Modeling advanced disaggregate demand as MILP

Michel Bierlaire

Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
Ecole Polytechnique Fédérale de Lausanne

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Back to Belgium!

I want to open a bar

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But there is a strong competition...

- La Cour St-Jean
- Le Mad Murphy
- Le Lausanne Express
- La Guimbarde
- ...

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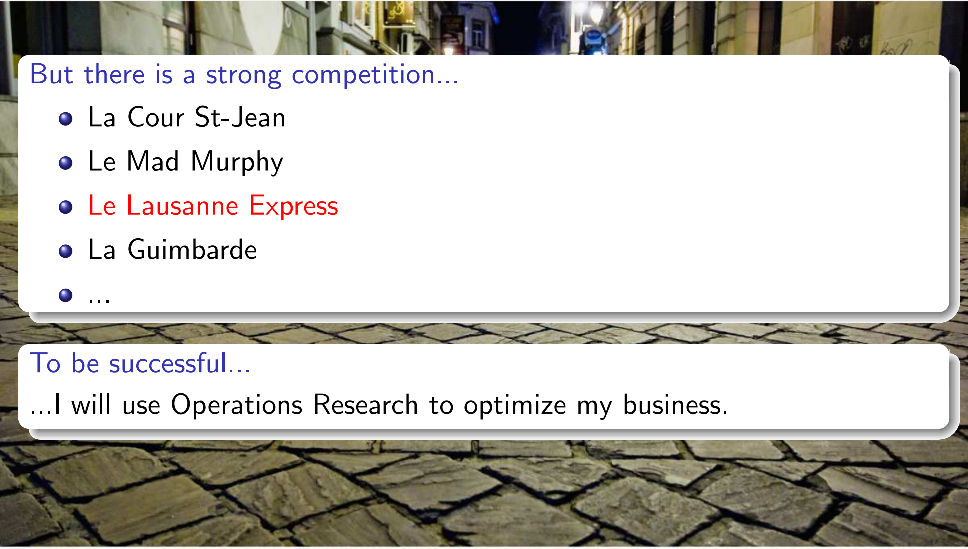
I want to open a bar



But there is a strong competition...

- La Cour St-Jean
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- ...

To be successful...

...I will use Operations Research to optimize my business.

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Demand analysis

Aggregate demand

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- 22000 students in the University of Liège

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- 45 bars in the "Carré"
- I should sell about 2000 liters of beer per week
- Jupiler 25cl at 4 €: total revenues = 32000 € per week.

Decisions

Assortment

Decisions

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Assortment

Decisions

Assortment and prices

4 €

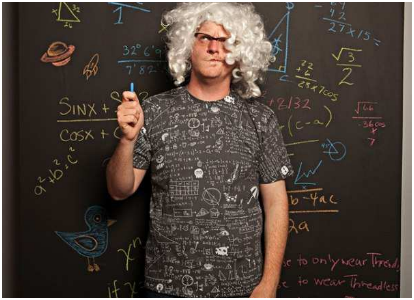
6 €

8 €

Customers are different

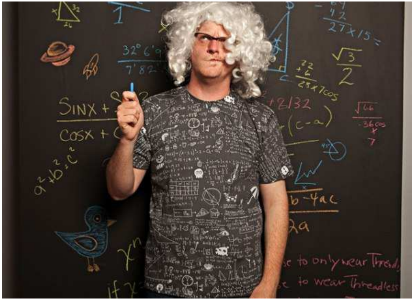
Customers are different

Mathematics



Customers are different

Mathematics



HEC

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Disaggregate demand analysis

Disaggregate demand analysis

Customers behavior

- Customers have different tastes
- Customers have different willingness to pay

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Customers choice

Outline

1. Choice models
2. MILP
3. Outlook

Variables: $x_{in} = (p_{in}, z_{in}, s_n)$

Attributes of alternative i: z_{in}

- Price (p_{in})
- Brand
- Color
- Percentage of alcohol
- etc.

Characteristics of customer n: s_n

- Income
- Age
- Sex
- Type of student
- etc.

