Integrated berth allocation and yard assignment problem using column generation

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May 2, 2012

Agenda

1. Introduction
   - Berth Allocation Problem
   - Yard Assignment Problem
   - Motivation
2. Problem Definition
3. Branch and Price
4. Results
5. Conclusion

Berth Allocation Problem

Figure: Lacon ltd.'s plan for extension of the Riga's port, Latvia

Yard Assignment Problem

Figure: Port of Weipa, Queensland, Australia

Motivation

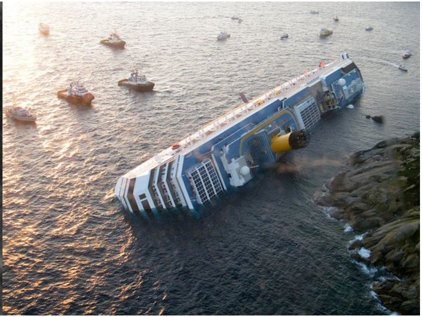
Figure: Vessels queueing at Newcastle port, Australia (queue hits 60 vessels as max)

Input – Vessel

Information

- Number of Vessels
- Arrival Time
- Length
- Draft (omitted)
- Cargo
  - Quantity
  - Cargo Type

Input – Draft Omitted



Input – Port

Information

- Number of Sections
  - Length
  - Draft (omitted)
  - Coordinates
  - Resources
- Number of Cargo Locations
  - Coordinates
  - Neighbouring Locations

Input – General Data

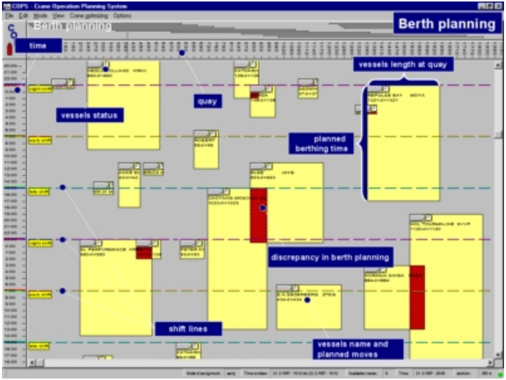
Information

- Time Horizon
- Number of Cargo Types
- Incompatible Cargo Types
- Distances
- Transfer Rate
- Crane Handling Rate
- Bulk Ports (No Containers)

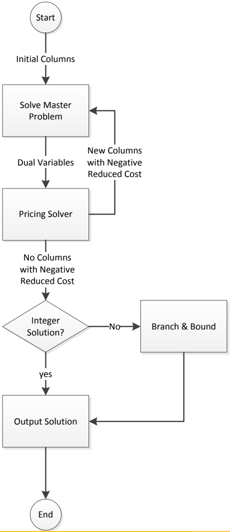
Output

Minimize

- Handling Time + Delay = Service Time
  - Paralell Handling

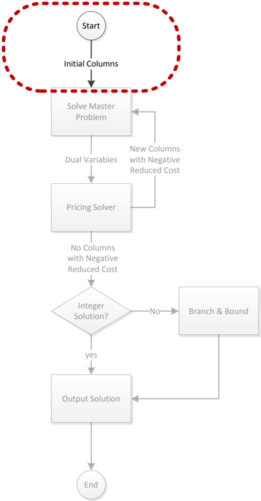


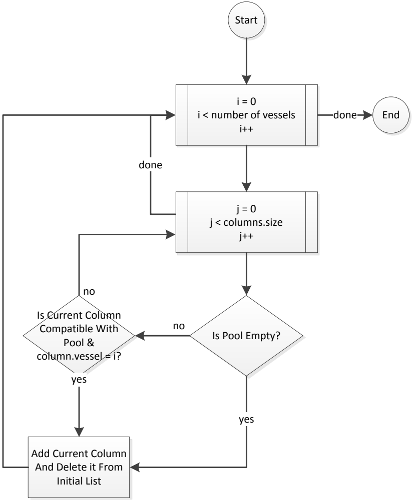
Framework



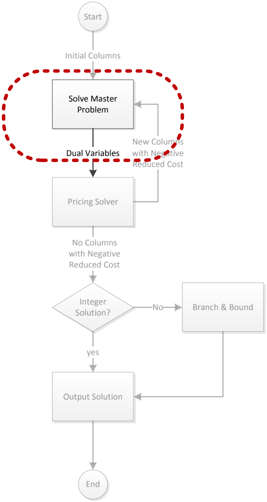
- Initial Solution
- Column Generation – Lower Bound
- Branch and Bound – Optimal Integer Solution

