Discrete choice models: a glance at the state-of-the-art and the state-of-practice

Michel Bierlaire

michel.bierlaire@epfl.ch

Transport and Mobility Laboratory

Outline

- Introduction
- State-of-practice: typical applications in transportation
- State-of-the-art: current challenges in research

Choice

"It is our choices that show what we truly are, far more than our abilities" Albus Dumbledore, Prof. at Hogwarts

"Liberty, taking the word in its concrete sense, consists in the ability to choose." Simone Weil, French philosopher

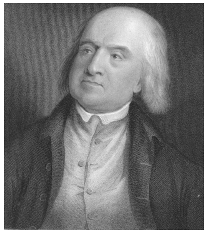
Field :	Type of behavior:
▶ Marketing	▶ Choice of a brand
▶ Transportation	▶ Choice of a transportation mode
▶ Politics	▶ Choice of a president
▶ Management	▶ Choice of a management policy
▶ Finance	▶ Choice of investments

Choice

Choice

Introduction

Homo Economicus (source: D. McFadden)

Jeremy Bentham (1789)	My notion of man is that . . . he aims at happiness . . . in every thing he does.	
Frank Taussig (1912)	The fact that [the consumer] is willing to give up something in order to procure an article proves once for all that for him it has utility	
Herb Simon (1956)	The rational man of economics is a maximizer, who will settle for nothing less than the best.	

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Introduction

Daniel
L.
McFadden

1937–

- UC Berkeley 1963, MIT 1977, UC Berkeley 1991
- Laureate of *The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2000*
- Owns a farm and vineyard in Napa Valley
- "Farm work clears the mind, and the vineyard is a great place to prove theorems"

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Discrete choice models

- Finite and discrete set of alternatives
  - Choice of brand: Nestlé, Leonidas, Lindt, Suchard, Toblerone, etc.
  - Choice of transportation mode: car, bus, etc.
  - Choice of university: ETHZ, EPFL, etc.
- Individual n associates a utility to alternative i
- Represented by a random function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + \varepsilon_{in}$$

For instance

$$U_{\text{Leonidas,mb}} = \beta_1 \text{sugar} + \beta_2 \text{bitterness} + \beta_3 \text{Belgian} + \ldots + \varepsilon_{\text{Leonidas,mb}}$$

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Transportation mode choice

Simple example

- Two modes: car and public transportation
- Two variables: cost and time

Mode choice

- Utility:

$$\begin{aligned} U_1 &= -\beta t_1 - \gamma c_1 \\ U_2 &= -\beta t_2 - \gamma c_2 \end{aligned}$$

  where β, $\gamma > 0$, t_i travel time, c_i cost
- Utility maximization
- 1 is chosen if $U_1 > U_2$
- 2 is chosen if $U_1 < U_2$

Mode choice

$$\begin{aligned} U_1 &= -\beta t_1 - \gamma c_1 \\ U_2 &= -\beta t_2 - \gamma c_2 \end{aligned}$$

with β, $\gamma > 0$

$$U_1 \geq U_2 \text{ if } -\beta t_1 - \gamma c_1 \geq -\beta t_2 - \gamma c_2$$

