The Ideal Train Timetabling Problem

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Liberalisation – 01.01.2010

Purely commercial rail passenger services in Europe

Red	Market closed for commercial national rail passenger services.
Yellow	Open access, but no external RUs providing commercial national rail passenger services .
Green	Open access with external RUs providing commercial national rail passenger services.
Yellow/green hatched	AT and CZ: commencing end of 2011, external RUs providing purely commercial national rail passenger services.

Liberalisation – Overview

Liberalisation time line

1 January 1993
Access for international groupings providing international services and for international combined transport goods service providers

15 March 2003
Access to the Trans-European Rail Freight Network for international freight services

1 January 2006
Access to the entire EU rail network for international freight services

1 January 2007
Access to the entire EU rail network for all types of rail freight (including domestic)

1 January 2010
Access to the infrastructure in all EU Member States for the purpose of operating international passenger services (cabotage permitted)

? December 2019
Access to the infrastructure in all EU Member States for all rail services, including domestic passenger services

Public Sector – Accessibility/Mobility

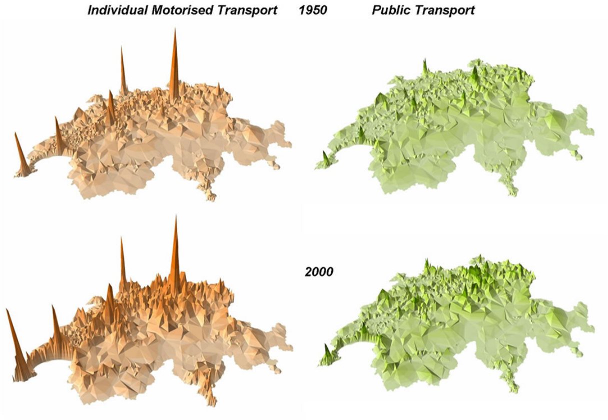


Figure : Mobility evolution in Switzerland[1]

[1] – *source: Entwicklung der MIV und OV Erreichbarkeit in der Schweiz: 1950-2000; Ph. Frohlich, M. Tschopp and K.W. Axhausen*

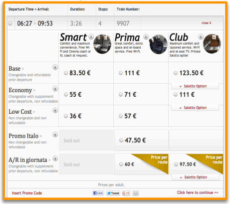
Private Sector

Market Settings

Travel Time is the same

Serve Different Destinations

Better Quality



Departure Time > Arrival:	Duration:	Stops:	Train Number:
06:27 > 09:53	3:26	4	9907

close X

	Smart — Comfort and maximum convenience. Free Wi-Fi and Cinema coach or XL coach at request.	Prima — Great comfort, extra space and on-board service. Free Wi-Fi.	Club — Maximum comfort and taylored service. Wi-Fi and at-seat TV. Private Salotto option
Base > Changeable and refundable prior departure	83.50 €	111 €	123.50 € (Salotto Option)
Economy > Changeable with supplement prior departure, non refundable	55 €	71 €	111 € (Salotto Option)
Low Cost > Non changeable and non refundable	36 €	57 €	
Promo Italo > Non changeable and non refundable	Sold out	47.50 €	
A/R in giornata > Changeable with supplement prior departure, non refundable	Sold out	60 € (Price per route)	97.50 € (Price per route) (Salotto Option)

Prices per adult.

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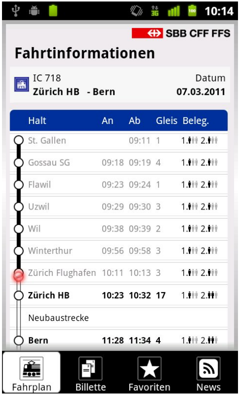
Better Price

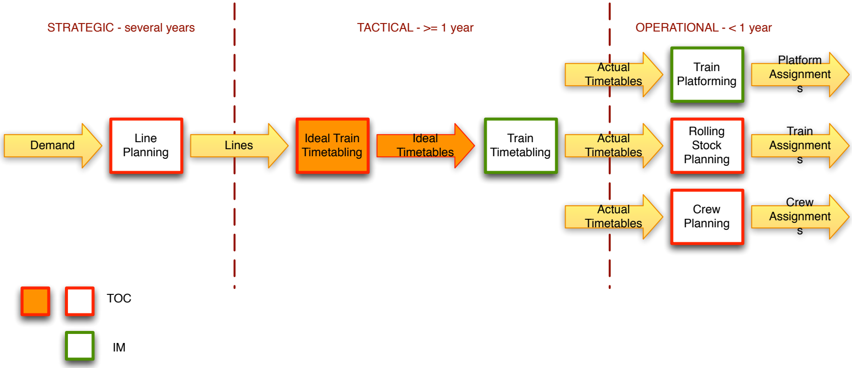
Better Departure Times

Goal: Better Timetables!

