Demand-based discrete optimization

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Abstract

We propose a methodological framework to include a wide variety of discrete choice models in (mixed) integer optimization problems. We succeed in obtaining a specification that is linear in the decision variables, allowing to use exact methods to solve the problem. This opens the door to solving integrated supply and demand models in many fields. This working paper describes the methodology and proposes the full model.

1 Introduction

During the last decade there has been a growing literature on combining customer behavior models in optimization. Several applications can be found in facility location problems (Haase and Müller, 2014a; Zhang et al., 2012; Benati and Hansen, 2002a) as well as application of revenue management networks in different contexts such as transportation and hotel management (See, Talluri and Van Ryzin, 2004; van Ryzin and Vulcano, 2014; Haase and Müller, 2014a; Haensel and Koole, 2010)

The main advantage of these integrated models is to enable policy makers to have a better understanding about the preferences of their clients while planning for their systems. The preferences of customers are formalized using a specific predefined choice model (see Gilbert et al., 2014a; Gilbert et al., 2014b for examples of discrete choice models in assortment optimization).

Even though at some cases the integrated choice-based optimization models are solvable easily, they are computationally complex. The challenge goes back to the type of the choice model integrated inside the optimization model which creates a great source of nonconvexity. Most of the time, a simple logit model is used, where customers are assumed to be homogeneous in their observable characteristics. Many techniques have been employed in order to convexify and linearize such models, however, many of such models fail to solve real case or large size problems (Azadeh et al., 2015).

In this paper, we are interested in discrete optimization models where supply and demand closely interact, typically appearing in transportation problems such as airline scheduling. Our objective is to incorporate state-of-the-art advanced discrete choice models in optimization problem. Our method is directly derived from the theory of utility maximization which can address two main issues in the choice-based optimization problems.

- We eliminate the nonconvex representation of choice probabilities which makes the optimization models computationally complex.

- We can consider a wide class of discrete choice models, including modern models such as multivariate extreme value models, latent variable and latent class models.

In fact, the general methodology leads to an integrated supply and demand model based on discrete choice that is linear in its decision variables. In this working paper, we present the state of the art mathematical model where a supplier needs to decide to offer some services, and to decide about the price levels of offered alternatives in order to maximize its revenues.

2 Modeling the demand

We consider a population composed of N individuals (or groups of individuals with an homogenous behavior). The set of products in the market is denoted by $\mathcal{C}$. Note that we assume without loss of generality that the market is closed, that is that every customer chooses exactly one product. It is always possible to include an artificial "opt-out" product to capture customers leaving the market. In the considered market, each individual n has to choose one alternative within a set $\mathcal{C}_n \subseteq \mathcal{C}$ of products that are available to her. Note that this is the first level of heterogeneity: the set of available products may vary from one customer to the next.

Discrete choice models are based on the assumption that each individual n associates a score, called *utility*, with each alternative i in the choice set $\mathcal{C}_n$, that is denoted by U_{in}. This utility is a function of several variables, describing the attributes of the alternative i, as well as the socio-economic characteristics of the individual n and interactions between the two. The main behavioral assumption is that alternative i is chosen by individual n if the utility associated by n with i is the highest within the choice set $\mathcal{C}_n$. Assume that there is no tie, that is for each n and $i, j \in \mathcal{C}_n$, either $U_{in} > U_{jn}$ or $U_{in} < U_{jn}$, and define the indicator

$$w_{in} = \begin{cases} 1 & \text{if } n \text{ chooses } i, \\ 0 & \text{otherwise.} \end{cases} \quad \forall n, \forall i \in \mathcal{C}. \tag{1}$$

Note that $w_{in} = 0$ if $i \notin \mathcal{C}_n$. Define also the indicator

$$y_{in} = \begin{cases} 1 & \text{if } i \in \mathcal{C}_n \\ 0 & \text{otherwise} \end{cases} \quad \forall n, \forall i \in \mathcal{C}. \tag{2}$$

Therefore, we have for each n and each i

$$w_{in} = 1 \iff y_{in} = 1 \text{ and } U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n. \tag{3}$$

In practice, the analyst does not have access to the exact specification of the utility, and must consider it as a random variable. The most common specification is

$$U_{in} = V_{in} + \varepsilon_{in} \tag{4}$$

where V_{in} is the deterministic part of the utility function, and ε_{in} is the error term, capturing everything that the analyst has not included explicitly in the model. Note that we assume here that V_{in} is linear in the variables involved in the optimization problem. This assumption is not necessary for the derivation of the choice model but, in our context, is important for its integration in the discrete optimization model. With this specification, the model becomes probabilistic and (3) is now written

$$\Pr(w_{in} = 1) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n). \tag{5}$$

Note that the probabilistic nature of the model associates a zero probability with ties, so that they can be safely ignored, as assumed above.

