Demand-based scheduling

Michel Bierlaire Stefan Binder Yousef Maknoon Tomáš Robenek

Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
Ecole Polytechnique Fédérale de Lausanne

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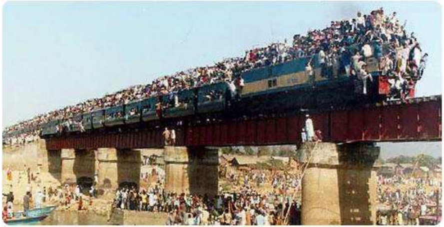
Outline

1. Demand and supply
2. Measuring satisfaction
3. Ideal timetable
4. Disposition timetable
5. Conclusion

Demand models

- Supply = infrastructure
- Demand = behavior, choices
- Congestion = mismatch

Demand models



- Usually in OR:
- optimization of the supply
- for a given (fixed) demand

Aggregate demand

- Homogeneous population
- Identical behavior
- Price (P) and quantity (Q)
- Demand functions: $P = f(Q)$
- Inverse demand: $Q = f^{-1}(P)$

Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.

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Demand-supply interactions

Operations Research

- Given the demand...
- configure the system

Johnson City Enterprise.

Published Every Saturday,

$1. per year—Advance Payment.

SATURDAY, APRIL 7, 1883.

TIME TABLE

E. T., V. & G. R. R.

PASSENGER,	ARRIVES,
No. 1, West,	6:37, a. m.
No. 2, East,	9:45, p. m.
No. 3, West,	11:51, p.m.
No. 4, East,	3:56, a. m.
LOCAL FREIGHT,	ARRIVES,
No. 5,	7:20, a. m.
No. 8,	6:20, p. m.

JNO. W. EAKIN, Agent.

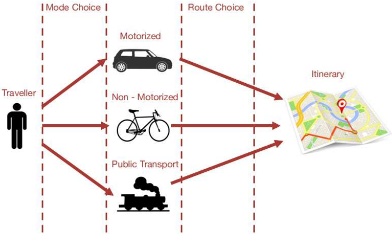
E. T. & W. N. C. R. R.

Passenger, leaves,	7, a. m.
" arrives,	6, p. m.

J. C. HARDIN, Agent.

Behavioral models

- Given the configuration of the system...
- predict the demand



Demand-supply interactions

Multi-objective optimization

Minimize costs

Maximize satisfaction

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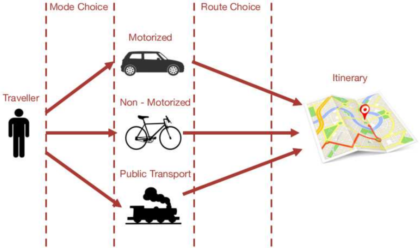
Measuring satisfaction



Behavioral models

- Demand = sequence of choices
- Choosing means trade-offs
- In practice: derive trade-offs from choice models

Choice models

Theoretical foundations

- Random utility theory
- Choice set: $\mathcal{C}_n$
- Logit model:

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}}}{\sum_{j \in \mathcal{C}_n} e^{V_{jn}}}$$

2000

Decision rules

Neoclassical economic theory

Preference-indifference operator $\succsim$

1. reflexivity

$$a \succsim a \quad \forall a \in \mathcal{C}_n$$

2. transitivity

$$a \succsim b \text{ and } b \succsim c \Rightarrow a \succsim c \quad \forall a, b, c \in \mathcal{C}_n$$

3. comparability

$$a \succsim b \text{ or } b \succsim a \quad \forall a, b \in \mathcal{C}_n$$

Decision rules

Utility

$$\exists\ U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a) \text{ such that}$$

$$a \gtrsim b \Leftrightarrow U_n(a) \geq U_n(b) \quad \forall a, b \in \mathcal{C}_n$$

Remarks

- Utility is a latent concept
- It cannot be directly observed

Example

Two transportation modes

$$\begin{aligned} U_1 &= -\beta t_1 - \gamma c_1 \\ U_2 &= -\beta t_2 - \gamma c_2 \end{aligned}$$

with β, $\gamma > 0$

$$U_1 \geq U_2 \text{ iff } -\beta t_1 - \gamma c_1 \geq -\beta t_2 - \gamma c_2$$