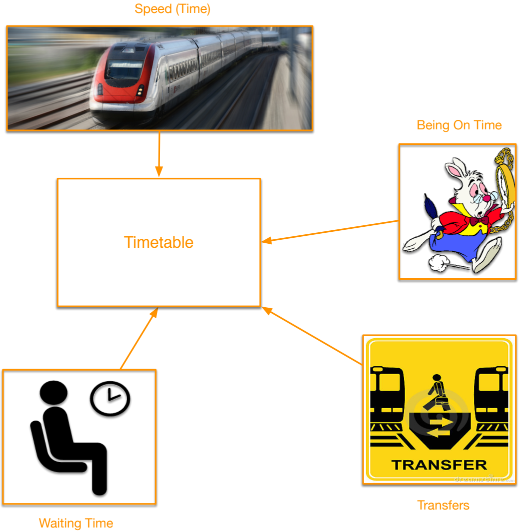


- How to measure goodness of a timetable?
- Timetable design in the literature
  - non-cyclic: using so called "ideal timetables"
  - cyclic: does not take into account anything
- In the industry – historical

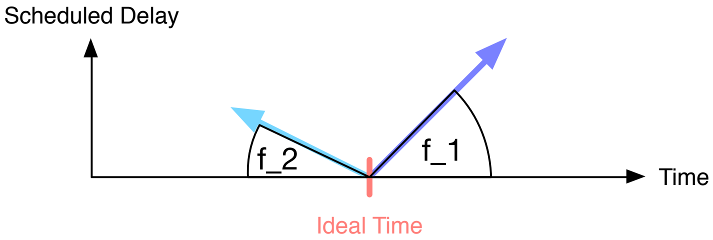
Update of Planning



How to Measure Quality of a Timetable?



Being on Time



- Scheduled delay times value of time (Arnott et al. (1990))

The Rest

Speed (Time)

- Running time multiplied by the value of time (Axhausen et al. (2008))

Waiting Time

- Waiting time multiplied by the value of waiting time (Wardman (2004))

Transfers

- Minimum transfer time multiplied by the number of transfers and the value of waiting time (Wardman (2004))

References

Arnott, R., de Palma, A. and Lindsey, R. (1990). Economics of a bottleneck, *Journal of Urban Economics* **27**(1): 111 – 130.

Axhausen, K. W., Hess, S., König, A., Abay, G., Bates, J. J. and Bierlaire, M. (2008). Income and distance elasticities of values of travel time savings: New swiss results, *Transport Policy* **15**(3): 173 – 185.

Wardman, M. (2004). Public transport values of time, *Transport Policy* **11**(4): 363 – 377.

Ideal Timetable

The ideal timetable consists of such train departures that the passengers' global costs are minimized, i.e. the fastest most convenient path to get from the origin to the destination traded-off by a timely arrival to the destination for every passenger.

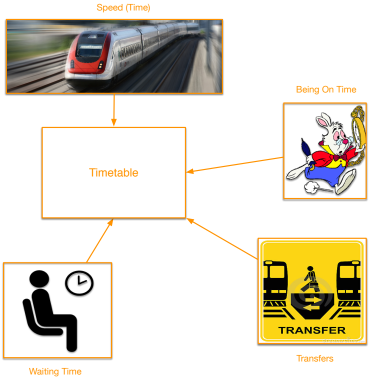
Inputs

$i \in I$	–	set of origin-destination pairs
$t \in T$	–	set of time steps t in the planning horizon
$t' \in T^i$	–	set of ideal times for OD pair i
$l \in L$	–	set of operated lines
$v \in V^l$	–	set of available vehicles on line l
$p \in P^i$	–	set of possible paths between OD pair i
$l \in L^p$	–	set of lines in the path p
r_i^{pl}	–	running time between OD pair i on path p using line l
h_i^{pl}	–	time to arrive from the starting station of the line l to the origin of the pair i
$D_i^{t'}$	–	demand between OD i with ideal time t'
m	–	minimum transfer time
c	–	cycle
q_1	–	value of the waiting time
q_2	–	value of the in vehicle time
f_1	–	coefficient of being early
f_2	–	coefficient of being late