Concrete operational models can be derived from specific assumptions about the distribution of the error terms ε_{in}. The most common one is the assumption that ε_{in} are independent (across both i and n) and identically distributed, with an Extreme Value distribution. In this case, it can be shown that the model (5) is written

$$\Pr(w_{in} = 1) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}. \tag{6}$$

It is called the *logit* model. Note that, this formulation is non linear as a function of the utilities. It is also non linear in the variables y_{in}, but linear reformulations have been proposed in the literature (Benati and Hansen, 2002b, Zhang et al., 2012, Haase and Müller, 2014b). Other assumptions about the distribution of ε_{in} lead to other models, such as the *nested logit*, the *cross nested logit* or the *logit mixtures* model, to cite just a few.

The demand within the market, that is the number of individuals choosing alternative i, for each $i \in \mathcal{C}$, is then given by

$$D_i = \sum_{n=1}^{N} \Pr(w_{in} = 1). \tag{7}$$

3 A linear formulation

The demand model (7) is in general non linear. Various ways to linearize it have been proposed in the literature. We propose here a different approach, derived directly from (4) and (5). For each i and n, we rely on simulation to generate R draws $\xi_{in1}, \ldots, \xi_{inR}$ from the distribution of ε_{in}. Note that this can be done for a wide variety of distributions, so that the demand model is not restricted to logit. Each draw corresponds to a behavioral scenario. Once the draws have been generated, the probabilistic nature of the model can be captured by simulation in the following way.

We denote by

$$U_{inr} = V_{in} + \xi_{inr} = \sum_{k} \beta_k x_{ink} + f(z_{in}) + \xi_{inr}, \tag{8}$$

the utility associated by individual n with alternative i, in the rth scenario. Note that we distinguish between the part of V_{in} that is linear in the variables x_{ink}, and the part that depends on other variables z_{in}, in a possibly non linear way defined by f. The variables x_{ink} are those involved in the optimization problem, and the variables z_{in} are additional exogenous variables. Note also that U_{inr} is not a random variable.

For the sake of generality, in addition to the variables y_{in}, we introduce the variables y_{inr} that characterize the availability of service i to individual n, in scenario r. While y_{in} is a decision of the operator, independent on the choices of the customers, these new variables may account for the possible unavailability of an alternative due to excess of demand, as illustrated in Section 4. They are related in the following way:

$$y_{inr} \leq y_{in}, \forall i, n, r. \tag{9}$$

We introduce also variables μ_{ijnr} that characterize the largest between the utilities of i and j, that is, for each n, i, j and r:

$$\mu_{ijnr} = \begin{cases} 1 & \text{if } U_{inr} \geq U_{jnr}, \\ 0 & \text{if } U_{inr} < U_{jnr}. \end{cases} \tag{10}$$

Note that it is possible that $\mu_{ijnr} = \mu_{jinr}$ if the two utilities happen to be equal, although in practice it should happen rarely. Moreover, we have the following valid inequality that may be included in the model:

$$\mu_{ijnr} + \mu_{jinr} \leq 1, \forall i, j, n, r, \tag{11}$$

Defining the constant M_{nr} such that

$$|U_{inr} - U_{jnr}| \leq M_{nr}, \forall i, j, \tag{12}$$

the definition (10) is characterized by the following constraint:

$$(\mu_{ijnr} - 1)M_{nr} \leq U_{inr} - U_{jnr} \leq \mu_{ijnr} M_{nr}, \forall i, j, n, r. \tag{13}$$