that is

$$-\frac{\beta}{\gamma} t_1 - c_1 \geq -\frac{\beta}{\gamma} t_2 - c_2$$

or

$$c_1 - c_2 \leq -\frac{\beta}{\gamma}(t_1 - t_2)$$

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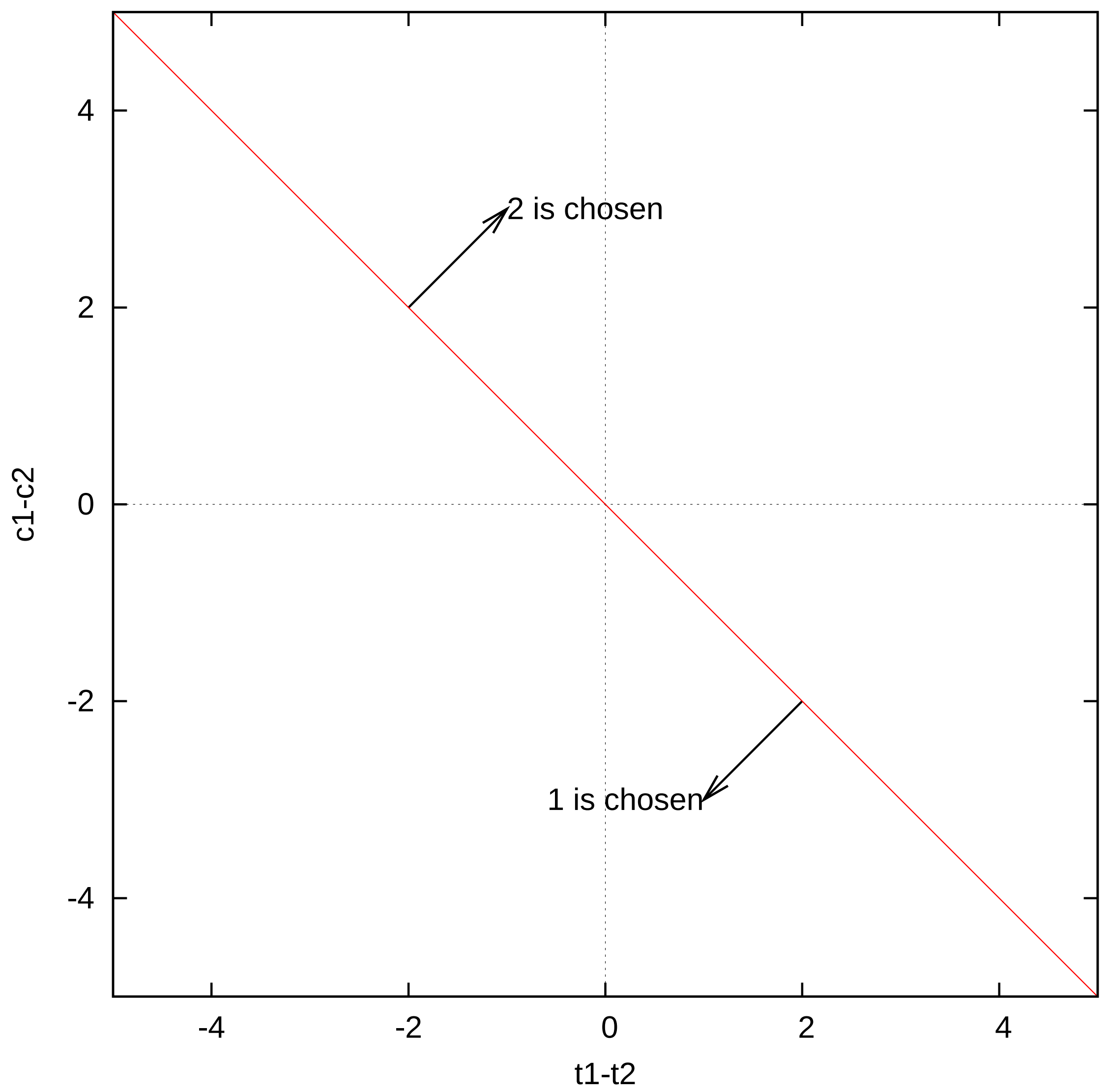
Mode choice

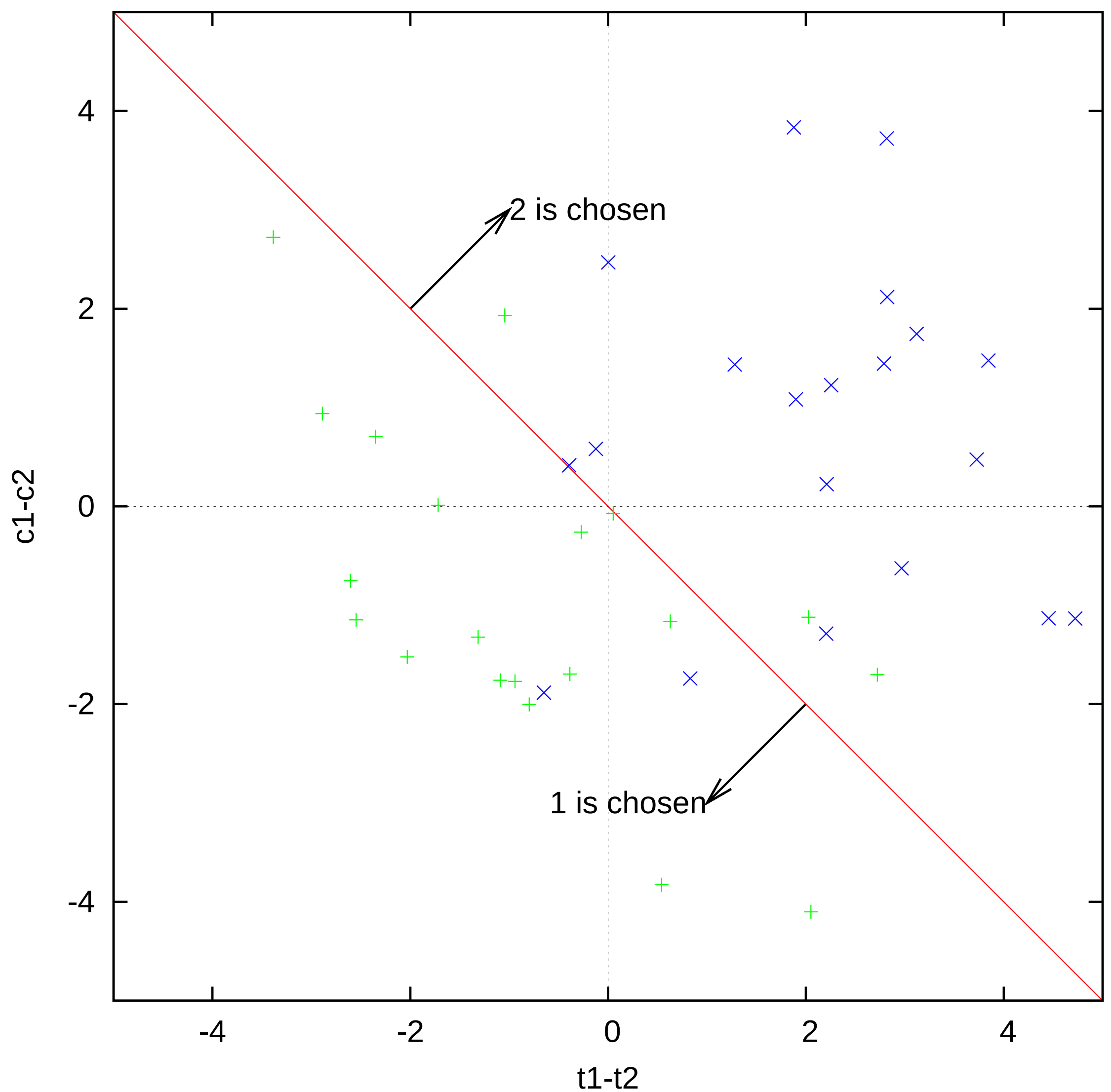
Obvious cases:

- $c_1 \geq c_2$ and $t_1 \geq t_2$: 2 dominates 1.
- $c_2 \geq c_1$ and $t_2 \geq t_1$: 1 dominates 2.
- Trade-offs in over quadrants

Mode choice



Mode choice



Mode choice

- Some observations are inconsistent with the model
- Assumptions are too strict and not realist
- Solution: include an error term.

$$
\begin{aligned}
U_1 &= -\beta t_1 - \gamma c_1 + \varepsilon_1 \\
U_2 &= -\beta t_2 - \gamma c_2 + \varepsilon_2
\end{aligned}
$$

Dependent variables

Unknown parameters

Independent or explanatory variables

Random term

Choice model

Linear model:

$$U_i = -\beta t_i - \gamma c_i + \varepsilon_i$$

But... U_i is not observable, is latent.

Linear regression does not apply here

Choice model

Decision-maker: n

- choice set: $\mathcal{C}_n$ (e.g. bus and car)
- explanatory (independent) variables:
  - characteristics of n and the choice context: $S_n =$ (age, income, sex, monthly pass, driving license, weather, trip purpose,. . .)
  - attributes of the alternatives: for each $i \in \mathcal{C}_n$: $z_{in} =$ (travel time, costs, frequency, comfort, . . .)
- We put everything together $x_{in} = (z_{in}, S_n)$

  $x_{in} =$ (travel time, costs, frequency, comfort, . . . , age, income, sex, monthly pass, driving license, weather, trip purpose,. . .)

Choice model

- Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \beta_1 x_{in1} + \beta_2 x_{in2} + \ldots + \varepsilon_{in}$$

- Choice model :

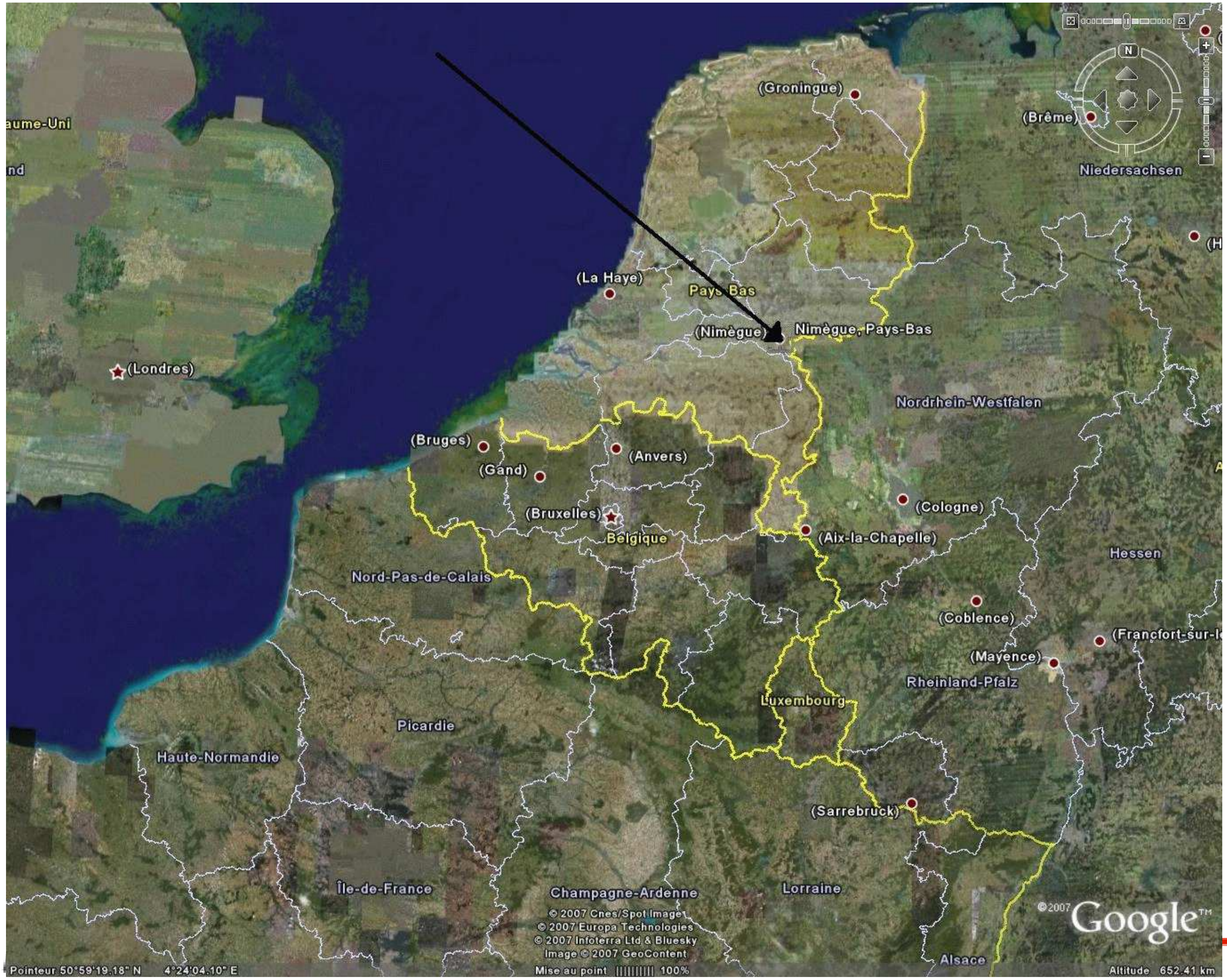
$$P_n(i|\mathcal{C}_n) = \Pr(U_{in} \geq U_{jn} \; \forall j \in \mathcal{C}_n)$$

