同济大学交通运输工程学院
COLLEGE OF TRANSPORTATION ENGINEERING
TONGJI UNIVERSITY

Further Development of Multiple Discrete-Continuous Extreme Value (MDCEV) Model and Comparisons with OASIS Framework

Speaker: Mengyi Wang

Supervisor: Michel Bierlaire

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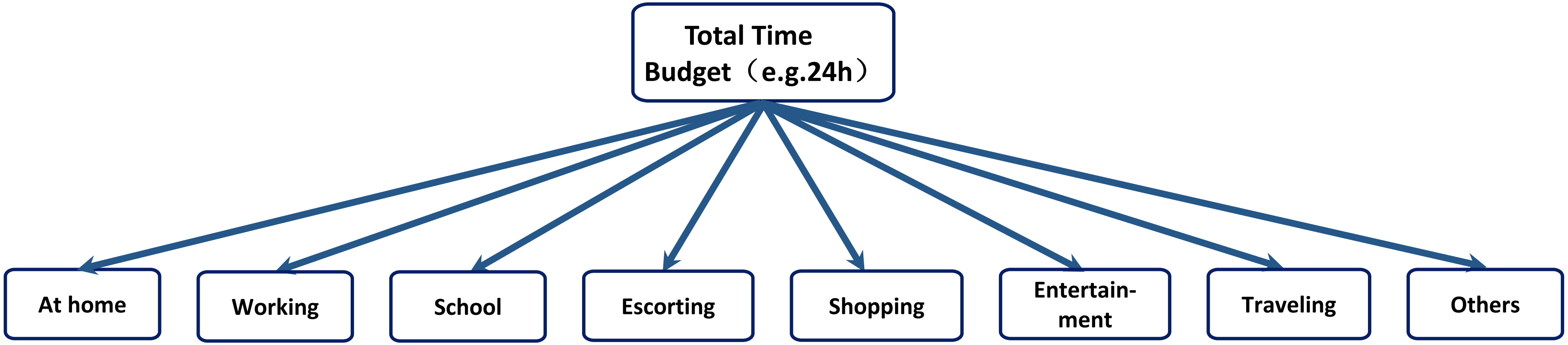
CONTENTS

Further Development of MDCEV model

Time Use in Activity-based Modeling (ABM)

- **Time use** can reflect the relationship between people's quality of life and transportation.
- Help us understand the micro-mechanism of **travel demand** generation.
- As **input information** for subsequent modules in ABM.



Existing Methods in Time Use

Time Use Models

- Fractional Logit Model
- Hazard-based Duration Model
- Structural Equation Model
- Tobit Model
- Multiple Discrete-Continuous Extreme Value (MDCEV) Model

MDCEV:

- Multiple activity participation and corresponding durations
- Random utility theory with diminishing marginal utilities
- Time budget constraint
- Efficient estimation

Traditional MDCEV Model (Bhat, 2008)

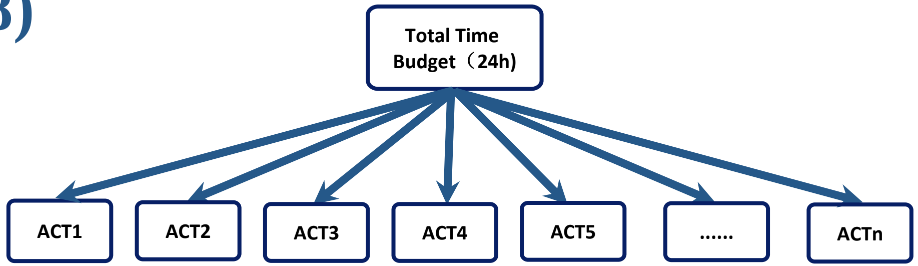
- Model Structure (k is the index of activity type; Individual subscript n is omitted):

$$\max\ U(\boldsymbol{t}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k} \psi_k \left\{ \left(\frac{t_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right\}$$

$$\text{s.t.} \sum_{k=1}^{K} t_k = T \quad (T > 0)$$

Where：ψ_k— baseline marginal utility；
γ_k and α_k— satiation parameters.
（$\psi_k > 0$，$\gamma_k > 0$，$0 < \alpha_k < 1$ for $\forall k$）

$$\psi(\boldsymbol{z}_k, \varepsilon_k) = \exp(\boldsymbol{\beta}' \boldsymbol{z}_k + \varepsilon_k)$$



To get maximum utility

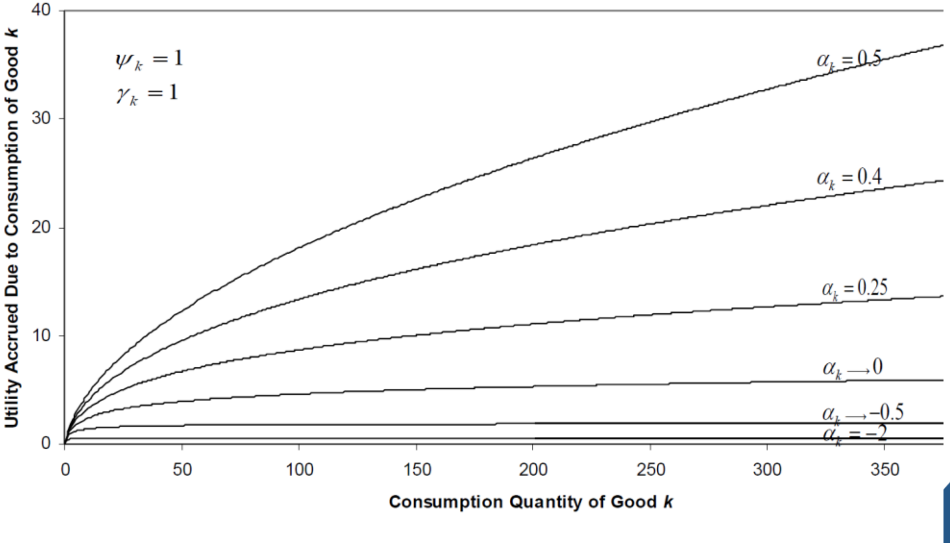


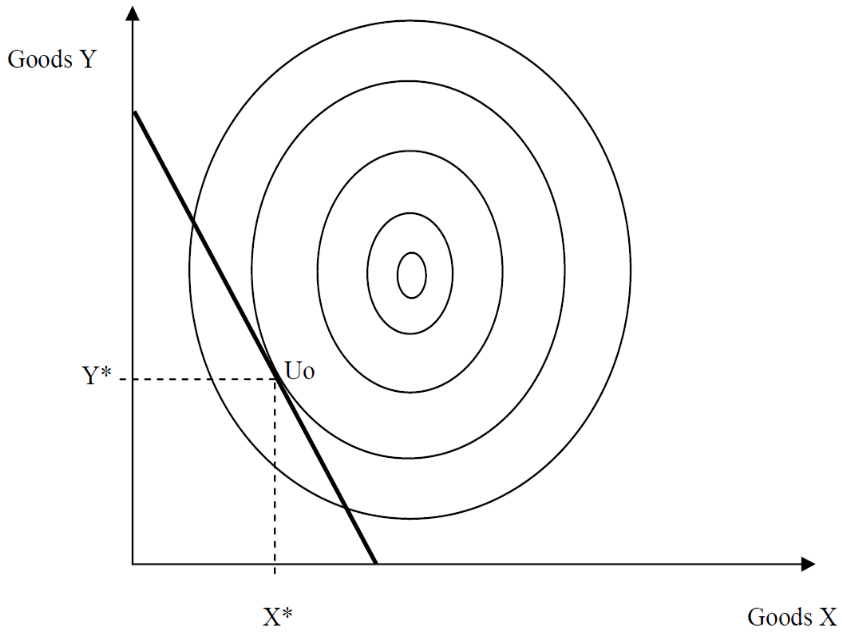
Figure 3. Effect of α_k Value on Good k's Subutility Function Profile