To account for the availability of the alternatives, we have to consider the above constraints only when both alternatives are available, that is when $y_{inr} = 1$ and $y_{jnr} = 1$. Therefore, we define the variables η_{ijnr}, which is 1 if both i and j are available to individual n in scenario r, that is, for each i, j, n, r,

$$y_{inr} + y_{jnr} \leq 1 + \eta_{ijnr}, \tag{14}$$

$$\eta_{ijnr} \leq y_{inr}, \tag{15}$$

$$\eta_{ijnr} \leq y_{jnr}. \tag{16}$$

Therefore, we write (13) as follows:

$$M_{nr}\eta_{ijnr} - 2M_{nr} \leq U_{inr} - U_{jnr} - M_{nr}\mu_{ijnr} \leq (1 - \eta_{ijnr})M_{nr}, \forall i, j, n, r. \tag{17}$$

To verify the above formulation, we consider four cases:

- $\eta_{ijnr} = 1$ and $\mu_{ijnr} = 1$. Then (17) is written

  $$0 \leq U_{inr} - U_{jnr} \leq M_{nr}, \forall i, j, n, r. \tag{18}$$

  The first inequality imposes that $U_{inr} \geq U_{jnr}$, which is consistent with $\mu_{ijnr} = 1$, and the second inequality is always verified, from (12).

- $\eta_{ijnr} = 1$ and $\mu_{ijnr} = 0$. Then, (17) is written

  $$-M_{nr} \leq U_{inr} - U_{jnr} \leq 0, \forall i, j, n, r. \tag{19}$$

  The first inequality is always verified, from 12, and the second imposes that $U_{inr} \leq U_{jnr}$, which is consistent with $\mu_{ijnr} = 0$.

- $\eta_{ijnr} = 0$ and $\mu_{ijnr} = 1$. Then, (17) is written

  $$-M_{nr} \leq U_{inr} - U_{jnr} \leq 2M_{nr}, \forall i, j, n, r, \tag{20}$$

  and is always verified from (12). Note that this configuration will be forbidden by another constraint.

- $\eta_{ijnr} = 0$ and $\mu_{ijnr} = 0$. Then, (17) is written

  $$-2M_{nr} \leq U_{inr} - U_{jnr} \leq M_{nr}, \forall i, j, n, r, \tag{21}$$

  and is always verified from (12).

Note that (17) is linear in the utility functions U_{inr}, and linear in μ_{ijmr} and η_{ijnr}. Therefore, for each variable x_{ink} such that U_{inr} is linear in x_{ink}, (17) is also linear in x_{ink}. We also impose that

$$\mu_{ijnr} \leq y_{inr}, \forall i, j, n, r, \tag{22}$$

so that i cannot be preferred to j if i is not available.

For each n, i and r, we define the choice variable w_{inr} using the maximum utility paradigm using the following constraints.

- The chosen alternative is the one with the largest utility:

$$w_{inr} \leq \mu_{ijnr}, \forall i, j, n, r. \tag{23}$$

- An available alternative is chosen

$$w_{inr} \leq y_{inr}, \forall i, n, r. \tag{24}$$

  Note that this constraint is not necessary as it is implicitly imposed by (22) and (23).

- Exactly one choice is performed by each individual in each scenario:

$$\sum_{i \in \mathcal{C}} w_{inr} = 1, \forall n, r. \tag{25}$$

The above model specification is pretty general and is linear in the following variables:

- any variable appearing linearly in the utility function,
- the choice variables w_{inr},
- the preference variables μ_{ijnr},
- the availability variables y_{inr}.

The demand within the market, that is the number of individuals choosing alternative i, for each $i \in \mathcal{C}$, is then given by

$$D_i = \frac{1}{R} \sum_{n=1}^{n} \sum_{r=1}^{R} w_{inr}. \tag{26}$$

4 Demand based revenues maximization

We illustrate the use of the framework described above using the following example. Consider an operator selling services to a market, and each service can be offered at a given price to a finite number of customers, called the *capacity* of the service. The demand is price elastic and heterogenous, in the sense that each group of customers may have a different behavior. Typical examples are airlines, where the service is a connection between two airports, or film distributors offering movies in various theaters.

We are interested in finding the best strategy in terms of capacity allocation and pricing, in order to maximize the revenues of the operator. The operator is proposing J services, each service i being accessible to a maximum of c_i customers. For the airline example, the service i can be Boston-Chicago, business class, where c_i is the number of seats available in the aircraft times the number of flights per day, say. For the movie theater example, the service i can be *Mad Max: Fury Road* at AMC Loews Boston Common, and c_i the total number of seats in the theater times the number of shows per day. In addition, we consider a "service" with index $i = 0$. Customers who do not buy any service from the operator, either because they do not buy any service at all, or they buy the service from a competing operator, are assigned to this service.