- Most popular model: LOGIT

$$P_n(i|\mathcal{C}_n) = \frac{e^{V_{in}}}{\sum_{j \in \mathcal{C}_n} e^{V_{jn}}}$$

Assumes that ε_{in} are independent and identically distributed

Example : case study in Nimègue, The Netherlands,



Mode choice in Nimègue

- Trips from Nimègue to the metropolitan area (Amsterdam, Rotterdam, La Haye)
- About 2 hours by train or car
- Project for the Dutch Railway
- 235 individual surveyed, 228 usable observations
  - made a trip during the last 3 months to one of the 3 cities
  - do not possess a yearly pass (to be able to measure the impact of cost)
  - have access to a car
  - have access to a train (no multiple destination, no heavy luggage, etc.)

Mode choice in Nimègue

Available data :

- mode actually used (train or car)
- trip purpose
- cost by car, cost by train
- in-vehicle travel time
- access and egress time
- number of transfers (train)
- socio-economic characteristics (e.g., age, sex, etc.)

Mode choice in Nimègue

Simple model, estimated from this data (cost in euros and time in hours) :

$$\begin{aligned} V_{\text{car}} &= -0.798 \quad -0.110 \cdot \text{cost}_{\text{car}} \quad -1.33 \cdot \text{temps}_{\text{car}} \\ V_{\text{train}} &= \qquad\qquad\;\; -0.110 \cdot \text{cost}_{\text{train}} \quad -1.33 \cdot \text{temps}_{\text{train}} \end{aligned}$$

Example :

- Car : 2h, 7 euros
- Train : from 0 to 15 euros, from 1h to 4h

$$P(\text{train}) = \frac{e^{V_{\text{train}}}}{e^{V_{\text{car}}} + e^{V_{\text{train}}}}$$

