

# Activity-based models: an optimization approach

Janody Pougala   Tim Hillel   Michel Bierlaire

Transport and Mobility Laboratory  
School of Architecture, Civil and Environmental Engineering  
Ecole Polytechnique Fédérale de Lausanne

June 2, 2022



# Outline

- 1 Motivation
- 2 Assumptions
- 3 Model
- 4 Parameter estimation
- 5 Applications





# Introduction

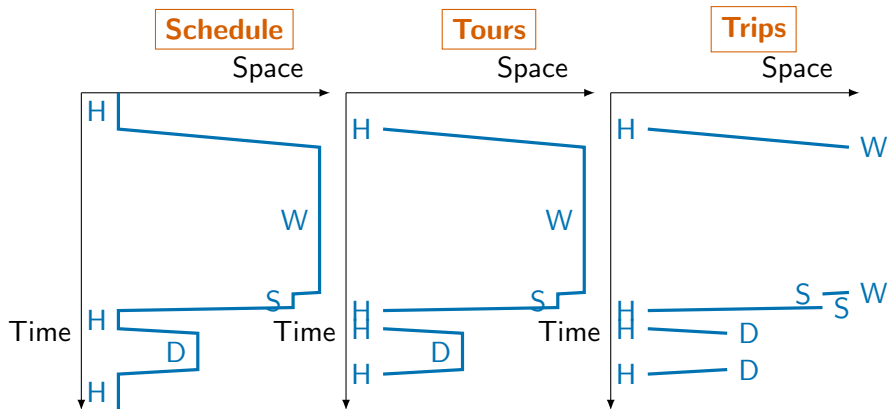


- Travel demand is derived from activity demand.
- Activity demand is influenced by socio-economic characteristics, social interactions, cultural norms, basic needs, etc. [Chapin, 1974]
- Activity demand is constrained in space and time [Hägerstrand, 1970].

## Activity-based models



# Travel demand models



H: Home, W: Work, S: Shop, D: Dining out [Source: M. Ben-Akiva]



# Literature

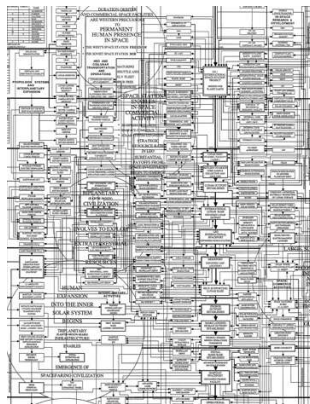
## Econometric models

Handwritten mathematical formulas on a blackboard background:

- $$\bar{S}_1 = \frac{1}{n} \sum_{i=1}^n s_i$$
- $$V_{S_1} = \text{VAR}(S_1) = \frac{1}{n^2} \sum_{i=1}^n (s_i - \bar{S}_1)^2$$
- $$V_{S_2} = \frac{1}{n} \sum_{i=1}^n s_i^2$$
- $$V_{S_2} = \text{VAR}(S_2) = \frac{1}{n^2} \sum_{i=1}^n (s_i - \bar{S}_2)^2$$
- $$\text{COV}(S_1, S_2) = \frac{1}{n^2} \sum_{i=1}^n (s_i - \bar{S}_1)(s_i - \bar{S}_2)$$
- $$\text{CORR}(S_1, S_2) = \frac{\text{COV}(S_1, S_2)}{\sqrt{\text{VAR}(S_1) \times \text{VAR}(S_2)}}$$

Additional notes include:  $\text{VAR}(S_1) = \frac{1}{n} \sum_{i=1}^n (s_i - \bar{S}_1)^2$ ,  $\text{VAR}(S_2) = \frac{1}{n} \sum_{i=1}^n (s_i - \bar{S}_2)^2$ , and  $\text{CORR}(S_1, S_2) = \frac{\text{COV}(S_1, S_2)}{\sqrt{\text{VAR}(S_1) \times \text{VAR}(S_2)}}$ .

## Rule-based models



# State of the art: econometric approach

[Pinjari et al., 2011]

- ... individuals make their activity-travel decisions to maximize the utility derived from the choices they make.
- These model systems usually consist of a series of ... discrete choice models ... that are used to predict ... individuals' activity-travel decisions.
- these model systems employ econometric systems of equations ... to capture relationships between ... socio-demographics and ... attributes on the one hand and the observed activity-travel decision outcomes on the other.



