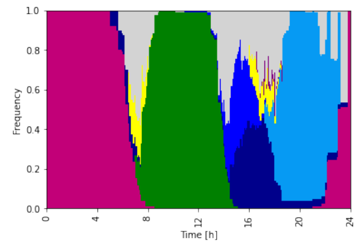
Figure 4: Generated schedules and location sequences of Sara, David, Alice, and the car in the example of family of 3



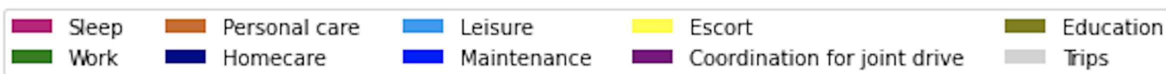
(a) Simulated distribution of activities for Sara in the example of independent individual



(b) Simulated distribution of activities for Sara in the example of family of 2

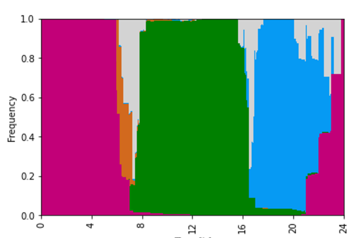


(c) Simulated distribution of activities for Sara in the example of family of 3

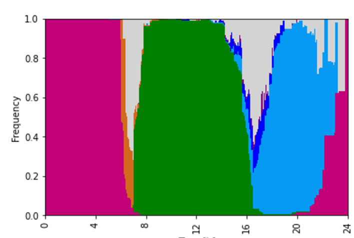


(d) Bar plot color guide

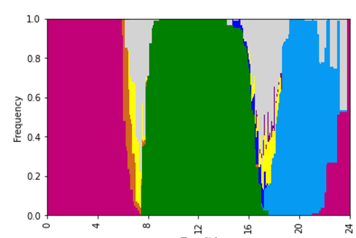
Figure 5: Comparison of the distribution of simulated activity schedules for Sara



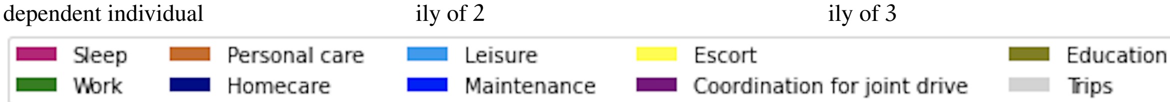
(a) Simulated distribution of activities for David in the example of independent individual



(b) Simulated distribution of activities for David in the example of family of 2

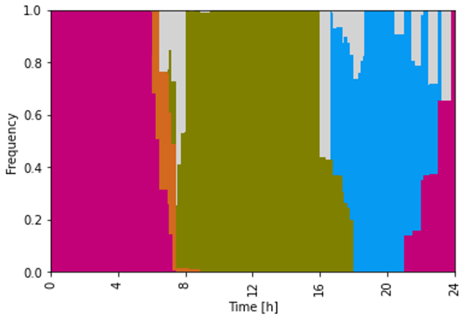


(c) Simulated distribution of activities for David in the example of family of 3

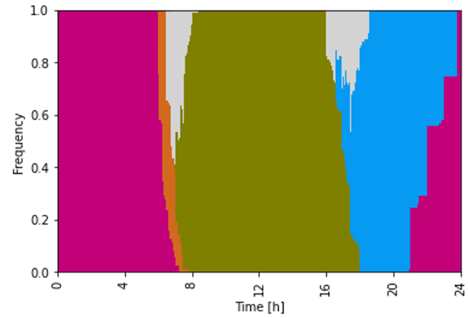


(d) Bar plot color guide

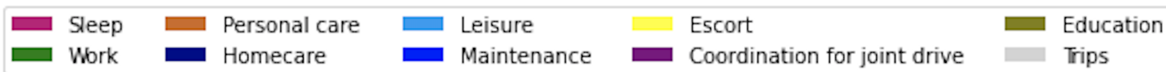
Figure 6: Comparison of the distribution of simulated activity schedules for David