Initial Solution





Master Problem – Parameters



Parameters

- Ω – set of all feasible assignments
- $\Omega_1 \subset \Omega$ – current pool of columns
- c_a – cost of assignment $a \in \Omega\,(\Omega_1)$
- $N = \{1..n\}$ – set of vessels
- $K = \{1..m\}$ – set of sections
- $W = \{1..w\}$ – set of cargo types
- $T = \{1..h\}$ – set of time steps
- $L = \{1..q\}$ – set of locations
- ct_w – number of vessels carrying cargo type w

Master Problem – Parameters

Decision Varibles

- $\lambda_a \in (0, 1)$ – 1 if assignment a is selected, 0 otherwise
- $\mu_w^l \in (0, 1)$ – 1 if location l is storing cargo of type w, 0 otherwise
- relaxed

Idea

- obtain berth and yard schedule
  - section
    - used by vessel at time
  - location
    - stored cargo
    - used by vessel at time

Master Problem – Parameters

Parameters

$$A_a^i = \begin{cases} 1 & \text{if vessel } i \text{ is assigned in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$$

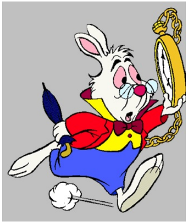
$$B_a^{kt} = \begin{cases} 1 & \text{if section } k \text{ is occupied at time } t \text{ in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$$

$$C_a^{lw} = \begin{cases} 1 & \text{if cargo } w \text{ is stored at location } l \text{ in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$$

$$D_a^{lt} = \begin{cases} 1 & \text{if cargo location } l \text{ is handling assignment } a \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Master Problem – Objective Function

$$minimize \sum_{a \in \Omega} c_a \cdot \lambda_a \tag{1}$$



Master Problem – Constraints
All Vessels Served

$$\sum_{a \in \Omega_1} A_a^i \cdot \lambda_a = 1, \qquad \forall i \in N, \tag{2}$$

Master Problem – Constraints
Section Occupation

$$\sum_{a \in \Omega_1} B_a^{kt} \cdot \lambda_a \leq 1, \qquad \forall k \in K, \forall t \in T, \tag{3}$$

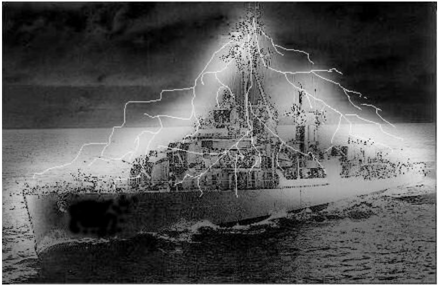


Figure: Illustration of Philadelphia Experiment

Master Problem – Constraints
Location Occupation

$$\sum_{a \in \Omega_1} D_a^{lt} \cdot \lambda_a \leq 1, \qquad \forall l \in L, \forall t \in T, \tag{4}$$

Master Problem – Constraints
One Cargo per Location

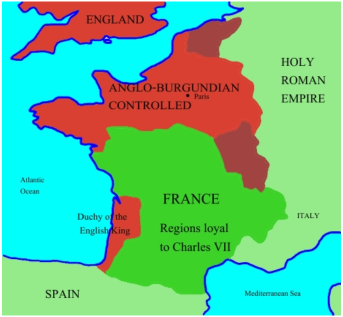
$$\sum_{a \in \Omega_1} C_a^{lw} \cdot \lambda_a - ct_w \cdot \mu_w^l \leq 0, \qquad \forall l \in L, \forall w \in W, \tag{5}$$

$$\sum_{w \in W} \mu_w^l \leq 1, \qquad \forall l \in L, \tag{6}$$

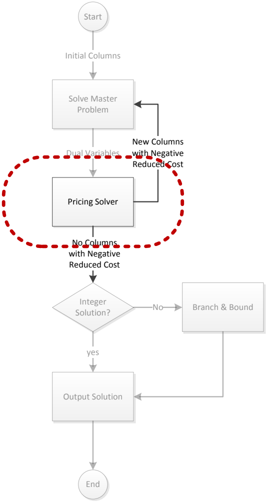


Master Problem – Constraints
Compatible Neighbours

$$\mu_w^l + \mu_{\overline{w}}^{\overline{l}} \leq 1, \qquad \forall l \in L, \forall \overline{l} \in \overline{L}, \forall w \in W, \forall \overline{w} \in \overline{W}, \tag{7}$$