# State of the art: econometric approach

[Bhat, 2005]

- Multiple Discrete Continuous Extreme Value
- Based on first principles.
- Decision-maker solves an optimization problem, with a time budget.
- Several alternatives may be chosen.
- Model derived from KKT conditions.





# State of the art: rule-based approach

[Rasouli and Timmermans, 2014]

- Rule-based models depict decision heuristics... by which individuals organize their daily activities
- Preferences drive the choice of activity participation, jointly with prior commitments and constraints.
- the scheduling process is based on a priori assumptions of the researchers
- the approach does not explicitly model the underlying decision processes and behavioral mechanisms that lead to observed activity-travel decisions.
- Examples: ALBATROSS [Arentze and Timmermans, 2000], TASHA [Roorda et al., 2008], ADAPTS [Auld and Mohammadian, 2009]

# Research question: can we combine the two?

	Econometric	Rule-based
Micro-economic theory	X	—
Parameter inference	X	—
Testing/validation	X	—
Joint decisions	—	X
Complex rules	—	X
Complex constraints	—	X

# Integrated approach

## Assumptions

- Individuals **are** utility maximizers.
- Sequence of models is most of the time arbitrary. All decisions are made together.
- Decisions are subject to complex constraints and interactions.
  - Time constraint: to increase the activity duration, another activity is impacted.
  - Interaction constraints: if I leave home by bus, driving my car is not an option until I come back home.
  - Resource constraints: if my wife uses the only car in the household, driving the car is not an option for me.



# Integrated approach

## Integrate the econometric and the rule-based approaches

- Utility associated with activity participation, duration, etc.
- Disutility associated with traveling.
- Complex interactions and constraints are captured by rules.

## Mathematical programming

- Individuals are solving an optimization problem.
- Decisions: activity participation and scheduling.
- Objective function: utilities.
- Constraints: complex rules.



# Outline

- 1 Motivation
- 2 Assumptions**
- 3 Model
- 4 Parameter estimation
- 5 Applications



# First principles



- Each individual  $n$  has a time-budget (a day).
- Each activity  $a$  considered by  $n$  is associated with a utility  $U_{an}$ .
- Individuals schedule their activities as to **maximize** the total utility, subject to their time-budget constraint.

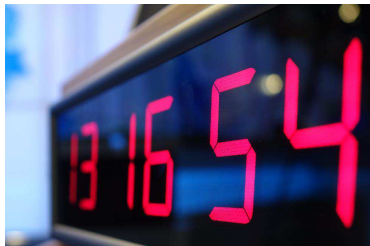
## Further assumptions



### Individuals are **time sensitive**

- Have a desired start time, duration and/or end time for each activity
- Deviations from their desired times in the scheduling process decrease the utility function

# Time



- Time horizon: 24 hours.
- Discretization:  $T$  time intervals.
- Trade-off between model accuracy and computational time.



# Space



- Discrete and finite set  $S$  of locations, indexed by  $s$ .
- For each  $(s_o, s_d)$ ,  $\rho^m(s_o, s_d)$  is the travel time with mode  $m$ .
- Extensions to include route choices are possible.

# Activities

## Definition: Activity

An activity requires a trip to/from a given location.



# Activities



- Set  $A$  of activities.
- Location  $s_a$ .
- Transportation mode:  $m_a$ .
- Starting time  $x_a$ ,  $0 \leq x_a \leq T$ .
- Duration:  $\tau_a \geq 0$ .
- Feasible time interval:  $[\gamma_a^-, \gamma_a^+]$  (e.g. opening hours).

# Activities

## Modeling location choice

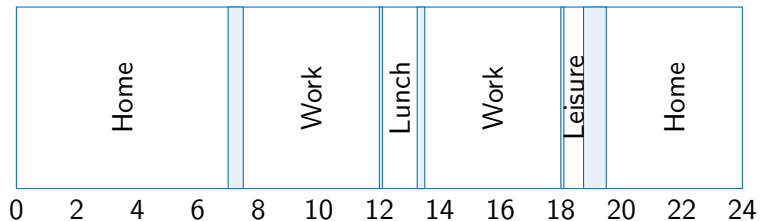
- “Dinner at home” and “dinner at a restaurant”
- are considered two different activities.
- Impose that maximum one of them is selected.

## Modeling mode choice

- Having dinner and coming back by car or taxi
- are considered two different activities.
- Impose that maximum one of them is selected.