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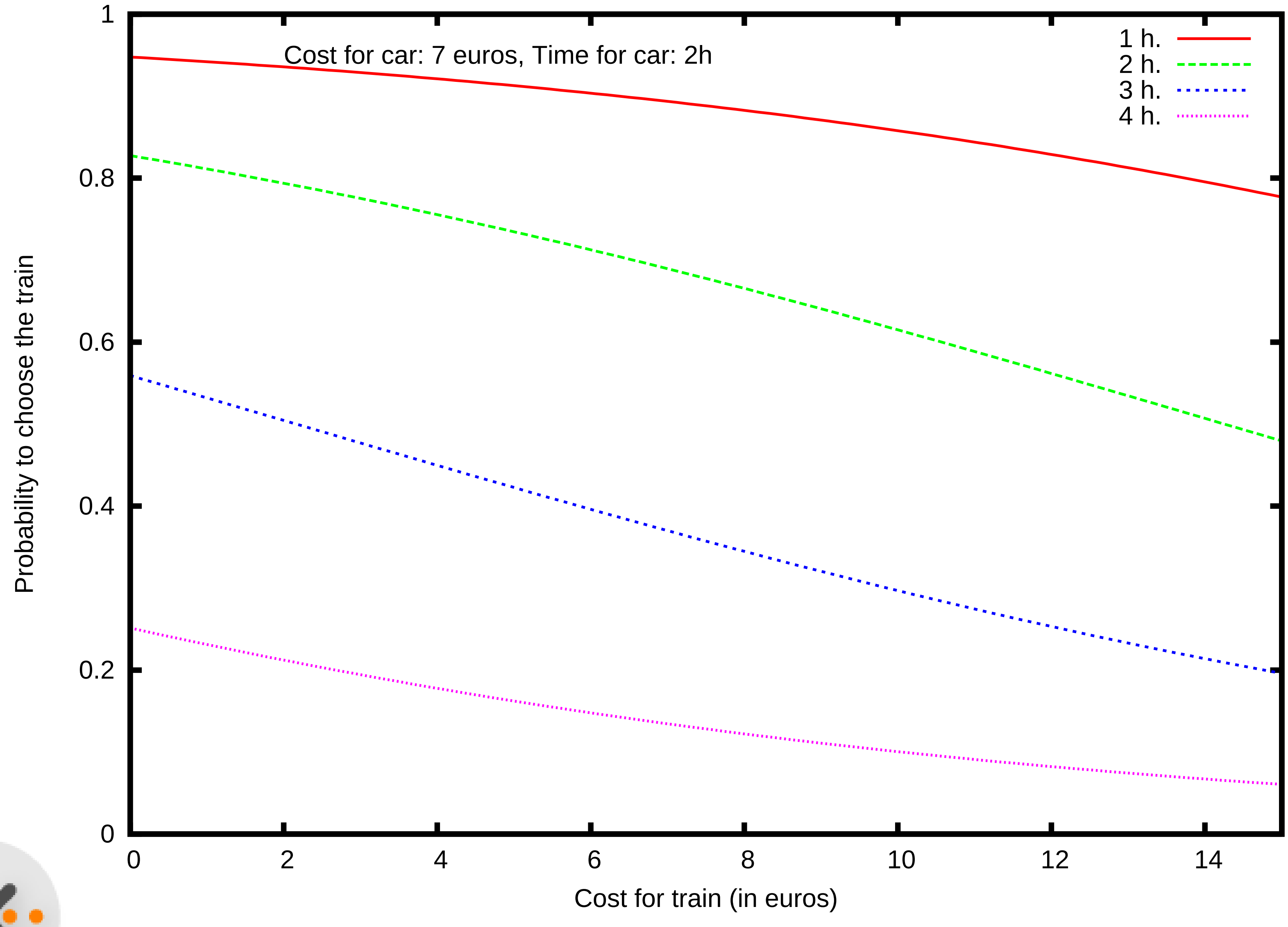
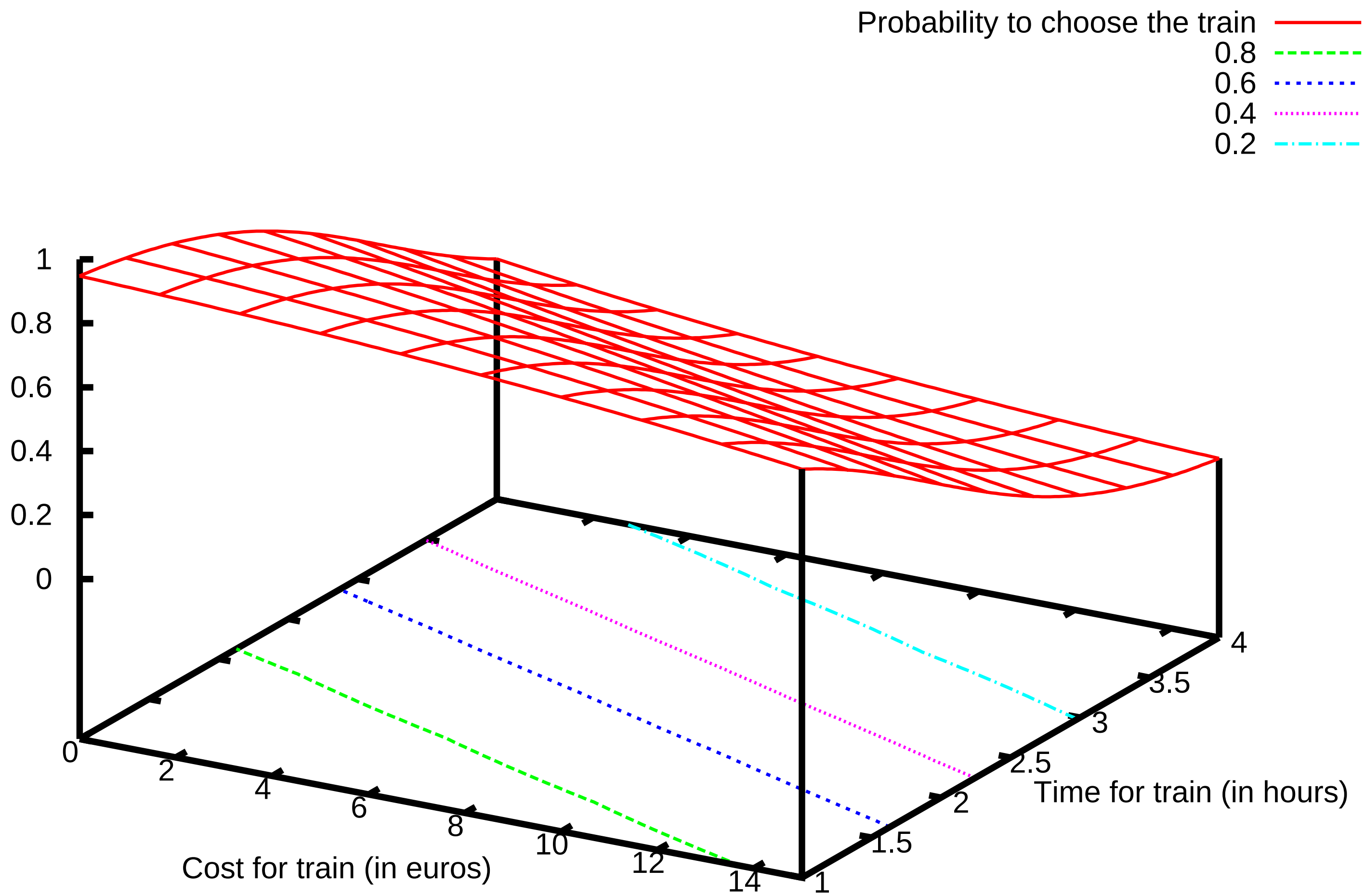
Mode choice in Nimègue