Sub-Problem – Parameters



Idea

- run for each vessel separately
- get n columns (one per vessel)

Sets

- $K = \{1..m\}$ – set of sections
- $W = \{1..w\}$ – set of cargo types
- $T = \{1..h\}$ – set of time steps
- $L = \{1..q\}$ – set of locations

Dual Variables

- $\alpha, \beta_{kt}, \gamma_{lt}, \delta_{lw}$

Sub-Problem – Objective Function

$$minimize\ (c + s - a) - (\alpha + \sum_{k\in K}\sum_{t\in T}\beta_{kt}\cdot beta_{kt} + \sum_{l\in L}\sum_{t\in T}\gamma_{lt}\cdot gamma_{lt} + \sum_{l\in L}\sum_{w\in W}\delta_{lw}\cdot delta_{lw}) \quad (8)$$

Parameters

- a – arrival time

Decision Variables

- $c \geq 0$ – handling time
- $s \geq 0$ – start time of service
- related to duals:
  - $beta_{kt} \in (0,1)$ – 1 if vessel occupies section k at time t, 0 otherwise
  - $gamma_{lt} \in (0,1)$ – 1 if vessel uses location l at time t, 0 otherwise
  - $delta_{lw} \in (0,1)$ – 1 if cargo type w is stored at location l, 0 otherwise

Sub-Problem – Constraints

$$s - a \geq 0, \tag{9}$$

$$c \geq ht_k \cdot fraction_{jk} - M \cdot (1 - ss_j)\,, \qquad \forall k, j \in K, \tag{10}$$

Parameters

- $fraction_{jk}$ – fraction of cargo handled at section k, if the starting section of the vessel is section j
- M – large enough number (set to 1 000 000, could be the largest quantity multiplied by the longest service time)

Decision Variables

- $ht_k \geq 0$ – handling time of section k
- $ss_j \in (0, 1)$ – 1 if section j is the starting section of the vessel

Sub-Problem – Constraints

$$\sum_{j \in K} ss_j = 1, \tag{11}$$

$$\sum_{j \in K} ss_j \cdot sc_j + length \leq ql, \tag{12}$$

Parameters

- sc_j – starting coordinate of section j
- $length$ – length of the vessel
- ql – quay length

Sub-Problem – Constraints

$$\sum_{l \in L} split_l \leq Z, \tag{13}$$

Parameters

- Z – maximum number of locations used by vessel

Decision Variables

- $split_l \in (0, 1)$ – 1 if vessel uses location l

Sub-Problem – Constraints

$$split_l \leq delta_{lw}, \qquad \forall l \in L, \tag{14}$$

$$\sum_{l \in L} cs_l = quantity, \tag{15}$$

$$cs_l \leq split_l \cdot quantity, \qquad \forall l \in L, \tag{16}$$

$$split_l \leq cs_l, \qquad \forall l \in L, \tag{17}$$

Parameters

- w – cargo type carried on the vessel

Decision Variables

- $cs_l \geq 0$ – quantity of cargo stored at location l

Sub-Problem – Constraints

$$td_k = \left(\sum_{l \in L} d_{kl} \cdot cs_l \right) / quantity, \qquad \forall k \in K, \qquad (18)$$

$$ht_k = F / cranes_k + V_w \cdot td_k, \qquad \forall k \in K, \qquad (19)$$

Parameters

- d_{kl} – distance between section k and location l
- $cranes_k$ – number of cranes in section k
- F – crane handling rate, V_w – cargo transfer rate