Behavioral assumptions

Choice set: $\mathcal{C}_n$

$y_{in} = 1$ if $i \in \mathcal{C}_n$, 0 otherwise

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$$U_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$$

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$$U_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$$

Choice

$$P_n(i|x;\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn})$$

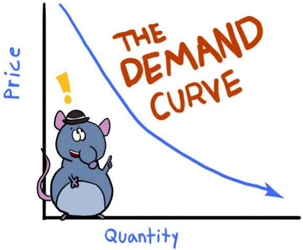
Choice models

Logit model

$$U_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$$
$$= V_{in} + \varepsilon_{in}$$
$$P_n(i|x;\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}$$

2000

Demand curve



Disaggregate model

$$P_n(i|p_{in}, z_{in}, s_n)$$

Total demand

$$D(i) = \sum_n P_n(i|p_{in}, z_{in}, s_n)$$

Difficulty

Non linear and non convex in p_{in} and z_{in}

Example

Choice set: Jupiler

- Lausanne Express $i = 0$
- La Cour St-Jean $i = 1$

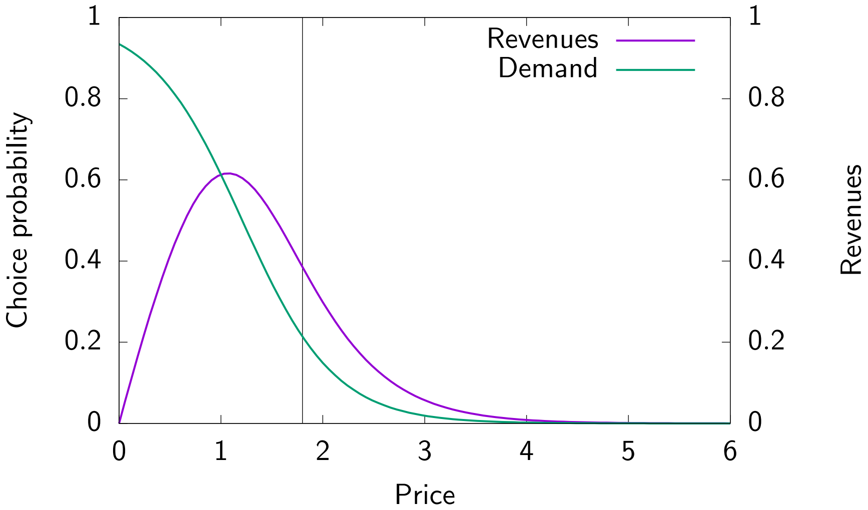
Utility functions

$$V_{0n} = -2.2 p_0 - 1.3$$
$$V_{1n} = -2.2 p_1$$

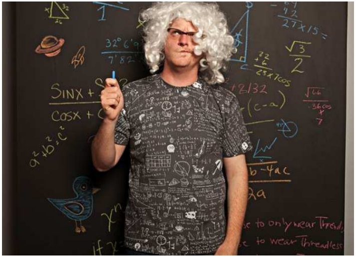
Prices

- Lausanne Express: [0 – 6 €]
- La Cour St-Jean: 1.8 €

Demand and revenues



Heterogeneous population