Mode choice in Nimègue

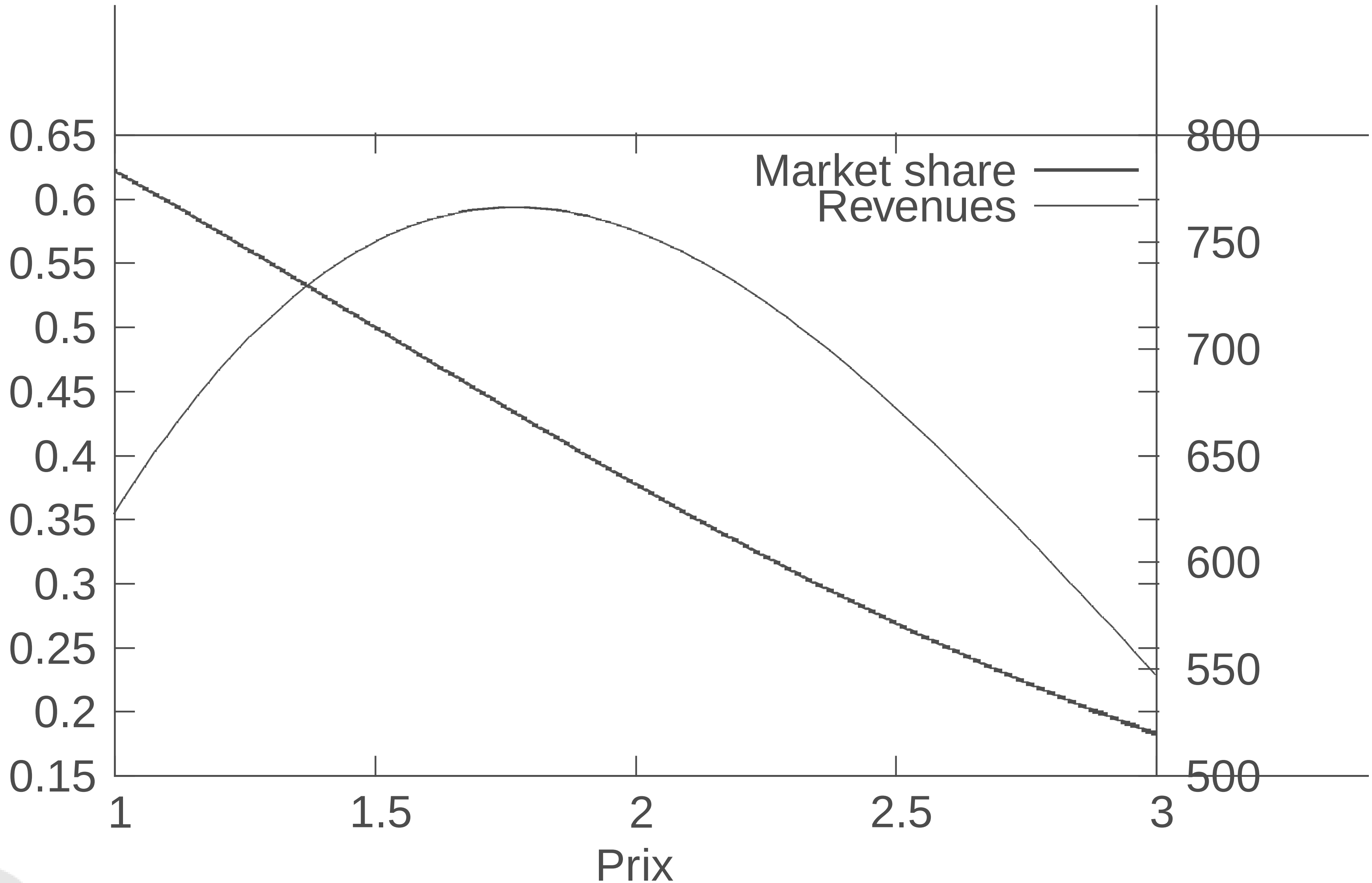


Application: price optimization

Scenario :

- New train connection between A and B
- Question: what price to apply?
- Choice model: $P(\text{train}|p)$
  - probability to take the train
  - given (namely) the price p of the ticket
- Let N be the number of travelers between A and B
- Nbr of travelers using the train at price p: $N \cdot P(\text{train}|p)$
- Revenues for the company: $N \cdot P(\text{train}|p) \cdot p$.