Inspirations From Economic Field

Examples:

- **Non-monotonic preferences** on consumption has been surveyed and confirmed;
- **Non-monotonic functions** for microeconomic analysis of sufficiency economy has been proposed;
- **Non-monotonic parabolic utility function** to describe the relationship between happiness and consumption has been proposed.

$$U = \alpha X + \mu Y - \frac{1}{2}\left(\beta X^2 + \theta Y^2\right)$$



Indifference curves (IC)

Parabolic MDCEV Model

- Model Structure (k is the index of activity type; Individual subscript n is omitted):

$$\max \ U(\boldsymbol{t}) = \sum_{k=1}^{K} -0.5\psi_k (t_k - m_k)^2$$

$$\text{s.t.} \sum_{k=1}^{K} t_k = T \quad (T > 0)$$

Where： ψ_k— opening size parameter；
m_k—extreme point parameter.
（$\psi_k > 0$ for $\forall k$）

$$m_k = \boldsymbol{\beta}_k{}' \boldsymbol{x}_k + \varepsilon_k \qquad \psi_k = \exp(\boldsymbol{\gamma}_k{}' \boldsymbol{z}_k)$$

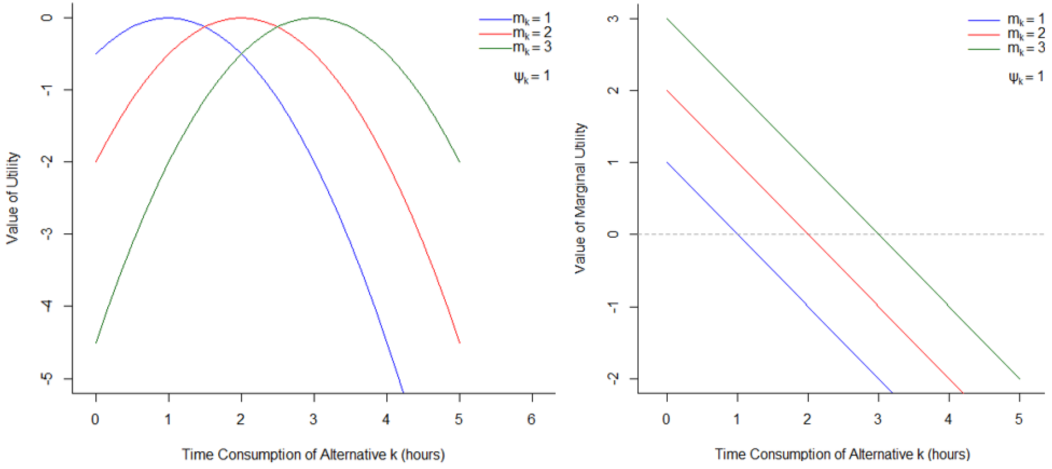


FIGURE 1 Effect of m_k on utility (left) and marginal utility (right)

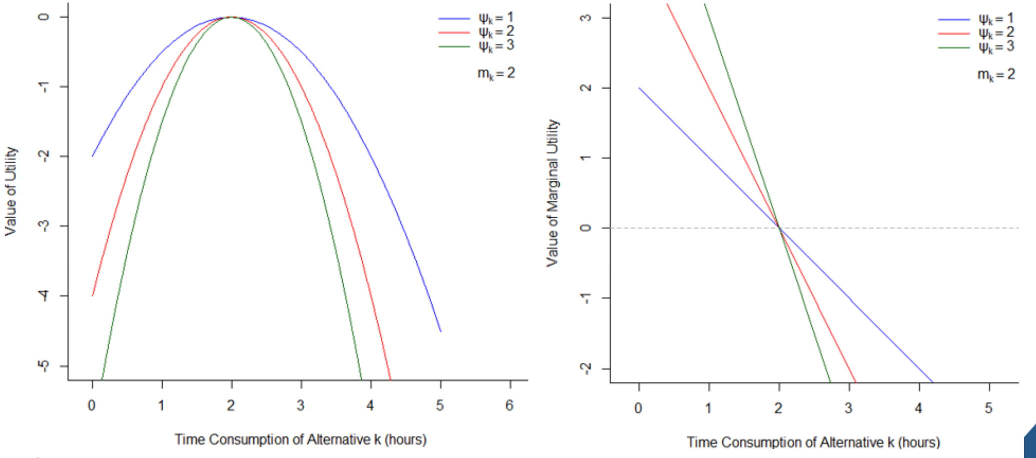
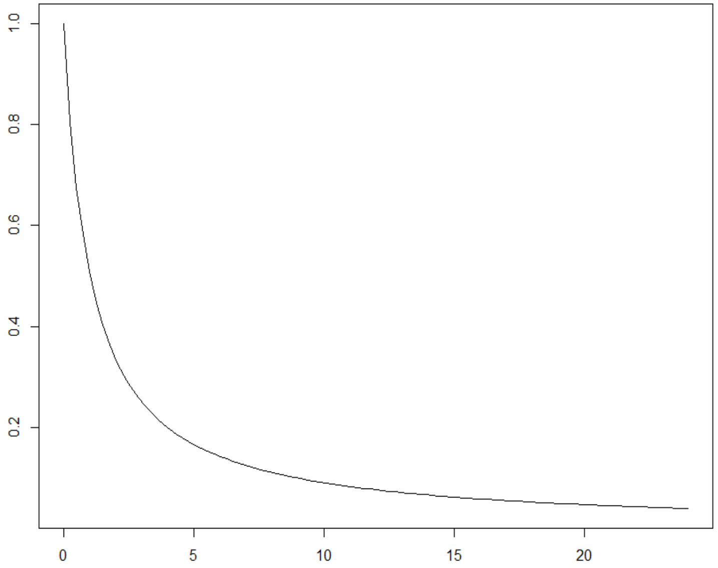


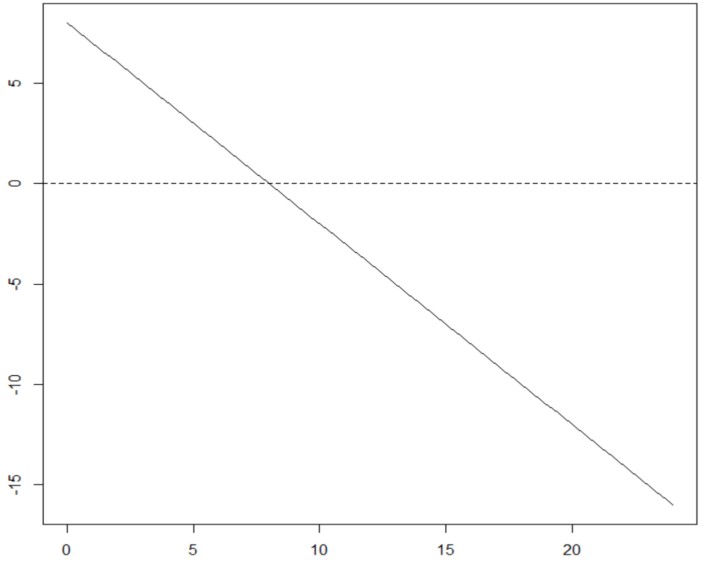
FIGURE 2 Effect of ψ_k on utility (left) and marginal utility (right)