(a) Simulated distribution of activities for Alice in the example of independent individual



(b) Simulated distribution of activities for Alice in the example of family of 3



(c) Bar plot color guide

Figure 7: Comparison of the distribution of simulated activity schedules for Alice

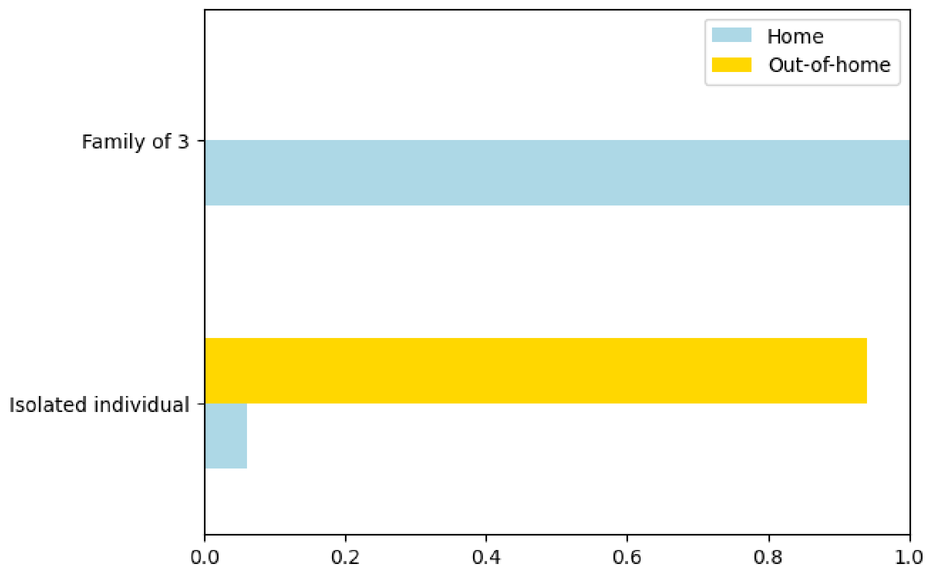
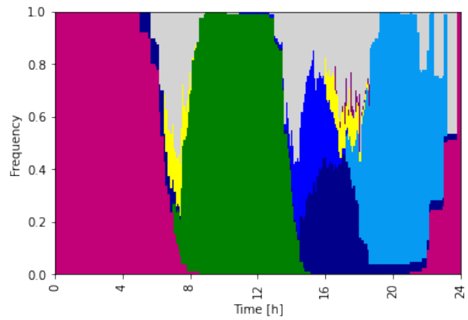


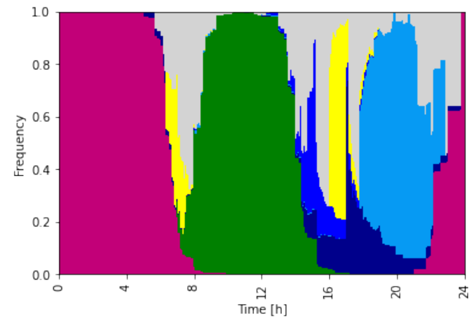
Figure 8: Proportion of location choice for the leisure activity for Alice in the simulated examples

4.2.5 Scenario analysis

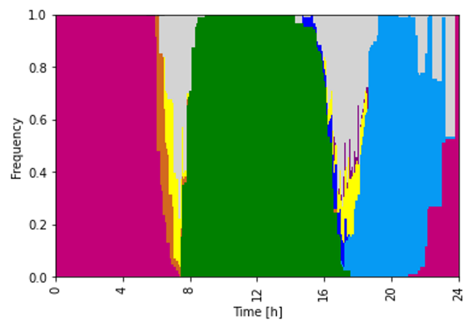
We illustrate a scenario analysis to demonstrate the importance of accounting for household private vehicle ownership when simulating the schedules of its members. Consider the example of Sara, David, and Alice living together in their family of 3 where the household has 1 car which can be used by all members. Now think of a scenario where the household has no cars. The distributions of simulated schedules for these scenarios are presented in Figure 9. As observed, the activity patterns especially the peaks for the more flexible activities such as maintenance and leisure are different in these two scenarios. The travel patterns are also affected accordingly. For example, escorting duties take longer in the scenario with no cars. This thus affects the travel patterns and activity schedules, as well. Therefore, ignoring the private vehicle ownership of the household can lead to wrong analyses of travel and activity patterns of agents which can be utilised by analysts for estimating the transport



