A Lagrangian relaxation technique for the demand-based benefit maximization problem

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Outline

Discrete choice models and optimization

- Disaggregate demand modeling
- Behavioral realism
- Complex formulations

- Supply decisions
- Linearity and/or convexity
- MILP models

Bridging the gap

- Linear characterization of a discrete choice model
- Simulation to address stochasticity
- Demand-based benefit maximization problem (MILP example)
- General framework that can be applied with an existing choice model

Linearization of the discrete choice model

Choice set $\mathscr{C}$ (i)

Population N (n)

$$U_{in} = V_{in} + \varepsilon_{in} \xrightarrow{\text{draw distribution } (R)} U_{inr} = V_{in} + \xi_{inr}$$

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Demand representation

Choice w_{inr}

$$w_{inr} = \begin{cases} 1 & \text{if } U_{inr} \geq U_{jnr}, \forall j \in \mathscr{C}_n, j \neq i \\ 0 & \text{otherwise} \end{cases} \xrightarrow{\text{demand}} D_i = \frac{1}{R} \sum_r \sum_n w_{inr}$$

Demand representation

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Benefit maximization problem (1)

- Set of alternatives $\mathscr{C}$ $(i > 0)$
- Opt-out option $i = 0$
- Population N $(n \geq 1)$

- Price $a_{in} \leq p_{in} \leq b_{in}$
- Capacity levels c_{iq} (Q levels, each with a certain cost)

Benefit maximization problem (2)

obj. fun.	$\sum_{i>0} \text{Revenue}_i - \text{Cost}_i$
availability	operator level and scenario level
disc. utility	variable capturing availability and utility
choice	linearization of the highest utility
price	linearization of the product $w_{inr} p_{in}$ (revenue)
capacity	relation with the availability at scenario level

Computational results

- Parking choices: mixtures of logit model
- Distributed parameters (and correlated)
- $R = 50$ draws and $N = 50$ customers
- $|\mathscr{C}| = 3$: PSP, PUP and FSP (opt-out)

- Several experiments
  - Price calibration (discrete and continuous prices)
  - Price differentiation by population segmentation
- Computational times up to 34 hours!

Motivation

Customer (n)

- Maximization of own utility
- Objective function and capacity constraints

Draw (r)

- Behavioral scenario
- Objective function

Why Lagrangian relaxation?

obj. fun.

easy constraints

hard constraints

obj. fun. $+\alpha$ hard constraints

easy constraints

Lagrangian decomposition

obj. fun.

constraints

$x = y$

obj. fun. (x)

constraints (x)

$+$

obj. fun. (y)

constraints (y)

Revenue maximization + unlimited capacity

obj. fun.	$\sum_{i>0} \text{Revenue}_i$
availability	
utility	no need for discounted utility (no availability)
choice	linearization of the highest utility
price	linearization of the product $w_{inr} p_{in}$ (revenue)
capacity	

Lagrangian decomposition

⚠ Price p_{in} is the same across draws $\Rightarrow$ **no** decomposition by n and r

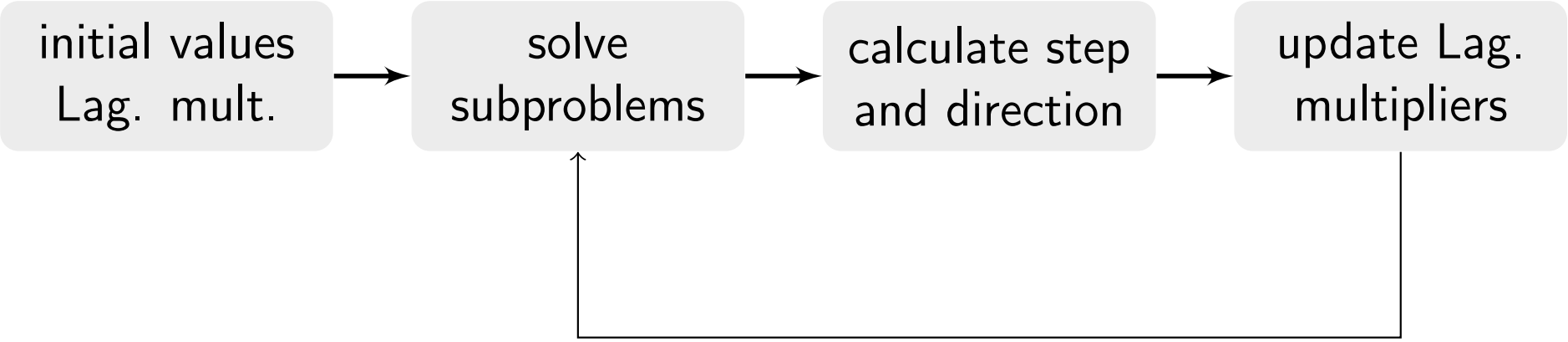
$$p_{in1} = p_{in2} = \cdots = p_{inR} = p_{in1}$$

