MICROSIMULATION FRAMEWORK FOR URBAN PRICE-TAKER MARKETS

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ABSTRACT

In the context of integrated transportation and other urban engineering infrastructure systems, there are many examples of markets, where consumers exhibit price-taking behaviour. While this behaviour is ubiquitous, the underlying mechanism can be captured in a single framework. Here, we present a microsimulation framework of a price-taker market that recognizes this generality and develop efficient algorithms for the associated market clearing problem. By abstracting the problem as a specific graph theoretic problem (i.e. maximum weighted bipartite graph), first we are able to exploit algorithms that are developed in graph theory. We then explore their appropriateness in terms of large-scale integrated urban microsimulations. Based on which, we further develop a generic and efficient clearing algorithm that takes advantage of the features specific to urban price-taker markets. This clearing solution is then used to operationalize two price-taker markets, from two different contexts, within a microsimulation of urban systems. The initial validation of results against the observed data generally shows a close match.

1. INTRODUCTION

The importance of microsimulation as a framework, to analyse integrated urban infrastructure systems, has been emphasized in recent integrated transportation and land use modelling literature (Wegner, 1995; Miller and Roorda, 2003; Miller, 2008; Hunt, Kriger, and Miller, 2005). Microsimulation provides a comprehensive and flexible framework for modelling the behaviour of individual agents as well as representing the various processes that drive urban evolution (Orcutt, 1957, 1990). In the microsimulation of urban systems, two important dimensions to capture are the decision making of individual agents (or groups of agents[1]), and their interactions with other agents in the markets. In the past forty years, modelling and analysis of decision making (e.g. households and firms' location, mode, and vehicle choice decisions, etc.) in the urban context have received considerable importance from economics, transportation, environment, energy, real estate, and urban planning literature. However, the modelling of inter-agent interactions within urban markets (e.g. housing, freight, airline seats auctions, etc.) remains relatively unexplored (Miller *et al.*, 2004, Zhang and Levinson, 2004).

Farooq (2011) conceptualized urban markets as the encapsulation of interactions between seller/producer and buyer/consumer agents that result in the exchange of a service/good and a monetary transaction. The goal of both buyers and sellers within this interaction is to achieve some desirable gain in terms of their profit/utility. Based on how the monetary value is formulated in this interaction, these markets can be categorized as either *Price-Taker* or *Price-Formation* markets. In both price-taker and price-formation markets there exist producer/seller and consumer/buyer agents that are profit/utility maximizers with varying levels of information about the market. Producers list their good at a certain asking price in the market. Consumers

[1] Decision Making Unit (DMU) is the generic term used for individual or group of agents that are involved in decision making (Miller, 2005a, b).

form their choice sets from the available options in the market. In the price-taker case consumers are assumed to accept the asking price as it is and determine the gain in their utility/profit at that price. By comparing the relative gains among the choices available to it, a consumer may decide on choosing one option. In terms of microsimulation, the modelling of price-taker market clearing problem thus becomes a matching problem in which the modeller is interested in finding out *"who gets what"*. The price determination and choice set formation models are exogenous to the clearing process. At a given exogenously determined price surface for the stock and choice sets of the buyer agents, the sequence of individual level clearing in the market thus guides the matching process.

This can be contrasted with price-formation markets, in which prices do not remain fixed during the clearing process but rather are determined within the market clearing process. In terms of microsimulation, the modelling of price-formation market clearing problem is a matching problem in which the modeller is interested in finding out *"who gets what at what price"*. Examples of classic approaches that used market equilibrium to formulate prices are Alonso (1964), Putman (1983), Echenique *et al.* (1990), Martinez (1992), Anas and Arnott (1993), (1994), and de la Barra (1995). Farooq (2011) proposed a disequilibrium-based comprehensive microsimulation framework for modelling urban price-formation markets and operationalized it for the owner-occupied housing market in the Greater Toronto and Hamilton Area within the ILUTE (Miller *et al.*, 2011) modelling system. Other examples of operational urban price-formation markets can be found in Waddell *et al.* (2003), Ettema *et al.* (2006), and Devisch *et al.* (2008).

The focus of this paper is on the urban price-taker markets. Here we present a generalized clearing framework developed for the urban price-taker markets, by reducing the clearing problem of the market to a graph theoretic problem. Such a market is represented as a bipartite graph in which consumers and products/producers are the vertices of the two sides; choice sets are the edges; and the unidirectional/bidirectional preferences are the weights on the edges. By doing so, the algorithms developed for finding the solution for maximum weighted bipartite matching problem are directly used to find the clearing solution for the urban price-taker markets. The generalized nature of the formulation ensures that the proposed mechanism can be used for clearing various urban markets that comes under the category of price-taker market. The market clearing problem discussed here is equivalent to an *assignment problem*, which is a special case of a class of linear programming problems called the *transportation problem*. To find a solution for an assignment problem, the predominantly used algorithm for transportation problems, called transportation simplex, is inefficient (Winston, 1991). Thus, in the literature, alternative approaches are developed to find the solution for assignment problems. As an initial exploration, here we first employ the most commonly used algorithm of such category, in order to explore various features and appropriateness of these graph theoretic algorithms in the context of large-scale integrated urban microsimulations. Based on this analysis we then develop an algorithm for optimum allocation under the computational and memory constraints that may rise due to the very large size of the market, in the microsimulation of urban systems.

The rest of the paper is organized as follows. Section 2 lists the model's assumptions, introduces the market model structure, clearing problem, and develops the methodology for finding the clearing solution. Section 3 then presents our proposed solution which adopts a probabilistic individual utility maximization approach. In Section 4, we present the application of the proposed framework to two important urban markets. In the last section, we present our concluding remarks and future directions of the research.

2. PRICE-TAKER MARKETS: MODEL STRUCTURE

2.0 Key Assumptions and Definitions

There are two types of agents in the market: consumer agents (persons, households and firms, etc.) and producer agents (persons, households, airlines, builders, landlords, etc.). The assumptions concerning each of these agents are listed below.