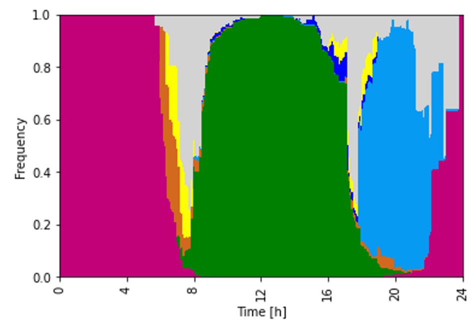
(a) Simulated distribution of activities for **Sara** in the example of family of 3 with 1 **car**



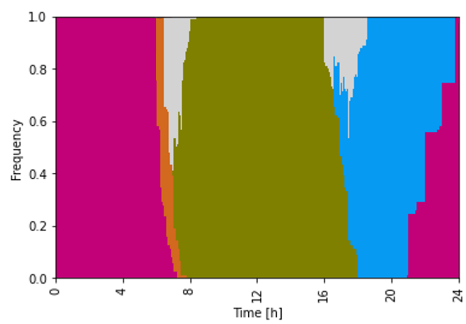
(b) Simulated distribution of activities for **Sara** in the example of family of 3 with **no cars**



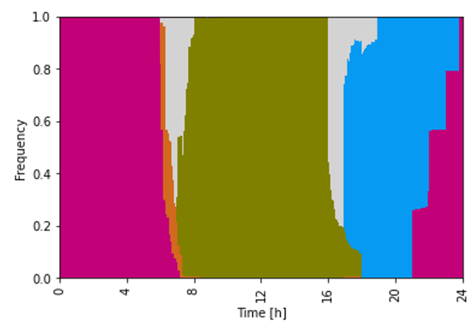
(c) Simulated distribution of activities for **David** in the example of family of 3 with 1 **car**



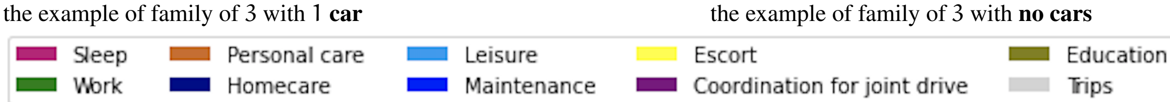
(d) Simulated distribution of activities for **David** in the example of family of 3 with **no cars**



(e) Simulated distribution of activities for **Alice** in the example of family of 3 with 1 **car**



(f) Simulated distribution of activities for **Alice** in the example of family of 3 with **no cars**



(g) Bar plot color guide

Figure 9: Scenario testing; distribution of simulated schedules for Sara, David, and Alice in a family of 3

and energy demands.

5 Conclusion and future work

This paper captures multiple interactions within a single activity-based model. Activity engagements of individuals are affected by various interactions dimensions such as the intra-household interplay. We reconstruct the daily activity schedules of individuals in the same household, considering both the individual- and household-level needs, preferences, and constraints. The model explicitly accommodates complex interactions among household members such as the allocation of the private vehicle to household members, escort duties, joint participation in activities, and sharing rides. Due to the flexible nature of the framework, interaction dimensions can be arbitrarily added. This methodology contributes to a more robust understanding of how intra-household dynamics influence the activity and travel behaviour of individuals. Our methodology builds on the optimization-based scheduling framework, OASIS Pougala et al. (2022*a*), which focuses on modelling the out-of-home activities at the level of isolated individuals. The main characteristics of our methodology are as follows:

- The activity scheduling is at the level of the household, rather than isolated individuals. It incorporates the group decision-making mechanism. The objective is to maximise the total utility of the household.
- It explicitly accounts for multiple interaction dimensions within the same framework. Therefore, it can be utilized for assessing policies aimed at groups.
- It captures resource constraints.
- Both in- and out-of-home activities are simulated within the same framework. The information can serve various purposes such as transport and energy demand-side management, as well as, evaluating the trade-offs between in-and out-of-home activities.
- Simultaneous simulation of different choice dimensions (activity participation, schedule, location, transport mode, participation mode, etc), which is more behaviourally realistic than the sequential models.
- It has a mechanism to incorporate behaviour change. Therefore, it allows planners to examine a wider range of policies such as changing the activity's timing constraints (e.g. flexi-time at work to encourage peak spreading), encouraging remote working, using online services and shopping, and land-use policies.

The proposed framework is a general framework applicable to different household compositions and available resources. This will address the limitation of current models applicable to specific cases. Besides the strengths of the current implementation of the framework, there also exist weaknesses. The model is flexible to extensions and various interactions, however, the speed and performance of the model searching for optimality can rapidly increase with the size of the activity choice set and the model complexity. More interplays and larger choice sets can be added in a straightforward manner but might increase the computational expense which can become prohibitive in practical applications. Moreover, a linear utility specification is assumed for the objective function, which might not necessarily be representative of the complex human behaviour.

There are further extensions and improvements of the current work, suggesting paths for future research. Day-to-day interactions in multi-day scheduling such as habit formation and activity frequencies is currently ignored in our framework. One of the interesting model extensions is capturing correlations between day-to-day scheduling for multi-day analysis within the framework. The framework can also be extended to accommodate other complex interaction dimensions such as interpersonal interactions beyond the household level known as social interactions. It is notable that the higher computational cost due to added complexities should be considered. Moreover, in the first operationalised version of the framework, we have assumed the value of the parameters to be known based on the literature. In order to represent the agents' behaviour more accurately, the model parameters should be estimated from the data. For example, the parameter estimation procedure using the maximum likelihood estimation technique proposed by Pougala et al. (2022*c*) can be used for this purpose. Another interesting research avenue is exploring the practical applications of the model in transportation and energy. The explicit consideration of inter-household interactions allows our framework to evaluate policies aimed at group travel such as high occupancy vehicle lanes and discounted transport fares for group travellers. Utilizing the proposed framework for various scenario analyses such as changes in built environment and lifestyles, or policy testings such as the effectiveness of High occupancy vehicle (HOV) would give us behaviourally credible insights. Using the framework for real-time rescheduling can be another interesting avenue for future research. Finally, dis-aggregated household-level time-use data collection would make a great contribution to studies on intra-household behaviour.

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A Appendix

A.1 Notations

In this appendix, we present a notation table, summarising the notations used in the framework illustrated in the original article.

Table A1: Notations used in the framework grouped by category

Notation	Name	Description
Main Variables		
n	Agent	An individual in the household having decision-making capabilities, $n \in \{1, 2, ...N_m\}$.
r	Resource	A household resource with no decision-making capabilities and purely used by the agents.
N_m	Household size	Number of agents in the household.
N_r	Number of household resources	The number of household resources of the same kind, which can be used by all its members upon availability.
O_r	Resource occupancy	The number of agents using resource r at the same time.
C_r	Resource capacity	Maximum number of agents that can use resource r at the same time
T	Time budget	The time period over which the schedules are simulated.
t	Time	The activity schedules are simulated over a time period T, with the start time at $t = 0$ until the end of the time horizon $t = T$.
A_n	Considered activity set	An activity set containing all activities that agent n considers performing within her time budget T.
E^r	Associated resource event set	An event set containing all possible events e_r that can be scheduled for resource r within the time budget T.
a_n	Activity	Activity a_n that can be performed by agent n.
e_r	Resource event	Event e_r that can be scheduled for resource r.
ℓ_{a_n}	Activity location	Location of activity a_n.