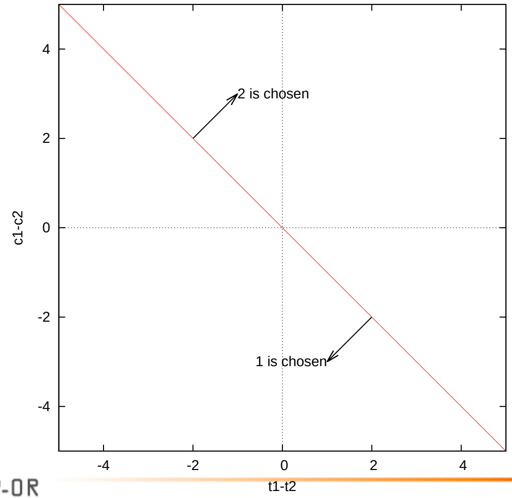
that is

$$-\frac{\beta}{\gamma} t_1 - c_1 \geq -\frac{\beta}{\gamma} t_2 - c_2$$

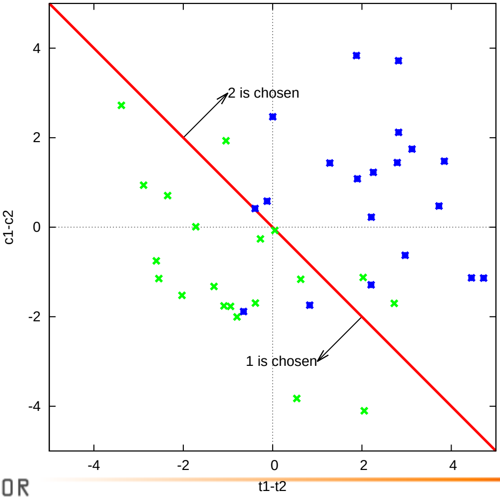
or

$$c_1 - c_2 \leq -\frac{\beta}{\gamma}(t_1 - t_2)$$

Example



Example



Assumptions

Decision-maker

- perfect discriminating capability
- full rationality
- permanent consistency

Analyst

- knowledge of all attributes
- perfect knowledge of $\gtrsim$ (or $U_n(\cdot)$)
- no measurement error

Must deal with uncertainty

- Random utility models
- For each individual n and alternative i

$$U_{in} = V_{in} + \varepsilon_{in}$$

and

$$P(i|\mathcal{C}_n) = P[U_{in} = \max_{j \in \mathcal{C}_n} U_{jn}] = P(U_{in} \geq U_{jn} \; \forall j \in \mathcal{C}_n)$$

Logit model

Utility

$$U_{in} = V_{in} + \varepsilon_{in}$$

Choice probability

$$P_n(i|\mathcal{C}_n) = \frac{e^{V_{in}}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}}}.$$

- Decision-maker n
- Alternative $i \in \mathcal{C}_n$

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Variables: $x_{in} = (z_{in}, s_n)$

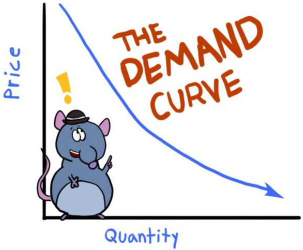
Attributes of alternative i: z_{in}

- Cost / price
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Characteristics of decision-maker n: s_n

- Income
- Age
- Sex
- Trip purpose
- Car ownership
- Education
- Profession
- etc.

Demand curve



Disaggregate model

$$P_n(i|c_{in}, z_{in}, s_n)$$

Total demand

$$D(i) = \sum_n P_n(i|c_{in}, z_{in}, s_n)$$

Difficulty

Inverse demand not analytically available

Willingness to pay

Attributes of alternative i: z_{in}

- Cost / price
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Willingness to pay for alternative i

- Value of travel time
- Value of waiting time
- Value of comfort
- Value of transfers
- Value of not being on time
- etc.

