Ambush Avoidance in Vehicle Routing

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Seventh Joint Operations Research Days 2009 September 3-4, Lugano, Switzerland





Ambushes

Avoiding the ambush

"The best way to survive an ambush is not to encounter it. In order for this to happen, vehicle movement mustn't be predictable in timing or route, and should avoid the most obvious routes."

Lt. Col. Kevin Stoddard, Soldiers handbook







Valuables distribution/collection



- Urban environment, street network
- Stable (ambush not pursuit)
- Depot and banks are safe locations
- Ambushes are prepared at nodes
- Un-escorted armored vehicle, multiple stops





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Both **deterministic** and **stochastic** in nature.





Relevant information is:

- *d*: the depot
- B: set of banks
- V: set of nodes $V = N \cup B \cup \{d\}$
- E: set of edges $(i,j), i \in N, j \in N$
- $\alpha_j, j \in N$: success rate of an ambush prepared at j
- c_{ij} , $(i,j) \in E$: length of edge (i,j)





The attacker as a feature of the environment (adaptation of Hazardous material transportation concepts) Erkut and Verter (1998) and List et al. (1991).

- Model the risk as an accumulating metric over the course of a vehicles route.
- Search for dissimilar routes, Carotenuto et al. (2007)





Risk function



Given a vehicle's route r

$$R_r = \sum_{j \in r} \sum_{i \in N} R_{ij}^t \cdot \alpha_j \text{ or } R_r = \max_{j \in r} \sum_{i \in N} R_{ij}^t \cdot \alpha_j$$

where

$$R_{ij}^t = \$_{ij} \cdot O_{ij}^t, \ O_{ij}^t = \beta_0 \frac{c_{ij}}{v} + \beta_1 p_{ij}^t$$





Predictability - time



• Historic data, symmetric fuzzy set

•
$$\sum_{l=j-1}^{H} \frac{\beta_3}{|t_j - t_l|} \beta_2$$





- Given two routes r_1 , r_2 , similarity is measured by the Levenshtein distance S_{r_1,r_2}
- The higher the distance the more "dissimilar" the orderings
- Model by $\beta_4 \cdot 1/S_{r_1,r_2}$





Advantages and critics of the environment-based modeling

- (+) All state-of-the-art techniques can be adopted from classic VRP.
- (+-) The model is deterministic.
- (-) Hazmat based predictability metrics are in fact predictable.





Modeling the attacker



Risk minimization models are acceptable against regular thieves





Modeling the attacker



ocean's 11 - ambush planning

What about a smart robber? We assume full knowledge!





We explicitly model the robber as a player in a 2-player non-cooperative game.

Assumptions:

- Complete information on the network, including the access to planning algorithms.
- Ambush sites and vehicle paths are chosen before the vehicle departs from the depot.
- The robber is rational, and will maximize payoff.
- One ambush site (a lonely gangster).

Bell (2004), Joseph (2005).





Modeling the attacker's incentive - single OD

Let p_{ij} : flow over the link $(i, j) \in E$. Success of ambush at node j:

$$r_j = \sum_{i \in N} (p_{ij}) \; \alpha_j$$

Robber's payoff at *j*:

$$R_j = \sum_{i \in N} (p_{ij}) \, \alpha_j \cdot \$_j$$

Robber's goal:

$$Z_r = \max_{j \in N} \sum_{i \in N} (p_{ij}) \, \alpha_j \cdot \$_j$$





MinMax formulation - single OD

Our goal:

$$Z_{p} = \min_{P} \left\{ \max_{j \in N} \sum_{i \in N} (p_{ij}) \, \alpha_{j} \cdot \$_{j} \right\}$$

p is a flow:

$$\begin{split} \sum_{i \in N} (p_{ij}) &= \sum_{k \in N} (p_{jk}) & \forall j \in N \\ \sum_{i \in V} (p_{ib}) &= 1 \sum_{j \in V} (p_{dj}) = 1 \\ 0 &\leq p_{ij} \leq 1 & \forall (i,j) \in E \end{split}$$





Given the optimal solution \mathbf{p}^* :

- p_{ij}^* is used as probability of traversing edge (i,j).
- A random path construction procedure is used to dynamically determine the vehicle's route: no critically vulnerable node exploitable by an intelligent robber.
- Flow circulation does not affect optimality (Joseph, 2005). Implementation: add ε · Σ_{i∈N} Σ_{j∈N} p_{ij}c_{ij} to the objective.





- Flows on network do not model temporal considerations.
- Route based formulation quickly explode in size w.r.t the size of the street network and loose one level of "stochasticity".

Two steps approach:

- Decide the order of the banks to be visited
- Adapt the flow-based model to multiple destinations (multi-commodity flow or layered network)





Multi-commodity flow

Given an ordering O for the banks to be visited, we define |B|+1 commodities, one for each OD pair in the vehicle's journey.

