Demand Based Timetabling of Passenger Railway Service

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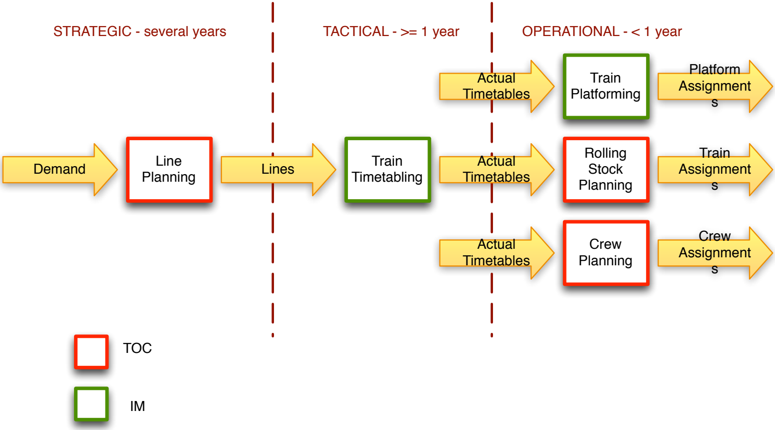
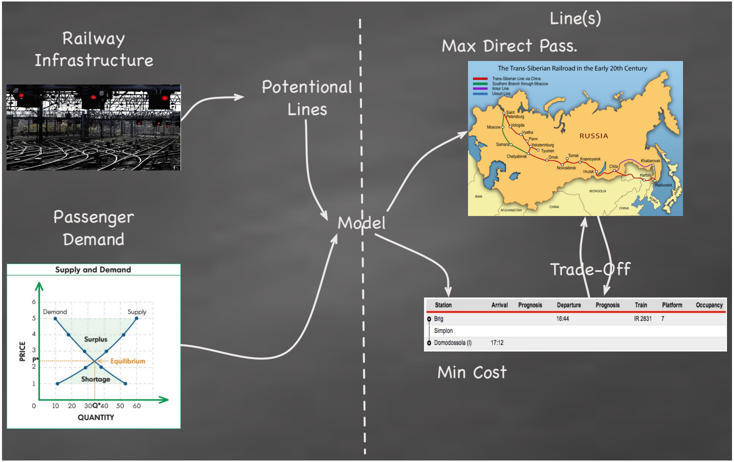
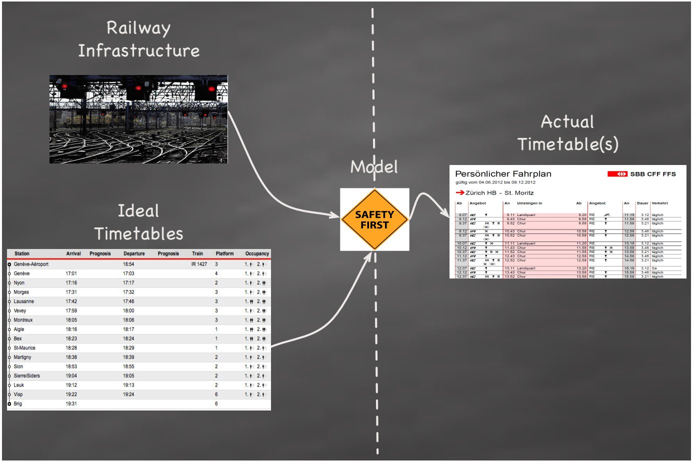
Railway Planning



Line Planning Problem



Train Timetabling Problem – Non-Cyclic



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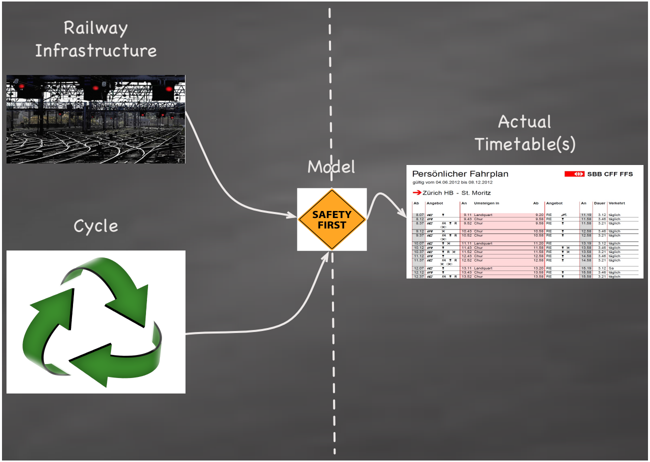
Train Timetabling Problem – Cyclic