Decision Variables

- $td_k \geq 0$ – total average distance for section k

Sub-Problem – Constraints

$$\sum_{t \in T} time_t = c, \tag{20}$$

$$t + M \cdot (1 - time_t) \geq s + 1, \qquad \forall t \in T, \tag{21}$$

$$t \leq s + c + M \cdot (1 - time_t), \qquad \forall t \in T, \tag{22}$$

Parameters

- M in 21 – minimum value is $s + 1$
- M in 22 – minimum value is $T - s + c$

Decision Variables

- $time_t \in (0, 1)$ – 1 if the vessel is at time t served, 0 otherwise

Sub-Problem – Constraints

$$beta_{kt} \geq x_k + time_t - 1, \qquad \forall k \in K, \forall t \in T, \qquad (23)$$

$$beta_{kt} \leq x_k, \qquad \forall k \in K, \forall t \in T, \qquad (24)$$

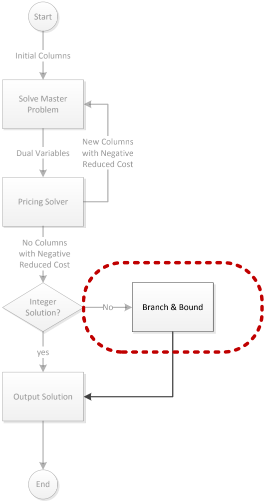
$$beta_{kt} \leq time_t, \qquad \forall k \in K, \forall t \in T, \qquad (25)$$

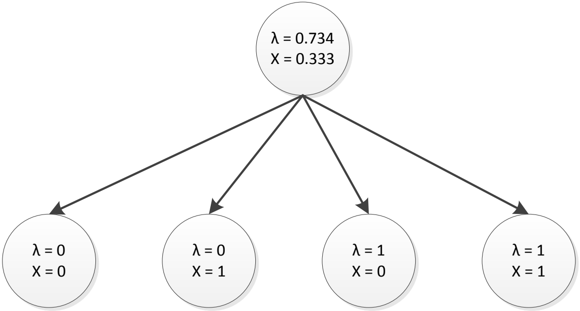
$$gamma_{lt} \geq split_l + time_t - 1, \qquad \forall l \in L, \forall t \in T, \qquad (26)$$

$$gamma_{lt} \leq split_l, \qquad \forall l \in L, \forall t \in T, \qquad (27)$$

$$gamma_{lt} \leq time_t, \qquad \forall l \in L, \forall t \in T, \qquad (28)$$

Branch and Bound

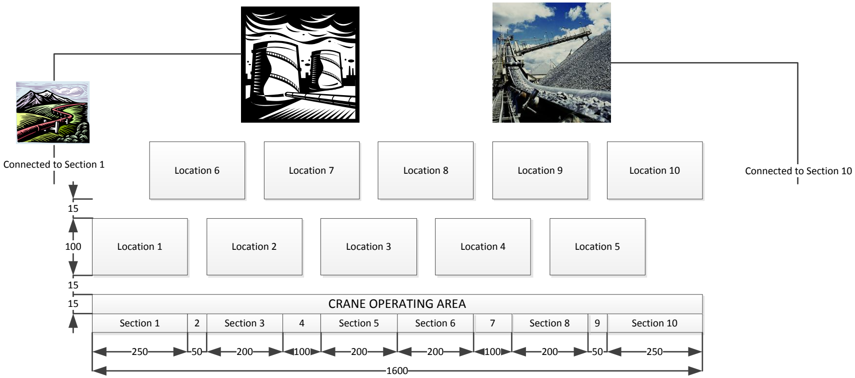




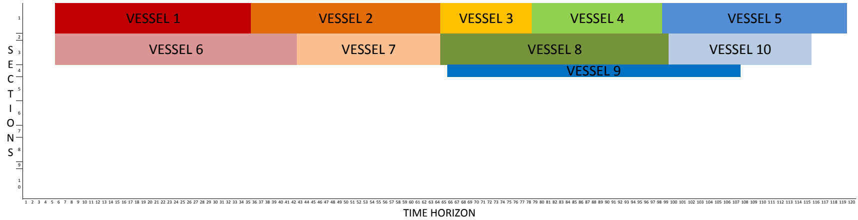
Status

- **Master problem:**
  - able to solve small instances
  - finished
- **Sub - problem:**
  - validated with Opl and the minimum handling time of generated assignments
  - running time < 3 sec.
- **Column Generation:**
  - without sub-problem, just reduced cost, able to solve small instances
  - strategies:
    - one column per turn – not able to solve in 3 hours (and still working)
    - all negative columns per turn – solved below 20 iterations
  - sub-problem
- **Branch and Bound:**
  - to be implemented
  - probably existing function in CPLEX