Generic assumptions:

- Agents maximize their individual profit/utility
- Agents are non-cooperative with varying degrees of information about the market
- The market perceptions (information) of agents are updated as they spend more time in the market
- Agents have the option to stay or leave the market at any time
- The utility function for both consumer and producer agents are exogenously defined

Consumer assumptions:

- Each consumer is looking for a single unit of good to purchase/lease
- There is an exogenous mechanism that generates a choice set for each consumer. This process models the choice set generation process of the consumer. The choice sets generated by this process for all the consumers will then be used by the clearing mechanism. There may be an indirect interaction between market clearing and choice set generation process. For instance: the shortage of certain type of good in short-term that is resulted by faster clearing of that type, may cause the choice set generation mechanism to adjust the choice set of the active consumers based on their reaction
- Due to changing market perceptions, buyers may update their choice sets over time

- The differences among the behaviour of consumers are captured in the utility function and the choice set generation mechanism

Producer assumptions:

- Each producer is offering a single unit of good for sale/lease
- Due to changing market perceptions, producers may adjust their valuation of a good

Definitions

Vertex [v in set V]: an object that may represent certain real life entity (for instance, person, household)

Edge [e in set E]: connects one vertex to another. It may convey the relationship between the two vertices that it connects together (for instance, two persons connected by a sibling edge)

Weight of an edge [w in set W]: It is an integer or a real value associated with an edge. It may convey the intensity of a relationship between the two vertices the edge is connecting

Association of an edge: It is the set of two vertices that an edge is connected to

Graph [G = (V; E, W)]: It is an ordered pair consisting of a set of vertices V connected by edges from set E having weights from set W

Adjacent vertices (v_i, v_j): Two vertices v_i and v_j in a graph that are directly connected by an edge (e_{ij}) of the same graph

Cardinality: Number of elements in the set. If $A = \{a, b, c, d\}$, then the cardinality $|A|$ is 4

Disjoint sets: Sets whose intersection is the empty set. There is no common element between the disjoint sets. If $A = \{a, b, c, d\}$ and $B = \{f, g, h\}$ then set A and B are disjoint

2.1 Model Structure

Suppose that in a price-taker market, there are *N* consumers interested in buying/leasing a differentiated good offered by *M* producers, offering one good each. Before making a selection, consumers generate a list of goods (i.e. choice set) that are of interest. They establish preferences for each good within their choice set which are based on their consumption behaviour and the attributes of the goods. Producers may also assign a value to the good that influences their preferences for each consumer. If the choice set and individual degree of preferences for all consumers/producers are known, we can express the market in the form of a bipartite graph (*G*). A bipartite graph is a type of graph that has two disjoint vertex sets, such that no two vertices in the same set are adjacent (Wilson, 1979; Gondran and Minoux, 1984; and Cormen *et al.*, 2001). Let the graph in Figure 1 be represented by $G = (C, P; E)$, where *C* and *P* are the two disjoint sets and $E \subseteq C \times P$ represents the set of edges between the vertices of the two sets. Note that the intersection of sets *C* and *P* is a null set and the cardinality of their union is the sum of their individual cardinalities. Also, if $E_1 \subset E$ is a set of edges strictly between vertices in set *C* and $E_2 \subset E$ is a set of edges strictly between vertices in set *P* then the cardinality of both E_1 and E_2 is strictly zero. An alternate way of defining this property is that every edge in set E is associated with one and only one vertex from each of the two disjoint vertex sets (*C* and *P*).

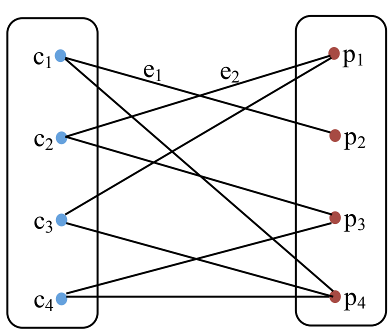


Figure 1: A Bipartite Graph G = (C, P; E)

In Figure 1, let set C represent the list of consumers and let set P represent the list of producers in the market. The preferences between C and P are represented by the edges between them, which belong to set E with the weights on each edge representing the individual degree of preference. These edges can be unidirectional or bidirectional. The unidirectional edges represent the case where only consumers generate a choice set and assign a certain degree of preference to each choice. An example of such a case is the rental housing market where the renters look at the available options in the market and form their choice sets and preferences. In the case of bidirectional edges, the mutual preferences are formed as a function of individual preferences of both consumers and producers to each other. An example of this case is potential couples that are matched in an abstract process which we define as a "marriage market". Both rental and marriage markets are discussed in more detail in Section 4.

2.2 Price-Taker Market Clearing as a Matching Problem of a Bipartite Graph

The clearing problem for an urban price-taker market requires using the available choice sets and the degrees of preferences to determine the one-to-one matching between consumers and producers. This matching problem, under the graph abstraction of the market defined in the previous section, can be restated as a problem of finding the maximum weighted bipartite matching. This approach provides the "best" possible matches that can be made for the market at hand. Suppose that for every edge e in set E, there is an associated weight w in set $W: C \times P \rightarrow \mathbb{R}$ then $G = (C, P; E, W)$ and the problem of finding maximum weighted bipartite matching can be defined as finding a graph $G^* = (C, P; E^*, W^*)$ such that the cardinality of E^* equals cardinality of C and P. Every vertex in set C is connected to one and only one vertex in set P by an edge in E^* and there are no more than one edge associated with each vertex. Moreover, there doesn't exist a graph $G^{**} = (C, P; E^{**}, W^{**})$ such that the sum of weights in W^{**} is greater than sum of weights in W^{*}

i.e. $\sum_{c \in C} W^{*}(c, e(c)) \geq \sum_{c \in C} W^{**}(c, e(c))$, where $e: C \rightarrow P$.

In the graph theory literature, the problem of maximum weighted matching or assignment has extensively been studied and various efficient algorithms have been developed for this purpose. The problem has proven to be a special case of the minimum cost flow problem, and thus can be solved using linear programming algorithms (Burkard *et. al.*, 2009). Hungarian algorithm is the most commonly used solution for the assignment problem and various variants of it are proposed in the literature. In ILUTE, we first used one such modified version of the Hungarian algorithm in order to implement the clearing process for the urban price-taker market.

