An optimisation framework for activity-based models

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About me

- 4th year PhD student
  - Transport and Mobility Lab (TRANSP-OR) @ EPFL, Switzerland
  - Supervisors: Prof. Michel Bierlaire, Dr. Tim Hillel

- Masters in Civil Engineering  (2019)

- UCL Visit until December
Introduction

Activity demand

Socio-economic characteristics
Social interactions
Cultural norms
Basic needs
...
(Chapin, 1974)

Travel demand

Time and space constraints
(Hägerstrand, 1970)
### Introduction

#### Utility-based models

*Decision is made by maximizing utility derived from activities*

- Bowman & Ben-Akiva, 2001
- Bhat et al, 2004

#### Rule-based models

*Decision is made by considering context-dependent rules*

- Gollegde et al., 1994
- Arentze & Timmermans, 2000
OASIS framework

- Optimisation-based Activity Scheduling Integrating Simultaneous choice dimensions
OASIS framework

- Optimisation-based Activity Scheduling **Integrating Simultaneous choice dimensions**
  - Activity participation, scheduling, mode, location choice
  - Explicitly capture **trade-offs** between choices
  - Combine econometric and rule-based approaches
OASIS framework

Data → Estimation → Parameters $\beta_n$ → Optimisation $\Omega$ → Schedule $S^n_\varepsilon$ → Indicators

Literature

Synthetic individual $n$

Disturbances $\varepsilon$
OASIS framework

Simulation module

Data → Estimation → Parameters $\beta_n$ → Synthetic individual $n$ → Disturbances $\varepsilon$ → Optimisation $\Omega$ → Schedule $S_{\varepsilon}^n$ → Indicators

Literature
Simulation

<table>
<thead>
<tr>
<th>Location</th>
<th>Mode</th>
<th>Start Time</th>
<th>Duration</th>
<th>Travel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Mode</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Location 2</td>
<td>Mode 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Location n</td>
<td>Mode m</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Feasible time interval

$\Omega_n = \max \sum U_{in}$

$U_i(x_i, \tau_i, \delta_{xin}, \delta_{tin}, t_i, \omega_{in})$

From an activity...

...to a utility function...

...to a maximisation problem
Activities

Definitions

- Start time
- Duration
- Travel
- Feasible time interval

Location
Mode

Start time

Duration

Travel

Time [h]
## Definitions

### Activities

- **Location 1**
  - **Mode 1**
  - e.g. Working from home

- **Location 2**
  - **Mode 2**
  - e.g. Working on campus, travelling by car

- **...**

- **Location n**
  - **Mode m**
  - e.g. Working on campus, travelling by PT
Utilities

People are time sensitive:
  • Preferences for start time, duration and/or end-time

\[ \delta_{\text{start}} \quad \delta_{\text{duration}} \]
Utilities

People derive a utility (satisfaction) when they perform activities

\[ U = f(\beta, X) \]

\[ U_{an} = U_{participation} + U_{start \ time} + U_{duration} + U_{travel} + \varepsilon_{an} \]
Utilities

People derive a utility (satisfaction) when they perform activities

\[ U_{an} = U_{participation} + U_{start\ time} + U_{duration} + U_{travel} + \varepsilon_{an} \]

Utility of doing the activity itself, regardless of when/how long

e.g. \( cst + \beta_{cost} c_a \)
Utilities

People derive a utility (satisfaction) when they perform activities

\[ U_{an} = U_{participation} + U_{start\ time} + U_{duration} + U_{travel} + \varepsilon_{an} \]

Start time deviations

\[ \beta_{early} \max(0, x^*_a - x_a) \]
\[ + \ \beta_{late} \max(0, x_a - x^*_a) \]
Utilities

People derive a utility (satisfaction) when they perform activities

\[ U_{an} = U_{participation} + U_{start\ time} + U_{duration} + U_{travel} + \varepsilon_{an} \]

Duration deviations

\[
\beta_{short} \max(0, \tau^*_a - \tau_a) + \\
\beta_{late} \max(0, \tau_a - \tau^*_a)
\]
Utilities

People derive a utility (satisfaction) when they perform activities

\[ U_{an} = U_{participation} + U_{start \ time} + U_{duration} + U_{travel} + \varepsilon_{an} \]

Travel from activity a to b
\[ \beta_{t,\text{time}}\rho_{ab} + \beta_{t,\text{cost}}c_{ab} \]
Utilities

People derive a utility (satisfaction) when they perform activities

\[ U_{an} = U_{participation} + U_{start\ time} + U_{duration} + U_{travel} + \varepsilon_{an} \]

Error terms with known distribution
Individuals maximise the **total utility**, subject to constraints:

\[ \Omega = \max \sum_a U_{an} \]

- Decision variables:
  - Activity participation
  - Start time
  - Duration
  - Succession between activities
Optimisation model