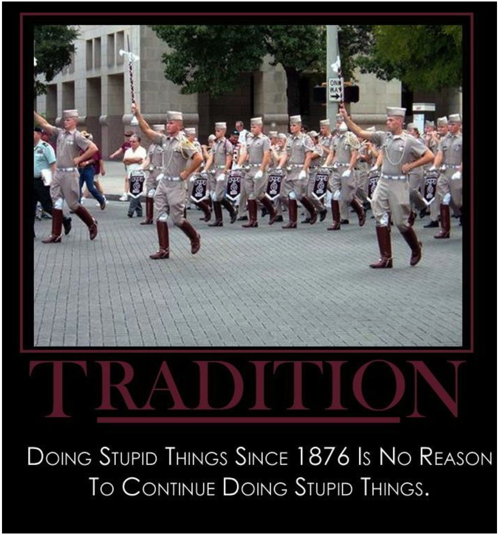
Arising Issues

Figure : Outside peak hour

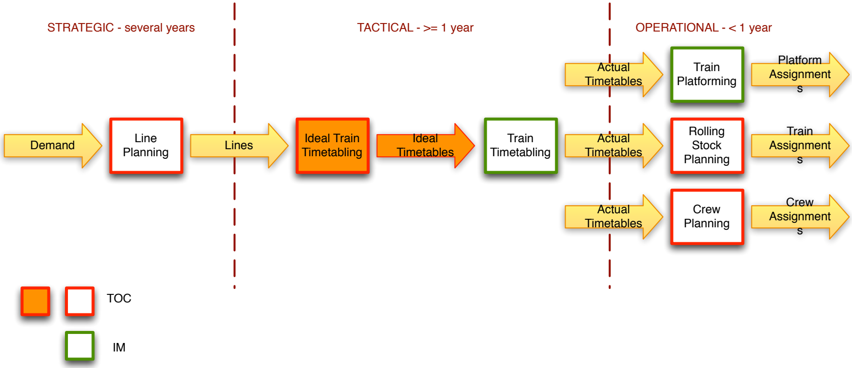
Figure : Inside peak hour

Figure : Train station in China

Do We Keep Traditions?

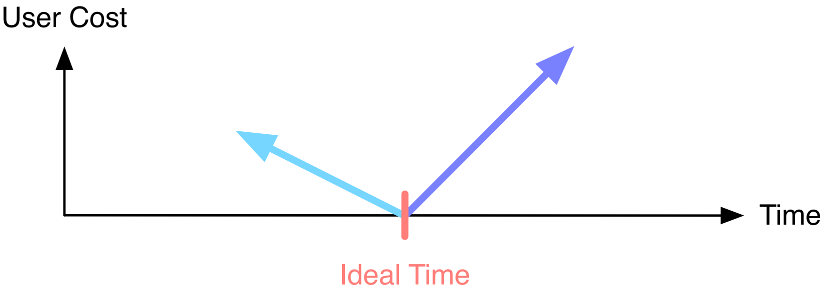


Railway Planning Improved

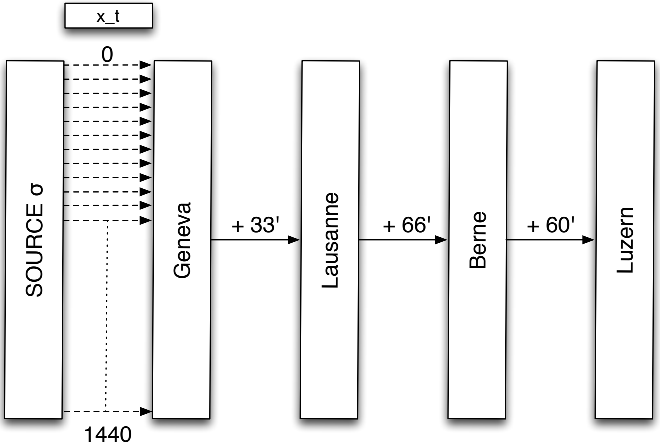


Agenda

Assumptions I



Assumptions II



Inputs

$t \in T$	–	set of time steps
$l \in L$	–	set of lines
f	–	fraction by which it is better to be early
d_t^l	–	demand captured along the line l, when scheduling a train at time t
$d_t^{ll'}$	–	connection demand captured along the line l and l', when scheduling a train at time t on the line l
n^l	–	number of trains available for line l
$h_l^{l'}$	–	relative headway to reach a connection point of lines l and l' from the first station on line l and l'
c^l	–	size of the cycle on line l
s	–	preferred start of the planning horizon
$M \in \mathbb{M}$	–	set of sufficiently large numbers

Primary Decision(s)

$$x_t^l = \begin{cases} 1 & \text{if a train on line } l \text{ is scheduled at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Secondary Decisions I

- $y_t^{lb} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l, when taking a closest train at t or before
- $y_t^{la} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l, when taking a closest train after t
- $y_t^{l} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l

