9th TRIENNIAL SYMPOSIUM ON TRANSPORTATION ANALYSIS (TRISTAN IX), Aruba

Data-driven characterization of pedestrian traffic

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Outline

Outline

Motivation

Background

Importance

- Understanding, reproducing and forecasting phenomena that characterize pedestrian traffic is necessary in order to provide services related to pedestrian safety and convenience

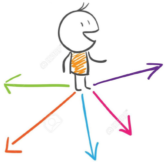
Vehicular traffic

- Well-established theory
- Regulated and separated by directions

Background

Pedestrian traffic

- Multidirectional, without strict rules for pedestrian to follow
- Pedestrians can occupy any part of the walkable area



Indicators

- Density k (ped/m^2), speed v (m/s) and flow q (ped/ms)
- Used to observe and to model the flows of pedestrians
- Consistent and unified approach to the definitions of the indicators is missing

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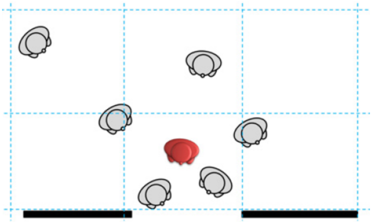
Grid-based (GB) method



$$k(A) = \frac{N}{|A|}$$

$$v(A) = \frac{\sum v_i}{N}$$

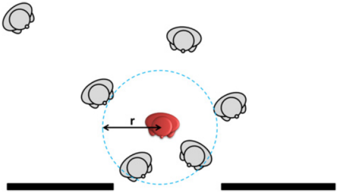
$$q(A) = k(A)v(A)$$

$$v_i = \sqrt{\left(\frac{\Delta x_i}{\Delta t}\right)^2 + \left(\frac{\Delta y_i}{\Delta t}\right)^2}$$

$$\Delta x_i = x_i(t + \Delta t) - x_i(t),\ \Delta y_i = y_i(t + \Delta t) - y_i(t)$$

[Duives et al., 2015]

Range-based (RB) method



$$k(A_r) = \frac{N}{|A_r|}$$

$$v(A_r) = \frac{\sum v_i}{N}$$

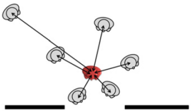
$$q(A_r) = k(A_r)v(A_r)$$

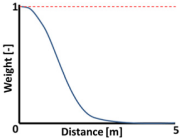
$$v_i = \sqrt{\left(\frac{\Delta x_i}{\Delta t}\right)^2 + \left(\frac{\Delta y_i}{\Delta t}\right)^2}$$

$$\Delta x_i = x_i(t + \Delta t) - x_i(t),\ \Delta y_i = y_i(t + \Delta t) - y_i(t)$$

[Duives et al., 2015]

Exponentially Weighted (EW) method





$$k(x,y,t) = \sum_{i=1} f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)$$

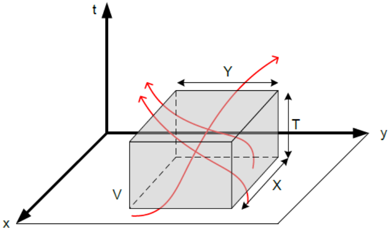
$$\vec{v}(x,y,t) = \frac{\sum_{i=1} \vec{v}_i(x,y,t) f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)}{\sum_{i=1} f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right)}$$

$$\vec{q}(x,y,t) = k(x,y,t)\vec{v}(x,y,t)$$

$$f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right) = \frac{1}{\pi R^2} \exp\left(- \frac{\left\| \begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \right\|^2}{R^2} \right)$$

[Helbing et al., 2007], [Duives et al., 2015]

