Price-based regulation of oligopolistic markets under discrete choice models of demand

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Abstract

We propose a framework to find optimal price-based policies to regulate markets characterized by oligopolistic competition and in which consumers make a discrete choice among a finite set of alternatives. With this framework, we can include general discrete choice models available in the literature to capture heterogeneous consumer behavior. In our work, consumers are utility maximizers and are modeled according to random utility theory. Suppliers are modeled as profit maximizers, according to the traditional microeconomic treatment. Market competition is modeled as a non-cooperative game, for which an ε-equilibrium solution is sought. Finally, the regulator can affect the behavior of all other agents by giving subsidies or imposing taxes to consumers. In transport markets, economic instruments might target specific alternatives, to reduce externalities such as congestion or emissions, or specific segments of the population, to achieve social welfare objectives. In public policy, different agents have different individual or social objectives, possibly conflicting, so value judgements are used to compare monetary and non-monetary objectives. We present a mixed integer optimization model to find optimal policies subject to supplier profit maximization and consumer utility maximization constraints. Then, we propose a model-based heuristic approach based on the fixed-point iteration algorithm that finds ε-equilibrium solutions for the market. Numerical experiments on an intercity travel case study show how the regulator can optimize its decisions for different policy instruments and for different objective functions.

Keywords: equilibrium, regulation, discrete choice modeling

1 Introduction

Public intervention in transport markets can be motivated by several phenomena. For decades it has been acknowledged that transport markets are often the source of negative externalities, two well-known cases of which are pollution and congestion. Policies to address these issues include road pricing (Button and Verhoef, 1998; Anas and Lindsey, 2011), taxes on fuel or on vehicle purchase (Fullerton and West, 2002) and creation of low emission zones (De Borger and Proost, 2013; Cullinane and Bergqvist, 2014), among others. More recently, much attention is given to the contribution of the transport sector to the increase of greenhouse gas emissions which are a leading cause of climate change (IPCC, 2014). Solutions that include a carbon tax are frequently proposed to reduce the negative impact of mobility on the environment. From a social perspective, a public entity might want to intervene in a transport market to incentivize mobility under certain circumstances. Indeed, improving mobility is often regarded as a means to increase economic output and enhance access to job opportunities or other activities (Van Goeverden et al., 2006; Guzman and Oviedo, 2018). Additionally, many transport markets, alike other network industries such as energy and telecommunications, are natural monopolies where suppliers benefit from large economies of scale and consumers place greater value on large networks than on small ones (Farsi et al., 2007).

Public intervention can take many forms. In this work, we look at regulation. Regulation is defined as an indirect public intervention aimed at orienting actors towards some welfare goals (Ponti, 2011). In this context, regulation can be seen as a middle way between a *command-and-control* approach and a pure *market competition* approach. Regulation can take various forms, which are generally framed within competition and antitrust laws that exist at local, national and international level and determine how a regulator can influence the market. One common approach to regulation is the use of economic instruments such as subsidies and taxes, which are the focus of this work.

In this paper, we propose a framework to find optimal policies to regulate oligopolistic transport markets where demand is modeled at a disaggregate level using discrete choice models, according to random utility theory. In markets characterized by imperfect competition between suppliers and by heterogeneous consumer demand, regulation affects the strategic decisions of suppliers, which in turn are influenced by the preferences of the customers and by the decisions of their competitors.

In the literature there exist methodologies for both discrete choice-based welfare optimization and competition modeling with welfare-maximizing regulator. However, to the best of our knowledge, contributions that can be classified in the former category do not explicitly account for strategic interactions between suppliers, while works belonging to the latter category do not allow for differentiated policies based on a disaggregate demand function. Therefore, we integrate the existing literature by proposing a methodology that accommodates discrete choice models into a game-theoretic framework of regulated competition.

Our approach allows to exploit an estimated discrete choice model and include it by means of simulation in a model of regulated competition featuring heterogeneous

demand, multi-product offer by suppliers and price differentiation. Only a few assumptions are made about the demand and the specification of the discrete choice model, in order to accommodate advanced choice models, such as mixtures of logit, multivariate extreme value models or hybrid choice models. The use of models that capture complex disaggregate choice behavior allows the regulator to account for product differentiation and consumer behavioral heterogeneity at the individual level, and therefore to better tailor its policies based on tradeoffs between different agents.
The remainder of this paper is organized as follows. Section 2 provides a literature review on social welfare approaches and on their use in presence of discrete choice models of user behavior and of imperfect competition. Section 3 presents our discrete choice-based optimization model for regulated competition. The model can be integrated in an algorithmic framework that finds ε-equilibrium solutions for the market. Section 4 illustrates numerical experiments performed on a case study representing an intercity travel market. Finally, Section 5 concludes the paper and provides directions for future research.

2 Literature review

Welfare economics is generally understood as the problem of achieving a social maximum derived from individual desires by comparing and ranking different social states (Arrow, 1951). Similarly, social choice theory aims at creating a framework that explores normative principles to support policy design and evaluation (Hausman et al., 2016).
If we accept the postulate that interpersonal comparisons of utilities are meaningful, then value judgements are required to define a relation between utilities of different individuals and to aggregate them into a mathematical formula measuring social welfare. The necessity and the appropriateness of comparing gains of certain individuals with losses of other individuals when evaluating economic policies are central in the seminal works by Pareto (1906), Bergson (1938) and Samuelson (1948).
Pareto (1906) proposes an ordinal approach to utilities and rejects the idea of welfare as an aggregation of individual cardinal utilities. In this approach, social states can only be compared in terms of preference and indifference relations for each individual. Then, a state is a Pareto-improvement over another state if and only if at least one individual is better off and, at the same time, no individual is worse off. Kaldor (1939) and Hicks (1939) propose weaker efficiency conditions by relying on hypothetical compensations which could be transferred from individuals who are better off to those who are worse off to move from an initial state to a Pareto-improving state, without imposing any distributional condition.
Bergson (1938) and Samuelson (1948) take a different point of view and introduce the concept of an individualistic social welfare function. Such function should allow the comparison of the utilities of different individuals and should take a form chosen according to ethical value judgements, which are to be agreed upon by society based on some sort of ethical assumptions. Later studies further develop these concepts by

looking at the interdependencies between an individual's social welfare function, representing ethical preferences, and her own utility function, representing personal tastes (Harsanyi, 1955; Sen, 1977), and formalize interpersonal comparability through social welfare functionals (d'Aspremont and Gevers, 2002; Sen, 2017).
Social welfare functionals are flexible enough to incorporate many approaches to public policy, allowing not only descriptive but normative analyses. Indeed, in the last decades the study of social welfare has expanded to include subjective well-being, distributional preferences and intergenerational equity as criteria to be considered during the decision-making process (Fleurbaey, 2009; Adler and Fleurbaey, 2016). In particular, the issue of climate change has been included in social welfare studies through the concept of social cost of carbon (SCC), defined as the monetary value of the damage caused by emitting one more unit of carbon at some point of time (Nordhaus, 1994; Pearce, 2003; Stern et al., 2006). The SCC is typically derived from integrated assessment models which require an assumption on the future path of CO_2 concentration in the atmosphere. The range of SCC estimates available in the literature is quite broad and is dependent on the values of the parameters used in each study, the most important of them being the marginal utility of consumption, the social discount rate assigned to future generations, and the equity parameter that assigns higher weight to the worse-off when computing the social welfare function (Adler et al., 2017). Nevertheless, this indicator is central in shaping climate policy and is extensively used within cost-benefit analyses.