In order to consider heterogenous demand, we assume that the market is composed of N individuals, or group of individuals of homogenous behavior. In the following, we refer only to "individuals". The demand is modeled using the methodology described in Sections 2 and 3. Each individual n associates a utility function with each service i:

$$U_{in} = V_{in} + \varepsilon_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}, \tag{27}$$

where $p_{in} \in \mathbb{R}$ is the price that individual n must pay to access service i, $z_{in} \in \mathbb{R}^K$ is a vector of variables describing the level of service of i for individual n, and ε_{in} is an error term gathering variables known by the decision-maker, but unknown to the analyst. The parameters β_{in} and the specification of the function f are given. For the airline example, z_{in} may for instance contain the leg room in the cabin, or the availability of free magazines. For the movie theater example, it may be the type of movie on show, or the availability of food and drinks in the theater. In our approach, the variable p_{in} is endogenous, as it is a decision variable for the operator. Note that the index n allows the operator to propose different prices to different groups of individuals. In this example, we consider all other variables z_{in} as exogenous. Therefore, the quantity $f(z_{sn})$ is a value that can be preprocessed, and it does not matter if f is linear or not in z_{in}.

Define the availability variables y_{in} as 1 if individual n considers and has access to service i, and 0 otherwise. Note that it is assumed that $y_{0n} = 1$, $\forall n$.

Also, some of these variables can be fixed to 0 before hand, if a market segment has only access to some of the services. For instance, it may be assumed that customers have no access to a movie theater in another city. Travelers have no access to a flight for another origin-destination pair.

The revenues obtained from service i by the operator can be derived directly from the demand function:

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}. \tag{28}$$

As p is an endogenous variable, (28) is nonlinear. It can be linearized by assuming that the price of service i and customer n can only take a finite number of predetermined different values: $p_{in}^1, p_{in}^2, \ldots, p_{in}^{L_{in}}$, so that

$$p_{in} = \sum_{\ell=1}^{L_{in}} \lambda_{in\ell} p_{in}^{\ell}, \tag{29}$$

where $\lambda_{in\ell} \in \{0, 1\}$, and

$$\sum_{\ell=1}^{L_{in}} \lambda_{in\ell} = 1, \forall i, n. \tag{30}$$

The revenues (28) can now be written

$$R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} \lambda_{in\ell} p_{in}^{\ell} \sum_{r=1}^{R} w_{inr}. \tag{31}$$

To linearize it, we introduce the variables $\alpha_{inr\ell} = \lambda_{in\ell} w_{inr}$, so that the formulation becomes

$$R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} \alpha_{inr\ell} p_{in}^{\ell}, \tag{32}$$

with

$$\lambda_{in\ell} + w_{inr} \leq 1 + \alpha_{inr\ell}, \forall i, n, r, \ell, \tag{33}$$

$$\alpha_{inr\ell} \leq \lambda_{in\ell}, \forall i, n, r, \ell, \tag{34}$$

$$\alpha_{inr\ell} \leq w_{inr}, \forall i, n, r, \ell. \tag{35}$$

Accounting for the costs of proposing the services, the objective function is

$$\frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} \alpha_{inr\ell} p_{in}^{\ell} \tag{36}$$

4.1 Dealing with capacities

Each service i cannot accommodate more than c_i customers. If the demand for service i is larger than its capacity, a selection must be done, to decide who has access to the service, and who has not. In a revenues maximization context, if nothing is explicitly specified, the optimization algorithm will favor customers who bring the largest amount of revenues to the operator. This is valid only if the operator can decide which customers can be served and which not.

In many situations, the customers arrive in a random order, and get served in a first-come-first-served basis. The model needs to know, for each pair of individuals n and m if n has priority over m, or the other way around.

A simple way to model it is to provide a priority list of individuals, where an individual is served only if all individuals before him in the list have been served. Note that the construction of this priority list can account for various aspects of the relationships between the operator and the customers, such as fidelity programs, VIP customers, etc. The priority list is supposed to be given.