Marginal Utilities

Traditional MDCEV model



Parabolic MDCEV model



Likelihood Function

Traditional MDCEV model

Parabolic MDCEV model

Construct the **Lagrangian function**

$$\mathcal{L} = \sum_k \frac{\gamma_k}{\alpha_k}[\exp(\beta' z_k + \varepsilon_k)]\left\{\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right\} - \lambda\left[\sum_{k=1}^{K} x_k - B\right]$$

$$\mathcal{L} = \sum_{k=1}^{K} -0.5\psi_k (t_k - m_k)^2 - \lambda\left[\sum_{k=1}^{K} t_k - T\right]$$

Apply the **KT first-order conditions** for solving the optimal time allocation, and get through some transformation:

$$V_k + \varepsilon_k = V_1 + \varepsilon_1 \text{ if } x_k^* > 0 \ \ (k = 2,3,\dots,K)$$

$$V_k + \varepsilon_k < V_1 + \varepsilon_1 \text{ if } x_k^* = 0 \ \ (k = 2,3,\dots,K)$$

Where:

$$V_k = \beta' z_k + (\alpha_k - 1)\ln\left(\frac{x_k^*}{\gamma_k}+1\right) \quad (k = 1,2,3,\dots,K)$$

$$V_k + \psi_k \varepsilon_k = V_1 + \psi_1 \varepsilon_1 \ \text{ if } t_k^* > 0 \ (k = 2,3,\dots,K)$$

$$V_k + \psi_k \varepsilon_k < V_1 + \psi_1 \varepsilon_1 \ \text{ if } t_k^* = 0 \ (k = 2,3,\dots,K)$$

Where:

$$V_k = -\psi_k \, t_k^* + \psi_k x_k \beta_k \quad (k = 1,2,3,\dots,K)$$

Assume that the ε_k of each alternative obeys the **Standard Gumbel Distribution** and **independent** of each other

$$P(x_1^*, x_2^*, x_3^*, \dots, x_M^*, 0,0,\dots,0) = \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M}\frac{1}{c_i}\right]\left[\frac{\prod_{i=1}^{M} e^{V_i}}{\left(\sum_{j=1}^{K} e^{V_j}\right)^M}\right](M-1)!$$

$$L = |J| \int_{-\infty}^{+\infty} g(\varepsilon_1)\cdot \prod_{i=2}^{M} g\left[\psi_i^{-1}(V_1 - V_i + \psi_1\varepsilon_1)\right]\cdot \prod_{s=M+1}^{K} G\left[\psi_s^{-1}(V_1 - V_s + \psi_1\varepsilon_1)\right] d\varepsilon_1$$

Model Estimation

Parabolic MDCEV model

$$\int_{-\infty}^{+\infty} g\left(\varepsilon_1\right) \cdot \prod_{i=2}^{M} g\left[\psi_i^{-1}\left(V_1 - V_i + \psi_1 \varepsilon_1\right)\right] \cdot \prod_{s=M+1}^{K} G\left[\psi_s^{-1}\left(V_1 - V_s + \psi_1 \varepsilon_1\right)\right] d\varepsilon_1$$

$$= \int_{-\infty}^{+\infty} \mathrm{e}^{-{\varepsilon_1}^2} \cdot \mathrm{e}^{{\varepsilon_1}^2} \cdot g\left(\varepsilon_1\right) \cdot \prod_{i=2}^{M} g\left[\psi_i^{-1}\left(V_1 - V_i + \psi_1 \varepsilon_1\right)\right] \cdot \prod_{s=M+1}^{K} G\left[\psi_s^{-1}\left(V_1 - V_s + \psi_1 \varepsilon_1\right)\right] d\varepsilon_1$$

$$\approx \sum_{i=1}^{n} w_i \cdot \mathrm{e}^{{x_i}^2} \cdot g\left(x_i\right) \cdot \prod_{i=2}^{M} g\left[\psi_i^{-1}\left(V_1 - V_i + \psi_1 x_i\right)\right] \cdot \prod_{s=M+1}^{K} G\left[\psi_s^{-1}\left(V_1 - V_s + \psi_1 x_i\right)\right]$$

Where： x_i— the node of the Gauss–Hermite quadrature；
w_i— the weight of the Gauss–Hermite quadrature;
n—the number of nodes and weights needed.

Model Simulation

If 'i' is a chosen alternative and 'j' is not a chosen alternative.

Traditional MDCEV model

$$\psi_i \left(\frac{t_i{}^*}{\gamma_i} + 1 \right)^{\alpha_i - 1} = \lambda, \; t_i{}^* > 0$$

$$\psi_j \left(\frac{t_j{}^*}{\gamma_j} + 1 \right)^{\alpha_j - 1} < \lambda, \; t_j{}^* = 0$$

$$\psi_j < \lambda < \psi_i$$

(Pinjari and Bhat, 2011)

KT conditions

Parabolic MDCEV model

$$-\psi_i (t_i{}^* - m_i) = \lambda, \; t_i{}^* > 0$$

$$-\psi_j (t_j{}^* - m_j) < \lambda, \; t_j{}^* = 0$$

$$\psi_j m_j < \lambda < \psi_i m_i$$

Model Simulation

Parabolic MDCEV model

$$t_i^* = m_i - \frac{\lambda}{\psi_i}, i = 1,2, \ldots, M \tag{19}$$

Substituting for t_i^* from above into **Equation 2**, the Lagrange multiplier can be obtained as:

$$\lambda = \frac{-T + \sum_{i=1}^{M} m_i}{\sum_{i=1}^{M} \psi_i^{-1}} \tag{20}$$

Therefore, a simple and efficient algorithm has been proposed as follows:

Step 1: Given the input data ($\boldsymbol{x}_k$, $\mathbf{z}_k$, T), model parameters ($\boldsymbol{\beta}_k$, $\boldsymbol{\gamma}_k$), random items (ε_k) drawn from standard Gumbel distribution, compute the product $\psi_k m_k$ values for all alternatives ($k = 1,2,3, \ldots, K$). Arrange all the K alternatives for the individual in the descending order of $\psi_k m_k$ values, and let this new ordering be indexed by i ($i = 1,2,3, \ldots, K$). Let $t_i^* = 0$ and $M = 1$. Go to step 2.

Step 2: Compute λ by **Equation 20**. Go to step 3.

Step 3: If $\lambda < \psi_{M+1} m_{M+1}$ and $M < K$, let $M = M + 1$, and go to step 2. Otherwise, go to step 4.

Step 4: Compute the optimal time consumptions for the first M alternatives by **Equation 19** and stop.

Simulation Experiments

$$m_1 = -0.5 + 0.2x_1 + \varepsilon_1$$

$$m_2 = -1.5 + 1.0x_2 + \varepsilon_2$$

$$m_3 = -1.0 + 0.4x_3 + \varepsilon_3$$

$$m_4 = 0.6 - 0.4x_4 + \varepsilon_4$$

$$\psi_1 = \exp(1.0 + 0.6z_1)$$

$$\psi_2 = \exp(1.5 + 0.5z_2)$$

$$\psi_3 = \exp(-0.8 + 1.0z_3)$$

$$\psi_4 = \exp(0.3z_4)$$

x_1, x_2, x_3, x_4 ~ U(0,10); z_1, z_2, z_3, z_4 ~ U(0,1)

T~U(0,24); $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ ~ G(0,1)

TABLE 1 Simulation results (Sample size=4000; Repetition number=30)