Welfare economics uses social welfare functions to aggregate individual consumer behavior, which is generally modeled as the choice of a bundle of continuous goods subject to a budget constraint. Complementing the continuous case, the theory of discrete choice modeling was developed to model behavioral situations in which an agent makes a choice from a finite set of discrete alternatives (Ben-Akiva and Lerman, 1985). Discrete choice models account for consumer behavioral heterogeneity at the disaggregate level. As such, these models allow for complex and precise representations of individual behavior by means of utility functions that capture tastes and socio-economic characteristics.
Small and Rosen (1981) discuss how conventional methods of applied welfare economics can be generalized to handle random utility models of discrete choices. The authors recognize that welfare judgments are of paramount interest when analyzing taxes and subsidies in some markets for which discrete choice models are used. They conclude that welfare effects can be derived directly from micro data using the utility functions, avoiding the explicit use of aggregate demand functions, which are not normally obtained in closed form. In particular, consumer surplus can be expressed in different forms depending on the random term distributional assumptions. One of these forms is the log sum metric, utilized in the case of multivariate extreme value distribution. Batley and Ibáñez (2013) analyze the assumptions underlying the approach by Small and Rosen (1981). In particular, they show that the consumer surplus measure requires income effects of price and income changes to be equal to zero, thus excluding the possibility to have heterogeneous marginal utilities of income,

which causes path dependence. Notwithstanding this limitation, the literature on welfare measurements using discrete choice models has largely followed the Marshallian framework, as noted by Hess et al. (2018). Alternative approaches that account for non-constant marginal utility of income rely on the Hicksian compensating variation (Hau, 1985; Jara-Díaz and Videla, 1990; McFadden, 1995; Morey et al., 2003; Batley and Dekker, 2019). However, both analytical and simulation-based methods come with a substantial computational burden, and this is the main reason for their limited use in practical applications to date. Finally, another method to account for different marginal utilities of income is proposed by Hau (1986), who modifies the approach by Small and Rosen (1981) to incorporate explicit value judgements by assigning distributional weights to segments of the population. The methodology is then use to carry out a transport infrastructure cost-benefit analysis where the population is stratified by income.

In prescriptive studies, we encounter some works whose goal is to design optimal fares, taxes or subsidies using a model of discrete choice. De Borger (2000) presents a model to determine a welfare-optimal two-part tariff under logit model of discrete choice, subject to budgetary constraints and distributional preferences. The fixed and the variable component of the tariff mimic the choices of ownership of a vehicle and quantity of consumption in terms of traveled kilometers. The expected value of the maximum utility for the logit model is obtained using the log sum welfare measure, interpreted as consumer surplus up to a constant, as in Small and Rosen (1981). It is shown that this methodology can be generalized to other classes of discrete choice models, for which there is no closed-form solution for the objective function, but no information is provided about computational tractability. A similar approach is followed in De Borger and Mayeres (2007), where nested constant elasticity of substitution utility functions are used. Borndörfer et al. (2012) propose a non-linear formulation to optimize fares on a public transport network. The demand functions take into account spatial heterogeneity in terms of origin-destination pairs and are based on a logit model to compromise between model accuracy and computability. Various objective functions are proposed to allow for the maximization of revenue, profit, demand, user benefit and social welfare.

The discrete choice literature dealing with welfare issues shares the assumption that consumers are the only agents who react to welfare-maximizing policies through their demand function. This assumption is realistic when studying a monopolistic market, but it is limiting when studying a regulated oligopolistic market where suppliers have market power and have objectives that conflict with those of the regulator. Indeed, in the latter case the outcome is determined jointly by all decision-makers, and the strategic behavior of firms in oligopolies is usually modeled using game theory (Fudenberg and Tirole, 1991; Osborne and Rubinstein, 1994). Relevant streams of literature that combine oligopolistic competition with social welfare analyses include the study of mixed oligopolies, that is, markets in which public enterprises interact with private firms to improve resource allocation in an imperfectly competitive market (De Fraja and Delbono, 1989; Cremer et al., 1991), and the study of network industries (Shapiro, 1998). However, none of these works incorporates discrete choice models of behavior.

To summarize, the literature review reveals that (i) discrete choice-based welfare analyses do not explicitly consider possible strategic interactions between suppliers, and (ii) game-theoretical competition models featuring a welfare maximizing public entity use aggregate demand functions that do not allow for heterogeneous policies. With our work, we present a comprehensive market model that integrates discrete choice models into a game-theoretic model of regulated competition, where the regulator aims to optimize a cardinal social welfare function which admits interpersonal comparisons of utility. In this context, the added value of the discrete choice model is twofold. First, it allows to generate a more precise representation of demand by modeling behavioral heterogeneity at the individual level. Secondly, it allows to develop policies that leverage on disaggregate demand models to target specific segments of the population.

3 A framework for regulated competition with discrete choice models

We consider a regulated competitive market where a number of different products are offered to a population by two or more suppliers that have market power.
On the demand side, let N represent the set of heterogeneous consumers (or groups of consumers), who are assumed to be utility maximizers, and let I indicate the discrete and finite set of alternatives available in the market. Utility functions U_{in} are defined for each consumer or group of homogeneous consumers $n \in N$ and alternative $i \in I$ in accordance with random utility theory, accounting for the socio-economic characteristics and tastes of the individual and for the attributes of the alternative.
On the supply side, let K represent the set of suppliers and let $I_k \subset I$ indicate the subset of alternatives controlled by each supplier $k \in K$. We impose that $\cup_{k \in K} I_k \subset I$, in order to allow customers to leave the market without purchasing. We assume that each supplier solves a discrete choice-based optimization problem, modeled in the form of a mixed integer optimization problem, aimed at finding the strategy that maximizes its profits. We define as $\mathcal{S}_k$ the set of strategies that can be selected by supplier k. A strategy consists in a vector (or bundle) p of decisions about all prices p_{in}, potentially differentiated for each (class of) consumer $n \in N$ and alternative $i \in I_k$.
The decisions of the suppliers affect the utility functions U of the consumers, whose deterministic part V can be generically expressed as

$$V_{in} = \beta_{p_{in}} p_{in} + q_{in}, \tag{1}$$

where $\beta_{p_{in}}$ is a pre-estimated parameter of the discrete choice model and q_{in} is the scalar product of the vectors of all the exogenous variables of the choice model and the corresponding pre-estimated parameters.
The peculiarity of our approach is the use of discrete choice models to inform the suppliers' strategic behavior. Pacheco Paneque (2020) and Bortolomiol et al. (2019)

provide a detailed discussion of the challenges associated with the integration of discrete choice models within mixed integer optimization models and within market equilibrium models and propose some methodologies to overcome them. In particular, Pacheco Paneque (2020) presents a mixed integer linear formulation which accommodates a discrete choice model of demand by relying on simulation to draw from the distribution of the error term of the utility function. Bortolomiol et al. (2019) introduce an algorithmic framework to find ε-equilibrium solutions of oligopolistic markets where demand is modeled at the disaggregate level using discrete choice models.

Next, we build on the aforementioned works to design a modeling framework that includes the role of the regulator. The regulator uses economic instruments, that is, subsidies and taxes, to influence the behavior of the other agents, thus modifying the equilibrium outcome of the market. In a general case, we assume that the regulator has a budget B which is available to finance some policies.

The policies set by the regulator affect the price paid by consumer (group) n for alternative i, which in return affect both the utilities U_{in} of the consumers, and the profits of the suppliers. Prices can be decomposed as

$$p_{in} = r_{in} + t_{in}, \tag{2}$$

where r_{in} is the revenue made by the supplier in case of purchase and t_{in} is the tax or the subsidy set by the regulator. If $t_{in} > 0$, then a tax is imposed on the purchase of alternative i by customer n. If $t_{in} < 0$, then a subsidy is offered for the same purchase. Then, we can write an optimization problem from the point of view of the regulator, whose goal is to maximize a social welfare function (SWF) at equilibrium. Equilibrium conditions require that no market agent has an incentive to deviate from the current status.

Let us now look in detail at the different components of the modeling framework.

3.1 Constraints

Three sets of conditions need to be enforced to model the common behavioral assumptions about consumers and suppliers: (i) utility maximization conditions; (ii) profit maximization conditions; (iii) equilibrium conditions. On top of them, other problem-specific constraints can be defined to model specific market features.