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Table A1 - Notations used in the framework grouped by category (Continued)

Notation	Name	Description
ℓ_{e_r}	Resource location	Resource location for event e_r.
L_{a_n}	Activity location choice set	A discrete and finite location choice set containing all locations that agent n considers for activity a_n.
M	Transport mode choice set	A discrete and finite list of considered transport modes.
m_{a_n}	Transportation mode	The mode to travel from the location of the current activity, ℓ_{a_n}, to the location of the following activity, ℓ_{a+1_n}.
$\rho(\ell_o, \ell_d, m)$	Travel time	The travel time between the locations ℓ_o and ℓ_d with mode m is characterized by $\rho(\ell_o, \ell_d, m)$.
$\rho^{\min}$	Minimum travel time	Minimum travel time between the locations in the case study of interest.
p_{a_n}	Activity participation mode	A binary variable, indicating whether activity a_n is performed jointly with other agent(s), 1, or is done solo, 0.
$\tau_{a_n}^{\min}$	Minimum activity duration	Minimum duration of activity a_n.
$[\tau_{a_n}^{*-}, \tau_{a_n}^{*+}]$	Desired activity duration range	A time range indicating the desired duration of activity a_n.
$[x_{a_n}^{*-}, x_{a_n}^{*+}]$	Desired activity start time range	A time range indicating the desired start time of activity a_n.
$[\gamma_{a_n}^{-}, \gamma_{a_n}^{+}]$	Feasible activity time range	A time range indicating the feasible time range during which activity a_n can take place.
G_{q_n}	Activity group	Each activity a_n is associated with a group G_{q_n}, which contains all possible combinations of locations, transport modes, and participation modes of that activity.
k_{a_n}	Activity scheduling flexibility	Specifies how sensitive activity a_n is to schedule deviations from the preference.
λ_{a_n}	Escort indicator	A binary variable, which specifies whether activity a_n is/needs escort, or not.
χ_{a_n}	Escort type	A variable indicating the escort type for escort activity a_n.
ϑ	Pick-up/drop-off duration	The stop time duration needed to pick-up or drop-off the passenger.

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Table A1 - Notations used in the framework grouped by category (Continued)

Notation	**Name**	**Description**
	Decision Variables	
ω_{a_n}	Activity participation	A binary variable equal to 1 if agent n participates in activity a_n, and 0 otherwise.
ω_{e_r}	Event occurrence	A binary variable equals to 1 if event e_r is scheduled for resource r, and 0 otherwise.
x_{a_n}	Activity start time	A positive continuous variable representing the start time of activity a_n.
x_{e_r}	Event start time	A positive continuous variable representing the start time of event e_r for resource r.
τ_{a_n}	Activity duration	A positive continuous variable representing the duration of activity a_n.
τ_{e_r}	Event duration	A positive continuous variable representing the duration of event e_r for resource r.
$z_{a_n b_n}$	Activity succession	A binary variable representing activity succession, equal to 1 if agent n schedules activity b_n immediately after activity a_n, and 0 otherwise.
$z_{e_r e'_r}$	Event succession	A binary variable representing resource event succession, which is 1 if event e'_r is scheduled immediately after event e_r for resource r, and 0 otherwise.
	Utility functions	
U_n	Schedule utility	The utility of the schedule of agent n
U_n^{gen}	General utility	A generic utility capturing characteristics of the whole schedule not directly linked with any specific activity.
U_n^{partic}	Participation utility	A utility term, purely associated with participation in activity a_n, irrespective of its timing and associated trips.
$U_{a_n}^{\text{start}}$	Start time utility	A utility term capturing the perceived penalty of deviation in start time from the desired start time.
$U_{a_n}^{\text{duration}}$	Duration utility	A utility term capturing the perceived penalty of deviation in duration of activity a_n from the preference.