# Scheduling



# Categories



- [Castiglione et al., 2014]: mandatory, maintenance, discretionary.
- Flexible, somewhat flexible, not flexible.

## Category

Activities that share the same preference profile.

# Preferences

## Preferences

- desired starting time  $x_a^*$ ,
- desired duration  $\tau_a^*$ .

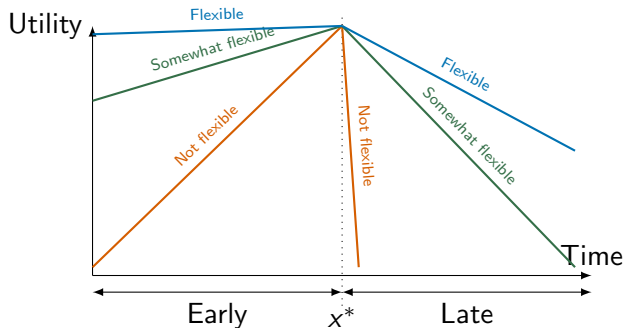
## Penalties

- Starting early [Small, 1982]:  
 $\theta_e \max(x_a^* - x_a, 0)$ .
- Starting late [Small, 1982]:  
 $\theta_l \max(x_a - x_a^*, 0)$ .
- Shorter activity:  $\theta_{ds} \max(\tau_a^* - \tau_a, 0)$ .
- Longer activity:  $\theta_{dl} \max(\tau_a - \tau_a^*, 0)$ .



# Preferences

Parameters depend on the category type





# Disutility of travel



## Traveling is part of the activity

- Travel (time and cost) from  $a$  to  $a^+$  negatively contributes to  $U_a$ :  $t_a, c_{t_a}$ .
- Exception: last activity of the day (home).

# Utility function

An individual  $n$  derives the following utility from performing activity  $a$ , with a schedule flexibility  $k$ :

$$\begin{aligned}U_{an} = & c_{an} + \theta_e \max(x_a^* - x_a, 0) \\ & + \theta_\ell \max(x_a - x_a^*, 0) \\ & + \theta_{ds} \max(\tau_a^* - \tau_a, 0) \\ & + \theta_{dl} \max(\tau_a - \tau_a^*, 0) \\ & + \theta_{tt} t_a + \theta_{tc} c_{t_a} \\ & + \theta_c c_a + \xi_{an},\end{aligned}$$

where  $\xi_{an}$  is a random term with a known distribution.

# Utility function



## Error terms

- Rely on simulation.
- Draw  $\xi_{anr}$ ,  $r = 1, \dots, R$ .
- Optimization problem for each  $r$ .
- Utility:  $U_{anr}$ .

# Households

## Assumptions

- Members of the households are altruist.
- Everybody makes decisions for the sake of the household.
- Objective function: sum of the utilities of each individual

## Model

- Similar model as for individuals.
- Resource constraints can easily be added.



# Outline

- 1 Motivation
- 2 Assumptions
- 3 Model**
- 4 Parameter estimation
- 5 Applications



## Decision variables for individual $n$ and draw $r$

For each (potential) activity  $a$ :

- Activity participation:  $w_{anr} \in \{0, 1\}$ .
- Starting time:  $x_{anr} \in \{0, \dots, T\}$ .
- Duration:  $\tau_{anr} \in \{0, \dots, T\}$ .
- Scheduling:  $z_{abnr} \in \{0, 1\}$ : 1 if activity  $b$  immediately follows  $a$ .
- Travel time:  $t_{anr}$ : travel time from  $a$  to the next activity.

# Objective function

Additive utility

$$\max \sum_n \sum_{a \in A} w_{anr} U_{anr}$$

# Constraints

## Time budget

$$\sum_a \tau_{anr} + t_{anr} = T, \forall n, r.$$

## Cost budget

$$\sum_a c_a w_{anr} + t_{canr} = B, \forall n, r.$$

## Time windows

$$0 \leq \gamma_a^- \leq x_{anr} \leq x_{anr} + \tau_{anr} \leq \gamma_a^+ \leq T, \forall a, n, r.$$



# Constraints

## Precedence constraints

$$z_{abnr} + z_{banr} \leq 1, \forall a, b, n, r.$$

## Single successor/predecessor

$$\sum_{b \in A \setminus \{a\}} z_{abnr} = w_{anr}, \forall a, n, r,$$

$$\sum_{b \in A \setminus \{a\}} z_{banr} = w_{anr}, \forall a, n, r.$$

# Constraints

## Travel time

$$t_{anr} = \sum_{b \in A} z_{abnr} \rho^{m_a}(s_a, s_b).$$

## Consistent timing

$$(z_{abnr} - 1)T \leq x_{anr} + \tau_{anr} + t_{anr} - x_{bnr} \leq (1 - z_{abnr})T, \forall a, b, n, r.$$