3. PRICE-TAKER MARKETS: CLEARING SOLUTIONS

3.1 Hungarian Algorithm

Kuhn (1955) used the König's matching theorem (König, 1931) and Egerváry's generalization of it to the weighted bipartite case (Egerváry, 1931) in order to derive the Hungarian algorithm for finding the maximum weight perfect matching in a bipartite graph (Frank, 2004). The Hungarian algorithm is based on a linear programming approach that involves transforming the problem into combinatorial optimization problem. Suppose the graph $G = (C, P; E, W)$ in Figure 2a is represented by *M* which is a $n \times n$ matrix. The rows in matrix *M* represent set *C* and its columns represent the set *P*. The value of each cell represents the weight of the edge between vertices. If there is no edge between the two pair of vertices, then the cell value is blank.

The steps of Hungarian algorithms are as follows [2] (Winston, 1991).

Step 0: Transform the problem into a minimization problem.

Step 1: For each row, subtract off the minimum cell value from rest of the cells. Each row will have at least one zero and all the values will be greater than or equal to zero.

Step 2: For each column, subtract off the minimum cell value from rest of the cells. Each row and column will have thus at least one zero.

Step 3: Go through the rows and columns and use lines to cover the zeros in the matrix in such a way that all the zeros are covered and that no more lines have been drawn than necessary. Use horizontal line for row and vertical for column.

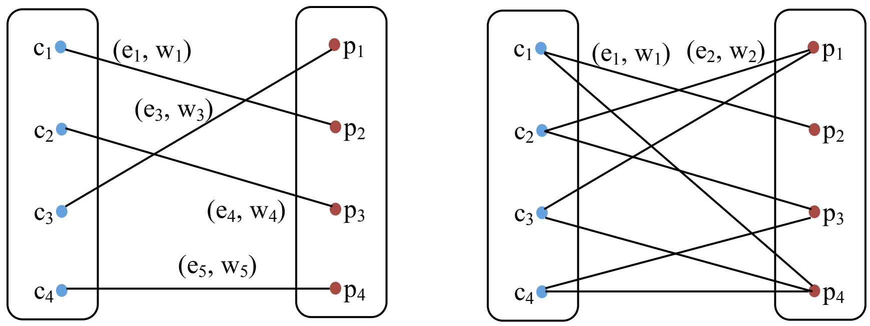
Step 4: Optimality test:

i. If the count of the lines is n, choose a combination from the modified matrix in such a way that the sum is zero

ii. If the number of the lines is less than n, go to Step 5.

Step 5. Find the smallest element which is not covered by any of the lines. Then subtract it from each entry which is not covered by the lines and add it to each entry which is covered by both a vertical and a horizontal line. Go back to Step 3.

[2] It is interesting to note that the original algorithm was proposed at the time when computers were not widely in use. It was thus designed for the matrix to be solved on a piece of paper.



a. G = (C, P; E, W) **b.** G* = (C, P; E*, W*)

Figure 2: Weighted bipartite graph

Note that Kuhn (1955) designed the algorithm for the case of a square matrix, but in the implementation of urban price-taker markets, this is rarely the case. A typical example is the rental housing market, where there can be more households than the number of dwellings available for rent (i.e. rows > columns). In that case, the rectangular matrix is padded with zero values columns so that it becomes a square matrix. This way, our solution will allow for consumer/producers not being able to buy/sell their good at the end of the clearing process. The original Hungarian algorithm proposed by Kuhn (1955) has a time complexity of $O(|E||C||P|)$[3]. In the worst case ($E = C \times P$) the complexity becomes a polynomial of order 4. Tomizawa (1971) proposed some modifications to the original algorithm to reduce the complexity to a polynomial of order 3. In the current implementation of ILUTE, we implemented the version proposed by Tomizawa.

3.2 Issues with a System Optimal Solutions

Winston (1991) showed that the solution computed by the Hungarian algorithm would provide the optimal solution to assignment type problems. This implies that a market clearing solution

[3] Time complexity *O()* of an algorithm explains how it's computational time will respond to the change in the size input (Cormen *et al.*, 2001). Hungarian algorithm's computational time is a product function of number of edges and total vertices.

that employs the Hungarian algorithm presents a system optimal solution, where the maximum sum of profits/utilities is obtained given a certain market. However, system optimal conditions may not be the best way to represent true urban market conditions, as can be observed in the case of user vs. system optimal flows in transportation networks or in modelling the marriages within demographic update module of an agent-based urban modelling system. Both consumers and producers are not expected to sacrifice their individual profits/utilities to improve society's overall utility. If that were the case, drivers would use longer individual routes just to keep the system level travel time low. Or brides and grooms may not marry their true love for the sake of the greater good of the society!

Furthermore, a deterministic system optimal solution leaves out the impact of uncertainty on the urban price-taker markets. Arguably, this stochasticity is an important driver for these markets' results. A landlord seeking to rent out an apartment will not wait until he has surveyed all possible tenants. Often instead the landlord settles on one of the first tenants that meets his asking price. In a similar vein, people who get married forgo the possibility of finding a better match had they remained single. Clearly, these two simple examples illustrate that the rather strong assumptions with a deterministic system optimal solution fails to capture important features of urban price-taker markets.

Finally, another shortcoming of the Hungarian algorithm for this application is the computational and memory size requirements that come from manipulating large matrices involved in the algorithm (i.e. operationalization for very large-scale markets). Gillett (1976) and Winston (1991) reported that in large-scale assignment problems, finding the minimum number of lines in the serial version of the Hungarian algorithm might not be computationally cost effective.

3.3 A Probabilistic Approach

To deal with these issues in the serial implementation, we propose a probabilistic approach that relaxes the assumptions made regarding the maximization of the market's overall utility by introducing some stochasticity in the approach. Our proposed method is more representative of the real life markets and is less burdensome with respect to computational time and memory requirements as well.