Two groups in the population

$$V_{0n} = -\beta_n p_0 + c_0$$

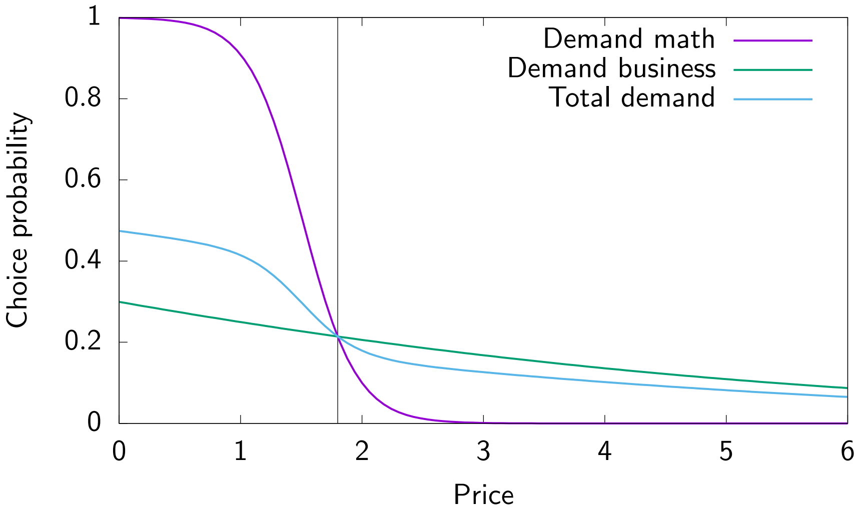
Mathematics: 25%

$\beta_1 = -4.5$,
$c_1 = -1.3$

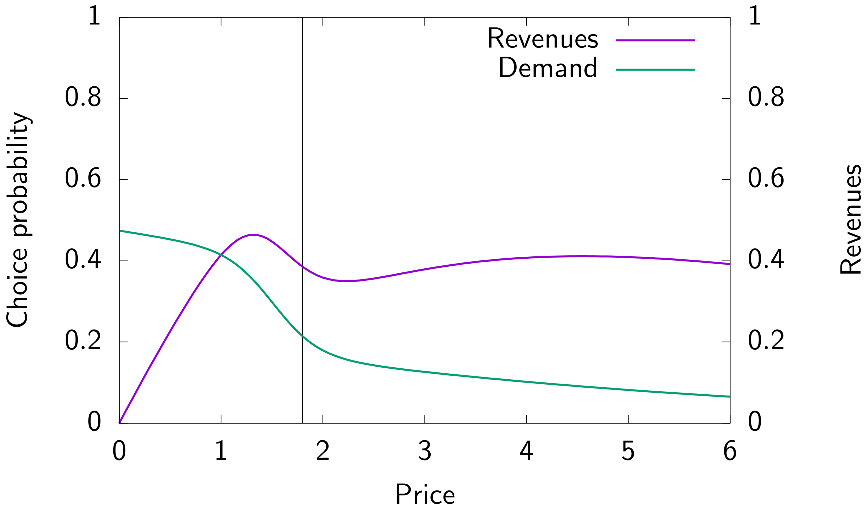
Business: 75%

$\beta_2 = -0.25$,
$c_2 = -1.3$

Demand per market segment



Demand and revenues



Optimization

Pricing

- Non linear optimization problem.
- Non convex objective function.
- Multimodal function.
- May feature many local optima.
- In practice, the groups are many, and interdependent.
- Optimizing each group separately is not feasible.

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- Non linear optimization problem.
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Assortment

What about assortment?

Heterogeneous population, two products

LE: Price Orval = 1.5 × price Jupiler

CSJ: Price Orval = 2 × price Jupiler

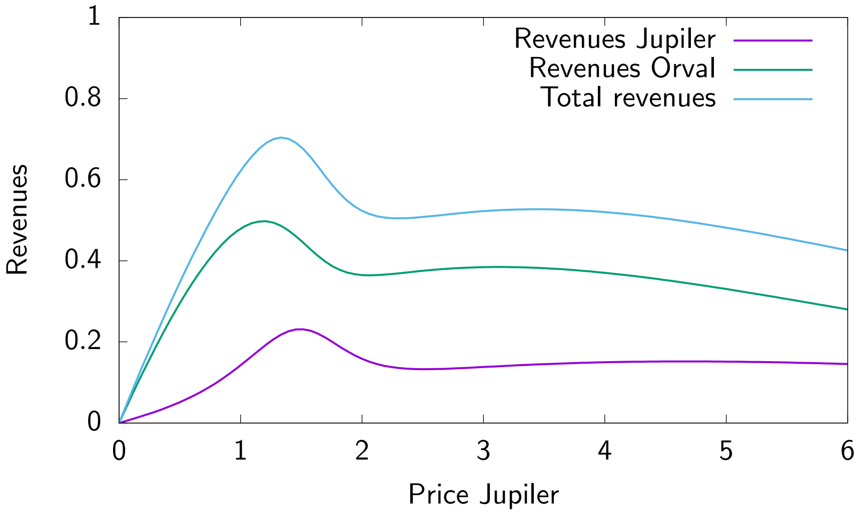
Utility functions: math

$$
\begin{aligned}
V_{\text{LE,Jupiler},m} &= -4.5 p_{\text{LE,Jupiler}} - 1.3 \\
V_{\text{LE,Orval},m} &= -4.5 p_{\text{LE,Orval}} - 1.3 + 3 \\
V_{\text{CSJ,Jupiler},m} &= -4.5 p_{\text{CSJ,Jupiler}} \\
V_{\text{CSJ,Orval},m} &= -4.5 p_{\text{CSJ,Orval}} + 3
\end{aligned}
$$