## Mutually exclusive duplicates

$$\sum_{a \in B_k} w_{anr} = 1, \forall k, n, r.$$

# Constraints

## Interaction constraint

- If I leave home by bus, driving my car is not an option until I come back home.
- $\delta_{anr}^{\text{car}} = 1$  if car is available for activity  $a$ .

$$\delta_{anr}^{\text{car}} \geq \delta_{bnr}^{\text{car}} + z_{abnr} - 1.$$

## Resource constraints

- Resource constraints: if my wife uses the only car in the household, driving the car is not an option for me.

$$\sum_n \delta_{anr}^{\text{car}} \leq \text{number of cars}, \forall a, r.$$

# Constraints: other examples

## Participation constraints

- Participation constraints: if I drop my children off, I need to pick them up later.
- Drop-off: activity  $a$ .
- Pick-up: activity  $b$ .
- Activity participation:  $w_{bnr} \geq w_{anr}$
- Timing:  $x_{bnr} \geq x_{anr}$ .

## Sequence constraints

- If I go grocery shopping I need to go back home before doing another activity.
- Shopping: activity  $a$ .
- Home: activity  $b$ .

$$z_{abnr} \geq w_{anr}.$$

# Integrated framework

## Mathematical programming

- Utility maximization.
- Scheduling problem.
- Rules are translated into additional constraints.
- Stochasticity is captured by simulation.



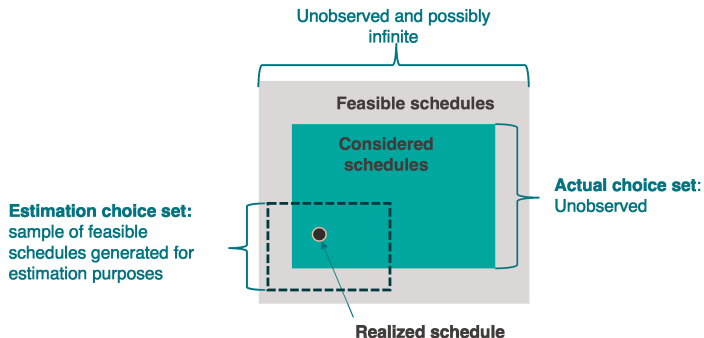
# Outline

- 1 Motivation
- 2 Assumptions
- 3 Model
- 4 Parameter estimation**
- 5 Applications



# Challenges

- The universal choice set cannot be enumerated.
- Traditional maximum likelihood estimators of parameters cannot easily be derived.



# Methodology

## Choice set generation

- Importance sampling with Metropolis-Hastings algorithm
- Bias the sampling towards “good” or “meaningful” schedule.

## Parameter estimation

- Maximum likelihood estimation of a random utility model.
- Choice set contains only feasible schedules for individual  $n$ .
- Constraints can be ignored for inference.
- Need for correction for importance sampling [Guevara and Ben-Akiva, 2013].



# Outline

- 1 Motivation
- 2 Assumptions
- 3 Model
- 4 Parameter estimation
- 5 Applications**



# Schedule simulation

## Data set

- 2015 Mobility and Transport Microcensus [ARE 2017]
- Nationwide travel survey conducted every 5 years
- Lausanne sample: 1118 individuals
  - Students: 236 individuals
  - Workers: 618 individuals



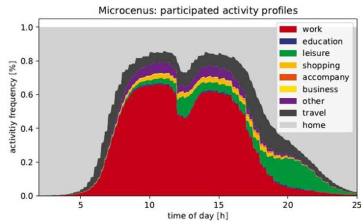
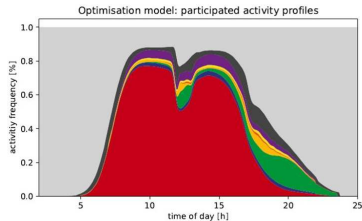
# Model 1 - Workers