3.1.1 Utility maximization

Concerning utility maximization, we apply the simulation-based linearization approach proposed by Pacheco Paneque (2020). A set R of independent draws are extracted from the known error term distribution of the discrete choice model for each $n \in N$ and $i \in I$, corresponding to different behavioral scenarios. For each scenario $r \in R$, the drawn error term parameter ξ_{inr} is included in the utility function as follows:

$$U_{inr} = V_{in} + \xi_{inr}, \tag{3}$$

and consumers deterministically choose the alternative with the highest utility. The expected maximum utility, corresponding to the utility of the chosen alternative, is equal to

$$EMU_{nr} = \max_{j \in I} U_{jnr}. \tag{4}$$

Then, we can define the binary variables P_{inr} to express the probability of n choosing i in scenario r as

$$P_{inr} = \begin{cases} 1 & \text{if } U_{inr} = EMU_{nr}, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

Expressions (3-5) can be included in a mixed integer linear optimization model through the following set of constraints:

$$U_{inr} = \beta_{p_{in}}(r_{in} + t_{in}) + q_{in} + \xi_{inr} \quad \forall i \in I, \forall n \in N, \forall r \in R, \tag{6}$$

$$U_{inr} \leq EMU_{nr} \quad \forall i \in I, \forall n \in N, \forall r \in R, \tag{7}$$

$$EMU_{nr} \leq U_{inr} + M_U(1 - P_{inr}) \quad \forall i \in I, \forall n \in N, \forall r \in R, \tag{8}$$

$$\sum_{i \in I} P_{inr} = 1 \quad \forall i \in I, \forall n \in N, \forall r \in R, \tag{9}$$

$$P_{inr} \in \{0, 1\} \quad \forall i \in I, \forall n \in N, \forall r \in R. \tag{10}$$

Over a sufficiently large number of draws, we obtain the choice probabilities

$$P_{in} = \frac{\sum_{r \in R} P_{inr}}{\mid R \mid} \tag{11}$$

and the expected maximum utilities

$$EMU_n = \frac{\sum_{r \in R} EMU_{nr}}{\mid R \mid}. \tag{12}$$

Numerical experiments by Pacheco Paneque (2020) show that good approximations of the choice probabilities and of the expected maximum utility can be obtained with a fairly low number of draws.

3.1.2 Profit maximization

The profit maximization problem of each supplier can be expressed through the following mixed integer linear optimization model:

$$\max_{p_k} \quad \pi_k = \sum_{i \in I_k} \sum_{n \in N} \theta_n P_{in} r_{in}, \tag{13}$$

$$\text{s.t.} \quad \text{constraints (6-10).} \tag{14}$$

In (13), r_{in} is the revenue obtained from the sale of product i to consumer n (see Equation 2), P_{in} is the probability that consumer group n chooses alternative $i \in I_k$ (see Equation 11), and θ_n is the size of group n, i.e. the number of individuals with homogeneous socio-economic characteristics. The supplier optimization problem constitutes lower-level optimization constraints for the regulator optimization problem.

3.1.3 Equilibrium conditions

Consistently with Bortolomiol et al. (2019), we use ε-equilibrium conditions to identify stationary states of the system in which no competitor can increase its profit by more than $1+\varepsilon$ times its current payoff by unilaterally changing its strategy. Formally, let us consider a market state $S = (p, t)$, which is defined by the strategies p_k of all suppliers $k \in K$ and the vector of all taxes (or subsidies) t_{in} set by the regulator. Let us define as $\pi_k(S)$ the expected profit of supplier k in state S and as $\pi_k^{max}(S_{-k})$ the expected profit obtained by supplier k when best responding to state S_{-k}, defined by the strategies of the regulator and all suppliers except k. Then, S is an ε-equilibrium if

$$\pi_k^{max}(S_{-k}) \quad \leq \quad (1+\varepsilon) \ \ \pi_k(S) \qquad \forall k \in K. \tag{15}$$

In the algorithmic framework that is outlined in Section 3.3, these equilibrium conditions on the suppliers' objective functions are verified by means of a heuristic based on fixed-point iterations.

3.1.4 Problem-specific constraints

Other problem-specific constraints can be integrated in this mixed integer formulation. For instance, many competition laws impose that the taxation or subsidization of products sold by competing suppliers must be fair, meaning that no competitive advantage must arise due to the intervention of the government in the market. On the supplier side, constraints can be included to ensure that demand for an alternative does not exceed capacity. This can be achieved by means of exogenous priority rules that simulate the arrival process of customers, as shown in Binder et al. (2017). With this technique, it is also possible to model price differentiation strategies based on the time of booking. On the consumer side, price bounds can be set to define limits for price discrimination across different population groups. All these sets of constraints reduce the feasible set of solutions in the optimization problems of the regulator and of the suppliers. Their effect in terms of computational time and resulting equilibrium is generally problem-dependent. In the case study presented in Section 4, we show how our modeling framework can integrate some of these strategic constraints. Other constraints, including capacity constraints and the related congestion effects, come with additional integrality constraints or non-linearities, which require the design of ad-hoc algorithms to be tackled (Pacheco Paneque, 2020).

3.2 Objective function

The objective of the market regulator is to maximize a cardinal social welfare function (SWF). We assume that this function takes into account all or some of the following components: (i) expected maximum utilities of the consumers; (ii) expected profits of the suppliers; (iii) market externalities; (iv) cost of policy implementation. To allow for comparability of different terms, it is necessary to monetize all values that are measured in non-monetary terms.

Expected utilities As shown in (12), individual expected maximum utilities can be derived directly from the discrete choice-based optimization model, thanks to the simulation technique used to linearize the utility functions of the consumers (Pacheco Paneque, 2020). The utility functions must be converted from preference space into the equivalent formulation in willingness-to-pay space, as discussed by Train and Weeks (2005). We consider linear income effects, thus adhering to the assumption outlined by Batley and Ibáñez (2013) under which the framework by Small and Rosen (1981) is consistent with economic theory. This means that consumer surplus can be obtained by dividing the expected maximum utility by a cost coefficient β_p, which needs to be constant across all alternatives in the choice set and corresponds to the constant marginal utility of income.
This component of the SWF can then be written as follows:

$$SWF_U = \sum_{n \in N} \theta_n \frac{EMU_n}{\beta_{p,n}}. \tag{16}$$

Expected profits The sum of the expected profits of the suppliers can also be obtained directly from the discrete choice-based optimization model. The expected profits of supplier k are defined in equation (13), so the sum of the expected profits is simply

$$SWF_\pi = \sum_{k \in K} \pi_k. \tag{17}$$

Externalities In transportation, externalities can be generally expressed as a function of demand, which is itself a derived from the choice probabilities, as follows:

$$d_i = \sum_{n \in N} \theta_n P_{in}. \tag{18}$$

In the case of environmental externalities, we may approximate CO_2 and other emissions to a linear function of demand:

$$SWF_E = -\sum_{i \in I} c_i d_i, \tag{19}$$

where d_i is the demand for alternative i and c_i is a parameter representing the monetized cost of emissions per person choosing alternative i, which can be expressed as

$$c_i = \ell_i \cdot e_i \cdot SCC, \tag{20}$$

where l_i is the distance traveled if alternative i is chosen [km], e_i is the CO_2 emissions produced per unit of distance when traveling with alternative i [ton/km] and SCC is the social cost per unit of carbon emission [monetary unit/ton].
In the case of negative externalities caused by road congestion, it is well-known that a non-linear relation exists between traffic volume and total travel time, which also affects the utility of the users. This requires a fixed-point iteration approach to be used in order to reach lower-level user equilibrium.

Public budget The monetary impact of the policy for the regulator is given by the difference between the taxes that are collected and the subsidies that are handed out from and to consumers, and is therefore conditional on their choices. We can write

$$SWF_R = \sum_{i \in I} \sum_{n \in N} \theta_n P_{in} t_{in}, \tag{21}$$

which can be expressed as the sum of products of the binary variables P_{inr}, whose relation with P_{in} is defined in (11), and the continuous variables t_{in}. This expression can be linearized using big-M constraints. Notice that SWF_R can be bounded by a budget constraint.