XY-T method



$$k(V) = \frac{\sum_{i}^{N} t_i}{XYT}$$

$$\vec{q}(V) = \begin{pmatrix} q_x(V) \\ q_y(V) \end{pmatrix} = \begin{pmatrix} \frac{\sum_{i=1}^{N} x_i}{XYT} \\ \frac{\sum_{i=1}^{N} y_i}{XYT} \end{pmatrix}$$

$$\vec{v}(V) = \begin{pmatrix} v_x(V) \\ v_y(V) \end{pmatrix} = \begin{pmatrix} \frac{q_x(V)}{k(V)} \\ \frac{q_y(V)}{k(V)} \end{pmatrix}$$

[van Wageningen-Kessels et al., 2014], [Saberi and Mahmassani, 2014]

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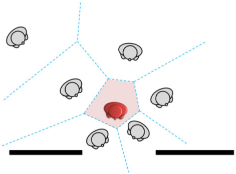
Voronoi-based (VB) method

A personal region A_i is assigned to each pedestrian i: each point p in the personal region of pedestrian i is closer to i than to any other, with respect of d_E

$$A_i = \{p | d_E(p, p_i) \leq d_E(p, p_j), \forall j\}$$



$$k(A_i) = \frac{1}{|A_i|}$$

$$v(A_i) = \sqrt{\left(\frac{\Delta x_i}{\Delta t}\right)^2 + \left(\frac{\Delta y_i}{\Delta t}\right)^2}$$

$$q(A_i) = k(A_i)v(A_i)$$

$$\Delta x_i = x_i(t + \Delta t) - x_i(t),\ \Delta y_i = y_i(t + \Delta t) - y_i(t)$$

[Steffen and Seyfried, 2010], [Duives et al., 2015]

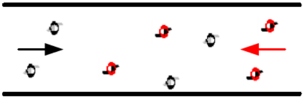
Arbitrary discretization

Sensitivity of results

The results might be very sensitive to minor changes

Unrealistic results

Velocity and flow vectors may cancel out when 2 equally sized streams of pedestrians walk with the same speed but in the opposite directions



How to define the discretization...

...independent of arbitrary chosen values?

One size and shape fits all?

It is all about adjustments...

Outline

Data-driven approach

Keep calm and let data speak!

Outline

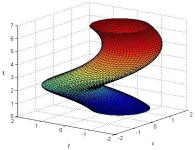
Data-driven discretization framework

Pedestrian trajectories

$$\Gamma_i : \{p_i(t) | p_i(t) = (x_i(t), y_i(t), t)\}$$

3D Voronoi diagrams associated with trajectories

Each trajectory Γ_i is associated with a 3D Voronoi 'tube' V_i



$$V_i = \{p | \min\{d_*(p, p_i) | p_i \in \Gamma_i\} \leq \min\{d_*(p, p_j) | p_j \in \Gamma_j\}, \forall j\}$$

$d_*(p, p_i)$ - spatio-temporal assignment rule

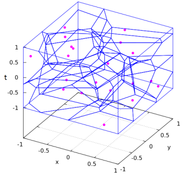
Data-driven discretization framework

Sample of points

$$\Gamma_i : \{p_{is} | p_{is} = (x_{is}, y_{is}, t_s)\}, t_s = [t_0, t_1, ..., t_f]$$

3D Voronoi diagrams associated with the points

Sequences of 3D Voronoi cells V_{is} are assigned to the sequence of points for each pedestrian



$$V_i = \{V_{is} | V_{is} = \{p | d_*(p, p_{is}) \leq d_*(p, p_{js})\}, \forall j\}$$

$d_*(p, p_i)$ - spatio-temporal assignment rule

Spatio-temporal assignment rules

Naive assignment rule (N-3DVoro)

$$d_N(p, p_i) = \begin{cases} \sqrt{(p - p_i)^T (p - p_i)}, & \Delta t = 0 \\ \infty, & \textit{otherwise} \end{cases}$$

Time-Transform assignment rules ($TT_{\{1,2,3\}}$-3DVoro)

$$\mathrm{d}_{TT_1}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + \alpha^2 (t - t_i)^2}$$

$$d_{TT_2}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2} + \alpha_i(t_i)|(t - t_i)|$$

$$d_{TT_3}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + \alpha_i^2(t_i)(t - t_i)^2}$$