Willingness to pay

Utility

$$U_{in} = \beta_c c_{in} + \beta_t t_{in} + \cdots$$

Value of time

$$\mathrm{VOT}_{in} = \frac{\partial U_{in}/\partial t_{in}}{\partial U_{in}/\partial c_{in}} = \frac{\beta_t}{\beta_c}$$

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Equivalence

Utility

$$U_{in} = \beta_c c_{in} + \beta_t t_{in} + \beta_w w_{in} + \beta_{\text{cft}} \text{cft}_{in} + \beta_T T_{in} + \beta_e e_{in} + \beta_\ell \ell_{in} + \cdots$$

Willingness to pay: cost per unit

- Travel time: β_t / β_c
- Waiting time: β_w / β_c
- Comfort: $\beta_{\text{cft}} / \beta_c$
- Transfers: β_T / β_c
- Being early: β_e / β_c
- Being late: β_ℓ / β_c

Travel time equivalent: hours per unit

- Cost: β_c / β_t
- Waiting time: β_w / β_t
- Comfort: $\beta_{\text{cft}} / \beta_t$
- Transfers: β_T / β_t
- Being early: β_e / β_t
- Being late: β_ℓ / β_t

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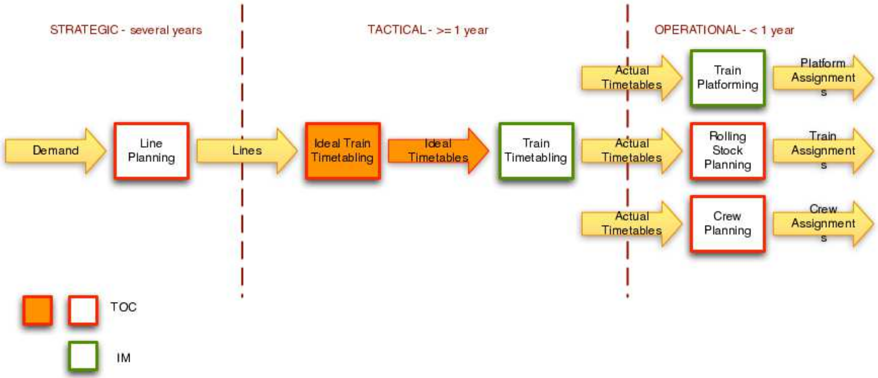
Planning of railway operations



Timetables

Objectives

- Minimize cost
- Maximize satisfaction

Constraints

- Cyclicity
- or not...

Modeling elements

Supply

- Line ℓ: sequence of stations served by the same train
- Train $v \in V_\ell$: service of a line at a given departure time

Demand

- Origin / destination i
- Ideal arrival time t
- Path $p \in P_i$: sequence of portions of lines to reach d from o
  - Access/egress time for path p (OD i)
  - Travel time for path p
  - Waiting time for path p

Model

Decision variables

- x_i^{tp}: 1 – if passenger with ideal time t between OD pair i chooses path p; 0 – otherwise
- y_i^{tplv}: 1 – if a passenger with ideal time t between OD pair i on the path p takes the train v on the line ℓ; 0 – otherwise
- d_v^ℓ: the departure time of a train v on the line ℓ (from its first station)
- u_v^ℓ: number of train units of a train v on the line ℓ
- α_v^ℓ: 1 – if a train v on the line ℓ is being operated; 0 – otherwise

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Model

Calculation variables

- C_i^t: total cost of a passenger with ideal time t between OD pair i
- w_i^t: total waiting time of a passenger with ideal time t between OD pair i
- s_i^t: value of the scheduled delay of a passenger with ideal time t between OD pair i
- z_v^l: dummy variable modeling the cyclicity corresponding to a train v on the line ℓ
- o_{vg}^{ℓ}: occupation of train v of line ℓ on segment g

Model

Problem constraints

- passenger cost $\leq \varepsilon$
- everyone uses at most one path
- link between path and trains: everyone boards one train of each line in the path
- cyclicity
- everyone uses only trains that are actually running
- train capacity
- maximum number of train units

Model

Calculation constraints

- Scheduled delay
- Waiting time
- Overall cost

Models

Current model

Departure times of trains are fixed, current values are used (cyclic).

