Activity-based models: an optimization approach

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Outline

1 Motivation
2 Assumptions
3 Model
4 Parameter estimation
5 Applications
Complexity of modern transportation systems requires complex travel demand models.
Travel demand is derived from activity demand.

Activity demand is influenced by socio-economic characteristics, social interactions, cultural norms, basic needs, etc. [Chapin, 1974]

Activity demand is constrained in space and time [Hägerstrøm, 1970].

Activity-based models
Travel demand models

Schedule

Tours

Trips

Space

Time

H: Home, W: Work, S: Shop, D: Dining out [Source: M. Ben-Akiva]
Econometric models

Rule-based models
Motivation

State of the art: econometric approach

[Pinjari et al., 2011]

- ... individuals make their activity-travel decisions to maximize the utility derived from the choices they make.
- These model systems usually consist of a series of ... discrete choice models ... that are used to predict ... individuals’ activity-travel decisions.
- these model systems employ econometric systems of equations ... to capture relationships between ... socio-demographics and ... attributes on the one hand and the observed activity-travel decision outcomes on the other.
State of the art: econometric approach

[Bhat, 2005]

- Multiple Discrete Continuous Extreme Value
- Based on first principles.
- Decision-maker solves an optimization problem, with a time budget.
- Several alternatives may be chosen.
- Model derived from KKT conditions.
State of the art: rule-based approach

[Rasouli and Timmermans, 2014]

- Rule-based models depict decision heuristics... by which individuals organize their daily activities
- Preferences drive the choice of activity participation, jointly with prior commitments and constraints.
- The scheduling process is based on a priori assumptions of the researchers
- The approach does not explicitly model the underlying decision processes and behavioral mechanisms that lead to observed activity-travel decisions.
- Examples: ALBATROSS [Arentze and Timmermans, 2000], TASHA [Roorda et al., 2008], ADAPTS [Auld and Mohammadian, 2009]
Research question: can we combine the two?

<table>
<thead>
<tr>
<th></th>
<th>Econometric</th>
<th>Rule-based</th>
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</thead>
<tbody>
<tr>
<td>Micro-economic theory</td>
<td>X</td>
<td>—</td>
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<tr>
<td>Parameter inference</td>
<td>X</td>
<td>—</td>
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<tr>
<td>Testing/validation</td>
<td>X</td>
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<td>Joint decisions</td>
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<td>X</td>
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<td>Complex rules</td>
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<td>X</td>
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<td>Complex constraints</td>
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<td>X</td>
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Assumptions

- **Individuals are utility maximizers.**
- **Sequence of models is most of the time arbitrary. All decisions are made together.**
- **Decisions are subject to complex constraints and interactions.**
  - Time constraint: to increase the activity duration, another activity is impacted.
  - Interaction constraints: if I leave home by bus, driving my car is not an option until I come back home.
  - Resource constraints: if my wife uses the only car in the household, driving the car is not an option for me.
Integrated approach

Integrate the econometric and the rule-based approaches

- Utility associated with activity participation, duration, etc.
- Disutility associated with traveling.
- Complex interactions and constraints are captured by rules.

Mathematical programming

- Individuals are solving an optimization problem.
- Decisions: activity participation and scheduling.
- Objective function: utilities.
- Constraints: complex rules.
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First principles

- Each individual $n$ has a time-budget (a day).
- Each activity $a$ considered by $n$ is associated with a utility $U_{an}$.
- Individuals schedule their activities as to maximize the total utility, subject to their time-budget constraint.
Further assumptions

Individuals are **time sensitive**

- Have a desired start time, duration and/or end time for each activity
- Deviations from their desired times in the scheduling process decrease the utility function
Time horizon: 24 hours.
Discretization: $T$ time intervals.
Trade-off between model accuracy and computational time.
Space

- Discrete and finite set $S$ of locations, indexed by $s$.
- For each $(s_o, s_d)$, $\rho^m(s_o, s_d)$ is the travel time with mode $m$.
- Extensions to include route choices are possible.
Definition: Activity

An activity requires a trip to/from a given location.
Activities

- Set $A$ of activities.
- Location $s_a$.
- Transportation mode: $m_a$.
- Starting time $x_a$, $0 \leq x_a \leq T$.
- Duration: $\tau_a \geq 0$.
- Feasible time interval: $[\gamma_a^-, \gamma_a^+]$ (e.g. opening hours).
Activities

Modeling location choice
- “Dinner at home” and “dinner at a restaurant”
- are considered two different activities.
- Impose that maximum one of them is selected.

