Incorporating behavioral model into transport optimization

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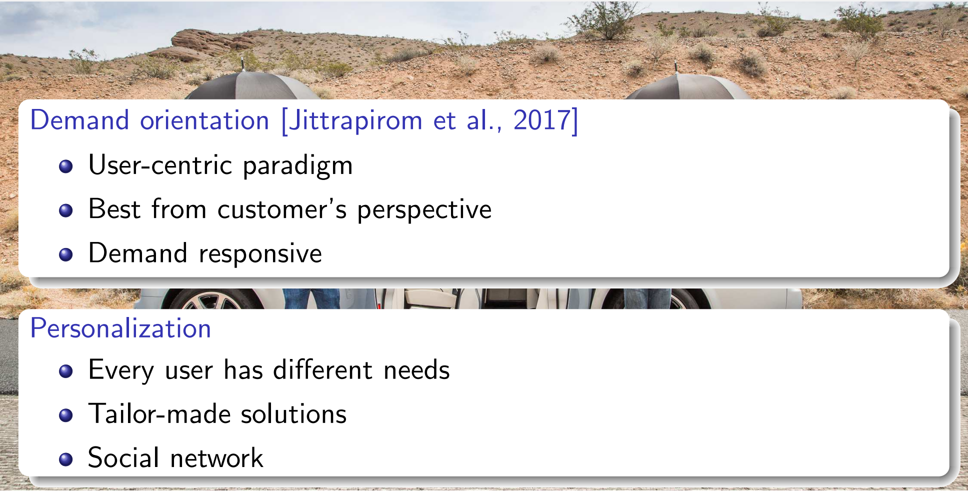
December 3, 2018

Outline

1. Demand and supply
2. Disaggregate demand models
3. A simple example
4. A generic framework
5. MILP
6. Conclusion

Mobility as a service

Mobility as a service



Demand orientation [Jittrapirom et al., 2017]

- User-centric paradigm
- Best from customer's perspective
- Demand responsive

Personalization

- Every user has different needs
- Tailor-made solutions
- Social network

Mobility as a service

Key challenges [Jittrapirom et al., 2017]

- Demand-side modeling
- Supply-side modeling
- Governance and business model to match supply and demand

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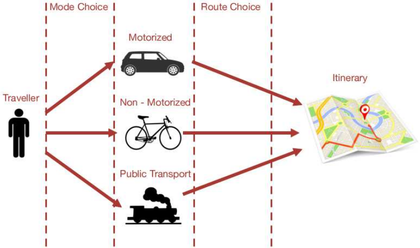
Choice models



Behavioral models

- Demand = sequence of choices
- Choosing means trade-offs
- In practice: derive trade-offs from choice models

Choice models

Theoretical foundations

- Random utility theory
- Choice set: $\mathcal{C}_n$
- $y_{in} = 1$ if $i \in \mathcal{C}_n$, 0 if not
- Logit model:

$$P(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}$$

2000

Logit model

Utility

$$U_{in} = V_{in} + \varepsilon_{in}$$

- Decision-maker n
- Alternative $i \in \mathcal{C}_n$

Choice probability

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}.$$

Variables: $x_{in} = (p_{in}, z_{in}, s_n)$

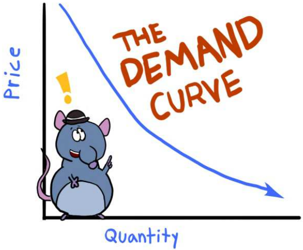
Attributes of alternative i: z_{in}

- Cost / price (p_{in})
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Characteristics of decision-maker n: s_n

- Income
- Age
- Sex
- Trip purpose
- Car ownership
- Education
- Profession
- etc.

