Integrating supply and demand within the framework of mixed integer optimization problems

Meritxell Pacheco Paneque
Shadi Sharif Azadeh, Michel Bierlaire

Transport and Mobility Laboratory (TRANSP-OR),
School of Architecture, Civil and Environmental Engineering (ENAC)
École Polytechnique Fédérale de Lausanne

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Outline

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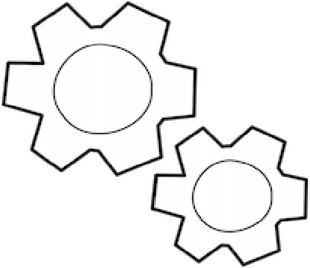
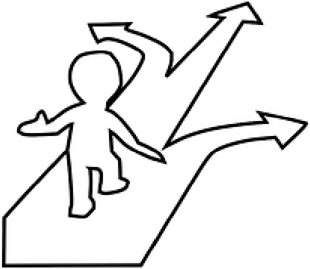
Motivation

Supply



- Demand is given
- Configuration of the system
- Maximize revenues
- **Here:** MILP

Demand



- System configuration is known
- Demand prediction
- Maximize satisfaction
- **Here:** discrete choice models

Integration of supply and demand

- Mathematical formulations of discrete choice models
  - Probabilistic
  - Nolinearity and nonconvexity
- Linear approach addressing
  - Nonconvex representation of probabilities
  - Wide class of discrete choice models
- High dimension of the problem: decomposition techniques

State of the art

- Two main integration paradigms
  - Exogeneous utility (decision variables are not in the utility)
  - Endogeneous utility $\Rightarrow$ introduces nonlinearity and nonconvexity to the optimization model
- The assumption of exogeneously given demand might be unrealistic
- Endogenous utility provides a better representation of the demand...
- ... but the complexity increases

Outline

Utility

Supply and demand

- Population of N individuals
- Set of alternatives $\mathcal{C}$
  - artificial *opt-out* alternative
- $\mathcal{C}_n \subseteq \mathcal{C}$ subset of available alternatives to individual n

Utility

$U_{in} = V_{in} + \varepsilon_{in}$: associated score with alternative i by individual n

- V_{in}: deterministic part
- ε_{in}: error term

Behavioral assumption: n chooses i if U_{in} is the highest in $\mathcal{C}_n$

Probabilistic model

Choice

$$w_{in} = \begin{cases} 1 & \text{if } n \text{ chooses } i \\ 0 & \text{otherwise} \end{cases}$$

$\forall i \in \mathcal{C}, n$

Availability

$$y_{in} = \begin{cases} 1 & \text{if } i \in \mathcal{C}_n \\ 0 & \text{otherwise} \end{cases}$$

$\forall i \in \mathcal{C}, n$

Probabilistic model

- $\Pr(w_{in} = 1) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n)$ and i available ($y_{in} = 1$)
- $D_i = \sum_{n=1}^{N} \Pr(w_{in} = 1)$
- D_i is in general non linear
- **Example:** $\Pr(w_{in} = 1) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}$ (logit model)

Outline

Simulation

Simulation

- Assume a distribution for ε_{in}
- Generate R draws $\xi_{in1} \dots \xi_{inR}$
- r behavioral scenario
- The choice problem becomes **deterministic**

Demand model

$$U_{inr} = V_{in} + \xi_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr} \quad (1)$$

Endogeneous part of V_{in}

- Decision variables x_{ink}
- Assumption: linear

Exogeneous part of V_{in}

- Other variables z_{in}
- f not necessarily linear

Availability of alternatives

Operator level

y_{in} decision of the operator

$$y_{in} = 0 \ \forall i \notin \mathcal{C}_n, n \tag{2}$$

Scenario level

y_{inr} availability at scenario level (*e.g.* demand exceeding capacity)

$$y_{inr} \leq y_{in} \ \forall i, n, r \tag{3}$$

Utility and availability

Auxiliary variables

$$\nu_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ l_{inr} & \text{if } y_{inr} = 0 \end{cases} \quad \forall i \in \mathcal{C}, n, r$$