Decisions

$\mathcal{C}_i^{t'}$	–	the total cost of the passengers with ideal time t' between OD pair i
$w_i^{t'}$	–	the total waiting time of the passengers with ideal time t' between OD pair i
$w_i^{t'p}$	–	the total waiting time of the passengers with ideal time t' between OD pair i using path p
$w_i^{t'pl}$	–	the waiting time of the passengers with ideal time t' between OD pair i on the line l that is part of the path p
$x_i^{t'p}$	–	1 – if the passengers with ideal time t' between OD pair i choose path p; 0 – otherwise
$s_i^{t'}$	–	the final scheduled of the passengers with ideal time t' between OD pair i
$s_i^{t'p}$	–	scheduled delay of the passengers with ideal time t' between OD pair i traveling on the path p
d_v^l	–	the departure time of a train v on the line l
$y_i^{t'plv}$	–	1 – if the passengers with ideal time t' between OD pair i on the path p take the train v on the line l; 0 – otherwise
z_v^l	–	frequency within cyclicity

Objective



$$\min \sum_{i \in I} \sum_{t' \in T^i} D_i^{t'} \cdot \mathcal{C}_i^{t'}$$

Pricing Constraints

$$
\begin{aligned}
\mathcal{C}_i^{t'} &= q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1) \\
&\quad + q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'}, && \forall i \in I, \forall t' \in T^i, \\
w_i^{t'} &\geq w_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'p} &= \sum_{l \in L^p \setminus 1} w_i^{t'pl}, && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'pl} &\geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
&\quad - M \cdot \left(1 - y_i^{t'pl'v'}\right) - M \cdot \left(1 - y_i^{t'plv}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
& && l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \\
w_i^{t'pl} &\leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
&\quad + M \cdot \left(1 - y_i^{t'pl'v'}\right) + M \cdot \left(1 - y_i^{t'plv}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
& && l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \\
s_i^{t'} &\geq s_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
s_i^{t'p} &\geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \\
s_i^{t'p} &\geq f_1 \cdot \left(t' - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},
\end{aligned}
$$

Pricing Constraints

$$
\begin{aligned}
C_i^{t'} &= q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1) \\
&+ q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'}, && \forall i \in I, \forall t' \in T^i, \\
w_i^{t'} &\geq w_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'p} &= \sum_{l \in L^p \setminus 1} w_i^{t'pl}, && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'pl} &\geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
&- M \cdot \left(1 - y_i^{t'pl'v'}\right) - M \cdot \left(1 - y_i^{t'plv}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
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s_i^{t'p} &\geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \\
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s_i^{t'p} &\geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \\
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\end{aligned}
$$

Pricing Constraints

$$\mathcal{C}_i^{t'} = q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1)$$

$$+ q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'}, \qquad \forall i \in I, \forall t' \in T^i,$$

$$w_i^{t'} \geq w_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,$$

$$w_i^{t'p} = \sum_{l \in L^p \setminus 1} w_i^{t'pl}, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,$$

$$w_i^{t'pl} \geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right)$$

$$- M \cdot \left(1 - y_i^{t'pl'v'}\right) - M \cdot \left(1 - y_i^{t'plv}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p :$$

$$l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'},$$

$$w_i^{t'pl} \leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right)$$

$$+ M \cdot \left(1 - y_i^{t'pl'v'}\right) + M \cdot \left(1 - y_i^{t'plv}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p :$$

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$$s_i^{t'p} \geq f_1 \cdot \left(t' - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},$$