True value	Min.	Max	Mean	S.D.	Mean (est.S.D.)
-0.5	-0.628	-0.353	-0.505	0.072	0.058
0.2	0.190	0.218	0.201	0.007	0.006
-1.5	-1.644	-1.384	-1.498	0.060	0.045
1.0	0.984	1.013	1.000	0.007	0.006
-1.0	-1.528	-0.570	-1.025	0.275	0.229
0.4	0.370	0.421	0.401	0.012	0.014
0.6	0.239	0.939	0.581	0.174	0.148
-0.4	-0.416	-0.377	-0.398	0.009	0.010
1.0	0.958	1.073	1.004	0.027	0.034
0.6	0.462	0.739	0.597	0.058	0.051
1.5	1.376	1.605	1.501	0.047	0.047
0.5	0.328	0.727	0.495	0.094	0.079
-0.8	-0.855	-0.746	-0.804	0.025	0.025
1.0	0.896	1.089	1.001	0.045	0.039
0.3	0.227	0.361	0.296	0.027	0.026

Case Study

- Data source: 2019 American Time Use Survey (ATUS)
- Research scope: time use of weekend outdoor non-mandatory activities
- Activity classification: shopping, socializing, recreation, personal and household affairs
- Number of cases: 4413
- Variables selected: individual and household demographics

ψ_k component		
Household demographics		
Annual household income < $25,000		
Socializing	-2.1609	-23.53
Recreation	3.0422	9.41
Individual demographics		
Male		
Shopping	0.1873	3.26
Recreation	-2.8884	-57.12
The highest degree is a bachelor or above		
Shopping	-2.9926	-15.11
Socializing	-4.9841	-27.30
Personal and household affairs	-2.5967	-11.91

Summary Statistics	
Number of cases	4413
Log-likelihood at convergence	-10083.88
Log-likelihood with only constants	-10183.09
Likelihood ratio test value	198.42

TABLE 3 Estimation results of the parabolic MDCEV model

Explanatory variables	Parameter	t-Statistic
m_k component		
Constants		
Shopping	-0.7371	-32.34
Socializing	0.2680	5.88
Recreation	-2.0346	-17.41
Household demographics		
Number of household members		
Socializing	0.0495	9.36
Recreation	0.1259	12.00
Personal and household affairs	0.0788	4.31
Presence of household children < 18 years old		
Shopping	-0.1561	-15.44
Recreation	-0.5067	-14.98
Personal and household affairs	-0.2652	-3.48
Annual household income < $25,000		
Shopping	0.1451	3.08
Recreation	-0.1725	-6.11
Personal and household affairs	0.1460	1.96
Household is from metropolitan areas		
Shopping	0.0553	3.59
Socializing	-0.0629	-2.95
Recreation	0.5524	5.12
Individual demographics		
Male		
Shopping	-0.0891	-8.44
Recreation	0.4197	4.68
Age 15-40 years		
Shopping	-0.0321	-3.00
Socializing	-0.1610	-5.03
Recreation	0.3290	12.53
Age 41-60 years		
Socializing	-0.2052	-5.34
Recreation	0.3597	10.71
Personal and household affairs	0.1184	2.01
The highest degree is a bachelor or above		
Shopping	0.2030	4.18
Socializing	-1.1566	-2.29
White		
Socializing	0.6393	30.20
Recreation	0.3726	21.30
Personal and household affairs	0.2084	3.86
Presence of spouse in the household		
Shopping	0.1854	20.78
Recreation	-0.3458	-21.28

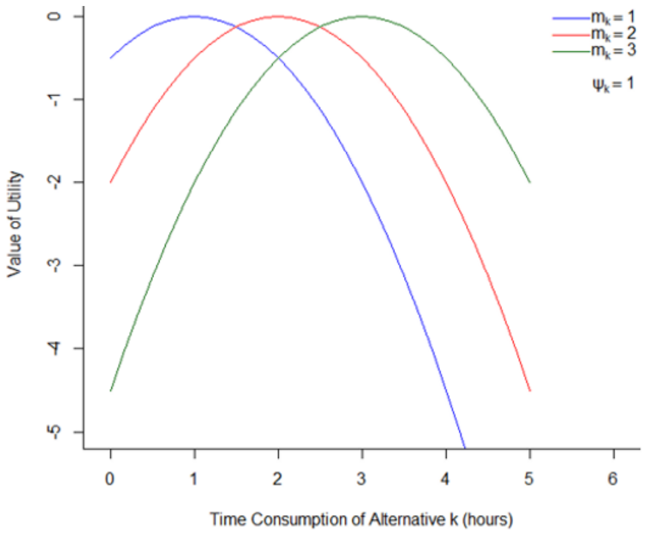
Variable Effects

- m_k Component

$$U_k = -0.5\psi_k(t_k - m_k)^2$$

$$m_k = \boldsymbol{\beta}_k{}'\boldsymbol{x}_k + \varepsilon_k$$

$$E(m_k) = \boldsymbol{\beta}_k{}'\boldsymbol{x}_k$$



- The larger the value of m_k, the larger the expected value of time allocation.

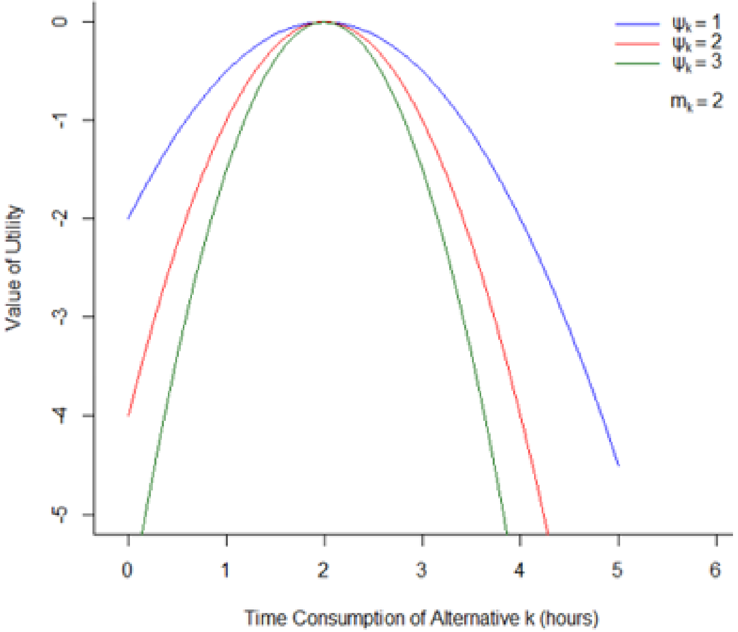
Explanatory variables	Parameter
Individual demographics	
Male	
Shopping	-0.0891
Recreation	0.4197
Age 15-40 years	
Shopping	-0.0321
Socializing	-0.1610
Recreation	0.3290
Age 41-60 years	
Socializing	-0.2052
Recreation	0.3597
Personal and household affairs	0.1184

Variable Effects

- ψ_k Component

$$U_k = -0.5\psi_k(t_k - m_k)^2$$

$$\psi_k = \exp(\boldsymbol{\gamma}_k{}'\boldsymbol{z}_k)$$



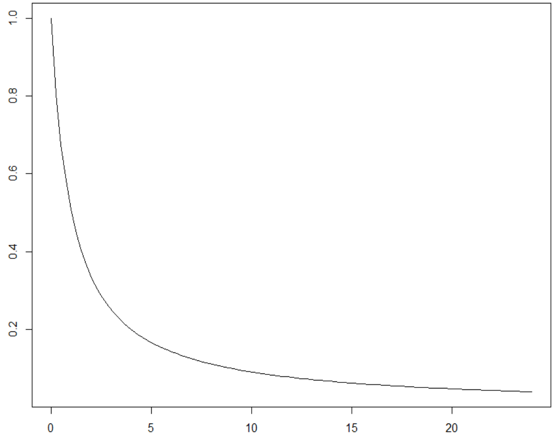
➢ The larger the value of ψ_k, the steeper the utility curve, indicating satisfaction or boredom is more easier to arise.