Linearizing constraints

$$l_{inr} \leq \nu_{inr} \; \forall i, n, r \tag{4}$$

$$\nu_{inr} \leq l_{inr} + M_{inr} y_{inr} \; \forall i, n, r \tag{5}$$

$$U_{inr} - M_{inr}(1 - y_{inr}) \leq \nu_{inr} \; \forall i, n, r \tag{6}$$

$$\nu_{inr} \leq U_{inr} \; \forall i, n, r \tag{7}$$

where $l_{inr} \leq U_{inr} \leq m_{inr}$ and $M_{inr} = m_{inr} - l_{inr}$

Choice

Highest utility

$$U_{nr} = \max_{i \in \mathcal{C}_n} \nu_{inr} \; \forall n, r$$

Choice

$$w_{inr} = \begin{cases} 1 & \text{if } U_{nr} = \nu_{inr} \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in \mathcal{C}, n, r$$

Linearizing constraints

$$\nu_{inr} \leq U_{nr} \; \forall i \in \mathcal{C}, n, r \tag{8}$$

$$U_{nr} \leq \nu_{inr} + M'_{inr}(1 - w_{inr}) \; \forall i \in \mathcal{C}, n, r, \tag{9}$$

$$\sum_{i \in \mathcal{C}} w_{inr} = 1 \; \forall n, r \tag{10}$$

where $M'_{inr} = \max_{j \in \mathcal{C}}\{m_{jnr}\} - l_{inr}$

Choice and availability

$$w_{inr} \leq y_{inr}, \forall i, n, r \tag{11}$$

Outline

Maximization of revenues

Application

- Operator selling services to a market, each service:
  - Price
  - Capacity (number of customers)
- Opt-out option denoted by $i = 0$
- Demand is price elastic and heterogenous
- **Goal:** best strategy in terms of capacity allocation and pricing

Revenues

- p_{in} price that individual n has to pay to access to service i (price as endogeneous variable in the utility function (1))

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}$$

- p_{in} endogenous variable $\Rightarrow R_i$ non linear

Pricing

Revenues per alternative

- Discretization of the price $\Rightarrow p_{in}^1, \ldots, p_{in}^{L_{in}}$
- Binary variables λ_{inl} such that $p_{in} = \sum_{l=1}^{L_{in}} \lambda_{inl} p_{in}^l$ and

$$\sum_{\ell=1}^{L_{in}} \lambda_{in\ell} = 1, \forall i > 0, n \tag{12}$$

- Revenues from alternative i:

$$R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{l=1}^{L_{in}} \lambda_{inl} p_{in}^l \sum_{r=1}^{R} w_{inr}$$

Linearization of $\alpha_{inrl} = \lambda_{inl} \cdot w_{inr}$

$$\lambda_{in\ell} + w_{inr} \leq 1 + \alpha_{inr\ell}, \forall i > 0, n, r, \ell, \tag{13}$$

$$\alpha_{inr\ell} \leq \lambda_{in\ell}, \forall i > 0, n, r, \ell, \tag{14}$$

$$\alpha_{inr\ell} \leq w_{inr}, \forall i > 0, n, r, \ell. \tag{15}$$

Capacity

Overview

- c_i capacity of service i
- The model favors customers bringing higher revenues
- ... but generally customers arrive in a random order
- Priority list of individuals is assumed to be known

Constraints

$$y_{inr} \geq y_{i(n+1)r} \; \forall i > 0, n, r \tag{16}$$

$$c_i(1 - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} + c_i(1 - y_{in}) \; \forall i > 0, n, r \tag{17}$$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n-1)(1 - y_{inr}) \; \forall i > 0, n > c_i, r \tag{18}$$

Full model

Objective function

$$\max \sum_{i>0} \frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} p_{in}^{\ell} \sum_{r=1}^{R} \alpha_{inr\ell}$$

Demand

$$D_i = \frac{1}{R} \sum_{r=1}^{R} \sum_{n=1}^{N} w_{inr}$$

Constraints

- Utility: (1)
- Availability of alternatives: (2) and (3)
- Utility and availability: (4), (5), (6) and (7)
- Choice: (8), (9), (10) and (11)
- Pricing: (12), (13), (14) and (15)
- Capacity allocation: (16), (17) and (18)