Continued on next page

Table A1 - Notations used in the framework grouped by category (Continued)

Notation	Name	Description
$U^{travel}_{a_n,b_n}$	Travel utility	A utility term associated with the trip from ℓ_{a_n} to ℓ_{b_n}.
$U^{location}_{a_n}$	Location utility	A utility term capturing the utility of different activity location choices
$U^{joint}_{a_n}$	Joint participation utility	A utility terms capturing the (dis)utility of joint activity engagement.
$U^{escort}_{a_n}$	Escort utility	A utility term capturing the (dis)utility of escorting other agent(s).
Parameters		
w_n	Priority parameter	Relative weight capturing the priority that is placed on the schedule utility of each individual.
$\alpha^{loc}_{a_n}$	Location specific parameter	A parameter associated with activity location ℓ_{a_n}.
$\alpha^{jnt}_{a_n}$	Joint participation parameter	A parameter capturing the (dis)utility of joint activity engagement.
θ^{esc}_{s}	Escort parameter	A penalty parameter associated with escort duration for individuals with employment status s.

A.2 Household-level schedule optimisation

Putting together the model objective function and constraints discussed in the original article, the resulting household-level optimisation-based scheduling problem is formulated as follows:

$$\max \sum_{n=1}^{n=N_m} w_n \, U_n \tag{A1}$$

subject to the following constraints:

$$\sum_{a_n} \sum_{b_n} (\omega_{a_n} \, \tau_{a_n} + z_{a_n b_n} \, \rho(\ell_{a_n}, \ell_{b_n}, m)) \; = \; T \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{A2}$$

$$\sum_{a_n \in G_{\text{Sleep\_morn}_n}} \omega_{a_n} \; = \; 1 \qquad \forall n \in \{1, ..., N_m\} \tag{A3}$$

$$\sum_{a_n \in G_{\text{Sleep\_night}_n}} \omega_{a_n} \; = \; 1 \qquad \forall n \in \{1, ..., N_m\} \tag{A4}$$

$$\tau_{a_n} \; \geq \; \omega_{a_n} \, \tau^{min}_{a_n} \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{A5}$$

$$\tau_{a_n} \leq \omega_{a_n} T \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{A6}$$

$$z_{a_n b_n} + z_{b_n a_n} \leq 1 \qquad \forall a_n, b_n \in A^n, a_n \neq b_n, \forall n \in \{1, ..., N_m\} \tag{A7}$$

$$\sum_{b_n \in G_{\text{Sleep\_morn}_n}} z_{a_n b_n} = 0 \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{A8}$$

$$\sum_{b_n \in G_{\text{Sleep\_night}_n}} z_{b_n a_n} = 0 \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{A9}$$

$$\sum_{a_n} z_{a_n b_n} = \omega_{b_n} \qquad \forall b_n \in A^n, b_n \neq \text{Sleep\_morn}_n, \forall n \in \{1, ..., N_m\} \tag{A10}$$

$$\sum_{b_n} z_{a_n b_n} = \omega_{a_n} \qquad \forall a_n \in A^n, a_n \neq \text{Sleep\_night}_n, \forall n \in \{1, ..., N_m\} \tag{A11}$$

$$(z_{a_n b_n} - 1) T \leq x_{a_n} + \tau_{a_n} + z_{a_n b_n} \rho(\ell_{a_n}, \ell_{b_n}, m) - x_{b_n} \qquad \forall a_n, b_n \in A^n, a_n \neq b_n, \forall n \in \{1, ..., N_m\} \tag{A12}$$

$$(1 - z_{a_n b_n}) T \geq x_{a_n} + \tau_{a_n} + z_{a_n b_n} \rho(\ell_{a_n}, \ell_{b_n}, m) - x_{b_n} \qquad \forall a_n, b_n \in A^n, a_n \neq b_n, \forall n \in \{1, ..., N_m\} \tag{A13}$$

$$\sum_{a_n \in G_{q_n}} \omega_{a_n} \leq 1 \qquad \forall q_n = 1, ..., Q_n, \forall n \in \{1, ..., N_m\} \tag{A14}$$

$$m^V_{a_n} \geq m^V_{b_n} + z_{a_n b_n} - 1 \qquad \forall a_n, b_n \in A^n, \ell_{b_n} \notin \{\text{Home}\}, \forall n \in \{1, ..., N_m\} \tag{A15}$$