α and α_i - conversion constants expressed in meters per second

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Spatio-temporal assignment rules

Predictive assignment rule (P-3DVoro)

$$d_P(p, p_i) = \begin{cases} \sqrt{(x_i(t) - x)^2 + (y_i(t) - y)^2}, & t - t_i \geq 0 \\ \infty, & \textit{otherwise}, \end{cases}$$

The anticipated position of pedestrian i at time t:
$x_i(t) = x_i(t_i) + (t - t_i)v_i^x(t_i), y_i(t) = y_i(t_i) + (t - t_i)v_i^y(t_i)$
The speed of pedestrian i at t_i in x and y directions: $v_i^x(t_i)$,$v_i^y(t_i)$

Mahalanobis assignment rule (M-3DVoro)

$$d_M(p, p_i) = \sqrt{(p - p_i)^T M_i (p - p_i)}$$

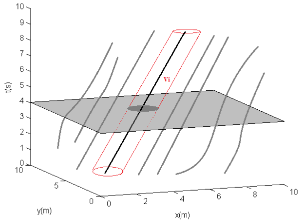
M_i - symmetric, positive-definite matrix that defines how distances are measured from the perspective of pedestrian i

Outline

Voronoi-based traffic indicators

The set of all points in V_i corresponding to a specific time t

$$V_i(t) = \{(x(t), y(t), t) \in V_i\} \sim [m^2]$$



Density indicator

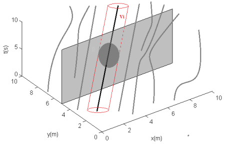
$$k(x, y, t) = \frac{1}{|V_i(t)|}, \text{ for } x, y \in V_i(t)$$

Voronoi-based traffic indicators

The set of all points in V_i corresponding to a given location x and y

$$V_i(x) = \{(x, y, t) \in V_i\} \sim [ms]$$

$$V_i(y) = \{(x, y, t) \in V_i\} \sim [ms]$$



Flow indicator

$$\vec{q}(x, y, t) = \begin{pmatrix} q^x(x, y, t) \\ q^y(x, y, t) \end{pmatrix} = \begin{pmatrix} \frac{1}{|V_i(x)|} \\ \frac{1}{|V_i(y)|} \end{pmatrix}$$

Velocity indicator

$$\vec{v}(x, y, t) = \begin{pmatrix} \frac{q^x(x,y,t)}{k(x,y,t)} \\ \frac{q^y(x,y,t)}{k(x,y,t)} \end{pmatrix} = \begin{pmatrix} \frac{|V_i(t)|}{|V_i(x)|} \\ \frac{|V_i(t)|}{|V_i(y)|} \end{pmatrix}$$

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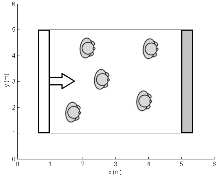
Performance of the approach

Synthetic data - unidirectional flow

- NOMAD simulation tool [Campanella, 2010]
- **Scenario I**: low congestion, homogenous population
- **Scenario II**: high congestion, heterogeneous population



Indicators

- Robustness w.r.t. the aggregation
- Robustness w.r.t. the sampling frequency

Characterization based on trajectories

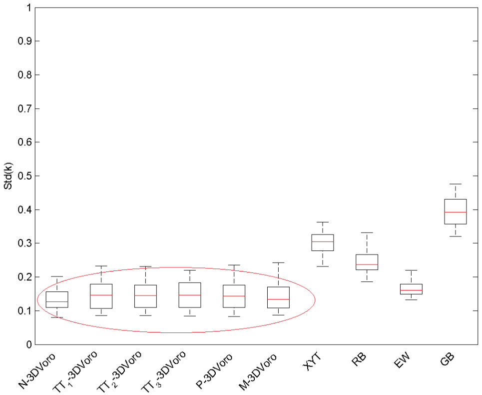
Robustness with respect to the aggregation