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s_i^{t'p} &\geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \\
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\end{aligned}
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w_i^{t'} &\geq w_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'p} &= \sum_{l\in L^p\setminus 1} w_i^{t'pl}, && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'pl} &\geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
&- M \cdot \left(1 - y_i^{t'pl'v'}\right) - M \cdot \left(1 - y_i^{t'plv}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
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w_i^{t'pl} &\leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
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s_i^{t'p} &\geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \\
s_i^{t'p} &\geq f_1 \cdot \left(t' - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},
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Pricing Constraints

$$
\begin{aligned}
\mathcal{C}_i^{t'} &= q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1) \\
&+ q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'}, && \forall i \in I, \forall t' \in T^i, \\
w_i^{t'} &\geq w_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'p} &= \sum_{l \in L^p \setminus 1} w_i^{t'pl}, && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'pl} &\geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
&- M \cdot \left(1 - y_i^{t'pl'v'}\right) - M \cdot \left(1 - y_i^{t'plv}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
& && l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \\
w_i^{t'pl} &\leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
&+ M \cdot \left(1 - y_i^{t'pl'v'}\right) + M \cdot \left(1 - y_i^{t'plv}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
& && l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \\
s_i^{t'} &\geq s_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
s_i^{t'p} &\geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \\
s_i^{t'p} &\geq f_1 \cdot \left(t' - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},
\end{aligned}
$$

Pricing Constraints

$$\mathcal{C}_i^{t'} = q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1)$$

$$+ q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'}, \qquad \forall i \in I, \forall t' \in T^i,$$

$$w_i^{t'} \geq w_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,$$

$$w_i^{t'p} = \sum_{l \in L^p \setminus 1} w_i^{t'pl}, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,$$

$$w_i^{t'pl} \geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right)$$

$$- M \cdot \left(1 - y_i^{t'pl'v'}\right) - M \cdot \left(1 - y_i^{t'plv}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p :$$

$$l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'},$$

$$w_i^{t'pl} \leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right)$$

$$+ M \cdot \left(1 - y_i^{t'pl'v'}\right) + M \cdot \left(1 - y_i^{t'plv}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p :$$

$$l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'},$$

$$s_i^{t'} \geq s_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,$$

$$s_i^{t'p} \geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},$$

$$s_i^{t'p} \geq f_1 \cdot \left(t' - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},$$

Pricing Constraints

$$
\begin{aligned}
\mathcal{C}_i^{t'} &= q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1) \\
&\quad + q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'}, && \forall i \in I, \forall t' \in T^i, \\
w_i^{t'} &\geq w_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'p} &= \sum_{l \in L^p \setminus 1} w_i^{t'pl}, && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
w_i^{t'pl} &\geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
&\quad - M \cdot \left(1 - y_i^{t'pl'v'}\right) - M \cdot \left(1 - y_i^{t'plv}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
& && l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \\
w_i^{t'pl} &\leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \\
&\quad + M \cdot \left(1 - y_i^{t'pl'v'}\right) + M \cdot \left(1 - y_i^{t'plv}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
& && l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \\
s_i^{t'} &\geq s_i^{t'p} - M \cdot \left(1 - x_i^{t'p}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
s_i^{t'p} &\geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t'\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|}, \\
s_i^{t'p} &\geq f_1 \cdot \left(t' - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) - M \cdot \left(1 - y_i^{t'p|L|v}\right), && \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},
\end{aligned}
$$

Feasibility Constraints

$$\sum_{p \in P^i} x_i^{t'p} = 1, \qquad \forall i \in I, \forall t' \in T^i,$$

$$\sum_{v \in V^l} y_i^{t'plv} = 1, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p,$$

$$d_v^l - d_{v-1}^l = c \cdot z_v^l, \qquad \forall l \in L, \forall v \in V : v > 1,$$

$$\textit{domain} \qquad \textit{constraints}$$

Feasibility Constraints

$$\sum_{p \in P^i} x_i^{t'p} = 1, \qquad \forall i \in I, \forall t' \in T^i,$$

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$$\textit{domain} \qquad \textit{constraints}$$