$$m^V_{b_n} \geq m^V_{a_n} + z_{a_n b_n} - 1 \qquad \forall a_n, b_n \in A^n, \ell_{b_n} \notin \{\text{Home}\}, \forall n \in \{1, ..., N_m\} \tag{A16}$$

$$\sum_{b_n \in A^n} (z_{a_n b_n} \rho(\ell_{a_n}, \ell_{b_n}, m_{a_n})) \geq \rho^{\text{min}} * \omega_{a_n} \qquad \forall a_n \in A^n, m_{a_n} \notin \{\textit{Null}\}, \forall n \in \{1, ..., N_m\} \tag{A17}$$

$$\sum_{b_n \in A^n} (z_{a_n b_n} \rho(\ell_{a_n}, \ell_{b_n}, m_{a_n})) = 0 \qquad \forall a_n \in A^n, m_{a_n} \in \{\textit{Null}\}, \forall n \in \{1, ..., N_m\} \tag{A18}$$

$$x_{a_n} \geq \gamma^-_a \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{A19}$$

$$x_{a_n} + \tau_{a_n} \leq \gamma^+_a \qquad \forall a_n \in A^n, \forall n \in \{1, ..., N_m\} \tag{A20}$$

$$\omega_{a_n} + m^V_{a_n} \leq N_V + 1 \qquad \forall a_n \in A^n, \forall n \in N_m \tag{A21}$$

$$z_{e_r e_r'} + z_{e_r' e_r} \leq 1 \qquad \forall e_r, e_r' \in E^r, e_r \neq e_r' \tag{A22}$$

$$\left(z_{e_r e_r'} - 1\right) T \leq x_{e_r} + \tau_{e_r} - x_{e_r'} \leq \left(1 - z_{e_r e_r'}\right) T \qquad \forall e_r, e_r' \in E^r \tag{A23}$$

$$\sum_{e_r \in E^r} \frac{\tau_{e_r}}{O_{e_r}} = T \tag{A24}$$

$$O_{e_r} \leq C_r \qquad \forall e_r \in E^r \tag{A25}$$

$$\omega_{e_r} = \omega_{a_n} \qquad \forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \forall n \in \{\text{Adults}\} \tag{A26}$$

$$x_{e_r} = x_{a_n} + \tau_{a_n} \qquad \forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \ell_{a_n} \in \{\text{Home}\}, \forall n \in \{\text{Adults}\} \tag{A27}$$

$$\tau_{e_r} = \sum_{b_n \in A^n} \left(z_{a_n b_n} \rho(\ell_{a_n}, \ell_{b_n}, \text{Driving})\right)$$
$$\forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \ell_{a_n} \in \{\text{Home}\}, \forall n \in \{\text{Adults}\} \tag{A28}$$

$$x_{e_r} = x_{a_n} \qquad \forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \ell_{a_n} \notin \{\text{Home}\}, \forall n \in \{\text{Adults}\} \tag{A29}$$

$$\tau_{e_r} = \tau_{a_n} + \sum_{b_n \in A^n} \left(z_{a_n b_n} \rho(\ell_{a_n}, \ell_{b_n}, \text{Driving})\right)$$
$$\forall e_r \in E^r \cap A^n, \forall a_n \in A^n \cap E^r, \ell_{a_n} \notin \{\text{Home}\}, \forall n \in \{\text{Adults}\} \tag{A30}$$

$$\sum_{n \in \text{Adults}} \omega_{\text{maintenance}_n} = 1 \tag{A31}$$

$$\omega_{a_n} = \omega_{a_{n'}} \qquad \forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \forall n, n' \in \{a, ..., N_m\} \tag{A32}$$

$$x_{a_n} = x_{a_{n'}} \qquad \forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \forall n, n' \in \{a, ..., N_m\} \tag{A33}$$

$$\tau_{a_n} = \tau_{a_{n'}} \qquad \forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \forall n, n' \in \{a, ..., N_m\} \tag{A34}$$