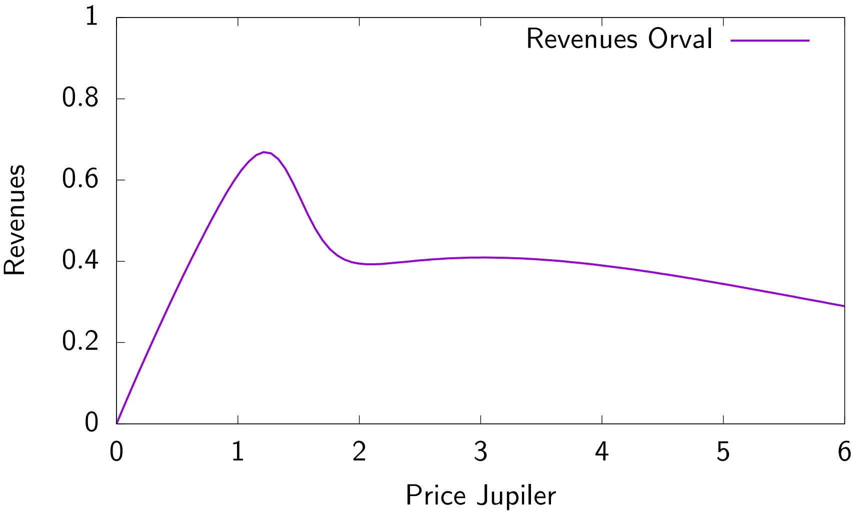
Utility functions: HEC

$$
\begin{aligned}
V_{\text{LE,Jupiler},b} &= -0.25 p_{\text{LE,Jupiler}} - 1.3 \\
V_{\text{LE,Orval},b} &= -0.25 p_{\text{LE,Orval}} - 1.3 + 1 \\
V_{\text{CSJ,Jupiler},b} &= -0.25 p_{\text{CSJ,Jupiler}} \\
V_{\text{CSJ,Orval},b} &= -0.25 p_{\text{CSJ,Orval}} + 1
\end{aligned}
$$

Total revenues



Orval only


Revenues Orval
Revenues
Price Jupiler
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
6

Optimization

Assortment and pricing

- Combinatorial problem
- For each potential assortment, solve a pricing problem
- Select the assortment corresponding to the highest revenues
- MINLP
- Non convex relaxation

Disaggregate demand models

Advantages

- Theoretical foundations
- Market segmentation
- Taste heterogeneity
- Many variables
- Estimated from data

Disadvantages

- Complex mathematical formulation
- Not suited for optimization
- Literature: heuristics

Research objectives

Observations

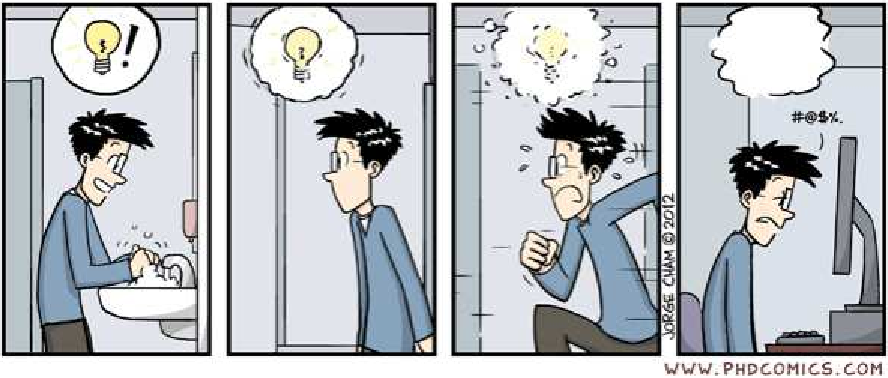
- Revenues is not the only indicator to optimize,
- e.g. customer satisfaction.
- Many OR applications need a demand representation

Goal

- Generic mathematical representation of choice models,
- designed to be included in MILP,
- linear in the decision variables.