As ambushes at j in different segments of the vehicle's journey are **mutually exclusive** events, the robber's goal can be modeled as:

$$Z_r = \max_{j \in N} \sum_{b \in B} \sum_{i \in N} (p_{ij}^b) \; \alpha_j^p \cdot \$_j^p$$

The minmax game is as follows:

$$Z_{p} = \min_{p} \left\{ \max_{j \in N} \sum_{b \in B} \sum_{i \in N} (p_{ij}^{b}) \alpha_{j}^{p} \cdot \$_{j}^{p} \right\}$$

s.t. \boldsymbol{p} is a multicommodity flow

$$\sum_{i \in V} (p_{ij}^b) = \sum_{k \in N} (p_{jk}^b) \qquad \forall j \in N, \forall b \in B$$

$$\sum_{i \in V} (p_{ij}^{b-1}) = 1, \sum_{i \in V} (p_{ji}^b) = 1 \qquad \forall b \in B, j \in O_b$$

$$\sum_{i \in V} (p_{dj}^0) = 1, \sum_{j \in V} (p_{id}^{|B|}) = 1$$

$$0 \le p_{ij}^b \le 1 \qquad \forall (i,j) \in E, \forall b \in B$$



Layered network



Both formulations have similar complexity

- |*B*| copies of the network are created
- Linked with directed links, where $p_{jj'} = 1, \ j \in O$
- Terminal arc $p_{jd} = 1, j \in O_{|B|}$, to impose flow circulation





Given the optimal solution \mathbf{p}^* :

• p_{ij}^b is used as probability of using edge (i, j) in the *b*-th part of the vehicle's journey.





We use an enumerative approach:

- Let Θ be the set of possible orderings, $|\Theta|=|B|!$
- In real world |B| is hardly bigger than 6, thus 6! = 720

Using a path based approach, as in Bell (2004) with Ω the set of possible paths, $|\Omega| = |V|!$, where |V| can be easily order of hundreds or thousands.

We add a stochastic decision level using a **mixed** strategy for the orderings with p_r , $r \in \Theta$ the probability of selecting the ordering r:

$$p_r^* = \frac{1 - \tilde{Z}_p(r)}{\sum_{k=1}^{\Theta} (1 - \tilde{Z}_p(k))}$$

where $\tilde{Z}_{p}(r)$ is the **normalized** payoff





Preliminary results on Cambridge network



- Considered by Joseph (2005), 50 nodes and 91 edges, LPs solved by GLPK, algorithm coded in C
- Performed experiments with 3 up to 6 banks, max runtime 120 seconds on a laptop





Preliminary results on Cambridge network

Equal α_j



- Payoffs between 0.333 and 0.376
- 72.3 (63, 80) edges with positive flow
- 47.1 (43, 49) nodes with positive inflow, 27.0 (20, 32) with max payoff





Preliminary results on Cambridge network

 α_i increasing with distance from banks and depot



- Payoffs between 0.262 and 0.504
- 62.0 (51, 70) edges with positive flow
- 43.6 (38, 48) nodes with positive inflow, 17.1 (11, 23) with max payoff





Advantages and critics of the minmax flow based approach

- (+) Unpredictable.
- (+) Does not suffer from dimensionality.
- (-) Deterministic constraints are difficult to model.





- Two models to deal with ambush avoidance for valuable transfer.
- Direct modeling of robber's incentives.
- Applicable approach to real world.

Outlook

- Relax restrictive hypoteses (rational robber, full information)
- Consider multiple vehicles
- Consider multiple ambush points
- Model restrictions on vehicle routes





The minmax approach **limits** the max payoff but this value **shades** the "magnitude" of the total payoff.

Multi-objective optimization, solved in two steps:

- Compute Z_p^* , with minmax
- Redistribute the flow:

$$\begin{split} U_{p} &= \min_{P} \sum_{j \in N} \sum_{b \in B} \sum_{i \in N} (p_{ij}^{b}) \ \alpha_{j}^{p} \cdot \$_{j}^{p} \\ s.t. \ \sum_{b \in B} \sum_{i \in N} (p_{ij}^{b}) \ \alpha_{j} \cdot \$_{j}^{p} \leq Z_{p}^{*} \qquad \forall j \in N, b \in B \\ \mathbf{p} \text{ is a flow} \end{split}$$





The formula proposed only approximates the best mixed strategy. Indeed, a 2-player game for bank ordering, would be better. We set up the following matrix game:

with $U(S_1, S_r) \le Z(S_1)$ and $U(S_r, S_1) \le Z(S_r)$. The game results in a system of linear equations order with $|B|^2$ variables.





If ambushes at j are **not** mutually exclusive, then robber incentive is:

$$Z_r = \max_{j \in \mathcal{N}} \sum_{k=1}^{B} \left((-1)^{k-1} \sum_{1 \le i_1 < i_2 < \cdots < i_k \le n} \prod_{i_1}^{i_k} (p_j^b) \; \alpha_j^p \cdot \$_j^p \right)$$

Unfortunately no more a nice LP!





Alternative formulation

Let:

- $\tilde{\alpha}_j = 1 \alpha_j$ as the **failure** rate
- $\tilde{p}_{ij} = 1 p_{ij}$ as probability of **not** passing by (i, j)

The robber will select j that minimize the failure

$$Z_r = \min_{j \in N} (\tilde{p}_j) \ \tilde{\alpha}_j$$

thus, for the multicommodity formulation,

$$Z_r = \min_{j \in N} \prod_B (\tilde{p}_j^b) \; \tilde{\alpha}_j$$

or, using "non" flow variables:

$$Z_r = \min_{j \in N} \prod_B \sum_{i \in N} (\tilde{p}_{ij}^b) \; \tilde{\alpha}_j$$



Alternative formulation

We obtain nash-equilibrium by maximizing the minimal failure

$$Z_{p} = \max_{P} \left\{ \min_{j \in N} \prod_{B} \sum_{i \in N} (\tilde{p}_{ij}^{b}) \ \tilde{\alpha}_{j} \right\}$$

todo what if we approximate Z_p by the following LP?

$$egin{aligned} & Z_p = \max_P \ z \ s.t. \ z &\leq \sum_{i \in \mathcal{N}} (ilde{p}_{ij}^b) \ ilde{lpha}_j & orall j \in \mathcal{N}, b \in B \ \mathbf{p} \ ext{is a flow} \end{aligned}$$





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