- Individuals maximise the **total utility**, subject to constraints:

\[
\Omega = \max \sum_a U_{an}
\]

- Constraints:
  - Time budget
  - No duplicates
  - Mode consistency
  - Resource availability
  - Participation constraints
  - Sequence constraints
Simulation

- Simulation procedure:
  - Draw $\beta^r$ from distribution of $\beta$
  - Draw $\varepsilon^r$ from distribution of $\varepsilon$
  - Solve $\Omega$ for $(\beta^r, \varepsilon^r)$
  - Repeat $N$ times
OASIS framework
Parameter estimation

OASIS framework

- **Data**
- **Literature**
- **Estimation**
- **Synthetic individual \( n \)**
- **Parameters \( \beta_n \)**
- **Disturbances \( \varepsilon \)**
- **Optimisation \( \Omega \)**
- **Schedule \( S^\varepsilon_n \)**
- **Indicators**
Estimation

How do we estimate the parameters of the model?

\[ U = f(\beta, X) \]

\[ U_{an} = U_{\text{participation}} + U_{\text{start time}} + U_{\text{duration}} + U_{\text{travel}} + \varepsilon_{an} \]
 Maximum likelihood estimation (MLE) of parameters in discrete choice models:

\[
\hat{\beta} = \arg \max L_n(\beta) \\
L_n = \prod_{n=1}^{N} \prod_{i \in C_n} P_n(i)^{y_{in}}
\]
Parameter estimation

- Maximum likelihood estimation (MLE) of parameters in discrete choice models:

\[ \hat{\beta} = \arg \max L_n(\beta) \]

\[ L_n = \prod_{n=1}^{N} \prod_{i \in C_n} P_n(i)^{y_{in}} \]

Enumeration over choice set \( C_n \)

- Common assumptions on choice set:
  - Universal across population
  - Fully observed or observable
Choice set generation

Unobserved and possibly infinite

Feasible schedules

Considered schedules

Actual choice set: Unobserved

Realised schedule

Estimation choice set: sample of feasible schedules generated for estimation purposes

Based on Shocker (1991)
Estimation

- **Choice set generation**
  - Metropolis-Hastings sampling of feasible schedules
Swiss Mobility and Transport Microcensus 2015 (BFS & ARE, 2017)

Sample
• Students living in Lausanne (236 individuals)

Choice set size
• N = 10 alternatives

Model 0:
• Deviation parameters from literature

Model 1 (12 parameters):
• Activity-specific constants
• Aggregated penalties (flexible vs. Non flexible)

Model 2 (20 parameters):
• Activity-specific constants
• Activity specific penalties
# Estimation

## Model 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Param. estimate</th>
<th>Rob. std err</th>
<th>Rob. t-stat</th>
<th>Rob. p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 F: early</td>
<td>-0.175</td>
<td>0.12</td>
<td>-1.46</td>
<td>0.145</td>
</tr>
<tr>
<td>2 F: late</td>
<td>-0.333</td>
<td>0.14</td>
<td>-2.38</td>
<td>0.0171</td>
</tr>
<tr>
<td>3 F: long</td>
<td>-0.105</td>
<td>0.0722</td>
<td>-1.45</td>
<td>0.146</td>
</tr>
<tr>
<td>4 F: short</td>
<td>-0.114</td>
<td>0.194</td>
<td>-0.585</td>
<td>0.559</td>
</tr>
<tr>
<td>5 NF: early</td>
<td>-1.14</td>
<td>0.367</td>
<td>-3.10</td>
<td>0.00191</td>
</tr>
<tr>
<td>6 NF: late</td>
<td>-0.829</td>
<td>0.229</td>
<td>-3.61</td>
<td>0.0003</td>
</tr>
<tr>
<td>7 NF: long</td>
<td>-1.20</td>
<td>0.393</td>
<td>-3.05</td>
<td>0.00231</td>
</tr>
<tr>
<td>8 NF: short</td>
<td>-1.19</td>
<td>0.468</td>
<td>-2.54</td>
<td>0.0011</td>
</tr>
<tr>
<td>9 Education: ASC</td>
<td>16.0</td>
<td>2.46</td>
<td>6.49</td>
<td>8.63e-11</td>
</tr>
<tr>
<td>10 Leisure: ASC</td>
<td>8.81</td>
<td>1.7</td>
<td>5.17</td>
<td>2.28e-07</td>
</tr>
<tr>
<td>11 Shopping: ASC</td>
<td>6.85</td>
<td>1.80</td>
<td>3.80</td>
<td>0.000146</td>
</tr>
<tr>
<td>12 Work: ASC</td>
<td>16.0</td>
<td>2.58</td>
<td>6.18</td>
<td>6.57e-10</td>
</tr>
</tbody>
</table>