Test Instance



Initial Berth Plan



Debugging


DEBUGGING
To catch a bug, you've got to learn to think like a bug

Thank you for your attention.

$$\min z \tag{1}$$

$$\text{s.t. } m_i - A_i \geq 0 \tag{2}$$

$$\sum_{k\epsilon M} s_k^i = 1 \tag{3}$$

$$\sum_{k\epsilon M} (s_k^i b_k) + L_i \leq L \tag{4}$$

$$\sum_{p\epsilon M} (\delta_{i\ell k} s_\ell^i) = x_{ik} \qquad \forall k\epsilon M \tag{5}$$

$$\sigma_t^{ik} \geq x_{ik} + \theta_{it} - 1 \qquad \forall k\epsilon M, \forall t\epsilon H \tag{6}$$

$$\sigma_t^{ik} \leq x_{ik} \qquad \forall k\epsilon M, \forall t\epsilon H \tag{7}$$

$$\sigma_t^{ik} \leq \theta_{it} \qquad \forall k\epsilon M, \forall t\epsilon H \tag{8}$$

$$(d_k - D_i) x_{ik} \geq 0 \qquad \forall k\epsilon M \tag{9}$$

$$c_i \geq h_{ik}^w \rho_{i\ell k} Q_i - B(1 - s_\ell^i) \quad \forall l\epsilon M, \forall k\epsilon M, \forall w\epsilon W_i \tag{10}$$

$$h_{ik}^w = \alpha_{ik}^w + \beta_{ik}^w \qquad \forall w\epsilon W_i, \forall k\epsilon M \tag{11}$$

$$\alpha_{ik}^w = T / n_{ik}^w \qquad \forall w\epsilon W_i, \forall k\epsilon M \tag{12}$$

$$\beta_{ik}^w = V_w r_k^i \qquad \forall w\epsilon W_i, \forall k\epsilon M \tag{13}$$

$$\sum_{l\epsilon L} \phi_{il} \leq 2 \tag{14}$$

$$r_k^i = \sum_{l\epsilon L} (r_k^l \lambda_{il}) / Q_i \qquad \forall k\epsilon M \tag{15}$$

$$\sum_{w\epsilon W} \pi_w^l \leq 1 \qquad \forall l\epsilon L \tag{16}$$

$$\phi_{il} \leq \pi_w^l \qquad \forall w\epsilon W_i, \forall l\epsilon L \tag{17}$$

$$\omega_t^{il} \geq \phi_{il} + \theta_{it} - 1 \qquad \forall l\epsilon L, \forall t\epsilon H \tag{18}$$

$$\omega_t^{il} \leq \phi_{il} \qquad \forall l\epsilon L, \forall t\epsilon H \tag{19}$$

$$\omega_t^{il} \leq \theta_{it} \qquad \forall l\epsilon L, \forall t\epsilon H \tag{20}$$

$$\sum_{t\epsilon H} \theta_{it} = c_i \tag{21}$$

$$t + B(1 - \theta_{it}) \geq m_i + 1 \qquad \forall t\epsilon H \tag{22}$$

$$t \leq m_i + c_i + +B(1 - \theta_{it}) \qquad \forall t\epsilon H \tag{23}$$

$$Q_i = \sum_{l\epsilon L} \lambda_{il} \tag{24}$$

$$\lambda_{il} \leq \phi_{il} Q_i \qquad \forall w\epsilon W_i, \forall l\epsilon L \tag{25}$$

$$\phi_{il} \leq B\lambda_{il} \qquad \forall l\epsilon L \tag{26}$$

$$\lambda_{il} \leq \sum_{w\epsilon W_i} \sum_{t\epsilon H} (R_w \omega_t^{il} + B(1 - \pi_w^l)) \qquad \forall l\epsilon L \tag{27}$$

(28)