Outline

The main idea



The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

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First principles

Each customer solves an optimization problem

The main idea

Linearization

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First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- For each scenario r, we can identify the largest utility.
- It corresponds to the chosen alternative.

Capacities

- Demand may exceed supply
- Each alternative i can be chosen by maximum c_i individuals.
- An exogenous priority list is available.
- Can be randomly generated, or according to some rules.
- The numbering of individuals is consistent with their priority.

Choice set

Variables

$y_i \in \{0,1\}$	operator decision
$y^d_{in} \in \{0,1\}$	customer decision (data)
$y_{in} \in \{0,1\}$	product of decisions
$y_{inr} \in \{0,1\}$	capacity restrictions

Constraints

$$y_{in} = y^d_{in} y_i \quad \forall i, n$$

$$y_{inr} \leq y_{in} \quad \forall i, n, r$$

Utility

Variables

U_{inr} utility

$z_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{nr} & \text{if } y_{inr} = 0 \end{cases}$ discounted utility

(ℓ_{nr} smallest lower bound)

Constraint: utility

$$U_{inr} = \overbrace{\beta_{in} p_{in} + q_d(x_d)}^{V_{in}} + \xi_{inr} \; \forall i, n, r$$

Utility (ctd)

Constraints: discounted utility

$$
\begin{aligned}
\ell_{nr} &\leq z_{inr} && \forall i, n, r \\
z_{inr} &\leq \ell_{nr} + M_{inr} y_{inr} && \forall i, n, r \\
U_{inr} - M_{inr}(1 - y_{inr}) &\leq z_{inr} && \forall i, n, r \\
z_{inr} &\leq U_{inr} && \forall i, n, r
\end{aligned}
$$

Choice

Variables

$$U_{nr} = \max_{i \in \mathcal{C}} z_{inr}$$

$$w_{inr} = \begin{cases} 1 & \text{if } z_{inr} = U_{nr} \\ 0 & \text{otherwise} \end{cases} \qquad \text{choice}$$

Constraints

$$z_{inr} \leq U_{nr} \qquad \forall i, n, r$$

$$U_{nr} \leq z_{inr} + M_{nr}(1 - w_{inr}) \qquad \forall i, n, r$$

$$\sum_i w_{inr} = 1 \qquad \forall n, r$$

$$w_{inr} \leq y_{inr} \qquad \forall i, n, r$$

Capacity

Capacity cannot be exceeded $\Rightarrow y_{inr} = 1$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1) y_{inr} + (n-1)(1 - y_{inr}) \; \forall i > 0, n > c_i, r$$

Capacity has been reached $\Rightarrow y_{inr} = 0$

$$c_i (y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr}, \; \forall i > 0, n, r$$

A case study

Challenge

- Take a choice model from the literature.
- It cannot be logit.
- It must involve heterogeneity.
- Show that it can be integrated in a relevant MILP.

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Parking choice

- [Ibeas et al., 2014]

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Parking choices [Ibeas et al., 2014]

Alternatives

- Paid on-street parking
- Paid underground parking
- Free street parking

Model

- $N = 50$ customers
- $\mathcal{C} = \{\text{PSP}, \text{PUP}, \text{FSP}\}$
- $\mathcal{C}_n = \mathcal{C} \quad \forall n$
- $p_{in} = p_i \quad \forall n$
- Capacity of 20 spots
- Mixture of logit models

General experiments

Uncapacitated vs Capacitated case

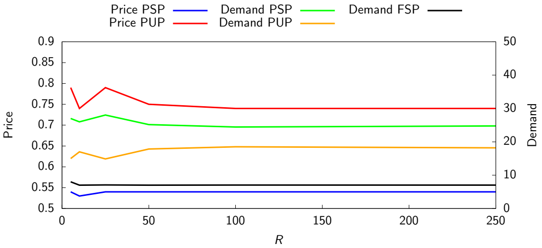
- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