- Ability of tolerating perturbations in data

- 100 sets of pedestrian trajectories synthesized per scenario
- Indicators k, v and q calculated for each set

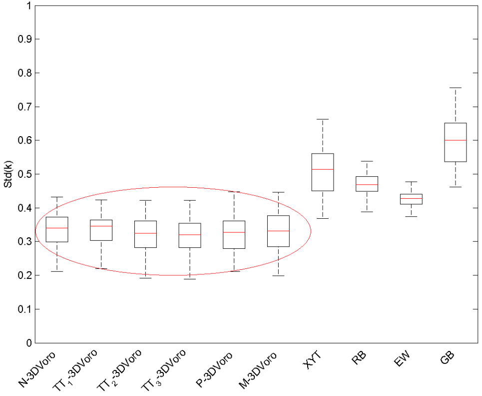
Robustness with respect to the aggregation

Standard deviation (1000 points) - Scenario I



Robustness with respect to the aggregation

Standard deviation (1000 points) - Scenario II



Characterization based on sampled data

Robustness with respect to the sampling frequency

- Ability of tolerating missing data

- Synthetic trajectories sampled using different sampling frequencies
- Indicators calculated via
  1. 3D Voro applied to the interpolated trajectories
  2. 3D Voro applied directly to the samples
- Comparison of the indicators at 1000 randomly selected points to the corresponding values obtained utilizing true trajectories

Robustness w.r.t the sampling frequency - Scenario I

High sampling frequency: $3.33s^{-1}$

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
N-3DVoro	1.17E-02	/	0	/	0	/	3.96E-02	/
TT_1-3DVoro	**2.70E-03**	6.70E-03	0	0	**3.00E-04**	2.30E-03	**7.30E-03**	1.02E-02
TT_2-3DVoro	5.80E-03	3.50E-02	0	2.80E-03	6.00E-04	2.08E-02	1.50E-02	6.69E-02
TT_3-3DVoro	5.40E-03	4.34E-02	0	8.00E-03	6.00E-04	2.83E-02	1.32E-02	9.22E-02
P-3DVoro	8.20E-03	5.36E-02	0	6.10E-03	2.40E-03	3.03E-02	1.30E-02	1.14E-01
M-3DVoro	4.50E-03	5.65E-02	0	6.80E-03	1.10E-03	4.55E-02	1.28E-02	1.04E-01

Low sampling frequency: $0.5s^{-1}$

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
N-3DVoro	1.64E-01	/	0	/	1.46E-01	/	3.02E-01	/
TT_1-3DVoro	2.54E-01	1.27E-01	1.35E-02	9.00E-03	1.16E-01	8.97E-02	3.41E-01	2.25E-01
TT_2-3DVoro	1.64E-01	1.22E-01	1.44E-02	1.06E-02	1.21E-01	**7.30E-02**	3.52E-01	2.33E-01
TT_3-3DVoro	1.89E-01	1.24E-01	1.84E-02	1.09E-02	1.24E-01	7.88E-02	3.40E-01	2.31E-01
P-3DVoro	3.19E-01	**1.21E-01**	3.26E-02	**6.20E-03**	1.43E-01	7.43E-02	3.36E-01	**2.10E-01**
M-3DVoro	1.97E-01	1.24E-01	3.48E-02	9.90E-03	1.41E-01	7.72E-02	3.21E-01	2.31E-01

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Robustness w.r.t the sampling frequency - Scenario II