Secondary Decisions II

$$z_t^l = \begin{cases} 1 & \text{if passengers wanting to travel at time } t \text{ on the line } l \text{ take the closest train after the time } t, \\ 0 & \text{otherwise.} \end{cases}$$

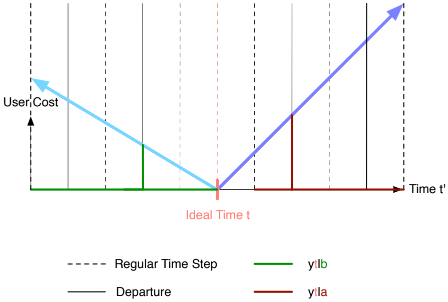
Objective

$$\min \sum_{l \in L} \sum_{t \in T} y_t^l \cdot d_t^l$$

Constraints I

$$y_t^{lb} \geq (t - t') / f \cdot \left(x_{t'}^l - \sum_{t''=t'+1}^{t} x_{t''}^l \right) \qquad \forall l \in L, \forall t, \forall t' \in T : t \geq t',$$

$$y_t^{la} \geq (t' - t) \cdot \left(x_{t'}^l - \sum_{t''=t+1}^{t'-1} x_{t''}^l \right) \qquad \forall l \in L, \forall t, \forall t' \in T : t < t',$$



Constraints II

$$y_t^{lb} \geq M_1 \cdot \left(1 - \sum_{t'=s}^{t} x_{t'}^{l}\right) \qquad \forall l \in L, \forall t \in T,$$

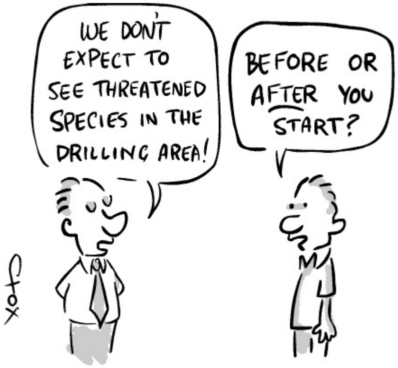
$$y_t^{la} \geq M_1 \cdot \left(1 - \sum_{t'=t}^{T} x_{t'}^{l}\right) \qquad \forall l \in L, \forall t \in T,$$

Constraints III

$$y_t^l \geq y_t^{lb} - z_t^l \cdot M_2 \qquad \forall l \in L, \forall t \in T,$$

$$y_t^l \geq y_t^{la} - \left(1 - z_t^l\right) \cdot M_2 \qquad \forall l \in L, \forall t \in T,$$

$$M_2 > M_1$$



Constraints IV

$$\sum_{t \in T} x_t^l \leq n^l \qquad \forall l \in L,$$

1 Motivation

2 Ideal Train Timetabling Problem
- Assumptions
- Inputs
- Decision Variables
- Objective
- Constraints
- Cyclicity
- Connections

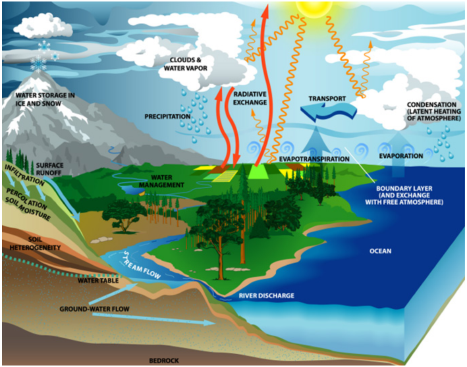
3 Conclusions

4 Future Work

Introducing Cyclicity

$$x^l_{t+c^l} = x^l_t \qquad \forall l \in L, \forall t \in T : t + c^l \leq T : t \geq s,$$

$$\sum_{t'=t+1}^{min(t+c^l, T)} x^l_{t'} \leq \left(1 - x^l_t\right) \cdot M_3 \qquad \forall l \in L, \forall t \in T : t \geq s,$$



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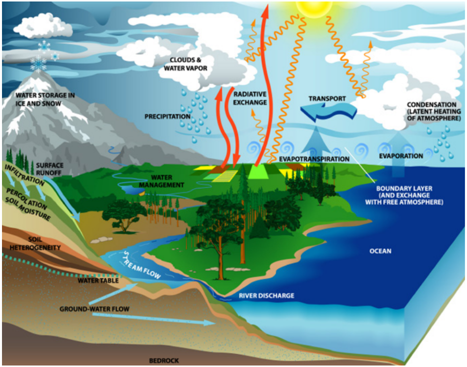