Price differentiation by population segmentation

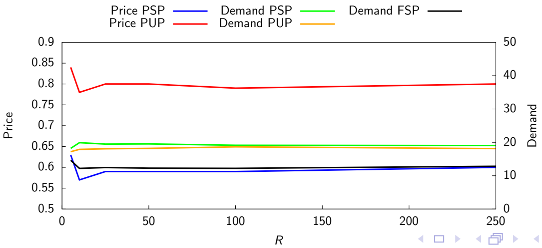
- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is forced to offer a reduced price

Uncapacitated vs Capacitated case

Uncapacitated



Capacitated



Computational time

	Uncapacitated case				Capacitated case			
R	Sol time	PSP	PUP	Rev	Sol time	PSP	PUP	Rev
5	2.58 s	0.54	0.79	26.43	12.0 s	0.63	0.84	25.91
10	3.98 s	0.53	0.74	26.36	54.5 s	0.57	0.78	25.31
25	29.2 s	0.54	0.79	26.90	13.8 min	0.59	0.80	25.96
50	4.08 min	0.54	0.75	26.97	50.2 min	0.59	0.80	26.10
100	20.7 min	0.54	0.74	26.90	6.60 h	0.59	0.79	26.03
250	2.51 h	0.54	0.74	26.85	1.74 days	0.60	0.80	25.93

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Outline

Linear formulation of choice models

Generic framework

- Not only logit: any choice model.
- Choice models from the literature can be used as such.
- Disaggregate: the choice of every individual for every draw is available.
- Many indicators can be derived.

Challenges

- Large scale
- Simulation noise
- Additional linearization may be necessary (e.g. revenue $= p \cdot w$)

Linear formulation of choice models

Opportunities: decomposition methods

- Lagrangian relaxation
- Decomposable by individual
- Decomposable by draw

Future work

- Game theory
- Parameter estimation (discrete maximum likelihood)
- Link with machine learning (SVM, random forests, etc.)

Thank you!

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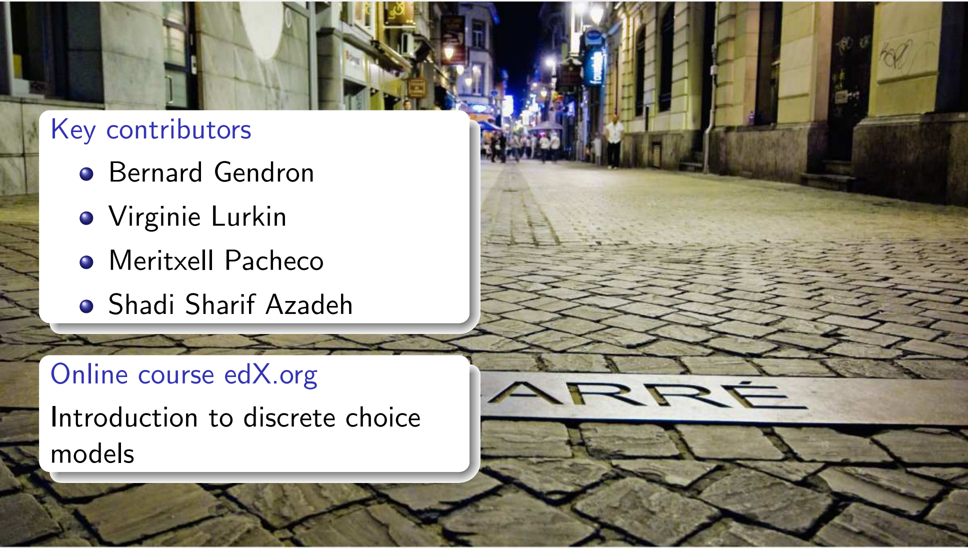

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Introduction to discrete choice models

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Merci

Dank u wel

Danke schön

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Bibliography

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Modelling parking choices considering user heterogeneity.
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