High sampling frequency: $3.33s^{-1}$

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
N-3DVoro	1.43E-02	/	0	/	**0**	/	2.64E-02	/
TT_1-3DVoro	**8.00E-03**	4.55E-02	0	0	8.00E-04	1.75E-02	**2.36E-02**	8.52E-02
TT_2-3DVoro	1.49E-02	1.07E-01	0	0	3.20E-03	5.72E-02	3.33E-02	2.21E-01
TT_3-3DVoro	1.24E-02	1.60E-01	0	0	3.50E-03	9.62E-02	2.98E-02	3.41E-01
P-3DVoro	2.10E-02	1.66E-01	0	0	4.20E-03	1.16E-01	5.27E-02	3.64E-01
M-3DVoro	1.31E-02	2.40E-01	0	0	2.50E-03	1.75E-01	2.91E-02	5.58E-01

Low sampling frequency: $0.5s^{-1}$

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
N-3DVoro	4.02E-01	/	0	/	2.49E-01	/	1.03E+00	/
TT_1-3DVoro	4.06E-01	**2.90E-01**	3.10E-01	2.48E-02	2.64E-01	**1.65E-01**	9.21E-01	**7.12E-01**
TT_2-3DVoro	3.92E-01	4.58E-01	2.85E-01	2.34E-01	2.48E-01	2.34E-01	9.30E-01	1.11E+00
TT_3-3DVoro	4.41E-01	5.07E-01	2.89E-01	5.89E-02	2.37E-01	3.06E-01	9.81E-01	1.17E+00
P-3DVoro	4.31E-01	3.71E-01	1.40E-03	**0**	2.58E-01	1.80E-01	9.43E-01	7.29E-01
M-3DVoro	4.34E-01	5.01E-01	3.16E-01	1.36E-01	2.75E-01	3.52E-01	9.96E-01	9.80E-01

Outline

Conclusion and future work

Conclusion

- A novel approach to pedestrian traffic characterization: data-driven discretization via 3D Voronoi diagrams
- Superior to existing methods w.r.t. robustness to the aggregation
- Robustness to the sampling frequency
  - TT_1-3DVoro: high sampling frequency or higher congestion
  - P-3DVoro: low sampling frequency and lighter traffic conditions

Future work

- Analysis of the performance for different scenarios
- Weighted assignment rules

Thank you

9^{th} TRIENNIAL SYMPOSIUM ON TRANSPORTATION ANALYSIS (TRISTAN IX), Aruba:
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Marija Nikolić, Michel Bierlaire

Help by S. S. Azadeh and F.Hänseler is appreciated.

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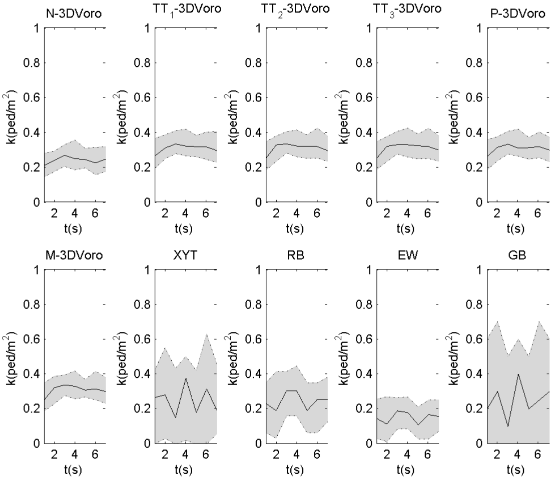
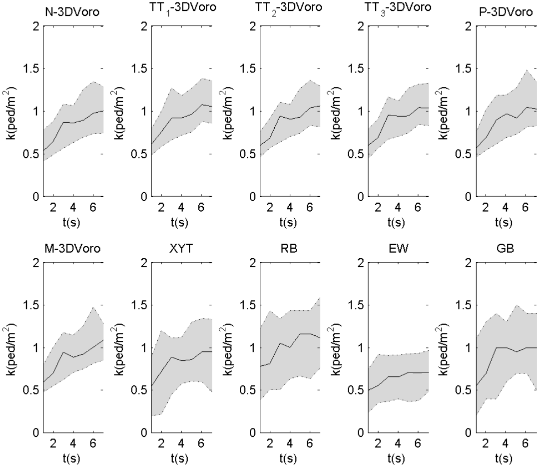
Robustness with respect to the aggregation