Cyclic model

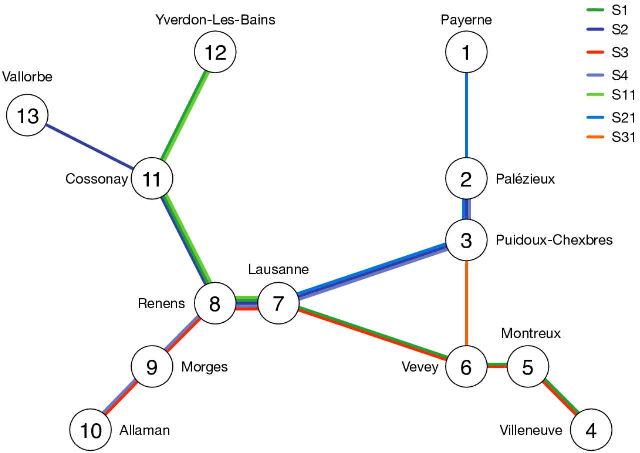
Departure times are optimized, cyclicity is enforced.

Non-cyclic model

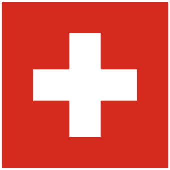
Departure times are optimized, cyclicity is not enforced.

Case Study – Switzerland

S-Train Network Canton Vaud, Switzerland



Case study: Switzerland



Context

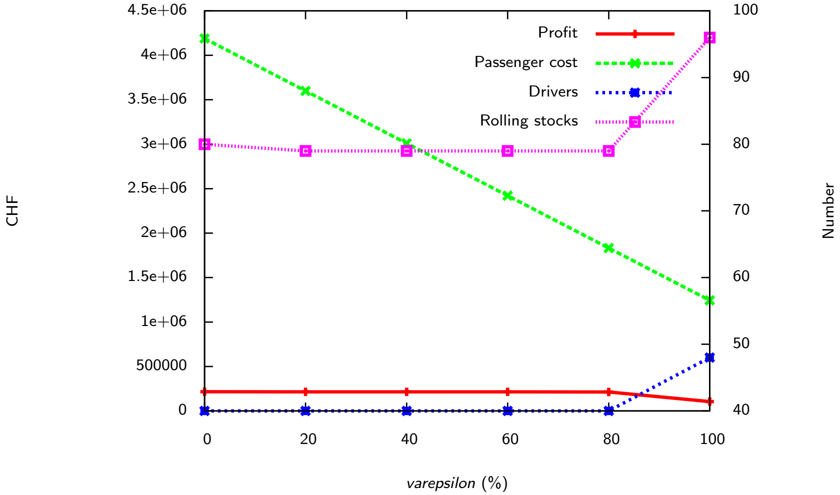
- SBB 2014 (5 a.m. to 9 a.m.)
- OD Matrix based on observation and SBB annual report
- 13 Stations
- 156 ODs
- 14 (unidirectional) lines
- 49 trains
- Min. transfer – 4 mins

Case study: Switzerland

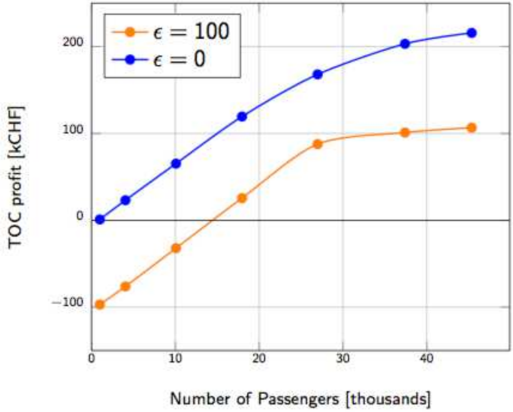
Willingness to pay from the literature

- Value of travel time: 27.81 CHF / hour
- Value of waiting time: 69.5 CHF /hour
- Value of comfort: —
- Value of transfers: 4.6 CHF / hour (10 min. travel time)
- Value of being late: 27.81 CHF / hour
- Value of being early: 13.9 CHF / hour
- etc.