Feasibility Constraints

$$\sum_{p \in P^i} x_i^{t'p} = 1, \qquad \forall i \in I, \forall t' \in T^i,$$

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$$d_v^l - d_{v-1}^l = c \cdot z_v^l, \qquad \forall l \in L, \forall v \in V : v > 1,$$

domain constraints

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Feasibility Constraints

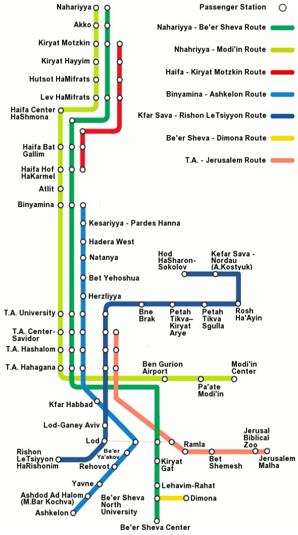
$$\sum_{p \in P^i} x_i^{t'p} = 1, \qquad \forall i \in I, \forall t' \in T^i,$$

$$\sum_{v \in V^l} y_i^{t'plv} = 1, \qquad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p,$$

$$d_v^l - d_{v-1}^l = c \cdot z_v^l, \qquad \forall l \in L, \forall v \in V : v > 1,$$

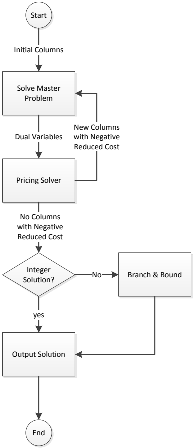
domain constraints

Case Study – Israel



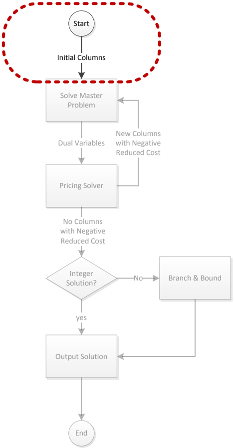
- OD Matrix for an average working day (Sunday to Thursday) in Israel during 2008
- 48 Stations
- 2256 ODs
- 36 (unidirectional) lines
- 389 trains

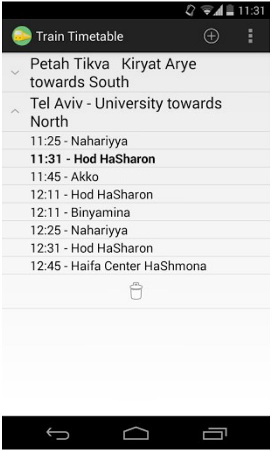
Too Heavy – Branch-and-Price Framework



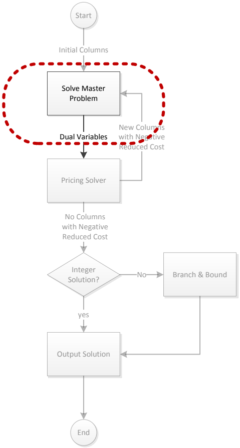
- Initial Solution
- Column Generation – Lower Bound
- Branch and Bound – Optimal Integer Solution

Initial Solution





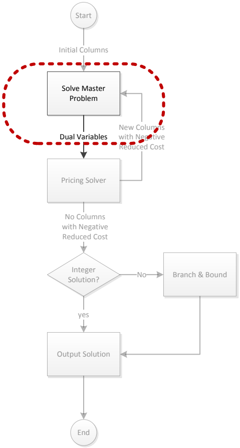
Master Problem (MP)



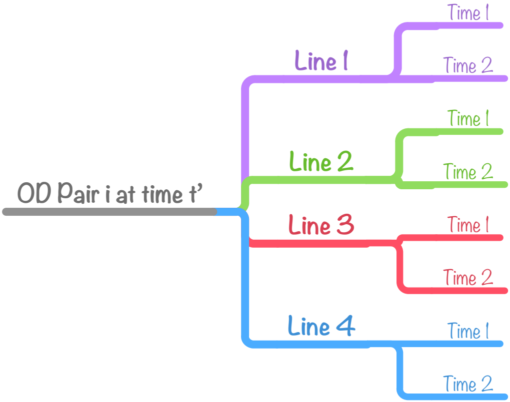
Idea

- Formulated as a Set Partitioning Problem
- Decision variables are relaxed, solution space restricted
- Consists of the feasibility constraints