Demand curve



Disaggregate model

$$P_n(i|p_{in}, z_{in}, s_n)$$

Total demand

$$D(i) = \sum_n P_n(i|p_{in}, z_{in}, s_n)$$

Difficulty

Non linear and non convex in p_{in} and z_{in}

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Example

Choice set: Jupiler

- 't Klooster $i = 0$
- Belvédère $i = 1$

Utility functions

$$V_{0n} = -2.2 p_0 - 1.3$$
$$V_{1n} = -2.2 p_1$$

Prices

- 't Klooster: [0 – 6 €]
- Belvédère: 1.8 €

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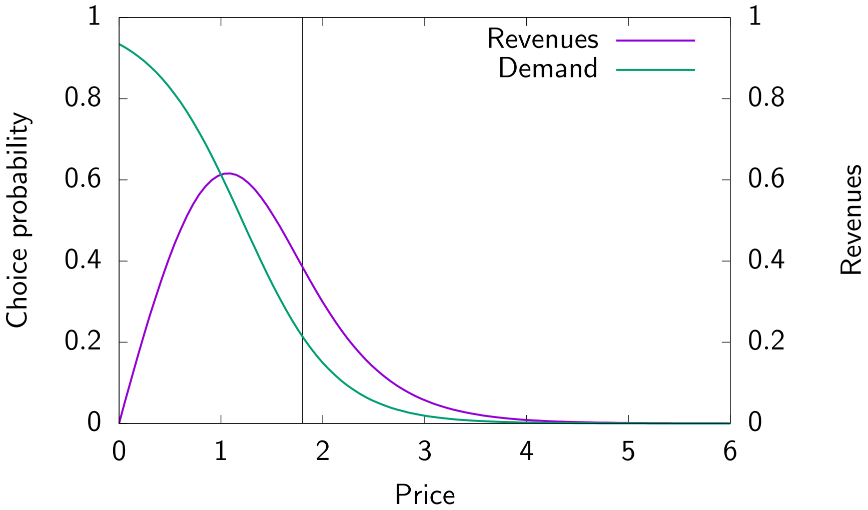
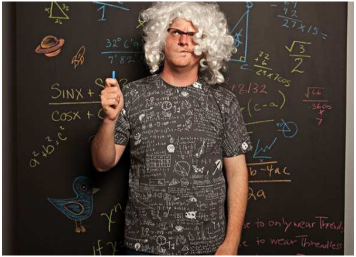
Demand and revenues



Heterogeneous population



Two groups in the population

$$V_{0n} = -\beta_n p_0 + c_0$$

Mathematics: 25%

$\beta_1 = -4.5$,
$c_1 = -1.3$

Business: 75%

$\beta_2 = -0.25$,
$c_2 = -1.3$

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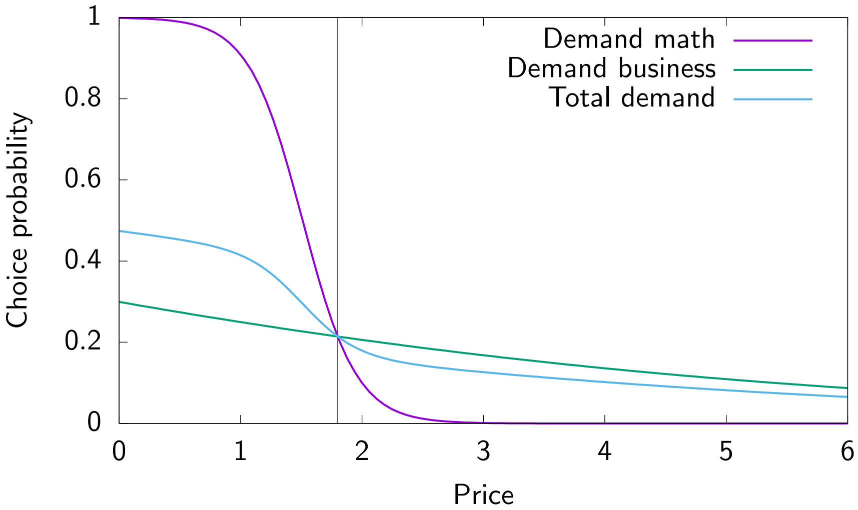
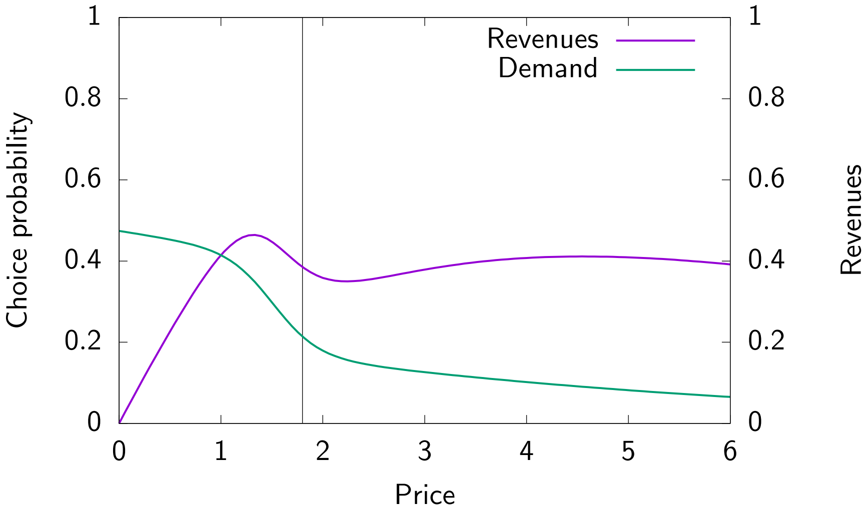
Demand per market segment