To summarize, expressions (16), (17), (19) and (21) capture different parts of social welfare which are relevant in policy-making for regulated competition. They can be integrated in a unique objective function, or alternatively they can be treated as different objectives in a multi-objective optimization problem. The resulting mixed integer optimization model that maximizes a given SWF as a function of the taxation and subsidization chosen by the regulator can be written as follows:

$$\max_{t} \quad SWF(t, r) \tag{22}$$

$$\text{s.t.} \quad -M^s_{in} \leq t_{in} \leq M^t_{in} \qquad \forall i \in I, \forall n \in N, \tag{23}$$

$$t'_{in} = t_{in} + M^s_{in} \qquad \forall i \in I, \forall n \in N, \tag{24}$$

$$0 \leq \gamma'_{inr} \leq (M^s_{in} + M^t_{in})P_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{25}$$

$$t'_{in} - (M^s_{in} + M^t_{in})(1 - P_{inr}) \leq \gamma'_{inr} \leq t'_{in} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{26}$$

$$-M^s_{in} \leq \gamma_{inr} \leq M^t_{in} P_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{27}$$

$$\gamma'_{inr} - M^s_{in} - (M^s_{in} + M^t_{in})(1 - P_{inr}) \leq \gamma_{inr} \leq \gamma'_{inr} - M^s_{in} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{28}$$

$$\sum_{i \in I} \sum_{n \in N} \sum_{r \in R} \gamma_{inr} \leq B \tag{29}$$

$$\pi_k = \max_{r_k} \sum_{i \in I_k} \sum_{n \in N} \theta_n P_{in} r_{in} \qquad \forall k \in K, \tag{30}$$

$$\text{s.t.} \quad U_{inr} = \beta_{p_{in}}(r_{in} + t_{in}) + q_{in} + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{31}$$

$$U_{inr} \leq EMU_{nr} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{32}$$

$$EMU_{nr} \leq U_{inr} + M_U(1 - P_{inr}) \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{33}$$

$$\sum_{i \in I} P_{inr} = 1 \qquad \forall i \in I, \forall n \in N, \forall r \in R, \tag{34}$$

$$P_{inr} \in \{0, 1\} \qquad \forall i \in I, \forall n \in N, \forall r \in R. \tag{35}$$

The objective function (22) maximizes the social welfare function defined by the regulator. Constraints (23) impose that the subsidies and taxes set by the regulator respect the given bounds. Constraints (24) define the non-negative variables t'_{in} by means of a transformation. Notice that the parameters M^s_{in} and M^t_{in}, which are upper bounds representing the maximum possible values for the subsidies and taxes, are necessary for modeling purposes and appear in constraints (24-28). This is needed to linearize the product of the binary choice variable P_{inr} and the continuous variable t_{in}, which is done in constraints (25-28). More specifically, constraints (25-26) say that each auxiliary variable γ'_{inr} is equal to t'_{in} if P_{inr} is equal to 1 and is equal to 0 if P_{inr} is equal to 0, while constraints (27-28) say that γ_{inr} is equal to the product $P_{inr} \cdot t_{in}$. Constraint (29) ensures that the budget of the regulator is respected. The expressions 30 represent the objective functions of the lower-level problem, enforcing the profit maximization condition on all suppliers. Finally, constraints (31-35) are the utility maximization constraints (6-10).

3.3 Model-based heuristic framework

We propose a model-based heuristic approach based on the fixed-point iteration algorithm to solve the problem described in Sections 3.1 and 3.2. From model (22-35), two mixed integer linear optimization models are derived: the supplier's profit maximization model (30-35), where the regulator's decisions as well as the prices of all other suppliers are fixed, and a modified regulator's welfare maximization model (22-29)+(31-35) where all supply prices are fixed and the optimization constraints (30) are not enforced. Algorithm 1 presents the pseudocode of the proposed solution.

Starting from an initial state $S = (p, t)$, the fixed-point iteration algorithm works as follows. First, the regulator solves model (22-29)+(31-35) to find t^* which maximizes the SWF given p, and the expected profits $\pi_k(S)$ are computed for state $S^* = (p, t^*)$. Then, each supplier solves model (30-35) given t^* and p_{-k} to find the potential increase in profits obtained by best responding to state S^*. If the equilibrium conditions are satisfied for all suppliers, then S^* is an ε-equilibrium solution for the problem. Finally, state S is updated to reflect best response solutions before restarting with a new iteration. Checks can be made to track visited states and diversify the search of the solution space within the algorithm. The algorithm can be terminated based on predefined stopping criteria, such as number of ε-equilibrium solutions found or number of iterations.

Notice that, when using demand functions based on general disaggregate choice models, which are highly non-linear and non-convex, there is no pure equilibrium existence condition for the resulting problem, and no analytical method can be exploited to find a pure equilibrium solution. More generally, the existence of a ε-equilibrium solution cannot be proved for any given ε. The threshold value ε can be tuned during the execution of the heuristic algorithm, for instance if no ε-equilibrium solution is found within a given time or number of iterations.

4 Case study

In this section, we illustrate the model-based algorithmic framework presented in Section 3.3 on a case study for which a non-trivial discrete choice model of demand is taken

Algorithm 1: Algorithmic solution

Input : A set I of alternatives
A set K of suppliers
A heterogeneous population
An estimated discrete choice model of consumer behavior

Output: A list E of ε-equilibrium solutions

$E \leftarrow \emptyset$

Define an initial state $S = (p, t)$

repeat

  Solve regulation model (22-29)+(31-35) to determine t^* that maximizes the SWF

  Define $S^* = (p, t^*)$ and calculate $\pi_k(S^*)$ for all $k \in K$

  for $k \in K$ **do**

    Solve supplier model (30-35) to find p_k that maximize expected profits given S^*_{-k} and obtain $\pi_k^{max}(S^*_{-k})$

  if $\pi_k^{max}(S^*_{-k}) \leq (1 + \varepsilon)\ \pi_k(S^*) \quad \forall k \in K$ **then**

    S^* is an ε-equilibrium solution of the problem

    Add S^* to list E

  Update $S = (p^*, t^*)$, where p^* is a vector of best response strategies p_k for all $k \in K$

until *stopping criterion is satisfied*

Alternative	0	1	2	3	4	5
Mode	Car	IC	Air	Air	HSR	HSR
Endogenous	No	No	Yes	Yes	Yes	Yes
Operator	-	-	2	2	1	1
Dep	-	23:00	7:30	9:30	4:30	8:30
Arr	-	9:00	9:00	11:00	10:30	14:30
TT	12h	10h	1h30'	1h30'	6h	6h
WT	-	-	1h	1h	-	-
Access	-	0-60'	30-60'	30-60'	0-60'	0-60'
Egress	-	0-30'	30-60'	30-60'	0-30'	0-30'
Price	120 €	60 €	p_2	p_3	p_4	p_5
Tax/subsidy	-	t_{IC}	t_{AIR}	t_{AIR}	t_{HSR}	t_{HSR}

Table 1: Attributes of all scheduled services for the analyzed problem instance.

from the literature (Cascetta and Coppola, 2012).

4.1 Data

We consider a competitive intercity travel market connecting two cities in a typical morning period. The distance between the two cities is assumed to be 1200 km, independent from the travel mode. The market is served by an airline, a high-speed rail company and an intercity train company operating under public service obligations. Additionally, we include the possibility that customers use a private means of transport, which is modeled as an opt-out alternative. We endogenously model the pricing strategies of the airline and of the high-speed rail operator, which must decide on the prices at which to sell tickets for each scheduled departure time. The price of the intercity train and of private transport are assumed to be fixed and exogenously given. We also endogenously model the policies of the regulator, which decides on taxes or subsidies that lead to a welfare-maximizing outcome.
Table 1 shows the supply data used for the tests. Travelers can choose among six different alternative services to go from origin to destination within a given time period. Car and intercity train alternatives are modeled as exogenous options, i.e. all their attributes are assumed to be parameters of the problem, while high-speed rail and air alternatives are modelled endogenously, that is, the two competing operators, each controlling two alternatives, strategically choose their prices in response to the conditions of the market. The attributes that are included in the customer utility functions for the different alternatives are cost, in-vehicle travel time, waiting time, access time to and egress time from terminals, early or late arrival at destination with respect to the desired arrival time of the traveler.