Explanatory variables	Parameter
Household demographics	
Annual household income < $25,000	
Socializing	-2.1609
Recreation	3.0422

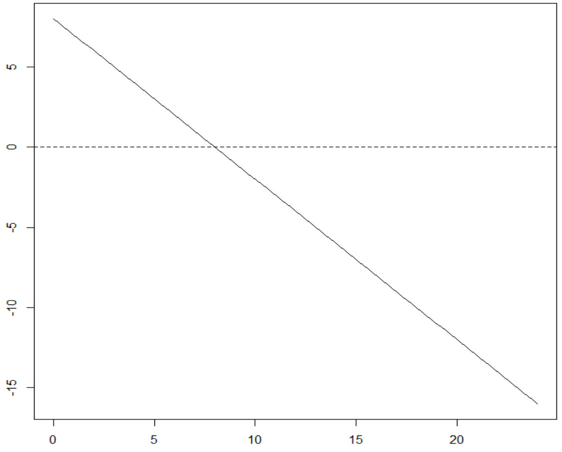
Further Work

Marginal utility curves:

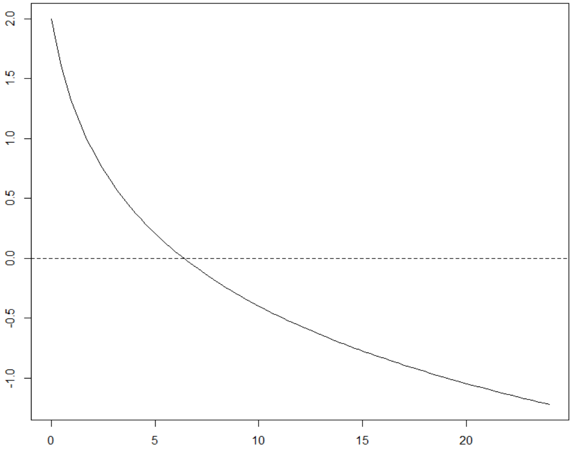
Traditional MDCEV model



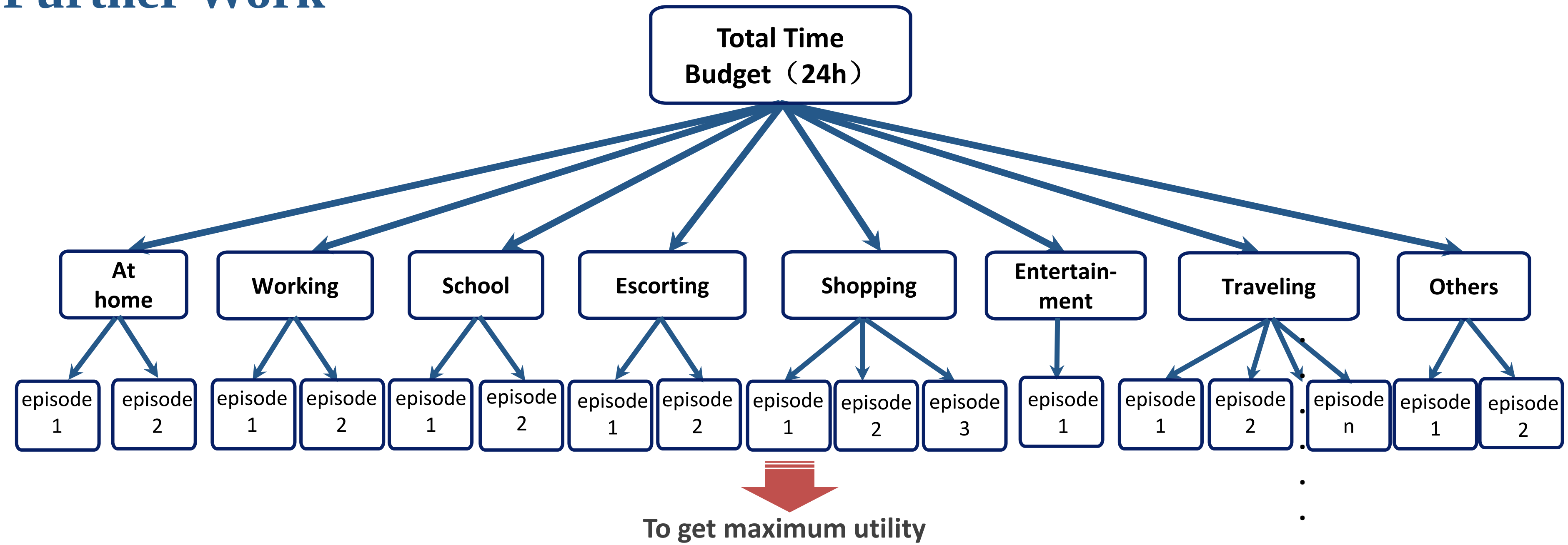
Parabolic MDCEV model



Future MDCEV model



Further Work



- To take a **single activity episode** as an alternative in the MDCEV model framework. The model should allocate time to each episode of different activity types but not to each activity type.
- The **frequency** of all kinds of activities and the **duration** of each episode can be obtained simultaneously.
- Provide information for **downstream travel behavior choices**, such as travel mode, destination and route, etc.

Comparisons between MDCEV and OASIS

Comparison List

Features		MDCEV	OASIS
Model Structure	Principle	Constrained optimization problem with random utility theory	
	Input	Individual and household attributes; Model parameters; Error terms	A set of considered activities with locations and transport modes; Scheduling preferences; Activity flexibility; Model parameters; Error terms
	Output	Time allocation of different activities	Activity schedule for whole day
	Modeling Dimensions	Activity participation; Activity duration , Activity frequency; Chronological order of episodes from the same activity category	Activity participation; Activity start time; Activity duration; Activity frequency; Activity sequence; Activity location; Travel mode; Travel time
	Utility Function	Non-linear	Linear
	Explanatory Variables	Individual and household attributes	Activity-travel attributes
	Choice Set	All activity categories	All valid schedules
Model Estimation		Maximum likelihood estimation	Maximum likelihood estimation relied on sampled choice set
Model Simulation		Specific algorithm by certain rules	Standard mathematical programming algorithm
Model Extension		In-home activities; Household level; Different distributions of error terms; Random coefficients	
Research Focus		Exploring time allocation mechanism by sophisticated utility functions; Describing influence factors by estimation results	Exploring daily scheduling mechanism by integrating all dimensions; Capturing trade-offs between different choices by model simulation

Model Structure—Principle

MDCEV

$$\max\ U(\boldsymbol{t}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k}\psi_k\left\{\left(\frac{t_k}{\gamma_k}+1\right)^{\alpha_k}-1\right\}$$

$$\text{s.t.} \sum_{k=1}^{K} t_k = T$$

Where: ψ_k— baseline marginal utility;

($\psi_k > 0,\ \psi(\boldsymbol{z}_k, \varepsilon_k) = \exp(\boldsymbol{\beta}'\boldsymbol{z}_k + \varepsilon_k)$)

γ_k and α_k— satiation parameters.