MP – Column



Start
Initial Columns
Solve Master Problem
Dual Variables
New Columns with Negative Reduced Cost
Pricing Solver
No Columns with Negative Reduced Cost
Integer Solution?
No
Branch & Bound
yes
Output Solution
End



OD Pair i at time t'
Line 1
Time 1
Time 2
Line 2
Time 1
Time 2
Line 3
Time 1
Time 2
Line 4
Time 1
Time 2

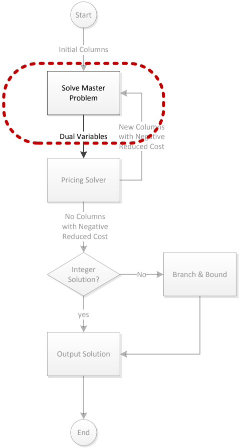
MP – Inputs

$a \in \Omega$	–	set of all possible assignments
$i \in I$	–	set of origin-destination pairs
$t \in T$	–	set of all time steps
$t' \in T^i$	–	set of times that there is a demand between OD i
$l \in L$	–	set of operated lines
c	–	cycle
$\mathcal{C}_a$	–	cost of the assignment a
D_a	–	demand using assignment a
n_l	–	number of available train units on line l

$$B_a^{it'} = \begin{cases} 1 & \text{if OD pair } i \text{ at time } t' \text{ is assigned in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$$

$$E_a^{lt} = \begin{cases} 1 & \text{if the assignment } a \text{ is using line } l \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

MP – Decisions



$$\lambda_a \quad - \quad \begin{cases} 1 & \text{if assignment } a \text{ is a part of the solution,} \\ 0 & \text{otherwise.} \end{cases}$$

$$x_l^t \quad - \quad \begin{cases} 1 & \text{if there is a train scheduled on line } l \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

MP

$$
\begin{aligned}
\min \sum_{a\in\Omega} D_a \cdot \mathcal{C}_a \cdot \lambda_a & \\
\sum_{a\in\Omega} B_a^{it'} \cdot \lambda_a = 1, & \qquad \forall i \in I, \forall t' \in T^i, \\
\sum_{a\in\Omega} E_a^{lt} \cdot \lambda_a \leq x_l^t, & \qquad \forall l \in L, \forall t \in T, \\
\sum_{t\in T} x_l^t \leq n_l, & \qquad \forall l \in L, \\
\sum_{t''=t}^{min(t+c,T)} x_l^{t''} \leq 1, & \qquad \forall l \in L, \forall t \in T, \\
\lambda_a \in \{0,1\}, & \qquad \forall a \in \Omega, \\
x_l^t \in \{0,1\}, & \qquad \forall l \in L, t \in T.
\end{aligned}
$$

MP

$$\min \sum_{a\in\Omega} D_a \cdot \mathcal{C}_a \cdot \lambda_a$$

$$\sum_{a\in\Omega} B_a^{it'} \cdot \lambda_a = 1, \qquad \forall i \in I, \forall t' \in T^i,$$

$$\sum_{a\in\Omega} E_a^{lt} \cdot \lambda_a \leq x_l^t, \qquad \forall l \in L, \forall t \in T,$$

$$\sum_{t\in T} x_l^t \leq n_l, \qquad \forall l \in L,$$

$$\sum_{t''=t}^{min(t+c,T)} x_l^{t''} \leq 1, \qquad \forall l \in L, \forall t \in T,$$

$$\lambda_a \in \{0,1\}, \qquad \forall a \in \Omega,$$

$$x_l^t \in \{0,1\}, \qquad \forall l \in L, t \in T.$$

MP

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\begin{aligned}
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\sum_{a\in\Omega} B_a^{it'} \cdot \lambda_a = 1, & \qquad \forall i \in I, \forall t' \in T^i, \\
\sum_{a\in\Omega} E_a^{lt} \cdot \lambda_a \leq x_l^t, & \qquad \forall l \in L, \forall t \in T, \\
\sum_{t\in T} x_l^t \leq n_l, & \qquad \forall l \in L, \\
\sum_{t''=t}^{min(t+c,T)} x_l^{t''} \leq 1, & \qquad \forall l \in L, \forall t \in T, \\
\lambda_a \in \{0,1\}, & \qquad \forall a \in \Omega, \\
x_l^t \in \{0,1\}, & \qquad \forall l \in L, t \in T.
\end{aligned}
$$