Furthermore, we generate a synthetic population of 1000 travelers for the given OD pair. Individuals are characterized by a trip purpose (business or other), an income level (high or low), and a specific origin location (urban or rural) which leads to

Group (n)	Size (θ_n)	Trip purpose	Reimbursement	Income	Origin
1	350	Other	-	Low	Rural
2	332	Other	-	Low	Urban
3	37	Other	-	High	Rural
4	39	Other	-	High	Urban
5	9	Business	No	Low	Rural
6	24	Business	Yes	Low	Rural
7	16	Business	No	Low	Urban
8	68	Business	Yes	Low	Urban
9	5	Business	No	High	Rural
10	30	Business	Yes	High	Rural
11	21	Business	No	High	Urban
12	69	Business	Yes	High	Urban

Table 2: Contingency table for the synthetic population according to socio-economic characteristics.

different access times to terminals. For the sake of the experiments, we assume that business travelers have a desired arrival time at destination between 9:00 and 12:00, which follows a non-uniform distribution: 50% of them desire to arrive between 9:00 and 10:00 (peak period), the rest between 10:00 and 12:00. Furthermore, we assume that all non-business travelers are indifferent to arrival time. The following demand patterns are to be mentioned: there is a higher proportion of high income and business travelers among urban travelers than among rural travelers; a part of business travelers are reimbursed and are therefore less price sensitive. We categorize the synthetic population in 12 groups of consumers, each having homogeneous socio-economic characteristics. Table 2 represents the contingency table of the synthetic population.

The discrete choice model is derived from Cascetta and Coppola (2012), where a nested logit model is estimated from a RP/SP survey dataset collected in Italy at the national level. Table 3 illustrates the parameters used in our experiments. Two separate sets of parameters are considered for business trips and other trip purposes. Additionally, the cost parameters are mode-specific and interact with income, producing different values of travel time savings, which are reported in Table 4. Two nests μ_{HSR} and μ_{Air} capture the correlation between the scheduled services of the high-speed train operator and of the airline. Non-starred values are taken as such from Cascetta and Coppola (2012). The $\beta_{cost_{car}}$ parameter for reimbursed business customers is derived by assuming that the ratio between the values of travel time of reimbursed and non-reimbursed business travelers by car is the same as by train. We have introduced an additional distinction between high income and low income travelers. This is done in order to test scenarios where government intervention is targeted to specific segments of the population. We have arbitrarily assumed that β_{cost} parameters of non-reimbursed business travelers and of other travelers from Cascetta and Coppola (2012) apply to our low income segment of the population. β_{cost} parameters for high income customers are derived by assuming that the ratio between the values of travel time of high income and low income customers is the same as in the SAMPERS long-distance model developed in Sweden and reported in Börjesson (2014).

We remark that the dataset used for the experiments and the derived results are

β		Business travelers		Other purpose travelers	
μ_{Air}		1.086		1.106	
μ_{HSR}		1.190		1.333	
Travel time (min)		-0.0133		-0.0054	
Access/egress time (min)		-0.00555		-0.0103	
Early schedule delay (min)		-0.00188		-0.00677	
Late schedule delay (min)		-0.0130		-0.00617	
	Reimbursed	High income	Low income	High income	Low income
Cost car (euro)	-0.0222*	-0.0296*	-0.0527	-0.0228*	-0.0405
Cost Air (euro)	-0.0109	-0.0113*	-0.0201	-0.0109*	-0.0194
Cost IC (euro)	-0.0158	-0.0212*	-0.0377	-0.0097*	-0.0172
Cost HSR (euro)	-0.0120	-0.0160*	-0.0284	-0.0144*	-0.0256

Table 3: Model coefficients derived from Cascetta and Coppola (2012).

Value of Travel Time	Reimbursed	High income	Low income	High income	Low income
Car (euro/h)	35.88*	26.95*	15.14	14.24*	8.00
Air (euro/h)	73.21	70.67*	39.70	29.73*	16.70
IC (euro/h)	50.51	37.68*	21.17	33.54*	18.84
HSR (euro/h)	66.50	50.02*	28.10	22.53*	12.66

Table 4: Values of travel time

hypothetical and do not represent real scenarios that are related to choices made by existing high-speed rail operators.

4.2 Numerical experiments

The framework described in Section 3 allows to model various policies and answer questions about the strategic behavior of all agents at equilibrium. Common regulatory policies include price-based instruments such as taxes and subsidies, but also other instruments such as emission allowances. With respect to demand, we can look at the effect of taxes and subsidies on the choices and the utilities for the population as a whole and for specific segments. With respect to supply, we can look at the changes in the pricing strategies and at the effect of regulation on profits. With respect to regulation and welfare, we can look at multiple objective functions to understand tradeoffs between environmental objectives, public expenditures and consumer utilities. We now illustrate how the framework allows to investigate some of these policies and to analyze their welfare effects.

4.2.1 Single objective optimization: emissions

We initially consider a single-objective optimization problem where the regulator's goal is the minimization of the total emissions produced by the travellers, which can be obtained by means of subsidies, taxes, or a combination of the two. For this analysis, we assume that emissions can be directly derived from the modal choices of the demand, even though the schedules, and therefore the actual vehicle emissions, are assumed to be exogenously given. The following mode-specific CO_2 emissions per passenger kilometer are taken from the 2014 estimates derived by European Environment Agency (2020):

244 g/pkm for air, 101 g/pkm for road, 28 g/pkm for rail. In Tables (5-13), the results of the benchmark scenario, that is, without any regulation, are indicated in the first row (experiment 0).

Taxation First, we assume that taxes are the only possible policy instrument for the regulator. Several tax caps are tested with a range from 0 € to 50 € per ticket. Table 5 shows ε-equilibrium prices, taxes, supplier revenues and emissions for all tax caps. As expected, total emissions tend to decrease when the tax on flights increases, although non-linearly, due to the strategic behavior of the two operators. In particular, the airline operator tends to decrease prices p_2 and p_3 for tax levels between 20 € and 30 € in order to retain its market share. For higher tax levels, the strategy changes and air prices are increased again, targeting lower market shares and higher margins. Finally, notice that for the high tax scenarios, the high-speed rail operator does also increase its price r_4 on the business peak alternative, as a consequence of the decreased price competitiveness of the air alternatives. Table 6 reports aggregate and disaggregate modal share, showing that the modal shift resulting from taxation occurs predominantly among low income and non-business travelers. For instance, if we compare the base scenario with the scenario where a 50 € tax is set on air tickets, the air modal share in the high income segments decreases from 51.4% to 45.1%, while in the low income segments it decreases more sharply from 53.0% to 36.5%. Table 7 shows market shares at the alternative level, highlighting the importance of arrival time among business customers, who largely ignore offpeak alternatives that do not match their schedules, irrespective from the level of market intervention.

Subsidization A similar analysis is performed by considering subsidization instead of taxation. Subsidies are set between 0 € to 50 € per ticket. At this stage, they are assumed to be financed through an unlimited budget. The ε-equilibrium results are presented in Tables 8, 9 and 10. We can see that rail subsidies yield results that are generally not too dissimilar to an equivalent aviation tax in terms of equilibrium prices (net of regulation), emissions, market shares and supplier revenues. This seems reasonable, since price enters the utility function linearly. The main difference between taxation and subsidization consists in the budget required to implement the two policies. In the case of taxation, the government gains the taxes of the air travelers who pay t_{AIR}. In the case of subsidization, the government pays out the subsidies t_{IC} and t_{HSR} to the rail travelers.

Revenue recycling Finally, we examine the scenario where the regulator does not have any budget available to implement its policies. As a consequence, the subsidies that are handed out to a subset of the consumers are to be collected in the form of taxation from another subset of consumers, following a revenue recycling approach. Here, we also use the disaggregate information about the travelers to set differentiated policies for two groups of consumers, based on their income category, that is, high income (H) and low income (L). Again, several tax caps are tested with a range from 0 € to 50

		Air Prices		HSR Prices		Regulation			Revenues		Tax rev.	Emissions
#	ε	r_2	r_3	r_4	r_5	t_{IC}	t_{AIR}	t_{HSR}	π_{AIR}	π_{HSR}	B	tCO_2
0	0.020	113.59	101.61	78.19	85.32	0.00	0.00	0.00	56697	28619	0	173.25
1	0.019	121.78	93.73	77.56	78.04	0.00	10.00	0.00	50497	30841	4747	159.79
2	0.020	112.31	86.78	78.29	81.63	0.00	20.00	0.00	46471	31241	9403	158.61
3	0.025	108.00	85.55	77.85	77.73	0.00	30.00	0.00	41304	32539	12872	148.33
4	0.026	119.31	101.78	107.97	83.52	0.00	40.00	0.00	45319	38270	16079	143.47
5	0.025	114.80	100.75	108.76	84.56	0.00	50.00	0.00	41429	40007	18918	136.97

Table 5: ε-equilibrium solutions, revenues and emissions for different tax caps.