($\gamma_k > 0$, $0 < \alpha_k < 1$)

OASIS

$$\max_{\omega,z,x,\tau} U(\omega, z, x, \tau, \epsilon) + \sum\nolimits_{a=0}^{A} \omega_a(V_a^1 + V_a^2 + V_a^3) + \sum\nolimits_{a=0}^{A} \sum\nolimits_{b=0}^{A} z_{ab}(V_{ab}^4 + V_{ab}^5).$$

$$\sum_a\sum_b(\omega_a\tau_a + z_{ab}\rho_{ab}) = T,$$

$$\sum_a\sum_b(\omega_a c_a + z_{ab}\kappa_{ab}) \le B,$$

$$\omega_{\text{dawn}} = \omega_{\text{dusk}} = 1,$$

$$\tau_a \ge \omega_a\tau_a^{\min}, \quad \forall a \in A,$$

$$\tau_a \le \omega_a T, \quad \forall a \in A,$$

$$z_{ab} + z_{ba} \le 1, \quad \forall a, b \in A, a \ne b,$$

$$z_{a,\text{dawn}} = z_{\text{dusk},a} = 0, \quad \forall a \in A,$$

$$\sum_a z_{ab} = \omega_b, \quad \forall b \in A, b \ne \text{dawn},$$

$$\sum_b z_{ab} = \omega_a, \quad \forall a \in A, a \ne \text{dusk},$$

$$(z_{ab} - 1)T \le x_a + \tau_a + z_{ab}\rho_{ab} - x_b, \quad \forall a, b \in A, a \ne b,$$

$$(1 - z_{ab})T \ge x_a + \tau_a + z_{ab}\rho_{ab} - x_b, \quad \forall a, b \in A, a \ne b,$$

$$\sum_{a\in G_k}\omega_a, \le 1 \quad k = 1, \ldots, K,$$

$$\alpha_a^m = 1 \quad \forall a \in G_{\text{home}}$$

$$\omega_a \le \alpha_a^m \quad \forall a \in A^m$$

$$\omega_a \ge \omega_b + z_{ab} - 1 \quad \forall a \in A, b \in A \setminus G_{\text{home}}$$

Where, the decision variables are:

ω_a—Whether activity a is selected;

z_{ab}—Whether activity b is after a;

x_a— Start time of activity a;

τ_a—Duration of activity a;

α_a^m—Whether private mode m is available for activity a.

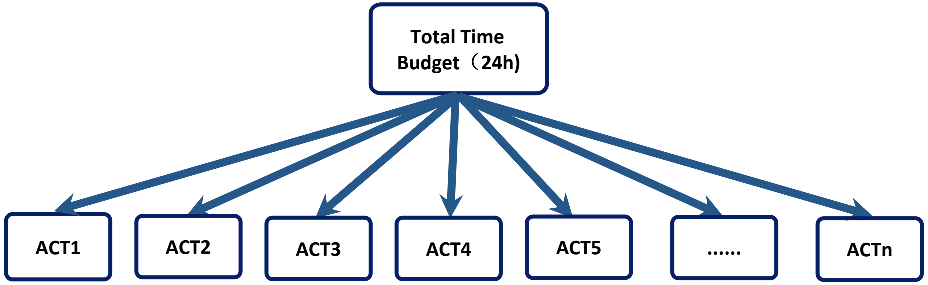
Model Structure

MDCEV

Input:

- Individual demographics: age, sex, education, etc.
- Household demographics: household size, household income, etc.

Output:



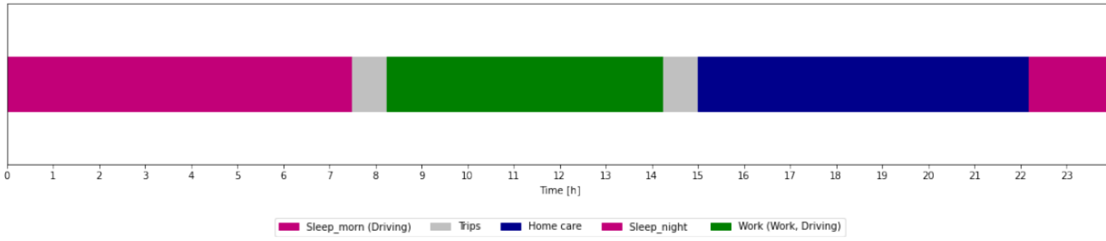
Individual number	Predicted (actual) time allocation in the four outdoor activity types (hours)			
	Shopping	Socializing	Recreation	Personal and household affairs
1	2.26 (1.5)	3.81 (4.25)	0 (0)	2.43 (2.75)
2	0 (0)	0 (0)	0 (0)	0.33 (0.33)
3	0 (0)	1.67 (1.33)	3.92 (4.42)	0.16 (0)
4	0 (0.17)	0 (0)	0 (0)	0.92 (0.75)
5	0 (0)	1.08 (1.08)	0 (0)	0 (0)

OASIS

Input:

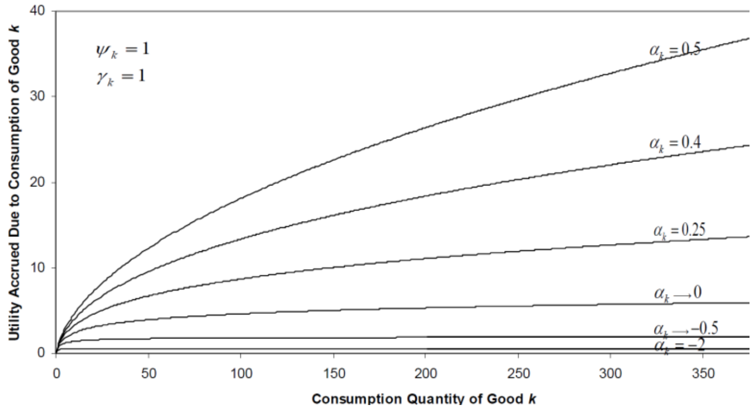
Person	Schedule	Activity	Start time (hh:mm)	Duration (hh:mm)	Location	Mode	Participation mode
Sara	Isolated indiv./ Fam. of 2/ Fam. of 3	Sleep_morn	00:00	6:30	Home	No trip	Solo
		Sleep_morn	00:00	6:30	Home	PT	Solo
		Sleep_morn	00:00	6:30	Home	Driving	Solo
		Personal care	6:30	2:00	Home	No trip	Solo
		Personal care	6:30	2:00	Home	PT	Solo
		Personal care	6:30	2:00	Home	Driving	Solo
		Work	8:30	6:00	Work	PT	Solo
		Work	8:30	6:00	Work	Driving	Solo
		Work	8:30	6:00	Home	No trip	Solo
		Work	8:30	6:00	Home	PT	Solo
		Work	8:30	6:00	Home	Driving	Solo
		Home care	14:30	7:40	Home	No trip	Solo
		Home care	14:30	7:40	Home	PT	Solo
		Home care	14:30	7:40	Home	Driving	Solo
		Sleep_night	22:10	1:50	Home	No trip	Solo

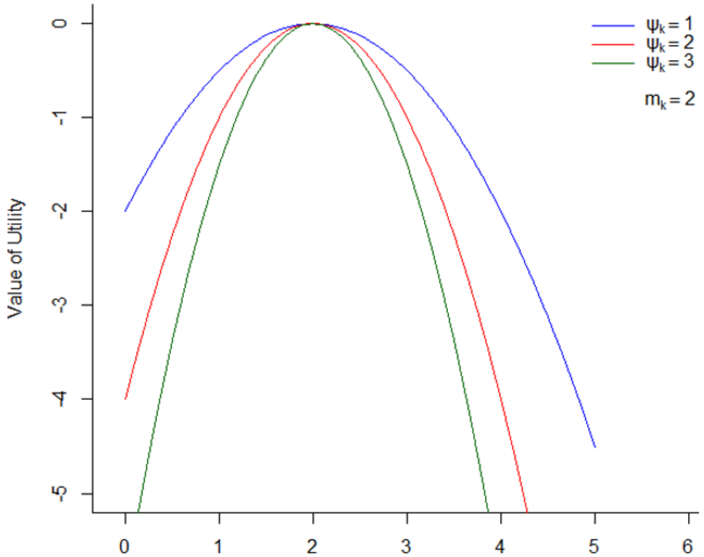
Output:



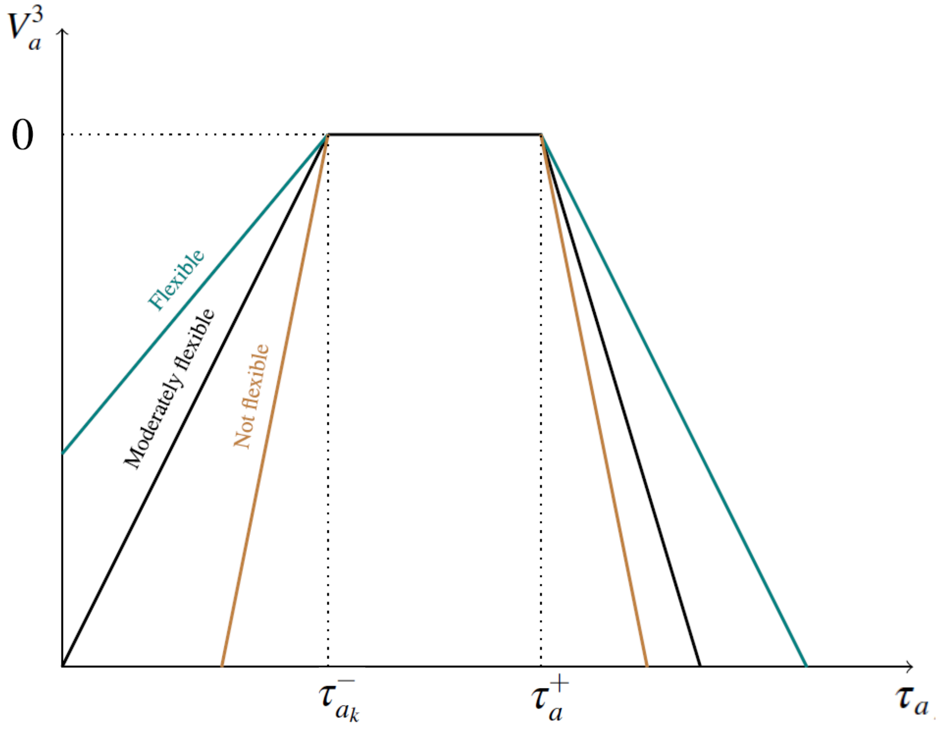
Model Structure——Utility Function

MDCEV





OASIS



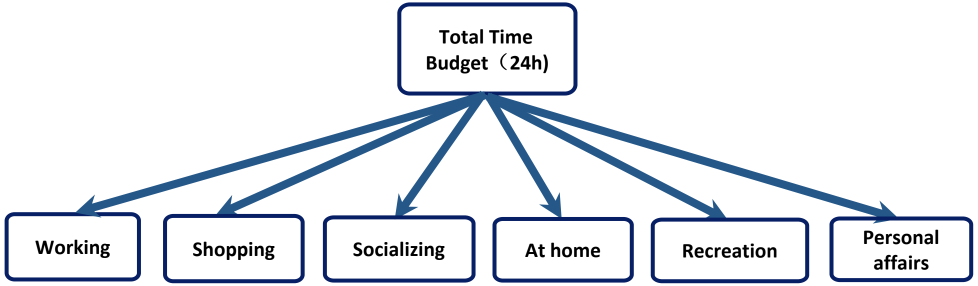
Model Structure

MDCEV

Explanatory Variables:

- Individual demographics: age, sex, education, etc.
- Household demographics: household size, household income, etc.

Choice set:

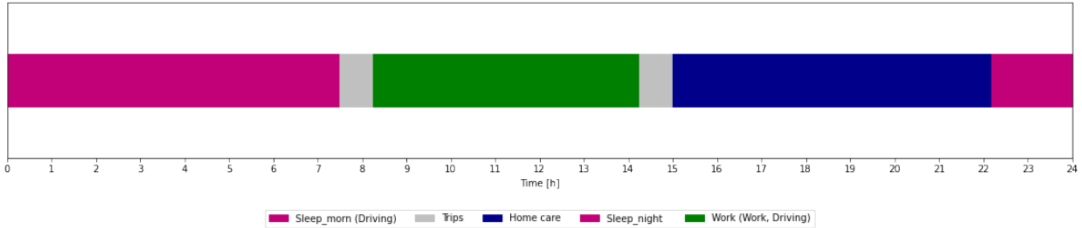


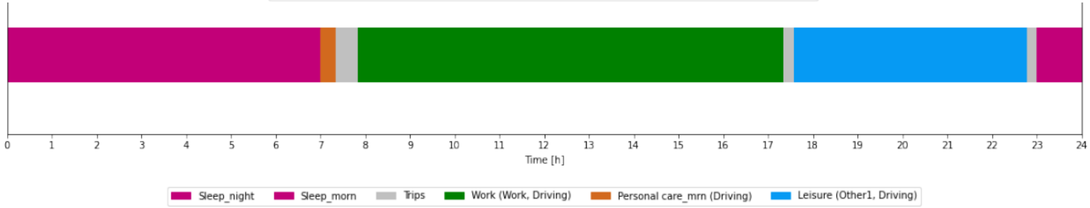
OASIS

Explanatory Variables:

- Activity specific attributes: participation cost, etc.
- Travel specific attributes: travel cost, travel time, etc.

Choice set:





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Model Estimation

MDCEV

$$\max\ U(\boldsymbol{t}) = \sum_{k=1}^{K} \frac{\gamma_k}{\alpha_k}\psi_k\left\{\left(\frac{t_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right\}$$

$$\text{s.t.} \sum_{k=1}^{K} t_k = T$$

$$\mathcal{L} = \sum_{k} \frac{\gamma_k}{\alpha_k}[\exp(\beta' z_k + \varepsilon_k)]\left\{\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right\} - \lambda\left[\sum_{k=1}^{K} x_k - B\right]$$

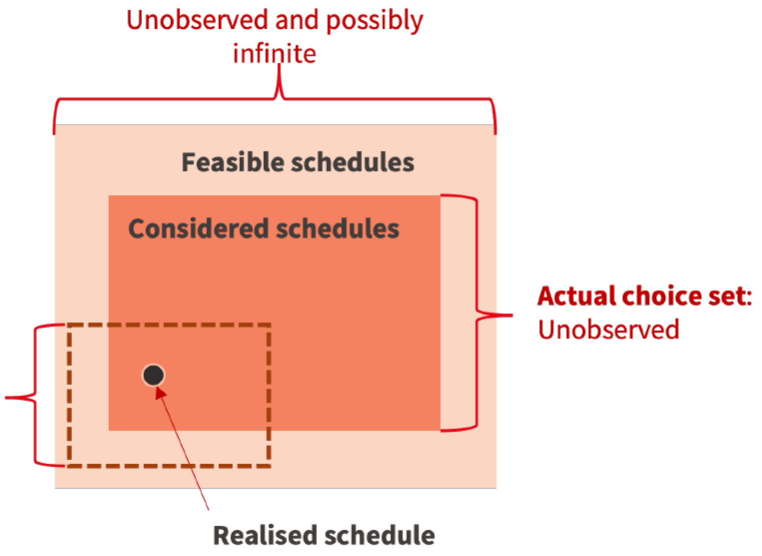
$$P(x_1^*, x_2^*, x_3^*, \dots, x_M^*, 0,0,\dots,0) = \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M} \frac{1}{c_i}\right]\left[\frac{\prod_{i=1}^{M} e^{V_i}}{\left(\sum_{j=1}^{K} e^{V_j}\right)^M}\right](M-1)!$$

Maximum likelihood estimation

OASIS

M-H algorithm

Estimation choice set: sample of feasible schedules generated for estimation purposes



$$P\left(i_n|\mathcal{C}_n\right) = \frac{\exp\left[V_{in} + \ln q(\mathcal{C}_n|i_n)\right]}{\sum_{j\in\mathcal{C}_n}\exp\left[V_{jn} + \ln q(\mathcal{C}_n|j)\right]}$$

Maximum likelihood estimation based on sampled choice set

Model Simulation

MDCEV

If 'i' is a chosen alternative and 'j' is not a chosen alternative.