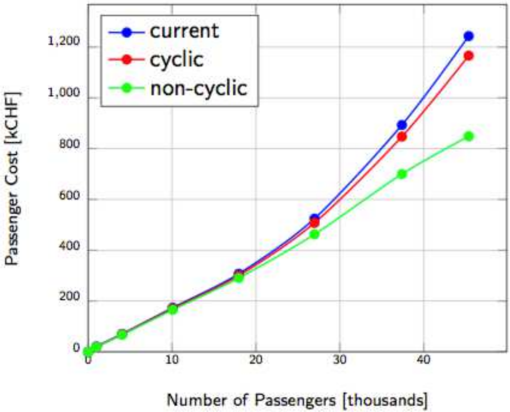
Current model



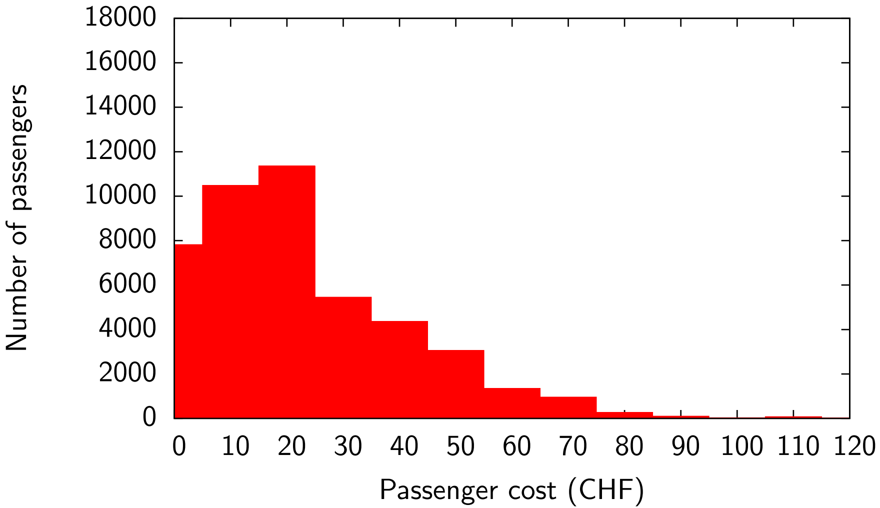
Impact of congestion



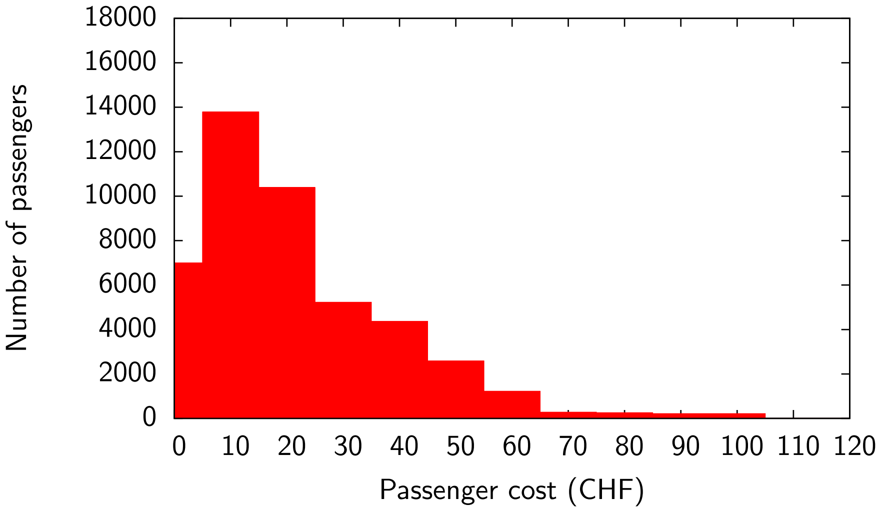
Impact of congestion



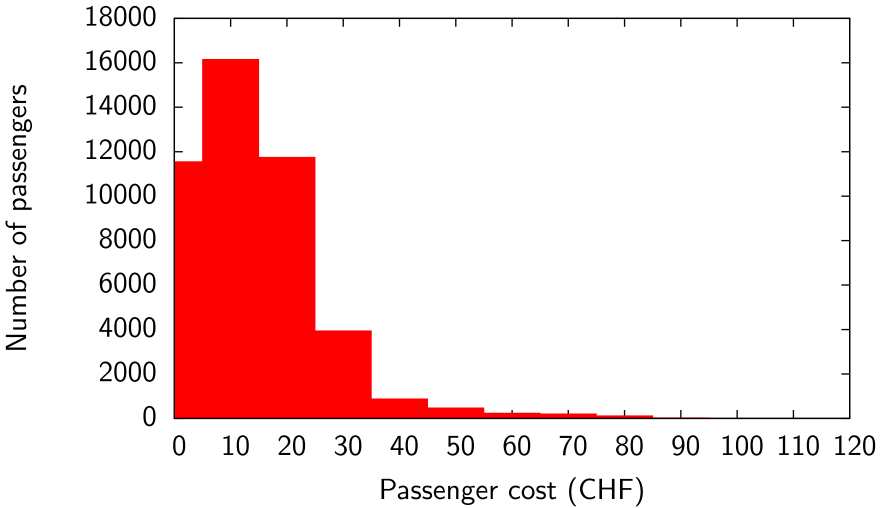
Passenger cost: highest demand, current model