The simple algorithm for finding $G^{*} = (C, P; E^{*}, W^{*})$ is as follows:

Step 1: With a predefined random distribution, pick between set C or P[4]

Step 2: From the selected set, choose a vertex v_1 using another predefined random distribution

Step 3: For v_1 choose v_2 such that $w_{12} \geq w_{1i} \forall\, V \rightarrow v_1$, where V is the set that was not chosen in Step 1

Step 4: Remove v_1 and v_2 and all the edges associated with them

Step 5: Stop if either C or P becomes null set. Else, go to Step 1

The probabilistic approach reduces the complexity of the matching processes to $O(max\ (|C|, |P|))$. Note that this algorithm results in linear complexity compared to cubic in the case of the Hungarian algorithm. Moreover, it is not dependent on the number of edges thus reducing the variability between the worst and best cases. The probabilistic approach does not guarantee perfect matching, but gives us an adequate solution that respects both individual profit/utility maximization and uncertainty. This approach is more representative of the real

[4] Examples: predefined random distribution can be uniform (0.5, 0.5) such that both sets have equal probability of being chosen. Or it can be restricted to choosing from only one set by setting the probability of selection to 1

price-taker markets in transportation, where due to the sequence of events and the limited amount of information available to the agents, the clearing of market doesn't always result in a perfect matching.

4. APPLICATIONS

4.1 Price-Taker Markets in ILUTE

In the urban systems modelling and microsimulation research, many urban markets can be expressed as the price-taker market formulation introduced in Section 2. A few examples of such markets include: labour, rental housing, airline seat auctions, bus routes, and spot-freight markets. Here we present the operationalization of two very important markets within ILUTE, using the price-taker market formulation.

The microsimulation modelling of activity-based travel demand and land use evolution requires maintaining the socio-economic characteristics of individual decision makers throughout the simulation horizon. This can be achieved by the implementation of a sophisticated demographic update mechanism within these systems. In ILUTE, the demographic update involves various processes that deal with a person's birth, education-level, driving licence, aging, death, marriage, divorce, and migration—the details of which can be found in Miller *et al.* (2008).

4.2 Marriage Market Model

For this paper, the process of managing agents' marriages in the simulation is of particular interest. In terms of the implementation of this process in a microsimulation framework, marriages can be abstracted as a market clearing problem in which currently single males and females are to be matched according to their mutual preferences. To achieve that, we reduce the process to a price-taker market formulation, which we call a *marriage market*. This market matches prospective husbands and wives together within a utility maximization framework.

At each time step in the ILUTE microsimulation, the decision of whether to look for a potential marital partner for all adults is first evaluated. This results in two pools of single men and single women. The marriage process then determines the choice set for every individual using predefined search criteria (e.g. spatial proximity, age difference, etc.). The random utility based model that was estimated by Choo *et al.* (2008) was adapted for ILUTE and is used to compute the utility of each potential couple. These utilities are based on the potential couple's income(s), education, and the male/female ratios in their respective geographic areas.

The two pools of males and females that are active in the marriage market here can be represented by the set *C* and *P* of the bipartite graph formulated in Section 2. A node in set *C* can represent a male in the pool of potential husbands, and a node in set *P* can represent a female in the pool of potential wives. The choice sets of all the individuals active in the marriage market can be expressed by the edges between sets *C* and *P*, while the mutual utility is represented by the weight on the edges between the sets' elements (i.e. the potential couple). This reduces the marriage market to the price-taker formulation suggested in Section 2. Moreover, the clearing of marriage market then becomes equivalent to the problem of finding the maximum bipartite graph under the conditions defined in Section 2. Note that the edges in the case of the marriage market are bidirectional, which represents the fact that the weight on each edge is a function of the utilities of both the potential bride and groom.

4.3 Marriage Market Operationalization within ILUTE

In the current version of ILUTE (ILUTE v1.0 which is under development), a generic class called the *StaticMarket* (Figure 3) is implemented as a super class representing the price-taker markets. This class encapsulates all the generic features of such a market, and it is based on the theoretical framework described in Section 2. The two clearing algorithms discussed in this paper, are at the moment, implemented in two separate versions of the *StaticMarket*. However, we intend to merge them in a single class that provides the option for the children of this class to select the exact clearing process. Various realizations of price-taker markets, including the *MarriageMarket* and *RentalMarket*, are then inherited from the *StaticMarket* and implement the specific features required by the markets they represent.

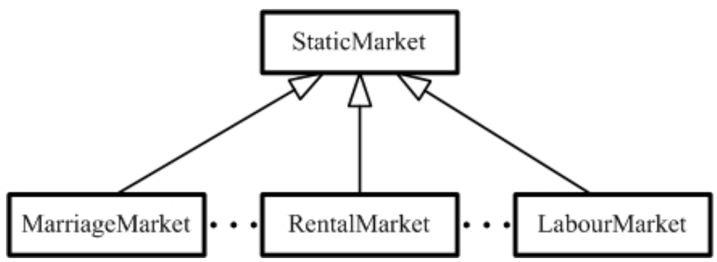


Figure 3: Class structure of price-taker markets within ILUTE

Figure 4 displays a sample relational class diagram for the *StaticMarket* superclass and the markets it represents. The superclass contains bidder and good objects that correspond to both sides of a market, as well as the necessary engines for market operation and clearing.

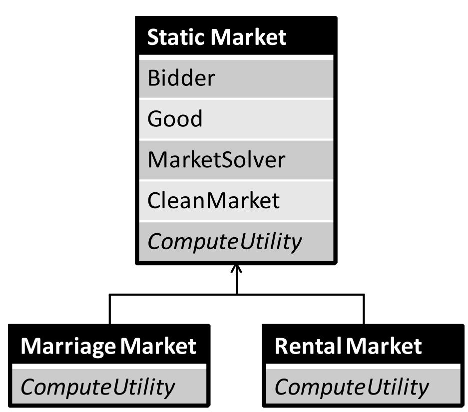


Figure 4: Sample relational class diagram

4.4 Marriage Market Simulation Results

This subsection presents results from the implementation of the *MarriageMarket* in the ILUTE model system. We simulated an initial population representative of the Greater Toronto Hamilton Area for a period of 15 years (1986–2001). Figure 5 shows the age distribution of married people in ILUTE as compared to a representative dataset for the GTHA in 2001. For the most part, the age distribution of married persons is reproduced fairly well. In addition to maintaining the marital age distribution throughout the simulation, it is also important to correctly model the age people decide to get married. Table 1 then shows the mean age of brides and grooms in ILUTE along with comparable historical data. The results are very strong for simulating the mean marrying age of single and divorced individuals. However, there is some divergence for the widowed class, which is not unreasonable due to the smaller market share of widowed persons and the results' inherent dependency on simulating deaths.