KT conditions

$$\begin{cases} \psi_i \left(\frac{t_i^*}{\gamma_i} + 1\right)^{\alpha_i - 1} = \lambda, \ t_i^* > 0 \\ \psi_j \left(\frac{t_j^*}{\gamma_j} + 1\right)^{\alpha_j - 1} < \lambda, \ t_j^* = 0 \end{cases}$$

$$\psi_j < \lambda < \psi_i$$

Specific algorithm by certain rules

OASIS

$$\max_{\omega,z,x,\tau} U(\omega, z, x, \tau, \epsilon) + \sum_{a=0}^{A} \omega_a (V_a^1 + V_a^2 + V_a^3) + \sum_{a=0}^{A} \sum_{b=0}^{A} z_{ab} (V_{ab}^4 + V_{ab}^5).$$

$$\sum_a \sum_b (\omega_a \tau_a + z_{ab} \rho_{ab}) = T,$$

$$\sum_a \sum_b (\omega_a c_a + z_{ab} \kappa_{ab}) \le B,$$

$$\omega_{\text{dawn}} = \omega_{\text{dusk}} = 1,$$

$$\tau_a \ge \omega_a \tau_a^{\min}, \quad \forall a \in A,$$

$$\tau_a \le \omega_a T, \quad \forall a \in A,$$

$$z_{ab} + z_{ba} \le 1, \quad \forall a, b \in A, a \ne b,$$

$$z_{a,\text{dawn}} = z_{\text{dusk},a} = 0, \quad \forall a \in A,$$

$$\sum_a z_{ab} = \omega_b, \quad \forall b \in A, b \ne \text{dawn},$$

$$\sum_b z_{ab} = \omega_a, \quad \forall a \in A, a \ne \text{dusk},$$

$$(z_{ab} - 1)T \le x_a + \tau_a + z_{ab}\rho_{ab} - x_b, \quad \forall a, b \in A, a \ne b,$$

$$(1 - z_{ab})T \ge x_a + \tau_a + z_{ab}\rho_{ab} - x_b, \quad \forall a, b \in A, a \ne b,$$

$$\sum_{a \in G_k} \omega_a, \le 1 \quad k = 1, \ldots, K,$$

$$\alpha_a^m = 1 \quad \forall a \in G_{\text{home}}$$

$$\omega_a \le \alpha_a^m \quad \forall a \in A^m$$

$$\omega_a \ge \omega_b + z_{ab} - 1 \quad \forall a \in A, b \in A \setminus G_{\text{home}}$$

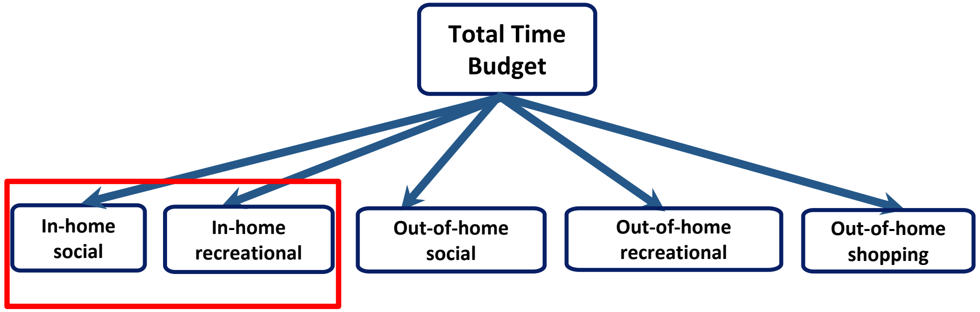
Where: ω_a, z_{ab}, x_a, τ_a, α_a^m are decision variables.

Standard mathematical programming algorithm

Model Extension

MDCEV

In-home activities:



Household level:

$$U_g = U_S^g + U_J^g + U_A^g + U_0^g$$

$$U_S^g = \sum_m^M \sum_s^{N^S} u_{sm}^{gS}(t_{sm}^{gS})$$

Error terms and parameters:

$$\varepsilon_k \sim N(0,1) \qquad \beta_k \sim N(\beta_0^k, \sigma^2)$$

OASIS

In-home activities:

Person	Schedule	Activity	Start time (hh:mm)	Duration (hh:mm)	Location	Mode	Participation mod
Sara	Isolated indiv./ Fam. of 2/ Fam. of 3	Sleep_morn	00:00	6:30	Home	No trip	Solo
		Sleep_morn	00:00	6:30	Home	PT	Solo
		Sleep_morn	00:00	6:30	Home	Driving	Solo
		Personal care	6:30	2:00	Home	No trip	Solo
		Personal care	6:30	2:00	Home	PT	Solo
		Personal care	6:30	2:00	Home	Driving	Solo
		Work	8:30	6:00	Work	PT	Solo
		Work	8:30	6:00	Work	Driving	Solo
		Work	8:30	6:00	Home	No trip	Solo
		Work	8:30	6:00	Home	PT	Solo
		Work	8:30	6:00	Home	Driving	Solo
		Home care	14:30	7:40	Home	No trip	Solo
		Home care	14:30	7:40	Home	PT	Solo
		Home care	14:30	7:40	Home	Driving	Solo
		Sleep_night	22:10	1:50	Home	No trip	Solo

Household level:

$$\max \sum_{n=1}^{n=N_m} \left(w_n \left(U_n^{gen} + \sum_{a_n \in A^n} U_{a_n} \right) \right)$$

$$= \max \sum_{n=1}^{n=N_m} \left(w_n \left(U_n^{gen} + \sum_{a_n \in A^n} \left(U_{a_n}^{partic} + U_{a_n}^{start} + U_{a_n}^{duration} + \sum_{b_n \in A^n} U_{a_n b_n}^{travel_m} \right) \right) \right)$$

$$U_{a_n}^{partic} = U_{a_n}^{social} + U_{a_n}^{joint} + U_{a_n}^{escort}$$

Comparison List

Features		MDCEV	OASIS
Model Structure	Principle	Constrained optimization problem with random utility theory	
	Input	Individual and household attributes; Model parameters; Error terms	A set of considered activities with locations and transport modes; Scheduling preferences; Activity flexibility; Model parameters; Error terms
	Output	Time allocation of different activities	Activity schedule for whole day
	Modeling Dimensions	Activity participation; Activity duration , Activity frequency; Chronological order of episodes from the same activity category	Activity participation; Activity start time; Activity duration; Activity frequency; Activity sequence; Activity location; Travel mode; Travel time
	Utility Function	Non-linear	Linear
	Explanatory Variables	Individual and household attributes	Activity-travel attributes
	Choice Set	All activity categories	All valid schedules
Model Estimation		Maximum likelihood estimation	Maximum likelihood estimation relied on sampled choice set
Model Simulation		Specific algorithm by certain rules	Standard mathematical programming algorithm
Model Extension		In-home activities; Household level; Different distributions of error terms; Random coefficients	
Research Focus		Exploring time allocation mechanism by sophisticated utility functions; Describing influence factors by estimation results	Exploring daily scheduling mechanism by integrating all dimensions; Capturing trade-offs between different choices by model simulation

Thank you for listening!

同济大学交通运输工程学院
COLLEGE OF TRANSPORTATION ENGINEERING
TONGJI UNIVERSITY