Extra Decisions I

- $y_t^{ll'b} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l, when taking a closest train at t or before and connecting to line l'
- $y_t^{ll'a} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l, when taking a closest train after t and connecting to line l'
- $y_t^{ll'} \in \mathbb{R}^+$ – cost of the passengers wanting to travel at time t on the line l and connecting to line l'

Extra Decisions II

$$z_t^{ll'} = \begin{cases} 1 & \text{if passengers wanting to travel at time } t \text{ on the line } l \text{ take the closest train after the time } t \text{ and connecting to line } l', \\ 0 & \text{otherwise.} \end{cases}$$

Objective

$$\min \sum_{l \in L} \sum_{t \in T} y_t^l \cdot d_t^l + \sum_{l \in L} \sum_{l' \in L} \sum_{t \in T} y_t^{ll'} \cdot d_t^{ll'}$$

Extra Constraints I

$$y_t^{ll'b} \geq (t - t') / f \cdot \left(x_{t'}^{l} - \sum_{t'''=t'+1}^{t} x_{t'''}^{l} \right) + \left(t'' - \left(t' + h_l^{l'} \right) \right) \cdot$$

$$\left(x_{t''}^{l'} - \sum_{t'''=t'+h_l^{l'}+1}^{t''-1} x_{t'''}^{l'} \right) - M_4 \cdot \left(1 - x_{t'}^{l} + \sum_{t'''=t'+1}^{t} x_{t'''}^{l} \right)$$

$$\forall l, \forall l' \in L : l \neq l',$$

$$\forall t, \forall t', \forall t'' \in T : t \geq t' \text{ and } t' + h_l^{l'} < t'',$$

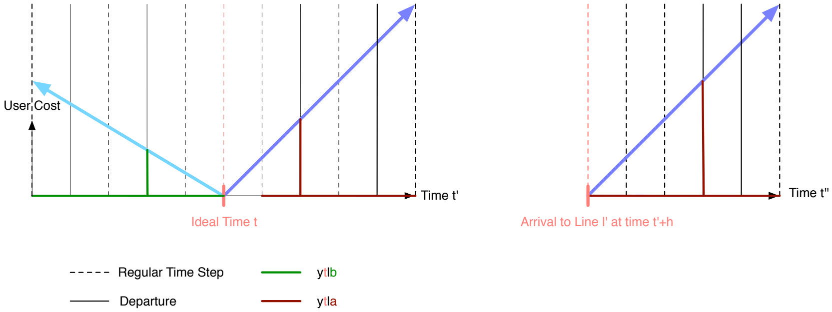
$$y_t^{ll'a} \geq (t' - t) \cdot \left(x_{t'}^{l} - \sum_{t'''=t+1}^{t'-1} x_{t'''}^{l} \right) + \left(t'' - \left(t' + h_l^{l'} \right) \right) \cdot$$

$$\left(x_{t''}^{l'} - \sum_{t'''=t'+h_l^{l'}+1}^{t''-1} x_{t'''}^{l'} \right) - M_4 \cdot \left(1 - x_{t'}^{l} + \sum_{t'''=t+1}^{t'-1} x_{t'''}^{l} \right)$$

$$\forall l, \forall l' \in L : l \neq l',$$

$$\forall t, \forall t', \forall t'' \in T : t < t' \text{ and } t' + h_l^{l'} < t'',$$

Extra Constraints II



Extra Constraints III

$$y_t^{ll'} \geq y_t^{ll'b} - z_t^{ll'} \cdot M_2 \qquad \forall l, \forall l' \in L : l \neq l', \forall t \in T,$$

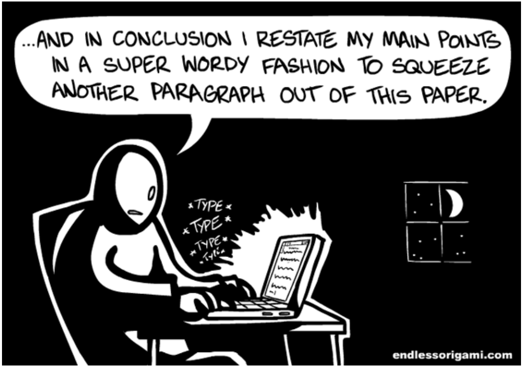
$$y_t^{ll'} \geq y_t^{ll'a} - \left(1 - z_t^{ll'}\right) \cdot M_2 \qquad \forall l, \forall l' \in L : l \neq l', \forall t \in T,$$

Constraints to add

- Beginning and the end of horizon, when no connections are possible

Conclusions

- New planning phase, based on the demand
- User cost rather than demand to capture (no need for discrete choice model)
- Can handle bot non- and cyclic timetables
- Connections are demand imposed



Future Work

- Methodology design (cyclic is tighter than the non-)
- Actually solving the problem
- Analysis of the general results
- Analysis of the connections

Thank you for your attention.