MP

$$\begin{aligned}
\min \sum_{a\in\Omega} D_a \cdot \mathcal{C}_a \cdot \lambda_a & \\
\sum_{a\in\Omega} B_a^{it'} \cdot \lambda_a = 1, & \qquad \forall i \in I, \forall t' \in T^i, \\
\sum_{a\in\Omega} E_a^{lt} \cdot \lambda_a \leq x_l^t, & \qquad \forall l \in L, \forall t \in T, \\
\sum_{t\in T} x_l^t \leq n_l, & \qquad \forall l \in L, \\
\sum_{t''=t}^{min(t+c,T)} x_l^{t''} \leq 1, & \qquad \forall l \in L, \forall t \in T, \\
\lambda_a \in \{0,1\}, & \qquad \forall a \in \Omega, \\
x_l^t \in \{0,1\}, & \qquad \forall l \in L, t \in T.
\end{aligned}$$

MP

$$
\begin{aligned}
\min \sum_{a\in\Omega} D_a \cdot \mathcal{C}_a \cdot \lambda_a & \\
\sum_{a\in\Omega} B_a^{it'} \cdot \lambda_a = 1, &\qquad \forall i \in I, \forall t' \in T^i, \\
\sum_{a\in\Omega} E_a^{lt} \cdot \lambda_a \leq x_l^t, &\qquad \forall l \in L, \forall t \in T, \\
\sum_{t\in T} x_l^t \leq n_l, &\qquad \forall l \in L, \\
\sum_{t''=t}^{min(t+c,T)} x_l^{t''} \leq 1, &\qquad \forall l \in L, \forall t \in T, \\
\lambda_a \in \{0,1\}, &\qquad \forall a \in \Omega, \\
x_l^t \in \{0,1\}, &\qquad \forall l \in L, t \in T.
\end{aligned}
$$

MP

$$\min \sum_{a\in\Omega} D_a \cdot \mathcal{C}_a \cdot \lambda_a$$

$$\sum_{a\in\Omega} B_a^{it'} \cdot \lambda_a = 1, \qquad \forall i \in I, \forall t' \in T^i,$$

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$$\sum_{t\in T} x_l^t \leq n_l, \qquad \forall l \in L,$$

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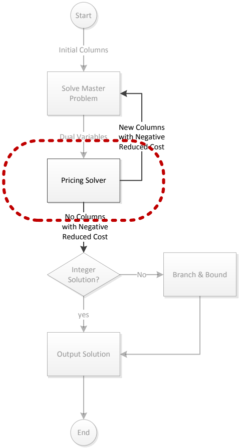
$$\lambda_a \in \{0,1\}, \qquad \forall a \in \Omega,$$

$$x_l^t \in \{0,1\}, \qquad \forall l \in L, t \in T.$$

MP

$$
\begin{aligned}
\min \; & \sum_{a \in \Omega} D_a \cdot \mathcal{C}_a \cdot \lambda_a \\
& \sum_{a \in \Omega} B_a^{it'} \cdot \lambda_a = 1, && \forall i \in I, \forall t' \in T^i, \\
& \sum_{a \in \Omega} E_a^{lt} \cdot \lambda_a \leq x_l^t, && \forall l \in L, \forall t \in T, \\
& \sum_{t \in T} x_l^t \leq n_l, && \forall l \in L, \\
& \sum_{t''=t}^{min(t+c,T)} x_l^{t''} \leq 1, && \forall l \in L, \forall t \in T, \\
& \lambda_a \in \{0,1\}, && \forall a \in \Omega, \\
& x_l^t \in \{0,1\}, && \forall l \in L, t \in T.
\end{aligned}
$$

Sub-Problem (SP)



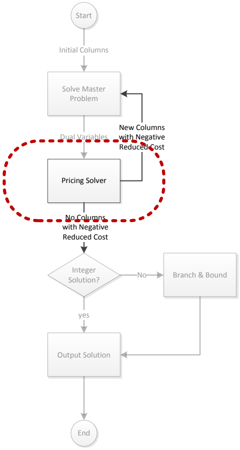
Idea

- run for each OD pair i, each ideal time $t' \in T^i$ and each path $p \in P^i$ separately
- Consists of the pricing constraints