Spread/point - Scenario I



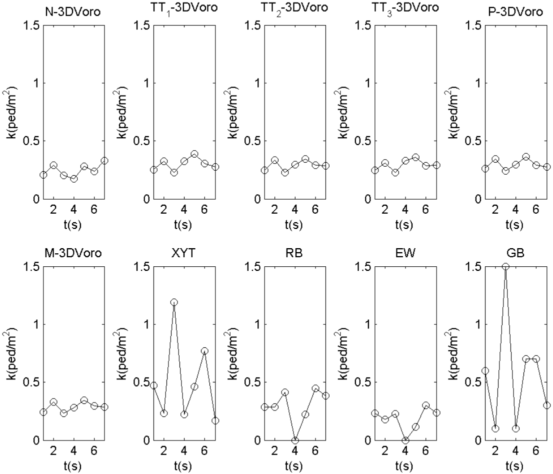
Robustness with respect to the aggregation

Spread/point - Scenario II



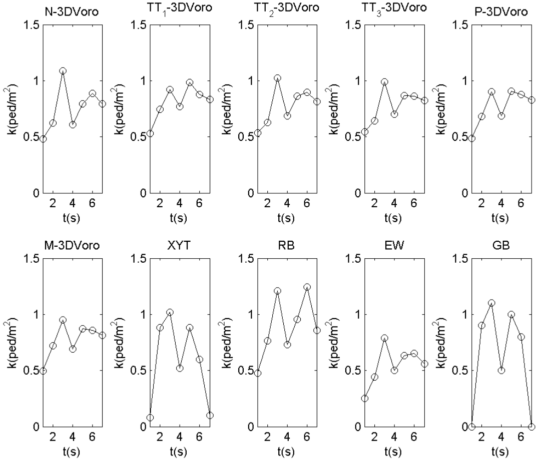
Robustness with respect to the aggregation

Smoothness/point - Scenario I



Robustness with respect to the aggregation

Smoothness/point - Scenario II



Mahalanobis distance

Directions of interest

$$p_{is} = (x_{is}, y_{is}, t_s), \; v_i(t_s) = \frac{1}{t_{(s+1)} - t_s} \begin{pmatrix} x_{i(s+1)} - x_{is} \\ y_{i(s+1)} - y_{is} \\ 1 \end{pmatrix}$$

$$d^1(t_s) = \frac{v_i(t_s)}{||v_i(t_s)||}, \; ||d^1(t_s)|| = 1$$

$$d^2(t_s) = \begin{pmatrix} d_x^1(t_s) \\ d_y^2(t_s) \\ 0 \end{pmatrix}, \; d^1(t_s)^T d^2(t_s) = 0, \; ||d^2(t_s)|| = 1$$

$$d^3(t_s) = \begin{pmatrix} 0 \\ 0 \\ t_{(s+1)} - t_s \end{pmatrix}, \; ||d^3(t_s)|| = t_{(s+1)} - t_s$$

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Mahalanobis distance

Change of coordinates

$$S_1(t_s,\delta) = p_{is} + (t_{(s+1)} - t_s)v_i(t_s) + \delta d^1(t_s)$$

$$S_2(t_s,\delta) = p_{is} - (t_{(s+1)} - t_s)v_i(t_s) - \delta d^1(t_s)$$

$$S_3(t_s,\delta) = p_{is} + \delta d^2(t_s)$$

$$S_4(t_s,\delta) = p_{is} - \delta d^2(t_s)$$

$$S_5(t_s,\delta) = p_{is} + \delta d^3(t_s)$$

$$S_6(t_s,\delta) = p_{is} - \delta d^3(t_s)$$

$$d_M = \sqrt{(S_j(t_s,\delta) - p_{is})^T M_{is}(S_j(t_s,\delta) - p_{is})} = \delta, j = 1,..,6$$

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