Assuming that the customers are numbered according to a priority list, we impose that

$$y_{inr} \geq y_{i(n+1)r}, \forall i, n, r. \tag{37}$$

For each scenario r the following constraint must be verified:

$$c_i(1 - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} + (1 - y_{in})c_{\max}, \forall i, n, r, \tag{38}$$

where $c_{\max} = \max_i c_i$. If $y_{inr} = 1$ then $y_{in} = 1$ (because of (9)), and this constraint becomes:

$$0 \leq \sum_{m=1}^{n-1} w_{imr},$$

that is always verified. If $y_{inr} = 0$ and $y_{in} = 1$, we obtain

$$c_i \leq \sum_{m=1}^{n-1} w_{imr},$$

meaning that the capacity has been reached due to the choices of individuals 1 to $n - 1$ in the priority list. It is the scenario when service i is available to n, but there is no room left due to the choice of other customers. Finally, if $y_{inr} = 0$ and $y_{in} = 0$, we obtain

$$c_i \leq \sum_{m=1}^{n-1} w_{imr} + c_{\max},$$

that is always verified, as all w_{imr} are equal to 0, because service i is not available.

The following constraint must also be verified

$$\sum_{m=1}^{n-1} w_{imr} + (1 - y_{in})c_{\max} \leq (c_i - 1)y_{inr} + K_n(1 - y_{inr}), \forall i, n > c_{\min}, r, \quad (39)$$

where $c_{\min} = \min_i c_i$ and

$$K_n = \max(n, c_{\max}). \quad (40)$$

If $y_{inr} = 1$ then $y_{in} = 1$ and we obtain

$$1 + \sum_{m=1}^{n-1} w_{imr} \leq c_i$$

imposing that the number of individuals up to and including n who have chosen service i must not exceed the capacity. If $y_{inr} = 0$ then $y_{in} = 1$, we obtain

$$\sum_{m=1}^{n-1} w_{imr} \leq K_n,$$

that is always verified as $\sum_{m=1}^{n-1} w_{imr} \leq n \leq K_n$. Finally, if $y_{inr} = 0$ then $y_{in} = 0$, we have

$$\sum_{m=1}^{n-1} w_{imr} + c_{\max} \leq K_n,$$

that is always verified, as $w_{imr} = 0$ because service i is not available. Note that no constraint is needed for individuals $n = 1, \ldots, c_{\min}$, as there is always enough capacity for these customers.

4.2 Capacities as decision variables

In some applications, the capacities c_i are not given, and must be decided. If c_i are decision variables, the above formulation becomes non linear due to the constraints (38) and (39). We propose to deal with this in the following way.

- Consider a service i, and denote by $c_i^1, \ldots, c_i^Q$ the Q possible capacities that this service can take;
- Use the above framework where the service i is replaced by q services instead of 1;
- Impose that maximum one of these q services is open:

$$\sum_{q=1}^{Q} y_{in}^q \leq 1. \quad (41)$$

The rest of the framework is identical.

4.3 The full model

Putting everything together, we have the following data

- a list of customers $n = 1, \dots, N$ or groups of customers, sorted according some priority rule,
- a list of services $\mathcal{C}$,
- for each n, a list of available services $\mathcal{C}_n$,
- for each service i and each customer n, the parameters β_k and the value $f(z_{in})$ for the utility function (8),
- for each service i, its capacity c_i,
- for each service i and each customer m, the list of possible prices p_{in}^{ℓ}, $\ell = 1, \dots, L_{in}$,
- for each customer i and each service i, a sequence of R draws ξ_{inr}, $r = 1, \dots, R$,
- for each customer n and each draw r, the bound M_{nr} verifying (12),

and the following decision variables:

- y_{in}, the availability of service s for customer n, $n = 1, \dots, N$, $i \in \mathcal{C}$,
- y_{inr}, the availability of service s for customer n and scenario r, $n = 1, \dots, N$, $r = 1, \dots, R$, $i \in \mathcal{C}$,
- w_{inr}, the choice of service s by customer n in scenario r, $n = 1, \dots, N$, $r = 1, \dots, R$, $i \in \mathcal{C}$,
- $\lambda_{in\ell}$, the selection of the level of price ℓ for service i, $i \in \mathcal{C}$, $\ell = 1, \dots, L_{in}$ and customer n.