	Total modal share			Modal share high inc.			Modal share low inc.			Modal share business			Modal share others		
#	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail
0	0.035	0.527	0.438	0.068	0.514	0.418	0.027	0.530	0.443	0.044	0.561	0.395	0.032	0.516	0.452
1	0.035	0.475	0.490	0.070	0.473	0.457	0.027	0.475	0.498	0.044	0.529	0.427	0.033	0.457	0.510
2	0.035	0.470	0.495	0.070	0.472	0.458	0.027	0.470	0.504	0.044	0.522	0.434	0.033	0.454	0.514
3	0.040	0.429	0.531	0.070	0.453	0.477	0.032	0.423	0.545	0.047	0.501	0.453	0.037	0.406	0.557
4	0.047	0.408	0.545	0.082	0.469	0.449	0.038	0.392	0.569	0.061	0.524	0.415	0.043	0.371	0.587
5	0.047	0.383	0.570	0.082	0.451	0.467	0.039	0.365	0.596	0.062	0.513	0.426	0.043	0.341	0.616

Table 6: Aggregate and segmented modal shares at ε-equilibrium for different tax caps.

	Market shares business						Market shares others					
#	Car	IC	Air1	Air2	HSR1	HSR2	Car	IC	Air1	Air2	HSR1	HSR2
0	0.044	0.002	0.397	0.164	0.379	0.015	0.032	0.108	0.223	0.293	0.203	0.141
1	0.044	0.004	0.354	0.176	0.406	0.016	0.033	0.121	0.170	0.288	0.213	0.176
2	0.044	0.004	0.358	0.164	0.414	0.016	0.033	0.132	0.179	0.275	0.218	0.164
3	0.047	0.004	0.345	0.156	0.432	0.016	0.037	0.148	0.160	0.246	0.219	0.190
4	0.061	0.006	0.368	0.156	0.390	0.020	0.043	0.191	0.169	0.201	0.140	0.256
5	0.062	0.006	0.362	0.151	0.399	0.021	0.043	0.205	0.155	0.186	0.150	0.261

Table 7: Segmented market shares for business and non-business customers at ε-equilibrium for different tax caps.

€. Tables 11, 12 and 13 show that a combination of taxes on carbon-intensive modes and subsidies on greener alternatives has higher effects on emissions than a tax-only or subsidy-only approach. Notice that all the formulations proposed until now do not consider consumer satisfaction and distributional effects within the regulator's objective function, even though these are relevant in policy-making to determine fairness and acceptability across the population.

4.2.2 Combining objectives

Next, we show the results of experiments in which other indicators of social welfare are considered alongside emissions in the objective function. This approach allows to endogenously evaluate tradeoffs between different objectives of public policy.

Emissions and cost of policy In a subsidization context, we evaluate an objective function that minimizes the sum of the cost of subsidies and the cost of emissions. The latter component is dependent on the social cost of carbon (SCC), for which notable divergences exist in the climate economics literature. Here, we test values ranging between 50 and 500 euros per ton of carbon. The results are presented in Table 14. We see that, in this instance, subsidies are not a cost-effective policy instrument if we consider a SCC lower than 300 €/ton. Such high value is largely due to the fact that the regulator must subsidizes all rail users, included those who would have chosen to travel by train regardless of subsidies. Additionally, for high values of the SCC, the optimal

		Air Prices		HSR Prices		Regulation			Revenues		Subsidies	Emissions
#	ε	r_2	r_3	r_4	r_5	t_{IC}	t_{AIR}	t_{HSR}	π_{AIR}	π_{HSR}	B	tCO_2
0	0.020	113.59	101.61	78.19	85.32	0.00	0.00	0.00	56697	28619	0	173.25
6	0.019	122.12	92.48	81.56	82.63	-10.00	0.00	-10.00	51820	31742	-4719	164.08
7	0.019	108.45	82.63	81.97	85.25	-20.00	0.00	-20.00	47033	32143	-9435	165.13
8	0.031	109.29	87.79	88.10	79.64	-30.00	0.00	-30.00	42267	36190	-16436	147.68
9	0.026	113.94	109.91	102.46	93.13	-40.00	0.00	-40.00	44163	43142	-23715	138.23
10	0.022	112.08	111.34	103.99	94.82	-50.00	0.00	-50.00	40145	48034	-30013	129.17

Table 8: ε-equilibrium solutions, revenues and emissions for different subsidy levels.

	Total modal share			Modal share high inc.			Modal share low inc.			Modal share business			Modal share others		
#	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail
0	0.035	0.527	0.438	0.068	0.514	0.418	0.027	0.530	0.443	0.044	0.561	0.395	0.032	0.516	0.452
6	0.035	0.491	0.474	0.068	0.484	0.448	0.027	0.493	0.480	0.044	0.536	0.420	0.032	0.477	0.491
7	0.029	0.498	0.474	0.060	0.498	0.443	0.021	0.497	0.481	0.038	0.542	0.421	0.026	0.483	0.491
8	0.029	0.430	0.541	0.057	0.465	0.478	0.021	0.422	0.557	0.038	0.517	0.446	0.026	0.403	0.572
9	0.029	0.394	0.577	0.058	0.443	0.499	0.021	0.381	0.597	0.039	0.502	0.459	0.026	0.359	0.615
10	0.028	0.359	0.613	0.053	0.409	0.538	0.021	0.347	0.632	0.036	0.469	0.495	0.025	0.324	0.651

Table 9: Aggregate and segmented modal shares at ε-equilibrium for different subsidy levels.

	Market shares business						Market shares others					
#	Car	IC	Air1	Air2	HSR1	HSR2	Car	IC	Air1	Air2	HSR1	HSR2
0	0.044	0.002	0.397	0.164	0.379	0.015	0.032	0.108	0.223	0.293	0.203	0.141
6	0.044	0.004	0.356	0.180	0.401	0.015	0.032	0.112	0.170	0.307	0.213	0.166
7	0.038	0.005	0.368	0.174	0.401	0.015	0.026	0.113	0.185	0.298	0.214	0.164
8	0.038	0.006	0.361	0.156	0.423	0.017	0.026	0.148	0.160	0.243	0.196	0.228
9	0.039	0.006	0.372	0.130	0.436	0.017	0.026	0.183	0.169	0.190	0.201	0.232
10	0.036	0.007	0.358	0.111	0.471	0.017	0.025	0.174	0.160	0.165	0.220	0.257

Table 10: Segmented market shares for business and non-business customers at ε-equilibrium for different subsidy levels.

solutions direct a higher share of subsidies to the low income group. These results suggest that subsidization might be more suitable for markets where the "desirable" modes have low initial market share and high potential modal shift due to high price elasticities of the population segments targeted by the policy.

Emissions and utilities In a taxation context, we evaluate an objective function with two components, capturing the cost of emissions and the monetized consumer utilities. Since only differences in utilities matter, we consider the utilities obtained for SCC = 100 €/ton as benchmark. Table 15 shows the results. Similarly to the previous case, for low values of the SCC, the component of the objective function capturing emissions is outweighed by the component capturing utilities. For SCC = 300 €/ton and above, the optimal solution imposes a maximum tax on the low income segment, which is more sensitive to price changes. This results in low income travelers largely shifting from air to train and car alternatives, with a notable decrease of utility for this group and a consequent reduction in CO_2 emissions. Obviously, such policy does not comply with the basic requirements of equity and fairness that are necessary to make it acceptable.

		Air Prices		HSR Prices		Regulation						Revenues		Emissions
#	ε	r_2	r_3	r_4	r_5	t^H_{IC}	t^H_{AIR}	t^H_{HSR}	t^L_{IC}	t^L_{AIR}	t^L_{HSR}	π_{AIR}	π_{HSR}	tCO_2
0	0.020	113.59	101.61	78.19	85.32	0.00	0.00	0.00	0.00	0.00	0.00	56697	28619	173.25
11	0.027	114.32	101.64	79.43	86.19	7.34	10.00	-10.12	-11.00	10.00	-6.72	47411	33249	150.20
12	0.028	105.78	79.79	88.36	82.72	8.54	20.00	-19.78	-10.87	20.00	-18.01	41042	35940	152.43
13	0.041	115.33	111.18	108.50	96.10	14.41	30.00	-16.16	5.16	30.00	-26.84	41781	47681	133.01
14	0.042	104.71	96.25	111.83	109.39	-35.61	40.00	-23.07	-0.64	40.00	-32.92	37396	49783	133.71
15	0.052	111.99	94.87	103.83	100.91	-47.76	50.00	-30.94	0.17	50.00	-28.82	32822	51577	119.42

Table 11: ε-equilibrium solutions, revenues and emissions for different tax caps and a revenue recycling approach.