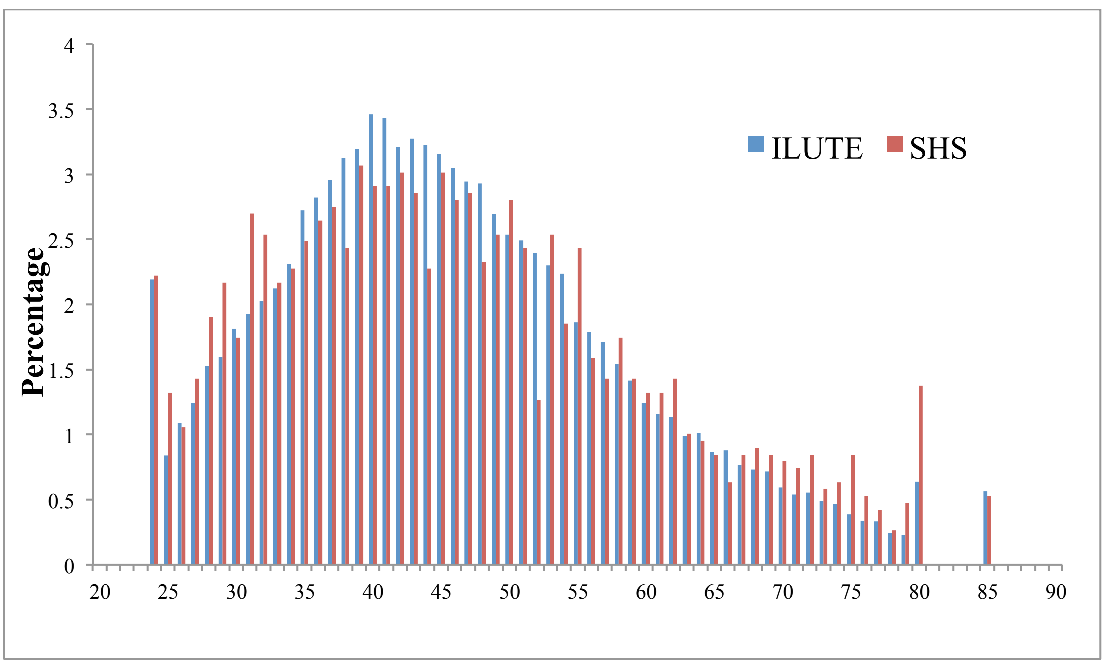


Figure 5: Comparison of results for age distribution of married-individuals, between ILUTE and Survey of Household Spending (SHS) for year 2001

Table 1: Average age by previous marital status of the newly married individuals in (2001)

Average Age of Newly Weds		Statistics Canada	ILUTE	% Error
Groom	Single	29.7	29.1	-2.1
	Widowed	62.4	50.9	-22.6
	Divorced	43.8	44.2	0.9
Brides	Single	27.6	27.3	-1.1
	Widowed	55.4	47.5	-16.6
	Divorced	40.3	41.7	3.4

Expanding on the results from Table 1, the marriage rates by age group (i.e. number of marrying persons divided by the size of the age group) for males and females are displayed on Figure 6. While the general trend is captured by the model, ILUTE shows systematically higher marriage rates than the census data.

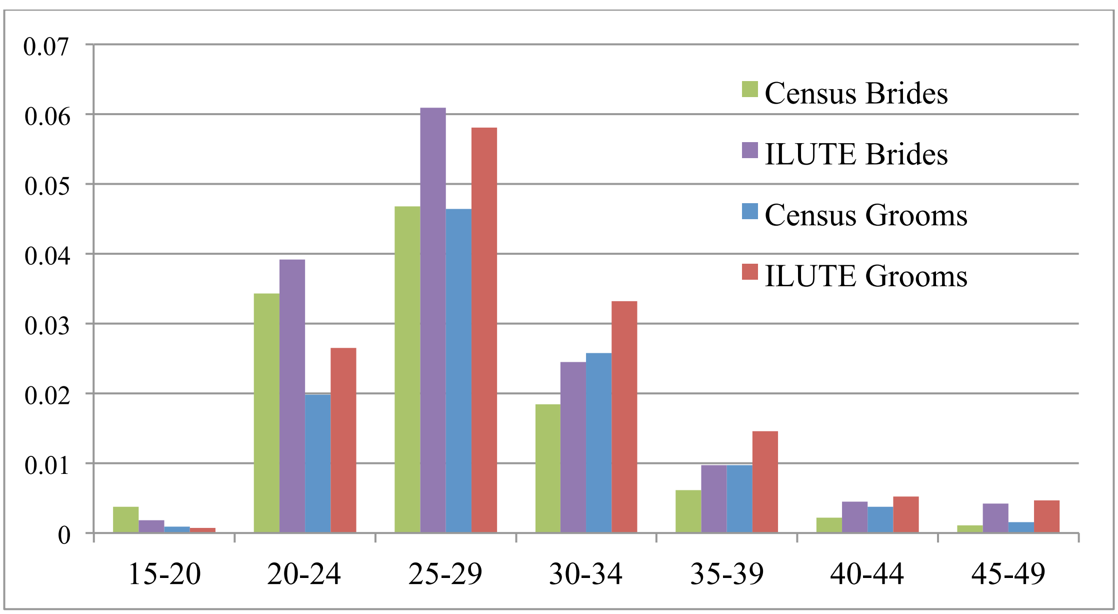


Figure 6: ILUTE and census marriage rates by age group (2001)

The previous results have centred on the decision to join the *MarriageMarket*. The next results now focus on how well the ILUTE *MarriageMarket* matches potential couples. Table 2 shows the distribution of newly wedded couples by age group. Again, the general trend is captured by the *MarriageMarket*, with majority of newly married persons being under 35. Note that historical GTHA values were not available, so national (i.e. Canadian) values were used as proxies. Besides age, income differences were used to pair up possible marriage matches. Figure 7 shows the distribution of income differences for married couples in the ILUTE simulation in 2001. These results display strong performance in comparison to census values.