Passenger cost: highest demand, cyclic model



Passenger cost: highest demand, non-cyclic model



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Motivation

Figure: Bray Head, Railway Accident, Ireland, 1867. The Liszt Collection.

Recovery

Research question

What are the impacts, in terms of passenger (dis-)satisfaction, of different recovery strategies in case of a severe disruption in a railway network?

Recovery strategies

- Train cancellation
- Partial train cancellation
- Global re-routing of trains
- Additional service (buses/trains)
- "Direct train"
- Increase train capacity

Assumptions

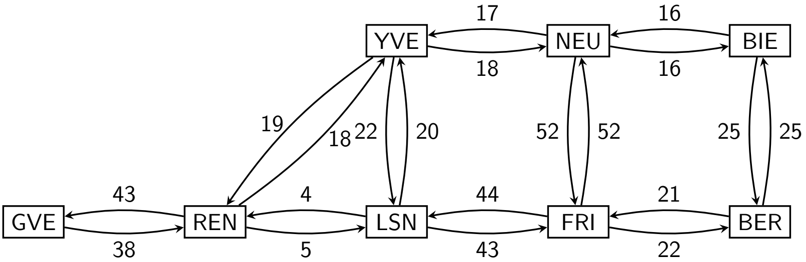
Supply side

- Homogeneity of trains
- Passenger capacity of trains / buses
- Depots at stations where trains can depart

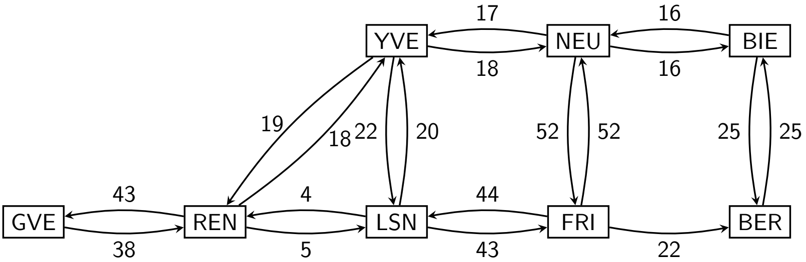
Demand side

- Disaggregate passengers : origin, destination and desired departure time
- Path chosen according to generalized travel time (made of travel time, waiting time and penalties for transfers and early/late departure)
- Perfect knowledge of the system
- No en-route re-rerouting

A sample network



A disrupted sample network



Time-expanded network

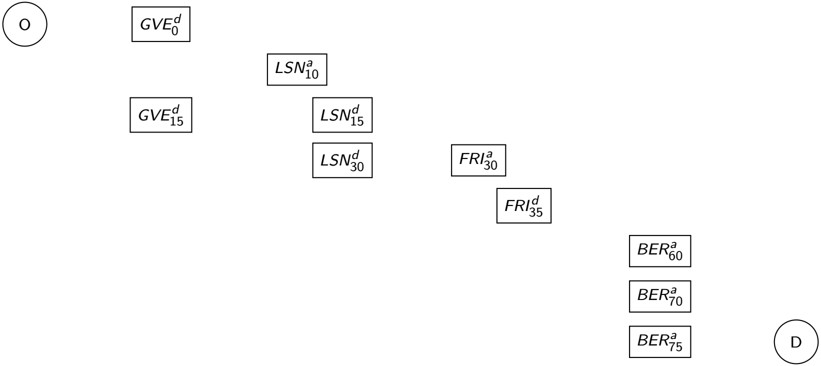
Nodes (N)

- s_t^a: train arrival event from station s at time t
- s_t^d: train departure event from station s at time t