Demand and revenues



Optimization

Pricing

- Non linear optimization problem.
- Non convex objective function.
- Multimodal function.
- May feature many local optima.
- In practice, the groups are many, and interdependent.
- Optimizing each group separately is not feasible.

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Assortment

What about assortment?

Heterogeneous population, two products

K: Price Orval = 1.5 × price Jupiler

B: Price Orval = 2 × price Jupiler

Utility functions: math

$$V_{\mathrm{K,Jupiler},m} = -4.5 p_{\mathrm{K,Jupiler}} - 1.3$$
$$V_{\mathrm{K,Orval},m} = -4.5 p_{\mathrm{K,Orval}} - 1.3 + 3$$
$$V_{\mathrm{B,Jupiler},m} = -4.5 p_{\mathrm{B,Jupiler}}$$
$$V_{\mathrm{B,Orval},m} = -4.5 p_{\mathrm{B,Orval}} + 3$$

Utility functions: HEC

$$V_{\mathrm{K,Jupiler},b} = -0.25 p_{\mathrm{K,Jupiler}} - 1.3$$
$$V_{\mathrm{K,Orval},b} = -0.25 p_{\mathrm{K,Orval}} - 1.3 + 1$$
$$V_{\mathrm{B,Jupiler},b} = -0.25 p_{\mathrm{B,Jupiler}}$$
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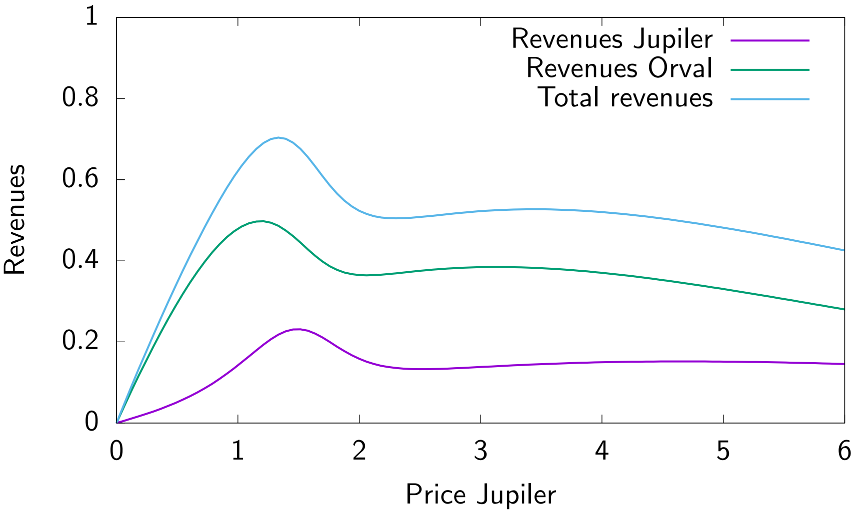
Total revenues



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In transportation

Assortment and pricing

- Airlines
- Deregulated railways
- Mobility as a service

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Optimization

Assortment and pricing

- Combinatorial problem
- For each potential assortment, solve a pricing problem
- Select the assortment corresponding to the highest revenues
- MINLP
- Non convex relaxation

Disaggregate demand models

Advantages

- Theoretical foundations
- Market segmentation
- Taste heterogeneity
- Many variables
- Estimated from data

Disadvantages

- Complex mathematical formulation
- Not suited for optimization
- Literature: heuristics

Research objectives

Observations

- Revenues is not the only indicator to optimize,
- e.g. customer satisfaction.
- Many transportation applications need a demand representation

Goal

- Generic mathematical representation of choice models,
- designed to be included in MILP,
- linear in the decision variables.