$p_{inr} - p_{in(r+1)} = 0 \Rightarrow$ Lagrangian multipliers $\alpha_{inr} \Rightarrow$ decomposition by n and r

Lagrangian subproblems

- Objective function:
  - Price of the chosen alternative
  - Lagrangian term: $(\alpha_{inr} - \alpha_{in(r-1)})p_{inr}$
- One alternative is chosen (based on the highest utility)
- Price of the chosen alternative specific for the draw

Subgradient method (1)



Subgradient method (2)

Input: UB: $Z(\overline{\alpha})$ with $\overline{\alpha}$ starting values, LB: Z^* (from a feasible solution)

```
while k < K or Z(α(k)) has not improved after some iterations do
    for r = 1...R do
        for n = 1...N do
            Lagrangian subproblem Z_nr(α(k)) (MILP);
            Obtain p_inr and Z_nr(α(k));
        end
    end
    Compute Z(α(k)) = Σ_r Σ_n Z_nr(α(k));
    k ← k + 1;
    Obtain ω(k) (step) and d_inr(k) (direction);
    Update the Lagrangian multipliers: α_inr(k) = α_inr(k−1) − ω(k) d_inr(k)
end
```

Case study

- $N = 20$ and $R = 100$
- Price bounds PSP: $[0.5, 1.0]$
- Price bounds PUP: $[0.7, 1.2]$
- Number of iterations: $K = 250$

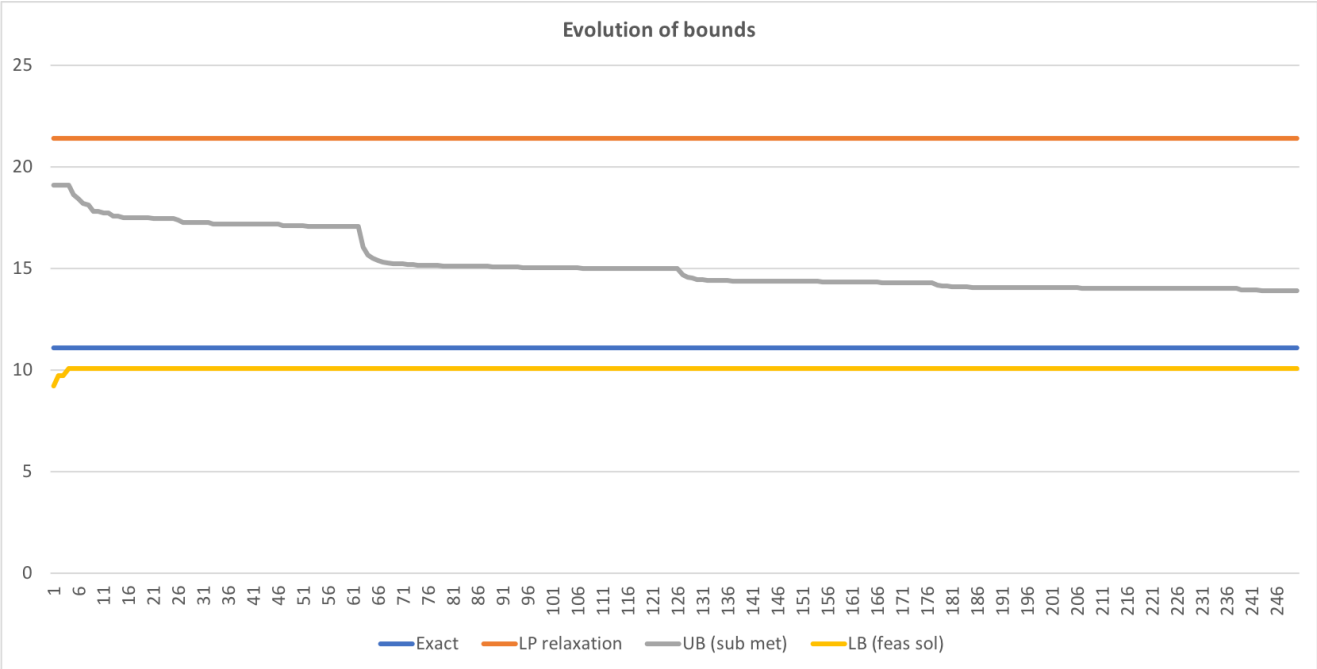
Evolution bounds

Computational time:

- Exact method: 32 min
- Subgradient method: 5.9 min (1.4 s/it)

Objective function:

- MILP: 11.0773
- LP relaxation: 21.4114



Valid inequalities

- Same optimal solution for the MILP
- Tighter formulation for the LP relaxation

$$\sum_i U_{inr} w_{inr} \geq U_{jnr} \ \forall j, n, r$$

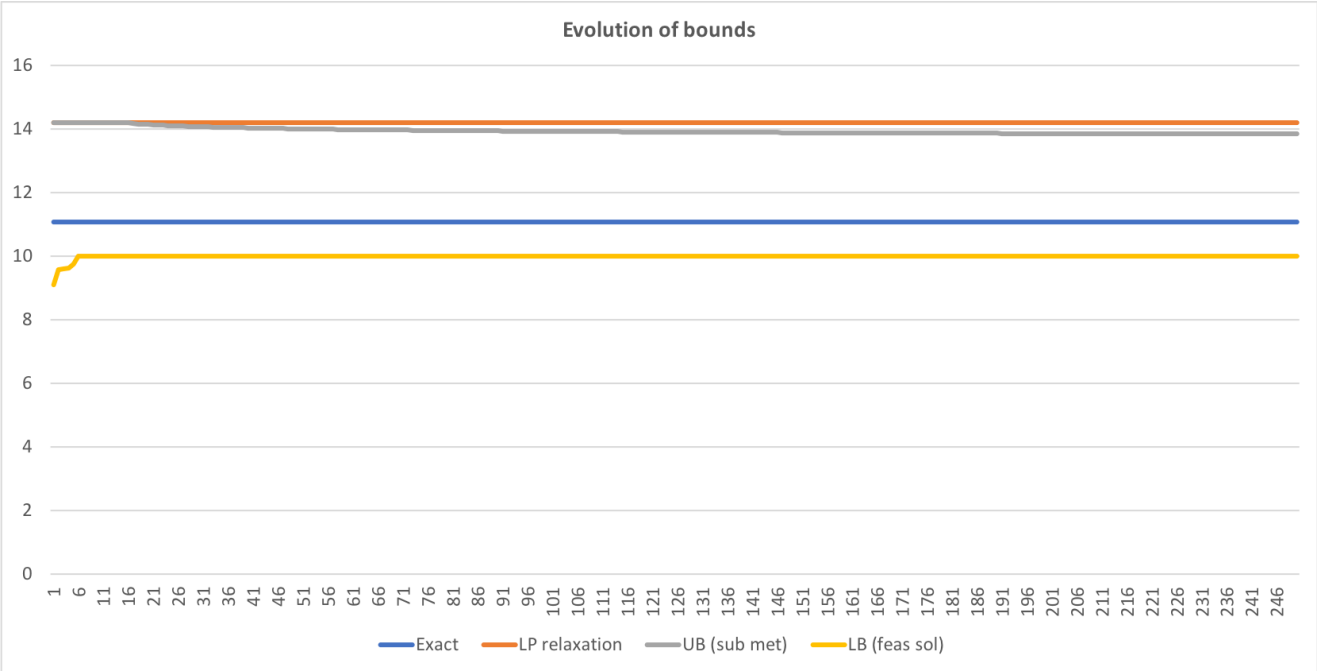
Evolution bounds (with valid inequality)

Computational time:

- Exact method: 11 min
- Subgradient method: 34 min (7 s/it)

Objective function:

- MILP: 11.0773
- LP relaxation: 14.1934



Observations

- Cheap iterations of the subgradient method
- LB provides a feasible solution
- Valid inequality
  - It helps to strength the LP bound (21.41 vs 14.19)
  - More expensive iterations but a smaller amount gives better bounds

Conclusions

- Efficient method to obtain lower and upper bounds + feasible solution
- Different configurations of parameters might help
- Valid inequalities

Future work

- Define other techniques to generate feasible solutions (LB)
- Try other valid inequalities: $(U_{nr} - U_{inr})w_{inr}$ in the objective function
  - $w_{inr} = 0 \Rightarrow$ term vanishes
  - $w_{inr} = 1 \Rightarrow U_{nr} = U_{inr} \Rightarrow$ term vanishes
- Evaluate other strategies (e.g., regularization term)
- Gradually include the complexity back (capacity, availability...)

Questions?

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Subgradient method: step size and direction

$$\alpha_{inr}(k) = \alpha_{inr}(k-1) - \omega(k) d_{inr}(k)$$

Step:

- $\omega(k) = \lambda(k) \frac{Z(\alpha(k-1)) - Z^*}{\|\gamma(k)\|^2}$
- $\lambda(0) \in [0,2)$
- $\gamma_{inr}(k) = p_{inr}(k) - p_{in(r-1)}(k)$ (subgradients)
- $\lambda(k)$ divided by ω_1 if $Z(\alpha(k))$ has not improved in ω_2 iterations

Direction:

- $d(k) = \gamma(k) + \theta d(k-1)$
- $\theta \in [0,1)$