Application: price optimization



Value of time

Value of time

- What is the monetary value of travel time?
- Important for costs-benefits analysis
- Costs : CHF
- Benefits : travel time savings

- Definition: price that a traveler is ready to pay to shorten the travel time. Concept: willingness-to-pay.
- Motivation: total time is limited, so travel time saved may be used for other activities.

Value of time

Choice model

$$\begin{aligned} U_1 &= -\beta t_1 - \gamma c_1 \\ U_2 &= -\beta t_2 - \gamma c_2 \end{aligned}$$

with β, $\gamma > 0$

$$U_1 \geq U_2 \text{ if } \underbrace{c_1 - c_2}_{\text{CHF}} \leq - \underbrace{\frac{\beta}{\gamma}}_{\frac{\text{CHF}}{\text{hour}}} \underbrace{(t_1 - t_2)}_{\text{hours}}$$

Value of time

- If utility is linear-in-parameters
- the value of time is the ratio between
  - the coefficient of the "time" variable, and
  - the coefficient of the "cost" variable.
- Warning: utility is not always linear-in-parameters
- Value of time varies with
  - trip purpose
  - transportation model
  - trip length
  - income

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Value of time

Nimègue :

$$\begin{aligned} V_{\text{car}} &= -0.798 \quad -0.110 \cdot \text{cost}_{\text{car}} \quad -1.33 \cdot \text{temps}_{\text{car}} \\ V_{\text{train}} &= \quad -0.110 \cdot \text{cost}_{\text{train}} \quad -1.33 \cdot \text{temps}_{\text{train}} \end{aligned}$$

Value of time = -1.33 / -0.110 $\approx$ 12 euros / h $\approx$ 0.20 euros / min

	Case 1	Case 2
Time	2 h	1.5 h
Cost	7 €	13 €
Utility for train	-3.43	-3.43

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Swiss value of time study

Axhausen, K., Hess, S., Koenig, A., Abay, G., Bates, J., and Bierlaire, M. (2008). Income and distance elasticities of values of travel time savings: new Swiss results, *Transport Policy* **15**(3):173-185.

Data collection:

- Recruiting source: survey “Kontinuierliche Erhebung zum Personenverkehr” (KEP) by CFF
- Stated preferences survey
- Questions derived from a real trip
- Three parts:
  - SP mode choice (car / bus or rail)
  - SP route choice (current mode or alternative mode)
  - Socio-economic characteristics, attributes of the real trip

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Swiss value of time study

Mode choice car – rail (main study version)

Travel costs:	18 Fr.	Travel costs:	23 Fr.
Total travel time:	40 minutes	Travel time:	30 minutes
... congested:	10 minutes	Headway:	30 minutes
... uncongested:	30 minutes	No. of changes:	0 times

☐ ← **Your choice** → ☐

Route choice rail (main study version)