and the technical variables

- μ_{ijnr} characterizing the largest utility between service i and j for individual n in scenario r, $i, j \in \mathcal{C}$, $i \neq j$, $n = 1, \dots, N$, $r = 1, \dots, R$,
- $\alpha_{inr\ell}$ for the linearization of the objective function, $i \in \mathcal{C}$, $n = 1, \dots, N$, $r = 1, \dots, R$, $\ell = 1, \dots, L_{in}$,
- η_{ijnr} identifying the common availability of two alternatives, $i, j \in \mathcal{C}$, $i \neq j$, $n = 1, \dots, N$, $r = 1, \dots, R$.

$$\max \sum_{i>0} \frac{1}{R} \sum_{r=1}^{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} \alpha_{inr\ell} p_{in}^{\ell}, \tag{42}$$

subject to

$$\begin{aligned}
U_{inr} &= \sum_{\ell=1}^{L_{in}} \beta_{in} \lambda_{in\ell} p_{in}^{\ell} + f(z_{in}) + \xi_{inr}, && \forall i, n, r, && (43)\\
M_{nr}\eta_{ijnr} - 2M_{nr} &\leq U_{inr} - U_{jnr} - M_{nr}\mu_{ijnr}, && \forall i, j, n, r, && (44)\\
U_{inr} - U_{jnr} - M_{nr}\mu_{ijnr} &\leq (1 - \eta_{ijnr})M_{nr}, && \forall i, j, n, r, && (45)\\
y_{inr} + y_{jnr} &\leq 1 + \eta_{ijnr}, && \forall i, j, n, r, i \neq j && (46)\\
\eta_{ijnr} &\leq y_{inr}, && \forall i, j, n, r, i \neq j && (47)\\
\eta_{ijnr} &\leq y_{jnr}, && \forall i, j, n, r, i \neq j && (48)\\
\mu_{ijnr} + \mu_{jinr} &\leq 1, && \forall i, j, n, r, i \neq j && (49)\\
\mu_{ijnr} &\leq y_{inr}, && \forall i, j, n, r, i \neq j && (50)\\
w_{inr} &\leq \mu_{ijnr}, && \forall i, j, n, r, i \neq j && (51)\\
w_{inr} &\leq y_{inr}, && \forall i, n, r, && (52)\\
\sum_{i \in \mathcal{C}} w_{inr} &= 1, && \forall n, r, && (53)\\
\lambda_{in\ell} + w_{inr} &\leq 1 + \alpha_{inr\ell}, && \forall i > 0, n, r, \ell, && (54)\\
\alpha_{inr\ell} &\leq \lambda_{in\ell}, && \forall i > 0, n, r, \ell, && (55)\\
\alpha_{inr\ell} &\leq w_{inr}, && \forall i > 0, n, r, \ell, && (56)\\
\sum_{\ell=1}^{L_{in}} \lambda_{in\ell} &= 1, && \forall i > 0, n && (57)\\
y_{in} &= 0, && \forall i \notin \mathcal{C}_n, \forall n, && (58)\\
y_{inr} &\leq y_{in}, && \forall i, n, r, && (59)\\
y_{inr} &\geq y_{i(n+1)r}, && \forall i, n, r, && (60)\\
c_i(1 - y_{inr}) &\leq \sum_{m=1}^{n-1} w_{imr} + (1 - y_{in})c_{\max}, && \forall i > 0, n, r, && (61)\\
\sum_{m=1}^{n-1} w_{imr} + (1 - y_{in})c_{\max} &\leq (c_i - 1)y_{inr} + K_n(1 - y_{inr}), && \forall i > 0, n > c_{\min}, r, && (62)\\
\sum_{n=1}^{N} w_{inr} &\leq c_i, && \forall i, r. && (63)
\end{aligned}$$

5 Conclusion

In this working paper, we presented a new mathematical model that integrates choice modeling with optimization in a linear way. In fact, with the help of utility maximization theory and simulation, we succeed to overcome the nonlinearity and non-convexity caused by choice probabilities. In addition, we can use general choice model assumption for individual customer behavior. When the number of alternatives, simulation draws and individuals grow the problem might take long time to be solved. However, as the individuals are independent from one another, we can use decomposition methods to solve the problem for large examples.

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