Modeling mode choice
- Having dinner and coming back by car or taxi
- are considered two different activities.
- Impose that maximum one of them is selected.
Scheduling

Activity-based models

Pougala, Hillel, Bierlaire (EPFL)
Categories

- [Castiglione et al., 2014]: mandatory, maintenance, discretionary.
- Flexible, somewhat flexible, not flexible.

Category

Activities that share the same preference profile.
Preferences

- desired starting time $x_a^*$,
- desired duration $\tau_a^*$.

Penalties

- Starting early [Small, 1982]:
  $\theta_e \max(x_a^* - x_a, 0)$.
- Starting late [Small, 1982]:
  $\theta_\ell \max(x_a - x_a^*, 0)$.
- Shorter activity: $\theta_{ds} \max(\tau_a^* - \tau_a, 0)$.
- Longer activity: $\theta_{d\ell} \max(\tau_a - \tau_a^*, 0)$.
Preferences

Parameters depend on the category type

- **Flexible**
- **Somewhat flexible**
- **Not flexible**

![Graph showing utility over time with categories Early and Late and a crossover point marked as X.*](graph.png)
Disutility of travel

- Traveling is part of the activity
  - Travel (time and cost) from \( a \) to \( a^+ \) negatively contributes to \( U_a: t_a, c_t_a \).
  - Exception: last activity of the day (home).
Utility function

An individual $n$ derives the following utility from performing activity $a$, with a schedule flexibility $k$:

$$U_{an} = c_{an} + \theta_{e} \max(x^*_a - x_a, 0) + \theta_\ell \max(x_a - x^*_a, 0) + \theta_{ds} \max(\tau^*_a - \tau_a, 0) + \theta_{dl} \max(\tau_a - \tau^*_a, 0) + \theta_{tt} t_a + \theta_{tc} c_t + \theta_c c_a + \xi_{an},$$

where $\xi_{an}$ is a random term with a known distribution.
Utility function

Error terms

- Rely on simulation.
- Draw $\xi_{anr}$, $r = 1, \ldots, R$.
- Optimization problem for each $r$.
- Utility: $U_{anr}$. 
Households

Assumptions

- Members of the households are altruist.
- Everybody makes decisions for the sake of the household.
- Objective function: sum of the utilities of each individual

Model

- Similar model as for individuals.
- Resource constraints can easily be added.
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Decision variables for individual $n$ and draw $r$

For each (potential) activity $a$:
- Activity participation: $w_{anr} \in \{0, 1\}$.
- Starting time: $x_{anr} \in \{0, \ldots, T\}$.
- Duration: $\tau_{anr} \in \{0, \ldots, T\}$.
- Scheduling: $z_{abnr} \in \{0, 1\}$: 1 if activity $b$ immediately follows $a$.
- Travel time: $t_{anr}$: travel time from $a$ to the next activity.
Objective function

Additive utility

$$\max \sum_{n} \sum_{a \in A} w_{anr} U_{anr}$$
Constraints

Time budget

$$\sum_a \tau_{anr} + t_{anr} = T, \ \forall n, r.$$ 

Cost budget

$$\sum_a c_a w_{anr} + t_{canr} = B, \ \forall n, r.$$ 

Time windows

$$0 \leq \gamma_a^- \leq x_{anr} \leq x_{anr} + \tau_{anr} \leq \gamma_a^+ \leq T, \ \forall a, n, r.$$
Constraints

Precedence constraints

\[ z_{abnr} + z_{banr} \leq 1, \quad \forall a, b, n, r. \]

Single successor/predecessor

\[
\sum_{b \in A \setminus \{a\}} z_{abnr} = w_{anr}, \quad \forall a, n, r, \\
\sum_{b \in A \setminus \{a\}} z_{banr} = w_{anr}, \quad \forall a, n, r.
\]
Constraints

Travel time

\[ t_{anr} = \sum_{b \in A} z_{abnr} \rho^{ma}(s_a, s_b). \]

Consistent timing

\[ (z_{abnr} - 1) T \leq x_{anr} + \tau_{anr} + t_{anr} - x_{bnr} \leq (1 - z_{abnr}) T, \quad \forall a, b, n, r. \]