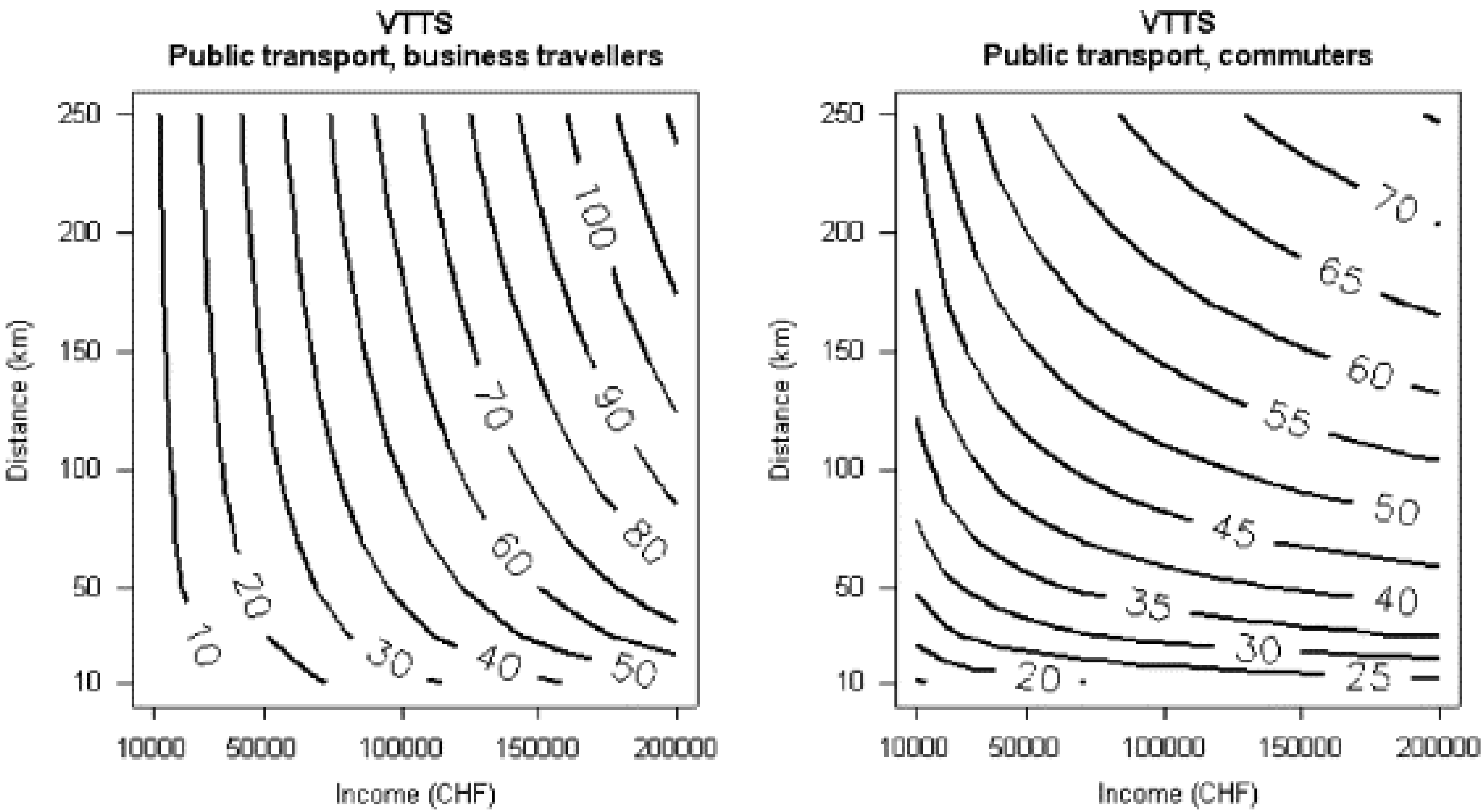
Travel costs:	20 Fr.	Travel costs:	23 Fr.
Travel time:	40 minutes	Travel time:	30 minutes
Headway:	15 minutes	Headway:	30 minutes
No. of changes:	1 times	No. of changes:	0 times

☐ ← **Your choice** → ☐

Swiss value of time study

	Business	Commute	Leisure	Shopping
Time TC (CHF/h)	49.57	27.81	21.84	17.73
Time car (CHF/h)	50.23	30.64	29.20	24.32
Headway (CHF/h)	14.88	11.18	13.38	8.48
CHF/transfer	7.85	4.89	7.32	3.52

Swiss value of time study



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Summary

- Multivariate nonlinear regression
- Try to capture causal effect rather than correlation
- Motivation: forecasting (stability over time)
- Modeling choice at the disaggregate level
- Allow to capture heterogeneity of the population
- Great deal of flexibility
- Success stories

State-of-the-art

- MEV models
- Unobserved heterogeneity: latent classes, random parameters
- Sampling biases
- Statistical tests
- Attitudes and perceptions: latent variables

MEV models

- Issue:
  - logit assumes independence across alternatives and individuals
  - when not verified may lead to wrong predictions
  - Example: red bus / blue bus.
- Solution: MEV models capture correlation, and are tractable.
- Research:
  - Reference paper: McFadden (1978)
  - Our work: Daly & Bierlaire (2006), Bierlaire (2006), Abbe, Bierlaire & Toledo (2007)

Heterogeneity

- Issue:
  - individuals are not alike
  - population must be segmented
  - segmentation can not always be done deterministically
- Solution: taste parameters can be distributed
- Research:
  - Reference paper: McFadden & Train (2000)
  - Our work: Fosgerau & Bierlaire (2007), Frejinger & Bierlaire (2007), Hess, Bierlaire & Polak (2007)

Sampling biases

- Issue:
  - sample may not be representative of the population
  - estimation biases, especially with choice-based sampling strategies
- Solution: use an appropriate estimator
- Research:
  - Reference papers: Manski & McFadden (1981), Cosslett (1981)
  - Our research: Bierlaire, Bolduc & McFadden (2008)

Statistical tests

- Issue:
  - verify modeling hypotheses
  - distinguish between real effects and noise
- Solution: perform formal statistical tests
- Research:
  - Reference paper: Ben-Akiva & Lerman (1985)
  - Our research: Fosgerau & Bierlaire (2007)

Attitudes and perceptions

- Issue:
  - integrate attitudes and perceptions to explain the choice
  - latent constructs
- Solution: latent class, latent variables models
  - Research:
    - Reference paper: Walker (2001)
    - Our research: Ben-Akiva, McFadden, Train, et al. (2002)

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Summary

- Active field of research
- Complex but tractable models
- Research motivated by concrete applications

Questions?