Table 2: Distribution of newly married couples by age group (2001)

ILUTE (GTHA Values, 2001)							
		Age of Husband					
		18-24	25-34	35-44	45-54	55-64	65-74
Age of Wife	18-24	17.1	8.8	1.7	0.2	0.0	0.2
	25-34	0.5	46.7	4.6	0.6	0.1	0.1
	35-44	0.0	1.1	9.6	1.1	0.2	0.1
	45-54	0.0	0.0	0.6	3.7	1.0	0.2
	55-64	0.0	0.0	0.0	0.3	0.6	0.3
	65-74	0.0	0.0	0.0	0.0	0.2	0.3
Statistics Canada (Canada Values, 2001)							
		Age of Husband					
		18-24	25-34	35-44	45-54	55-64	65-74
Age of Wife	18-24	11.5	12.5	0.8	0.1	0.0	0.0
	25-34	2.8	35.1	9.2	1.0	0.1	0.0
	35-44	0.1	3.1	8.8	3.8	0.6	0.1
	45-54	0.0	0.2	1.4	3.8	1.8	0.3
	55-64	0.0	0.0	0.1	0.4	1.1	0.7
	65-74	0.0	0.0	0.0	0.0	0.1	0.5

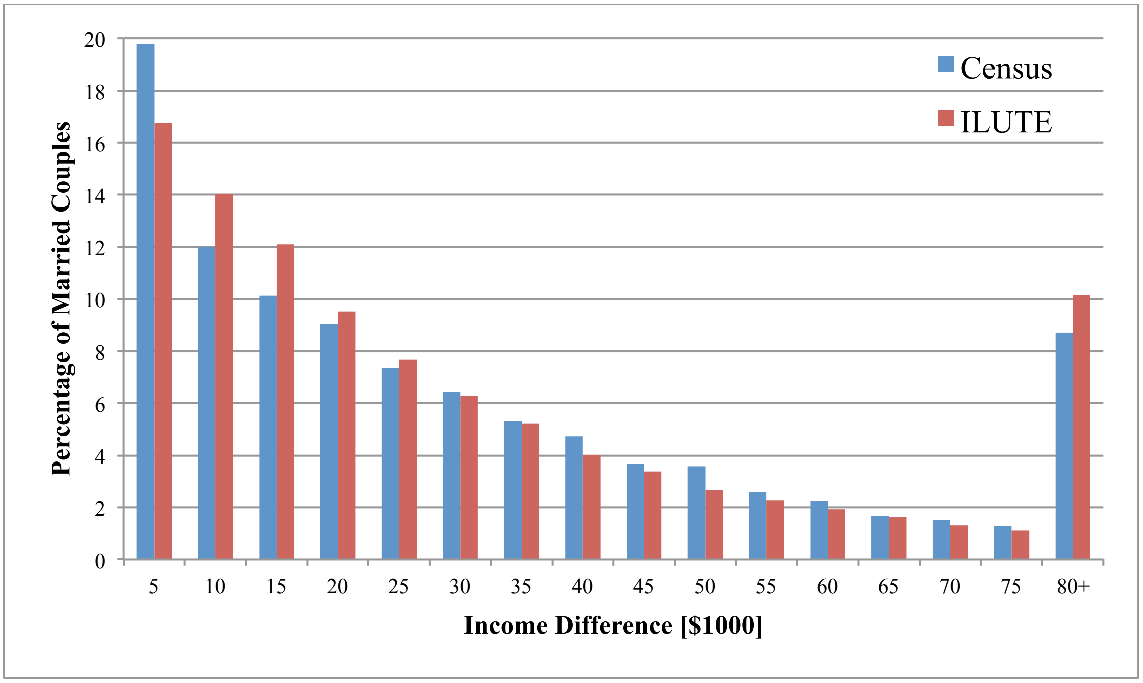


Figure 7: Distribution of income differences (2001)

4.5 Rental Housing Market

Another important application of urban price-taker markets is the market for rental housing. Housing (rental and owner-occupied) market models are important in the context of urban microsimulation, as they influence the spatial and temporal distributions of the population in the region. The two markets may have different dynamics due to regional economics, supply, space-time, and socio-demographics conditions. Though, the two markets influence each other through agent interactions and lagged signals. Conditions in both rental and owner-occupied housing markets play key roles in the location decisions of the households. Both market types are separately implemented in the current version of ILUTE: the owner-occupied housing market is modelled as a price-formation market (Farooq 2011), while rental housing is modelled as a price-taker market. This clear distinction between the operationalization of two markets in ILUTE enables it to capture the individual market dynamics while ensuring the interactions between them through the loose coupling the two. To our knowledge this is the first time that this distinction between the two markets has been implemented explicitly in an urban model. These next two subsections focus on the rental housing market in ILUTE, along with initial model results.

Each year in ILUTE, households evaluate the decision to change their existing locations. If a household decides to move, it is then faced with the tenure decision, i.e. whether to get active in the owner-occupied or rental housing market. This decision is based on probability distributions generated from the Canadian Census data for various income levels of the households (Giroux-Cook, 2010). Households that decide to rent a dwelling start the search process for a potential dwelling. In the current implementation, all the active dwellings in the market are available for consideration to all the active households. The other option that could have been used was to randomly choose the choice set for each renter. Elgar *et al.* (2011) investigated the choice set generation process for firm's location choice models and suggested that in forecasting mode, the model considering all the options out-performed the choice set generation process where a subset of choices were randomly chosen. They also suggested anchor points based choice set generation processes in the spatial context. With the availability of better datasets, a more realistic choice set generation process that is inspired by anchor point based approach can be developed for renting households.

Rents for the active dwellings are determined using the rent-setting model, developed using average rent data from the Canadian Mortgage and Housing Corporation (CMHC), supplemented by Canadian Census data. In the clearing process, the problem is reduced to finding a maximum weighted bipartite graph, using the formulation and solution suggested in Sections 2 and 3. In the current version, the income levels of the households determine the weight on the edges. Hence, these weights are unidirectional and represent the assumption that landlords give the dwelling to the interested household with the highest income. However, Giroux-Cook (2010) recommends that a random utility-based model be developed that incorporates the preferences of the households. Moreover, it is pointed out that landlords often screen out the potential renters due to discrimination against race, gender, class, etc. Giroux-Cook suggests that the utility function of the landlord that expresses this behaviour should also be included in the weight of the edges. In terms of our formulation, the edges will then correspond to a bidirectional weights.