$$\omega_{\text{Coord}_n} = \omega_{\text{Coord}_{n'}} = \omega_{a_n}$$
$$\forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \ell_{a_n} = \ell_{a_{n'}} \notin \{\text{Home}\},$$
$$m_{a_n} = m_{a_{n'}} = \text{Driving}, \forall n, n' \in \{a, ..., N_m\} \tag{A35}$$

$$z_{\text{Coord}, a_n} = z_{\text{Coord}, a_{n'}} = \omega_{a_n}$$
$$\forall a_n \in A^n \cap A^{n'}, \forall a_{n'} \in A^{n'} \cap A^n, p_{a_n} = p_{a_{n'}} = 1, \ell_{a_n} = \ell_{a_{n'}} \notin \{\text{Home}\},$$
$$m_{a_n} = m_{a_{n'}} = \text{Driving}, \forall n, n' \in \{a, ..., N_m\} \tag{A36}$$

$$\tau_{\text{Coord}_n} = \tau_{\text{Coord}_{n'}} = \mu\ \omega_{a_n}$$
$$\forall a_n \in A^n, \forall a_{n'} \in A^{n'}, p_{a_n} = p_{a_{n'}} = 1, \ell_{a_n} = \ell_{a_{n'}} \notin \{\text{Home}\},$$
$$m_{a_n} = m_{a_{n'}} = \text{Driving}, \forall n,\ n' \in \{a, ..., N_m\} \quad \text{(A37)}$$

$$\sum_{n \in \text{Adults}} \omega_{a_n} = \omega_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1 \quad \text{(A38)}$$

$$\sum_{n \in \text{Adults}} x_{a_n} = x_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1 \quad \text{(A39)}$$

$$\sum_{n \in \text{Adults}} \tau_{a_n} = \tau_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 0 \quad \text{(A40)}$$

$$\sum_{n \in \text{Adults}} \sum_{b_n \in A^n} (z_{b_n a_n}\ \ell_{b_n}) = \sum_{b_{\text{Passenger}} \in A^{\text{Passenger}}} \left(z_{b_{\text{Passenger}} a_{\text{Passenger}}}\ \ell_{b_{\text{Passenger}}}\right)$$
$$\forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 0 \quad \text{(A41)}$$

$$\sum_{n \in \text{Adults}} \sum_{b_n \in A^n} (z_{a_n b_n}\ \ell_{b_n}) = \sum_{b_{\text{Passenger}} \in A^{\text{Passenger}}} \left(z_{a_{\text{Passenger}} b_{\text{Passenger}}}\ \ell_{b_{\text{Passenger}}}\right)$$
$$\forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 0 \quad \text{(A42)}$$

$$\sum_{n \in \text{Adults}} \tau_{a_n} = \vartheta\ \omega_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 1 \quad \text{(A43)}$$

$$\sum_{n \in \text{Adults}} \sum_{b_n \in A^n} (z_{a_n b_n}\ \ell_{b_n}) = \sum_{b_{\text{Passenger}} \in A^{\text{Passenger}}} \left(z_{a_{\text{Passenger}} b_{\text{Passenger}}}\ \ell_{b_{\text{Passenger}}}\right)$$
$$\forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 1 \quad \text{(A44)}$$

$$\sum_{n \in \text{Adults}} \tau_{a_n} = \vartheta\ \omega_{a_{\text{Passenger}}} \qquad \forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 2 \quad \text{(A45)}$$

$$\sum_{n \in \text{Adults}} \sum_{b_n \in A^n} (z_{b_n a_n}\ \ell_{b_n}) = \sum_{b_{\text{Passenger}} \in A^{\text{Passenger}}} \left(z_{b_{\text{Passenger}} a_{\text{Passenger}}}\ \ell_{b_{\text{Passenger}}}\right)$$
$$\forall a \in A^{\text{Passenger}} \cap A^{\text{Adults}}, \lambda_{a_{\text{Passenger}}} = 1, \chi_{a_{\text{Passenger}}} = 2 \quad \text{(A46)}$$

Equations A2 to A20 ensure the validity of the schedules at the individual-level and Equations A21 to A46 explicitly capture their feasibility considering the intra-household interactions.