	Total modal share			Modal share high inc.			Modal share low inc.			Modal share business			Modal share others		
#	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail	Car	Air	Rail
0	0.035	0.527	0.438	0.068	0.514	0.418	0.027	0.530	0.443	0.044	0.561	0.395	0.032	0.516	0.452
11	0.035	0.438	0.528	0.065	0.451	0.484	0.027	0.435	0.539	0.042	0.507	0.451	0.032	0.416	0.552
12	0.034	0.447	0.519	0.064	0.460	0.477	0.027	0.444	0.530	0.042	0.515	0.443	0.032	0.425	0.544
13	0.045	0.368	0.587	0.078	0.442	0.480	0.037	0.350	0.614	0.055	0.479	0.466	0.042	0.333	0.625
14	0.045	0.371	0.585	0.075	0.428	0.497	0.037	0.357	0.606	0.054	0.471	0.476	0.042	0.339	0.619
15	0.044	0.316	0.640	0.070	0.350	0.580	0.037	0.308	0.655	0.054	0.387	0.559	0.041	0.294	0.666

Table 12: Aggregate and segmented modal shares at ε-equilibrium for different tax caps and a revenue recycling approach.

	Market shares business						Market shares others					
#	Car	IC	Air1	Air2	HSR1	HSR2	Car	IC	Air1	Air2	HSR1	HSR2
0	0.044	0.002	0.397	0.164	0.379	0.015	0.032	0.108	0.223	0.293	0.203	0.141
11	0.042	0.005	0.366	0.142	0.432	0.015	0.032	0.156	0.184	0.232	0.233	0.162
12	0.042	0.002	0.356	0.160	0.424	0.017	0.032	0.134	0.160	0.265	0.201	0.209
13	0.055	0.005	0.358	0.121	0.445	0.017	0.042	0.162	0.160	0.173	0.195	0.268
14	0.054	0.005	0.343	0.128	0.455	0.017	0.042	0.177	0.154	0.185	0.222	0.219
15	0.054	0.009	0.279	0.108	0.533	0.018	0.041	0.179	0.130	0.164	0.248	0.239

Table 13: Segmented market shares for business and non-business customers at ε-equilibrium for different tax caps and a revenue recycling approach.

		Air Prices		HSR Prices		Regulation					Objective function	
SCC	ε	r_2	r_3	r_4	r_5	t^H_{TRAIN}	t^H_{AIR}	t^L_{TRAIN}	t^L_{AIR}	tCO_2	SWF_E	SWF_R
50	0.025	114.59	99.03	77.75	84.49	0.00	0.00	0.00	0.00	172.94	-8647	0
100	0.018	111.74	100.23	77.79	88.11	0.00	0.00	-0.12	0.00	175.25	-17525	-43
150	0.021	125.19	100.74	76.87	78.64	0.00	0.00	0.00	0.00	164.52	-24678	0
200	0.019	128.55	101.72	78.36	83.17	-0.05	0.00	-0.73	0.00	164.35	-32870	-286
250	0.021	111.57	99.55	77.52	88.68	-0.33	0.00	-0.99	0.00	174.28	-43570	-382
300	0.022	111.13	98.59	79.76	85.68	-2.30	0.00	-0.68	0.00	175.22	-52566	-433
350	0.020	109.63	97.27	80.81	89.17	0.00	0.00	-6.76	0.00	172.28	-60298	-2488
400	0.027	109.55	92.01	80.69	81.93	-5.43	0.00	-11.29	0.00	166.60	-66640	-4780
450	0.031	115.56	93.48	82.29	87.31	-2.88	0.00	-17.80	0.00	158.27	-71222	-7658
500	0.034	122.29	98.57	91.75	83.44	-8.54	0.00	-25.78	0.00	146.54	-73270	-12631

Table 14: ε-equilibrium solutions for an objective function that minimizes the sum of the cost of subsidies and the cost of emissions, with different values of the SCC.

5 Conclusion

In this paper, we introduced a framework which exploits discrete choice models of demand to find optimal policies to regulate oligopolistic markets. Using a disaggregate representation of demand that captures demand heterogeneity allows to account for product differentiation and consumer behavioral heterogeneity at the individual level. The objective function is a social welfare function which can include measures of individual utility and collective welfare.

The proposed framework is very general and requires limited assumptions on the specification of the used discrete choice models. This means that it can accommodate

		Air Prices		HSR Prices		Regulation					Objective function		
SCC	ε	r_2	r_3	r_4	r_5	t^H_{TRAIN}	t^H_{AIR}	t^L_{TRAIN}	t^L_{AIR}	tCO_2	SWF_E	SWF^H_U	SWF^L_U
50	0.029	117.76	100.79	78.29	83.65	0.00	0.00	0.00	0.00	170.68	-8534	-141	-303
100	0.024	113.77	103.21	78.80	80.44	0.00	0.00	0.00	0.20	170.30	-17030	0	0
150	0.028	127.45	102.00	83.75	80.34	0.00	0.00	0.00	1.21	166.30	-24945	-872	-2503
200	0.025	118.51	104.32	77.99	86.99	0.00	0.00	0.00	0.00	168.74	-33748	-291	-1242
250	0.047	119.92	117.54	95.96	82.30	0.00	0.00	0.00	6.00	162.09	-40523	-1558	-7036
300	0.020	98.90	95.99	106.37	82.05	0.00	0.00	0.00	99.90	119.09	-35727	-258	-22298
350	0.030	94.25	92.35	106.70	82.45	0.00	0.69	0.00	99.18	122.52	-42882	37	-21780
400	0.028	95.28	92.50	106.11	83.00	0.00	0.00	0.00	99.91	121.40	-48560	47	-21985
450	0.023	95.70	94.73	101.87	83.17	0.00	0.28	0.00	99.54	120.51	-54230	83	-21473
500	0.028	91.90	92.04	106.15	82.74	0.00	0.00	0.00	99.83	123.45	-61725	257	-21651

Table 15: ε-equilibrium solutions for an objective function that maximizes the monetized consumer utilities minus the cost of emissions, with different values of the SCC.

a large variety of choice models available in the literature. The use of disaggregate demand models allows to design disaggregate policies that leverage on subsidization or taxation to obtain desirable outcomes from economic, social and environmental points of view.

The following research directions could be further investigated.

Decisions other than price could be included in the framework, both for the suppliers and for the regulator. Examples are assortment, capacity levels and quality changes, among others. Adapting the mathematical models is straightforward, if these variables appear as linear or integer variables in the utility functions. However, these extensions would come with additional computational complexity caused by the expanded solution space. Consequently, the applicability of our framework to large-scale problem depends on the capability to efficiently exploit the problem structure and find tight bounds or ad-hoc algorithms to such hard combinatorial problem.

A fundamental issue in public policy is the aggregation of individual utilities into a social welfare function. Different agents have different utilities and objectives that conjugate individual and social welfare. Therefore, multi-objective social welfare optimization problems cannot prescind from value judgements. In this work, we have followed the Marshallian approach which assumes constant marginal utility of income. However, even the 'neutral' assumption that assigns the same value to a single monetary unit, irrespective of the agent's status, is a value judgement (Sen, 1999; Fleurbaey, 2009). Our framework could be adapted to incorporate distributional preferences in the social welfare function. Indeed, it is well-known that the acceptability of any market-based instrument for public policy depends on the perceived fairness of the instrument (Maestre-Andrés et al., 2019). Further research could also be conducted to investigate how acceptability and perceived fairness can be included in our framework.

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References

M. Adler, D. Anthoff, V. Bosetti, G. Garner, K. Keller, and N. Treich. Priority for the worse-off and the social cost of carbon. *Nature Climate Change*, 7(6):443–449, 2017.