## Model 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Param. estimate</th>
<th>Rob. std err</th>
<th>Rob. t-stat</th>
<th>Rob. p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Education: ASC</td>
<td>18.7</td>
<td>3.17</td>
<td>5.89</td>
<td>3.79e-09</td>
</tr>
<tr>
<td>2 Education: early</td>
<td>-1.35</td>
<td>0.449</td>
<td>-3.01</td>
<td>0.00264</td>
</tr>
<tr>
<td>3 Education: late</td>
<td>-1.63</td>
<td>0.416</td>
<td>-3.91</td>
<td>9.05e-05</td>
</tr>
<tr>
<td>4 Education: long</td>
<td>-1.14</td>
<td>0.398</td>
<td>-2.86</td>
<td>0.00428</td>
</tr>
<tr>
<td>5 Education: short</td>
<td>-1.75</td>
<td>0.457</td>
<td>-3.84</td>
<td>0.000123</td>
</tr>
<tr>
<td>6 Leisure: ASC</td>
<td>8.74</td>
<td>1.94</td>
<td>4.50</td>
<td>6.79e-06</td>
</tr>
<tr>
<td>7 Leisure: early</td>
<td>-0.0996</td>
<td>0.119</td>
<td>-0.836</td>
<td>0.403</td>
</tr>
<tr>
<td>8 Leisure: late</td>
<td>-0.239</td>
<td>0.115</td>
<td>-2.07</td>
<td>0.0385</td>
</tr>
<tr>
<td>9 Leisure: long</td>
<td>-0.08</td>
<td>0.0617</td>
<td>-1.30</td>
<td>0.195</td>
</tr>
<tr>
<td>10 Leisure: short</td>
<td>-0.101</td>
<td>0.149</td>
<td>-0.682</td>
<td>0.495</td>
</tr>
<tr>
<td>11 Shopping: ASC</td>
<td>10.5</td>
<td>2.20</td>
<td>4.78</td>
<td>1.74e-06</td>
</tr>
<tr>
<td>12 Shopping: early</td>
<td>-1.01</td>
<td>0.287</td>
<td>-3.51</td>
<td>0.000443</td>
</tr>
<tr>
<td>13 Shopping: late</td>
<td>-0.858</td>
<td>0.237</td>
<td>-3.63</td>
<td>0.000284</td>
</tr>
<tr>
<td>14 Shopping: long</td>
<td>-0.683</td>
<td>0.387</td>
<td>-1.76</td>
<td>0.0779</td>
</tr>
<tr>
<td>15 Shopping: short</td>
<td>-1.81</td>
<td>1.73</td>
<td>-1.04</td>
<td>0.297</td>
</tr>
<tr>
<td>16 Work: ASC</td>
<td>13.1</td>
<td>2.64</td>
<td>4.96</td>
<td>7.16e-07</td>
</tr>
<tr>
<td>17 Work: early</td>
<td>-0.619</td>
<td>0.217</td>
<td>-2.85</td>
<td>0.00438</td>
</tr>
<tr>
<td>18 Work: late</td>
<td>-0.338</td>
<td>0.168</td>
<td>-2.02</td>
<td>0.0438</td>
</tr>
<tr>
<td>19 Work: long</td>
<td>-1.22</td>
<td>0.348</td>
<td>-3.51</td>
<td>0.000441</td>
</tr>
<tr>
<td>20 Work: short</td>
<td>-0.932</td>
<td>0.213</td>
<td>-4.37</td>
<td>1.23e-05</td>
</tr>
</tbody>
</table>
Estimation

Data

Model 0

Model 1

Model 2
Estimation

Education

Work
OASIS framework

Data → Estimation → Synthetic individual $n$

Literature → Estimation

Parameters $\beta_n$ → Optimisation $\Omega$

Disturbances $\varepsilon$ → Optimisation $\Omega$

Schedule $S^n_\varepsilon$ → Indicators
OASIS framework

Data → Estimation → Parameters $\beta_n$ → Optimisation $\Omega$ → Schedule $S^n_\varepsilon$ → Indicators

- Literature
- Synthetic individual $n$
- Disturbances $\varepsilon$

Applications
Applications

- **OPTIMS (OPTimisation of Individual Mobility Schedules)**
  - Collaboration with Swiss Federal Railways (SBB)
  - Integration of optimisation model into SBB’s forecasting framework

- [https://github.com/optims-org/optims-sbb](https://github.com/optims-org/optims-sbb)

Manser et al (2022)
Applications
Applications

- Sociological applications (e.g. mobility motifs)
- Energy demand
- Epidemiology
- ...

Schultheiss (2021)
Conclusion

Summary
- Optimisation framework to simulate activity schedules
  - Simultaneous estimation of all scheduling dimensions
  - Combining econometric and rule-based approaches
- Methodology to estimate the parameters
- Successful practical applications

Current challenges – future work:
- Validation
- Intra- and interpersonal interactions
Related publications


Thank you!

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