Dual Variables

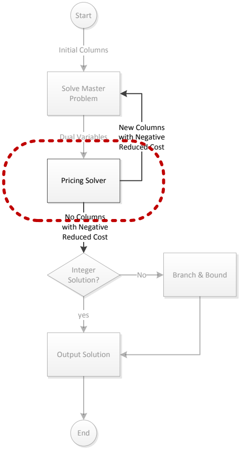
- $\alpha_i^{t'}, \beta_l^t$

SP – Inputs



i	–	the origin destination pair
t'	–	ideal travel time for OD pair i
p	–	path of the sub-problem
$t \in T$	–	set of all time steps
$l \in L^p$	–	the sequence of lines used to get from the origin to the destination
r_l	–	running time of line l
h_l	–	running time to get from the starting station of the line l to the first station on the same line included in the current path
m	–	the minimum transfer time
q_1	–	the value of time spent waiting
q_2	–	the value of time spent in vehicle
f_1	–	coefficient of being early
f_2	–	coefficient of being late

SP – Decision Variables



- $beta_l^t$ – $\begin{cases} 1 & \text{if line } l \text{ is used at time } t, \\ 0 & \text{otherwise.} \end{cases}$
- w – the total waiting time of the passengers
- w_l – the waiting time of the passengers when transferring to line l
- s – scheduled delay of the passengers
- $\mathcal{C}$ – the cost of the passengers

SP – Model

$$\min\ \mathcal{C} - \left(\alpha + \sum_{l\in L}\sum_{t\in T} \beta_l^t \cdot beta_l^t\right)$$

$$\mathcal{C} = \left(q_1 \cdot w + q_1 \cdot m \cdot (|L| - 1) + q_2 \cdot \sum_{l\in L} r^l + q_2 \cdot s\right)$$

$$\sum_{t\in T} beta_l^t = 1, \qquad l \in L^p,$$

$$w = \sum_{l\in L\setminus 1} w_l,$$

$$w_l \geq \left(t \cdot beta_l^t + h_l\right) - \left(t'' \cdot beta_{l-1}^{t''} + h_{l-1} + r_{l-1} + m\right), \qquad \forall l \in L^p : l > 1, \forall t, t'' \in T : t \geq t'' + h_{l-1} + r_{l-1}$$

$$w_l \leq \left(t \cdot beta_l^t + h_l\right) - \left(t'' \cdot beta_{l-1}^{t''} + h_{l-1} + r_{l-1} + m\right), \qquad \forall l \in L^p : l > 1, \forall t, t'' \in T : t \geq t'' + h_{l-1} + r_{l-1}$$

$$s \geq f_2 \cdot \left(\left(t \cdot beta_{|L|}^t + h_{|L|}\right) - t'\right), \qquad \forall t \in T,$$

$$s \geq f_1 \cdot \left(t' - \left(t \cdot beta_{|L|}^t + h_{|L|}\right)\right), \qquad \forall t \in T,$$

domain *constraints*

SP – Model

$$\min\ \mathcal{C} - \left(\alpha + \sum_{l \in L} \sum_{t \in T} \beta_l^t \cdot beta_l^t \right)$$

$$\mathcal{C} = \left(q_1 \cdot w + q_1 \cdot m \cdot (|L| - 1) + q_2 \cdot \sum_{l \in L} r^l + q_2 \cdot s \right)$$

$$\sum_{t \in T} beta_l^t = 1, \qquad l \in L^p,$$

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State of the work

- Original Formulation – currently processing
- Initial Solution – halfway
- Master Problem – ready
- Sub-Problem – ready
- Data Processing – halfway

Conclusions

- New planning phase based on the demand
- In line with the new market structure
- Can handle both non- and cyclic timetables
- Takes care of the connections, in the current practice:
  - non-cyclic – does not exist
  - cyclic – always imposed
- Returns ideal timetables, its cost and the routings of the passengers

SBB CFF FFS ICN RABDe 500 037 “Grock” gestaltet als “Clown” bei der Präsentation im SBB Unterhaltszentrum in Genf.
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Thank you for your attention.