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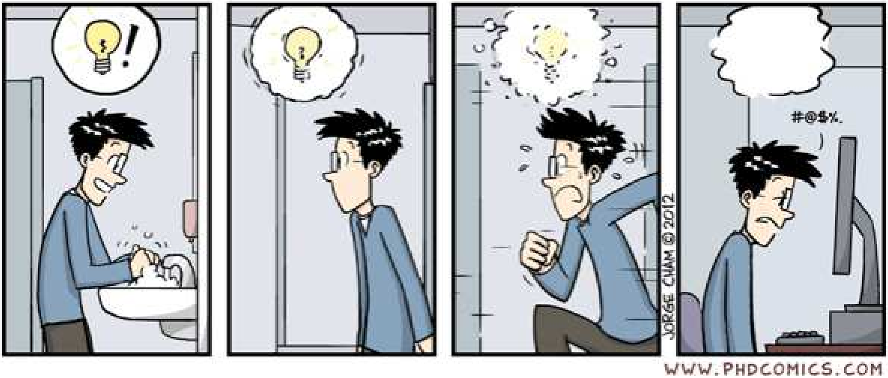
The main idea



The main idea

Linearization

- Hopeless to linearize the logit formula (we tried...)
- Anyway, we want to go beyond logit.

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First principles

Each customer solves an optimization problem

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First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- For each scenario r, we can identify the largest utility.
- It corresponds to the chosen alternative.

MILP (in words)

MILP

max	benefit
subject to	utility definition
	availability
	discounted utility
	choice
	capacity allocation
	price selection

A case study

Challenge

- Take a choice model from the literature.
- It cannot be logit.
- It must involve heterogeneity.
- Show that it can be integrated in a relevant MILP.

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Parking choice

- [Ibeas et al., 2014]

Parking choices [Ibeas et al., 2014]

Alternatives

- Paid on-street parking
- Paid underground parking
- Free street parking

Model

- $N = 50$ customers
- $\mathcal{C} = \{\mathrm{PSP}, \mathrm{PUP}, \mathrm{FSP}\}$
- $\mathcal{C}_n = \mathcal{C} \quad \forall n$
- $p_{in} = p_i \quad \forall n$
- Capacity of 20 spots
- Mixture of logit models

General experiments

Uncapacitated vs Capacitated case

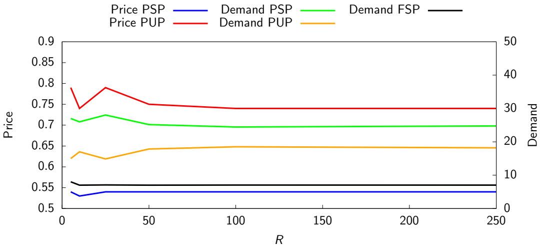
- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

Price differentiation by population segmentation

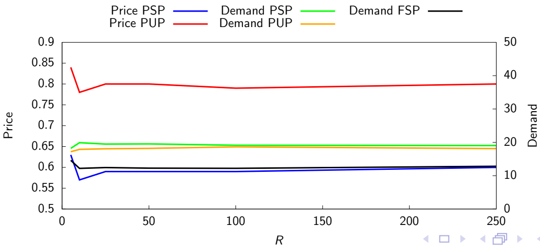
- Reduced price for residents
- Two scenarios
    1. Subsidy offered by the municipality
    2. Operator is forced to offer a reduced price

Uncapacitated vs Capacitated case

Uncapacitated



Capacitated



Computational time

	Uncapacitated case				Capacitated case			
R	Sol time	PSP	PUP	Rev	Sol time	PSP	PUP	Rev
5	2.58 s	0.54	0.79	26.43	12.0 s	0.63	0.84	25.91
10	3.98 s	0.53	0.74	26.36	54.5 s	0.57	0.78	25.31
25	29.2 s	0.54	0.79	26.90	13.8 min	0.59	0.80	25.96
50	4.08 min	0.54	0.75	26.97	50.2 min	0.59	0.80	26.10
100	20.7 min	0.54	0.74	26.90	6.60 h	0.59	0.79	26.03
250	2.51 h	0.54	0.74	26.85	1.74 days	0.60	0.80	25.93

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Summary

Demand and supply

- Supply: prices and capacity
- Demand: choice of customers
- Interaction between the two

Discrete choice models

- Rich family of behavioral models
- Strong theoretical foundations
- Great deal of concrete applications
- Capture the heterogeneity of behavior
- Probabilistic models

Optimization

Discrete choice models

- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation

- Linear in the decision variables
- Large scale
- Fairly general

Ongoing research

- Decomposition methods.
- Competitive markets: several suppliers.

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