Arcs (A)

- Train driving arcs
- Train waiting arcs
- Connection arcs
- Access & egress arcs

Time-expanded network: an example



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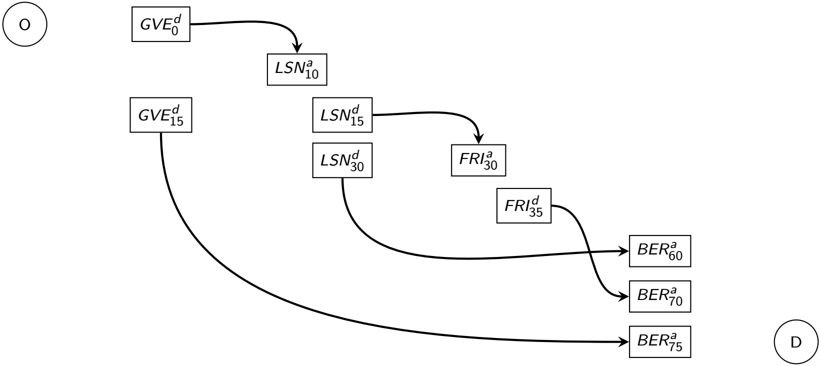
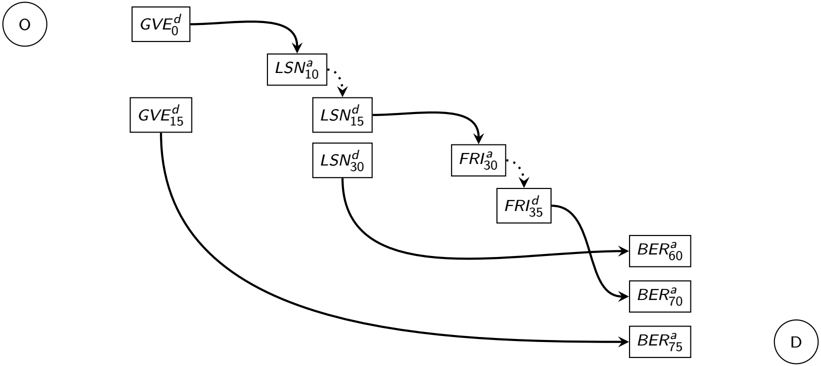


Time-expanded network: an example



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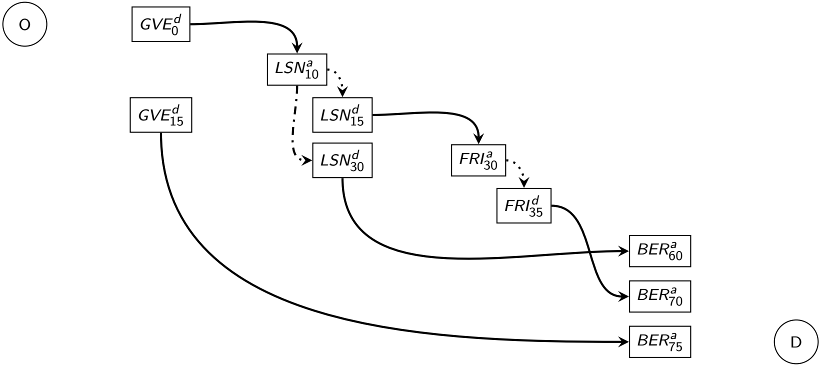

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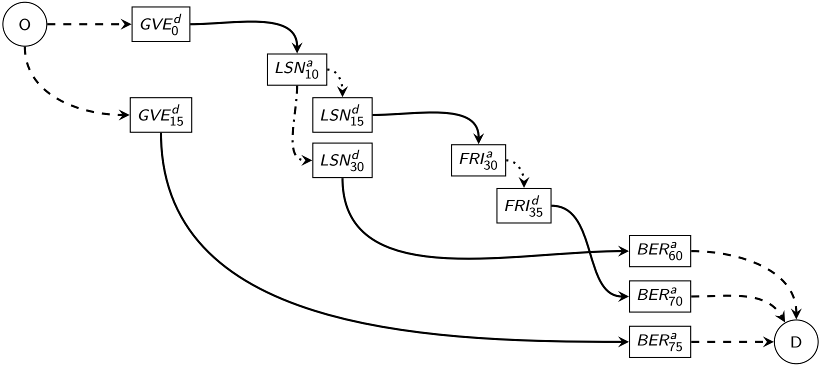


Capacitated passenger assignment algorithm

1. Assign passengers on the least expensive path according to path disutility function.
2. If an arc capacity is exceeded, decide which passengers need to be re-assigned. Otherwise, stop.
3. Re-assign unassigned passengers on a reduced network, then go to Step 2.