Mutually exclusive duplicates

\[ \sum_{a \in B_k} w_{anr} = 1, \quad \forall k, n, r. \]
Constraints

Interaction constraint

- If I leave home by bus, driving my car is not an option until I come back home.
- \( \delta_{anr}^{car} = 1 \) if car is available for activity \( a \).

\[
\delta_{anr}^{car} \geq \delta_{bnr}^{car} + z_{abnr} - 1.
\]

Resource constraints

- Resource constraints: if my wife uses the only car in the household, driving the car is not an option for me.

\[
\sum_n \delta_{anr}^{car} \leq \text{number of cars}, \forall a, r.
\]
Constraints: other examples

Participation constraints
- Participation constraints: if I drop my children off, I need to pick them up later.
- Drop-off: activity $a$.
- Pick-up: activity $b$.
- Activity participation: $w_{bnr} \geq w_{anr}$
- Timing: $x_{bnr} \geq x_{anr}$.

Sequence constraints
- If I go grocery shopping I need to go back home before doing another activity.
- Shopping: activity $a$.
- Home: activity $b$.

\[ Z_{abnr} \geq W_{anr}. \]
Integrated framework

Mathematical programming

- Utility maximization.
- Scheduling problem.
- Rules are translated into additional constraints.
- Stochasticity is captured by simulation.
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Challenges

- The universal choice set cannot be enumerated.
- Traditional maximum likelihood estimators of parameters cannot easily be derived.
Methodology

Choice set generation
- Importance sampling with Metropolis-Hastings algorithm
- Bias the sampling towards “good” or “meaningful” schedule.

Parameter estimation
- Maximum likelihood estimation of a random utility model.
- Choice set contains only feasible schedules for individual $n$.
- Constraints can be ignored for inference.
- Need for correction for importance sampling [Guevara and Ben-Akiva, 2013].
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Schedule simulation

Data set

- 2015 Mobility and Transport Microcensus [ARE 2017]
- Nationwide travel survey conducted every 5 years
- Lausanne sample: 1118 individuals
  - Students: 236 individuals
  - Workers: 618 individuals
## Model 1 - Workers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Param. estimate</th>
<th>Rob. std err</th>
<th>Rob. t-stat</th>
<th>Rob. p-value</th>
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</thead>
<tbody>
<tr>
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<td>-0.813</td>
<td>0.16</td>
<td>-5.09</td>
<td>3.53e-07</td>
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<tr>
<td>2 F late</td>
<td>-1.12</td>
<td>0.138</td>
<td>-8.08</td>
<td>6.66e-16</td>
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<tr>
<td>3 F long</td>
<td>-0.569</td>
<td>0.165</td>
<td>-3.45</td>
<td>0.554e-04</td>
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<tr>
<td>4 NF early</td>
<td>-0.827</td>
<td>0.160</td>
<td>-5.15</td>
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<td>0.236</td>
<td>-5.31</td>
<td>1.08e-07</td>
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<tr>
<td>6 NF long</td>
<td>-0.789</td>
<td>0.229</td>
<td>-3.45</td>
<td>0.57e-04</td>
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<tr>
<td>7 NF short</td>
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<tr>
<td>8 ASC_Education</td>
<td>10.8</td>
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<td>1.50e-05</td>
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<tr>
<td>9 ASC_Leisure</td>
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<td>10 ASC_Work</td>
<td>18.5</td>
<td>2.00</td>
<td>9.28</td>
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</tbody>
</table>
OPTIMs

OPTimization of Individual Mobility Schedules, [Manser et al., 2021a]

- Collaboration with Swiss Federal Railways.
- Integration of the optimization framework into their long-term travel demand forecasting tool (SIMBA MOBi).
Conclusions

Achievements so far

- Formulation of the model.
- Simulation of complex and valid activity schedules.
- Application to real case studies.
- Procedure for the estimation of the parameters.

Challenges

- Latent preferences (desired start times, durations...)
- Validation.
Summary

- Motivation: design operational activity-based models.
- Combine the econometric and the rule-based approaches.
- Methodological contribution: use mathematical programming and simulation.
- Simulation of activity schedule: [Pougala et al., 2022].
- Application with the Swiss Railways: [Manser et al., 2021b].
- Estimation of the parameters: ongoing.
- Main advantage of the framework: flexibility.
Bibliography I


Bibliography III