4.6 Rental Housing Market Simulation Results

For the validation of the rental market, small samples of 10,000 and 50,000 households were microsimulated from 1986 to 2006 and their evolution were compared with historical data. Table 3 compares the average rental prices in 2001 with census data. Currently the prices forecasted by ILUTE are lower and have lesser variance compared. This is due to the fact that the current rent model operational in ILUTE, is insensitive to neighbourhood characteristics, market conditions, and accessibility. We plan to replace it with a more detailed model, as soon as we have access to better datasets.

Table 3: Average rents in Year 2001

	ILUTE		Census	
	Average	St. Dev.	Average	St. Dev.
Total	610.19	241.13	848.03	392.42

Figure 8 presents the comparison between the spatial distribution of renter households in ILUTE and the 2001 census. The spatial trend produced by ILUTE generally seems to match the historical pattern, with a few exceptions, particularly in the Southwest (Hamilton region). Note that a more detailed discussion on the results from the operationalized rental market in ILUTE can be found in Giroux-Cook (2010).

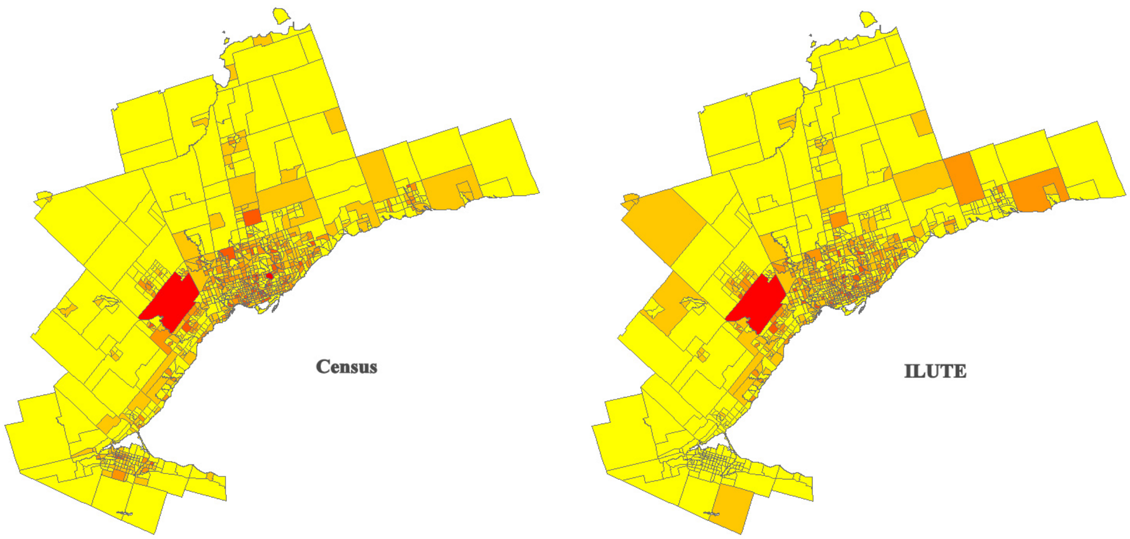


Figure 8: Spatial distribution of renter households in Year 2001: Census-ILUTE (darker shades representing higher densities)

5. CONCLUDING REMARKS AND FUTURE DIRECTION

In literature, few examples can be found on microsimulation modelling of specific price-taker markets, for instance: Waddell *et al*. (2003) presented a housing market model while Leombruni and Richiardi (2011) proposed a microsimulation labour market model. However in this paper, taking advantage of the similarities among these markets, we presented a single generic microsimulation framework for modelling the urban price-taker markets that can we used to model a wide range of markets. Core concepts from graph theory were used to abstract the market as a bipartite weighted graph. Commonly used algorithm was first explored for its appropriateness in the context of large-scale microsimulation of urban systems. Based on which, then an efficient algorithm was developed to find the solution for the market clearing problem.

We applied the proposed framework to marriage and rental housing markets within ILUTE modelling system. Due to unavailability of data on actual marriages, the validation of the results produced by the implemented marriage markets was performed using indirect means. A close match was found between the evolved simulation population and census. The choice set generation process and utility function needs to be revisited in the marriage market. Clear distinction was made between rental owner-occupied markets. In case of owner-occupied market the endogenous formation of price is a dominant characteristics, while in case of rental market the rent levels are very much predetermined. We thus modelled the rental market as a price-taker market. In case of rental market, the comparison between the simulation results and historical data demonstrated that the current rental model requires further modifications to improve the accuracy of its results. The three primary areas of improvement needed are (a) estimates of the number of households getting active in the renting market and (b) estimates of the dwelling asking rents (c) choice set generation for the renters.

The framework developed here is very rich in term of representing agents' behaviour and market characteristics—agents heterogeneity, differences in choice set formation process, market segments, supply and demand shocks, are some of the key features that can be represented by this framework. At the same time, it is highly efficient and scalable in terms of microsimulation operationalization of various urban markets that display price-taker behaviour by consumers. The proposed implementation has an order of complexity that is linear function of number of active consumers or producers (depending on whoever are more) in the market. Using the same framework, we are in the process of operationalizing the labour force market within ILUTE. Moreover, as a future research direction, we intend to further extend the application of the proposed framework in the areas like urban freight transportation and air-travel.

A full-scale microsimulation of the marriage market for the Greater Toronto and Hamilton Area requires dealing with approximately one hundred thousand agents (including all active males and females). Within an urban microsimulation system such as ILUTE this results in very high memory and computational requirements (as is commonly the case in any large scale microsimulation of urban systems). To overcome such challenges, as an ongoing research, we are exploring efficient use of readily available multi-core 64-bit computer architecture, by exploiting access to larger memories and speedup by parallelization. Because of the complex nature of interactions between the agents, the parallelization of any type of market is non-trivial. That requires careful partitioning of the problem, resolving various dependencies, and avoiding deadlocks. In future, we plan to develop specialized algorithms and data structures that are capable of handling.

REFERENCES

Alonso W. (1964) *Location and land use: towards a general theory of land rent.* Harvard University Press, Cambridge, Massachusetts.