Decision variables

Supply

$$x_{(i,j)} = \begin{cases} 1 & \text{if a train runs on arc } (i,j) \in A \\ 0 & \text{otherwise} \end{cases}$$

Demand

$$w^p_{(i,j)} = \begin{cases} 1 & \text{if passenger } p \text{ uses arc } (i,j) \in A_p \\ 0 & \text{otherwise} \end{cases}$$

Objective function

$$\min \sum_{p \in P} \sum_{(i,j) \in A_p} c^p_{(i,j)} \cdot w^p_{(i,j)} + \sum_{(i,j) \in A | i \in R} c_t \cdot x_{(i,j)}$$

Passenger Cost $c^p_{(i,j)}$

- In-vehicle-time
- Waiting time
- Number of transfers
- Departure time shift

Constraints

$$\sum_{j \in N} x_{(r,j)} \leq n_r \qquad \forall r \in R$$

$$\sum_{i \in V} x_{(i,k)} = \sum_{j \in V} x_{(k,j)} \qquad \forall k \in V$$

$$x_{(i,j)} = 0 \qquad \forall (i,j) \in A_D$$

$$\sum_{(i,j) \in A_p | i = o_p} w^p_{(i,j)} = 1 \qquad \forall p \in P$$

$$\sum_{(i,j) \in A_p | j = d_p} w^p_{(i,j)} = 1 \qquad \forall p \in P$$

$$\sum_{i \in V_p} w^p_{(i,k)} = \sum_{j \in V_p} w^p_{(k,j)} \qquad \forall k \in V_p, \forall p \in P$$

$$w^p_{(i,j)} \leq x_{(i,j)} \qquad \forall p \in P, \forall (i,j) \in A \cap A_p$$

$$\sum_{p \in P} w^p_{(i,j)} \leq cap_{(i,j)} \cdot x_{(i,j)} \qquad \forall (i,j) \in A \cap A_p$$

$$x_{(i,j)} \in \{0,1\} \qquad \forall (i,j) \in A$$

$$w^p_{(i,j)} \in \{0,1\} \qquad \forall (i,j) \in A_p, \forall p \in P$$

Framework

Adaptive large neighborhood search (ALNS)

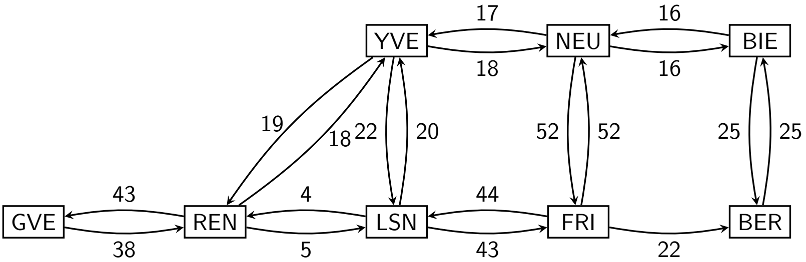
It combines

- Simulated annealing
- Destroy and repair operators

Case study in Switzerland

- **8 stations** : GVE, REN, LSN, FRI, BER, YVE, NEU, BIE
- **207 trains** : All trains departing from any of the stations between 5am and 9am
- **40'446 passengers** : Synthetic O-D matrices, generated with Poisson process
- **Disruption** : Track unavailable between BER and FRI between 7am and 9am

Case study network



Results

	Total passenger disutility	# disrupted passengers
Before ALNS	2'666'630.49	2'847
After ALNS	2'539'605.59	1'508
Improvement	4.8 %	47.0 %

Substantial improvements.

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Conclusions

Importance of demand

- Passenger satisfaction
- Choice behavior
- Willingness to pay
- Heterogeneity

Railway applications

- Ideal timetables
- Disposition timetables