M. D. Adler and M. Fleurbaey. *The Oxford handbook of well-being and public policy*. Oxford University Press, 2016.

A. Anas and R. Lindsey. Reducing urban road transportation externalities: Road pricing in theory and in practice. *Review of Environmental Economics and Policy*, 5(1): 66–88, 2011.

K. J. Arrow. *Social choice and individual values*. John Wiley & Sons, 1951.

R. Batley and T. Dekker. The intuition behind income effects of price changes in discrete choice models, and a simple method for measuring the compensating variation. *Environmental and Resource Economics*, 74(1):337–366, 2019.

R. Batley and J. N. Ibáñez. Applied welfare economics with discrete choice models: implications of theory for empirical specification. In *Choice Modelling*. Edward Elgar Publishing, 2013.

M. E. Ben-Akiva and S. R. Lerman. *Discrete choice analysis: theory and application to travel demand*, volume 9. MIT Press, 1985.

A. Bergson. A reformulation of certain aspects of welfare economics. *The Quarterly Journal of Economics*, 52(2):310–334, 1938.

S. Binder, Y. Maknoon, and M. Bierlaire. Exogenous priority rules for the capacitated passenger assignment problem. *Transportation Research Part B: Methodological*, 105:19–42, 2017.

M. Börjesson. Forecasting demand for high speed rail. *Transportation Research Part A: Policy and Practice*, 70:81–92, 2014.

R. Borndörfer, M. Karbstein, and M. E. Pfetsch. Models for fare planning in public transport. *Discrete Applied Mathematics*, 160(18):2591–2605, 2012.

S. Bortolomiol, V. Lurkin, and M. Bierlaire. A disaggregate choice-based approach to find ε-equilibria of oligopolistic markets. Technical report, Transport and Mobility Laboratory, EPFL, 2019.

K. Button and E. Verhoef. *Road pricing, traffic congestion and the environment*. Edward Elgar Publishing, 1998.

E. Cascetta and P. Coppola. An elastic demand schedule-based multimodal assignment model for the simulation of high speed rail (HSR) systems. *EURO Journal on Transportation and Logistics*, 1(1-2):3–27, 2012.

H. Cremer, M. Marchand, and J.-F. Thisse. Mixed oligopoly with differentiated products. *International Journal of Industrial Organization*, 9(1):43–53, 1991.

K. Cullinane and R. Bergqvist. Emission control areas and their impact on maritime transport. *Transportation Research Part D: Transport and Environment*, 28:1–5, 2014.

C. d'Aspremont and L. Gevers. Social welfare functionals and interpersonal comparability. *Handbook of social choice and welfare*, 1:459–541, 2002.

B. De Borger. Optimal two-part tariffs in a model of discrete choice. *Journal of public economics*, 76(1):127–150, 2000.

B. De Borger and I. Mayeres. Optimal taxation of car ownership, car use and public transport: Insights derived from a discrete choice numerical optimization model. *European Economic Review*, 51(5):1177–1204, 2007.

B. De Borger and S. Proost. Traffic externalities in cities: the economics of speed bumps, low emission zones and city bypasses. *Journal of Urban Economics*, 76: 53–70, 2013.

G. De Fraja and F. Delbono. Alternative strategies of a public enterprise in oligopoly. *Oxford Economic Papers*, 41(2):302–311, 1989.

European Environment Agency. Energy efficiency and specific CO2 emissions. `https://www.eea.europa.eu/data-and-maps/indicators/energy-efficiency-and-specific-co2-emissions/energy-efficiency-and-specific-co2-9`, 2020. [Online; accessed 31 December 2020].

M. Farsi, A. Fetz, and M. Filippini. Economies of scale and scope in local public transportation. *Journal of Transport Economics and Policy (JTEP)*, 41(3): 345–361, 2007.

M. Fleurbaey. Beyond gdp: The quest for a measure of social welfare. *Journal of Economic literature*, 47(4):1029–75, 2009.

D. Fudenberg and J. Tirole. *Game theory*. MIT Press, 1991.

D. Fullerton and S. E. West. Can taxes on cars and on gasoline mimic an unavailable tax on emissions? *Journal of Environmental Economics and Management*, 43(1): 135–157, 2002.

L. A. Guzman and D. Oviedo. Accessibility, affordability and equity: Assessing 'pro-poor'public transport subsidies in bogotá. *Transport Policy*, 68:37–51, 2018.

J. C. Harsanyi. Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility. *Journal of political economy*, 63(4):309–321, 1955.

T. D. Hau. Distributional cost-benefit analysis in discrete choice. *Journal of Transport Economics and Policy*, pages 313–338, 1986.

T. D.-K. Hau. A hicksian approach to cost-benefit analysis with discrete-choice models. *Economica*, 52(208):479–490, 1985.

D. Hausman, M. McPherson, and D. Satz. *Economic analysis, moral philosophy, and public policy*. Cambridge University Press, 2016.

S. Hess, A. Daly, and R. Batley. Revisiting consistency with random utility maximisation: theory and implications for practical work. *Theory and Decision*, 84(2):181–204, 2018.

J. R. Hicks. The foundations of welfare economics. *The Economic Journal*, 49(196): 696–712, 1939.

IPCC. *Climate Change 2014. Mitigation of Climate Change*. Cambridge University Press, 2014.

S. R. Jara-Díaz and J. I. Videla. Welfare implications of the omission of income effect in mode choice models. *Journal of Transport Economics and Policy*, pages 83–93, 1990.

N. Kaldor. Welfare propositions of economics and interpersonal comparisons of utility. *The Economic Journal*, pages 549–552, 1939.

S. Maestre-Andrés, S. Drews, and J. van den Bergh. Perceived fairness and public acceptability of carbon pricing: a review of the literature. *Climate policy*, 19(9): 1186–1204, 2019.

D. McFadden. Computing willingness to pay in random utility models. department of economics. Technical report, University of California-Berkeley, December, 1995.

E. R. Morey, V. R. Sharma, and A. Karlstrom. A simple method of incorporating income effects into logit and nested-logit models: theory and application. *American Journal of Agricultural Economics*, 85(1):248–253, 2003.

W. D. Nordhaus. *Managing the global commons: the economics of climate change*, volume 31. MIT press Cambridge, MA, 1994.

M. J. Osborne and A. Rubinstein. *A course in game theory*. MIT Press, 1994.

M. Pacheco Paneque. *A general framework for the integration of complex choice models into mixed integer optimization*. PhD thesis, École Polytechnique Fédérale de Lausanne, 2020.

V. Pareto. *Manuale di economica politica*. Societa Editrice Libraria, 1906.

D. Pearce. The social cost of carbon and its policy implications. *Oxford review of economic policy*, 19(3):362–384, 2003.

M. Ponti. Competition, regulation and public service obligations. In *A handbook of transport economics*. Edward Elgar Publishing, 2011.

P. A. Samuelson. *Foundations of economic analysis*. Harvard University Press, 1948.

A. K. Sen. Rational fools: A critique of the behavioral foundations of economic theory. *Philosophy & Public Affairs*, pages 317–344, 1977.

A. K. Sen. The possibility of social choice. *American economic review*, 89(3):349–378, 1999.

A. K. Sen. *Collective choice and social welfare*. Harvard University Press, 2017.

C. Shapiro. Exclusivity in network industries. *Geo. Mason L. Rev.*, 7:673, 1998.

K. A. Small and H. S. Rosen. Applied welfare economics with discrete choice models. *Econometrica: Journal of the Econometric Society*, pages 105–130, 1981.

N. H. Stern, S. Peters, V. Bakhshi, A. Bowen, C. Cameron, S. Catovsky, D. Crane, S. Cruickshank, S. Dietz, N. Edmonson, et al. *The economics of climate change: the Stern review*, volume 30. Cambridge University Press, 2006.

K. Train and M. Weeks. Discrete choice models in preference space and willingness-to-pay space. In *Applications of simulation methods in environmental and resource economics*, pages 1–16. Springer, 2005.

C. Van Goeverden, P. Rietveld, J. Koelemeijer, and P. Peeters. Subsidies in public transport. *European Transport*, 32:5–25, 2006.