	Parameter	Param. estimate	Rob. std err	Rob. <i>t</i> -stat	Rob. <i>p</i> -value
1	F early	-0.813	0.16	-5.09	3.53e-07
2	F late	-1.12	0.138	-8.08	6.66e-16
3	F long	-0.569	0.165	-3.45	0.554e-04
4	NF early	-0.827	0.160	-5.15	2.58e-07
5	NF late	-1.26	0.236	-5.31	1.08e-07
6	NF long	-0.789	0.229	-3.45	0.57e-04
7	NF short	-3.24	0.555	-5.84	5.30e-09
8	ASC_Education	10.8	2.50	4.33	1.50e-05
9	ASC_Leisure	15.3	1.38	11.1	0.0
10	ASC_Work	18.5	2.00	9.28	0.0

# OPTIMs

## OPTimization of Individual Mobility Schedules, [Manser et al., 2021a]

- Collaboration with Swiss Federal Railways.
- Integration of the optimization framework into their long-term travel demand forecasting tool (SIMBA MOBi).



# Conclusions

## Achievements so far

- Formulation of the model.
- Simulation of complex and valid activity schedules.
- Application to real case studies.
- Procedure for the estimation of the parameters.

## Challenges

- Latent preferences (desired start times, durations...)
- Validation.



# Summary

- Motivation: design operational activity-based models.
- Combine the econometric and the rule-based approaches.
- Methodological contribution: use mathematical programming and simulation.
- Simulation of activity schedule: [Pougala et al., 2022].
- Application with the Swiss Railways: [Manser et al., 2021b].
- Estimation of the parameters: ongoing.
- Main advantage of the framework: flexibility.



# Bibliography I



ARE: Office fédéral de la statistique and Office fédéral du développement Territorial (2017).

Comportement de la population en matière de transports. Résultats du microrecensement mobilité et transports 2015.

Technical report, Neuchâtel, Berne.



Arentze, T. and Timmermans, H. (2000).

Albatross: a learning based transportation oriented simulation system.  
Citeseer.



Auld, J. and Mohammadian, A. (2009).

Framework for the development of the agent-based dynamic activity planning and travel scheduling (adapts) model.

Transportation Letters, 1(3):245–255.

# Bibliography II



Bhat, C. R. (2005).

A multiple discrete-continuous extreme value model: formulation and application to discretionary time-use decisions.

[Transportation Research Part B: Methodological](#), 39(8):679 – 707.



Castiglione, J., Bradley, M., and Gliebe, J. (2014).

[Activity-Based Travel Demand Models: A Primer](#).

Transportation Research Board, Washington, D.C.



Chapin, F. S. (1974).

[Human activity patterns in the city: Things people do in time and in space](#), volume 13.

Wiley-Interscience.



## Bibliography III



Guevara, C. A. and Ben-Akiva, M. E. (2013).  
Sampling of alternatives in logit mixture models.  
[Transportation Research Part B: Methodological](#), 58:185–198.





Hägerstrand, T. (1970).  
What about people in regional science?  
[Papers in Regional Science](#).



Manser, P., Haering, T., Hillel, T., Pougala, J., Krueger, R., and Bierlaire, M. (2021a).  
Resolving temporal scheduling conflicts in activity-based modelling.  
Technical Report TRANSP-OR 211209, Transport and Mobility  
Laboratory, Ecole Polytechnique Fédérale de Lausanne, Lausanne,  
Switzerland.

## Bibliography IV

-  Manser, P., Haering, T., Hillel, T., Pougala, J., Krueger, R., and Bierlaire, M. (2021b).  
Resolving temporal scheduling conflicts in activity-based modelling.  
Technical Report TRANSP-OR 211209, Transport and Mobility Laboratory, Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland.
-  Pinjari, A. R., Bhat, C. R., et al. (2011).  
Activity-based travel demand analysis.  
[A Handbook of Transport Economics](#), 10:213–248.
-  Pougala, J., Hillel, T., and Bierlaire, M. (2022).  
Capturing trade-offs between daily scheduling choices.  
[Journal of Choice Modelling](#), 43(100354).  
Accepted on Mar 19, 2022.

# Bibliography V



Rasouli, S. and Timmermans, H. (2014).

Activity-based models of travel demand: promises, progress and prospects.

[International Journal of Urban Sciences](#), 18(1):31–60.



Roorda, M. J., Miller, E. J., and Habib, K. M. (2008).

Validation of tasha: A 24-h activity scheduling microsimulation model.

[Transportation Research Part A: Policy and Practice](#), 42(2):360–375.



Small, K. A. (1982).

The scheduling of consumer activities: work trips.

[American Economic Review](#).