Anas A., and R.J. Arnott (1993) Development and testing of the Chicago prototype housing market model. *Journal of housing Research,* 4(1), 72–129.

Anas A., and R. J. Arnott (1994) The Chicago prototype housing market model with tenure choice and its policy applications. *Journal of Housing Research,* 5(1), 23–90.

Burkard R., M. Dell'Amico, and S. Martello (2009) *Assignment problems*. SIAM, Society for Industrial and Applied Mathematics, Philadelphia.

Chingcuanco F. (2010) *ILUTE demographic updating module: current operational status*. Working paper, Urban Transportation Research and Advancement Centre, University of Toronto.

Choo E., S. Seitz, and A. Siow (2008) *The collective marriage matching model: identification, estimation and testing*. Working paper: 340, University of Toronto.

Cormen T.H., C.E. Leiserson, R.L. Rivest, and C. Stein (2001) *Introduction to algorithms*. Second Edition, MIT Press, Cambridge.

de la Barra T. (1995) *Integrated land use and transportation modeling: Decision chains and hierarchies.* Cambridge University Press, Cambridge, UK.

Devisch O.T.J., H.J.P. Timmermans, T.A. Arentze, and A.W.J. Borgers (2009) An agent-based model of residential choice dynamics in nonstationary housing markets. *Environment and Planning (Part A)*, 41, 1997–2013.

Echenique M., A.D.J. Flowerdew, J.D. Hunt, T.R. Mayo, I.J. Skidmore, and D.C. Simmonds (1990) The MEPLAN models of Bilbao, Leeds and Dortmund. *Transport Reviews,* 10(4), 309–322.

Egerváry J. (1931) On combinatorial properties of matrices. *Matematikai és Fizikai Lapok*, 38, 16–28.

Elgar I., Farooq B., and Miller E.J. (2011) *Simulation of firm location decisions: an attempt to replicate office location choices in the Greater Toronto Area.* Working paper, University of Toronto.

Ettema D., K. de Jong, H.J.P. Timmermans and A. Bakema. (2006) PUMA: multi-agent modeling of urban systems. In: *Land use modeling* (Koomen E., A. Bakema, J. Stillwell, and H. Scholten, Eds.), Springer.

Farooq B. (2011) *Dynamics of complex urban systems: behavioural modelling and microsimulation*. VDM Verlag Dr. Müller (ISBN: 978-3-639-34693-0).

Frank A. (2004) *On Kuhn's Hungarian method–a tribute from Hungary.* Technical Report: H1117, Egervary Research Group, Budapest.

Gillett B. (1976) *Introduction to operations research: a computer-oriented algorithmic approach*. McGraw-Hill, New York.

Giroux-Cook M. (2010) *Building a microsimulation of the rental sector in the Greater Toronto Area.* Master Thesis, University of Toronto.

Gondran M., and M. Minoux (1984) *Graphs and algorithms.* Wiley, New York.

König D. (1931) Graphs and matrices. *Matematikai és Fizikai Lapok*, 38, 116–119.

Martinez, F. (1992) The bid-choice land use model: An integrated economic framework. *Environment and Planning A*, 15, 871–885.

Miller E.J., and M.J. Roorda (2003) A prototype model of household activity/travel scheduling. *Transportation Research Record*, 1831, 114−121.

Miller E.J., J.D.H. Hunt, J.E. Abraham, and P.A. Salvini (2004) Microsimulating urban systems. *Computer, Environment and Urban Systems*, 28, 9–44.

Miller E.J. (2005a) An integrated framework for modelling short- and long-term household decision modelling. In: *Progress in Activity based Analysis* (Timmermans H.J.P., Ed.) Elsevier, Amsterdam, 175−201.

Miller, E.J. (2005b) Propositions for modelling household decision-making. In: *Behavioural Foundations of Integrated Land-use and Transportation Models: Assumptions ad New Conceptual Frameworks* (M. Lee-Gosselin, and S.T. Doherty, Eds.), Elsevier, New York.

Miller E.J. (2008) *Project final report*. Volume I in Development of an Operational Integrated Urban Model System. Technical report, Urban Transportation Research & Advancement Centre, University of Toronto.

Miller E.J., F. Chingcuanco, B. Farooq, K.M.N. Habib, and M.A. Habib (2008) *Demographic and labour market.* Volume IV in Development of an Operational Integrated Urban Model System. Technical report, Urban Transportation Research & Advancement Centre, University of Toronto.

Miller E.J., B. Farooq, F. Chingcuanco, and D. Wang (2011) Historical validation of an integrated transport–land use model system. Forthcoming in *Transportation Research Record*.

Hunt J.D., D.S. Kriger, and E.J. Miller (2005) Current operational urban land-use-transport modelling frameworks: a review. *Transport Review*, 25(3), 329−376.

Kuhn H.W. (1955) The Hungarian method for the assignment problem. *Naval Research Logistic Quarterly*, 2, 83–97.

Leombruni R., and M.G. Richiardi (2011) LABORsim. An agent-Based microsimulation of labour supply: an application to Italy. *Forthcoming: Computational Economics*.

Orcutt G.H. (1957) A new type of socio-economic system. *Review of Economics and Statistics*, 32(2), 116−123.

Orcutt G.H. (1990) The microanalytic approach for modeling national economies. *Journal of Economic Behavior and Organization*, 14, 20−41.

Tomizawa, N., (1971) On some techniques useful for solution of transportation network Problems. *Networks*, 1, 173–194.

Waddell P., A. Borning, M. Noth, N. Freier, M. Becke, and G.F. Ulfarsson (2003) Microsimulation of urban development and location choices: design and implementation of UrbanSim. *Networks and Spatial Economics*, 3(1), 43−67.

Wegner M. (1995) Operational urban models: state of the art. *Journal of the American Planning association*, 60, 17−29.

Wilson R.J. (1979) *Introduction to graph theory*. Academic Press, New York.

Winston W.L. (1991) *Introduction to mathematical programming: applications and algorithms*. PWS-Kent Publishing Company, Boston.

Zhang L., D. Levinson (2004) An agent-based approach to travel demand modeling: an exploratory analysis. *Transportation Research Record.* 1898. 28–36.