Outline

Parking choices

Original experiment

- [Ibeas et al., 2014] *Modelling parking choices considering user heterogeneity*
- Stated preferences data
- Analyze viability of an underground car park

Free on-Street Parking (FSP)

Free

Paid on-Street Parking (PSP)

Price levels: 0.6 and 0.8

Paid Underground Parking (PUP)

Price levels: 0.8 and 1.5

Choice model

Survey

- 197 respondents
- 8 scenarios: AT, TD, FEE

Mixed Logit model

- **Attributes:** time to reach the destination (TD)
- **Random parameters:** access time (AT) and price (FEE)
- **Socioeconomic characteristics:** residence, age of the vehicle
- **Interactions:** price and low income, price and residence

Price levels calculation

Algorithm

Data: Subset of N
Initialization: set a loose LB and UB for p_i^l;
while *improvement in the objective function* **do**
  divide the interval defined by the current LB and UB of p_i^l ;
  run the uncapacitated MILP for $R = 100$;
  define a new interval centered in the obtained p_i^l : $[0.5p_i^l, 1.5p_i^l]$;
end
Result: set of price levels for alternative i

Price levels

- PSP: 0.00, 0.33, 0.67, 1.00, 1.33, 1.67, 2.00, 2.33, 2.67, 3.00
- PUP: 0.00, 0.33, 0.67, 1.00, 1.33, 1.67, 2.00, 2.33, 2.67, 3.00

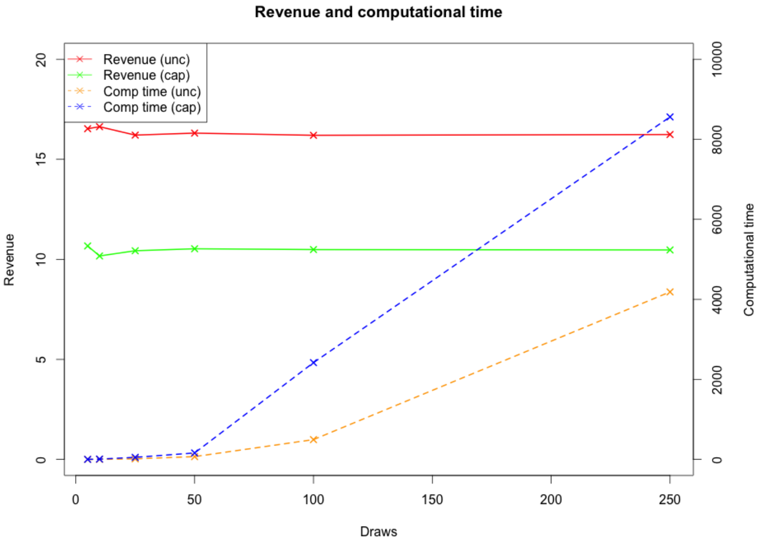
Computational results: overview

Assumptions

- Subset of 25 individuals
- General price levels
- Uncapacitated vs. capacitated case
- Capacity of 10 invidiuals for both PSP and PUP

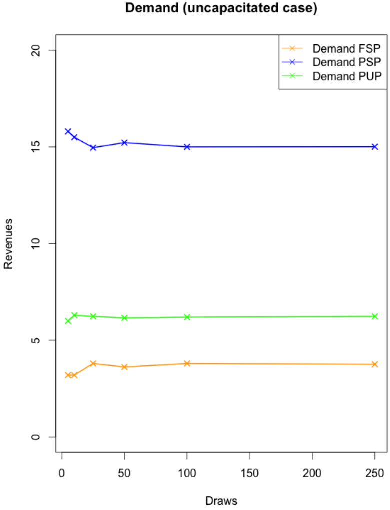
	FSP			PSP			PUP		
Scenario	AT	TD	FEE	AT	TD	FEE	AT	TD	FEE
5	15	15	0	10	10	0.6	5	10	1.5

Computational results: revenue and computational time

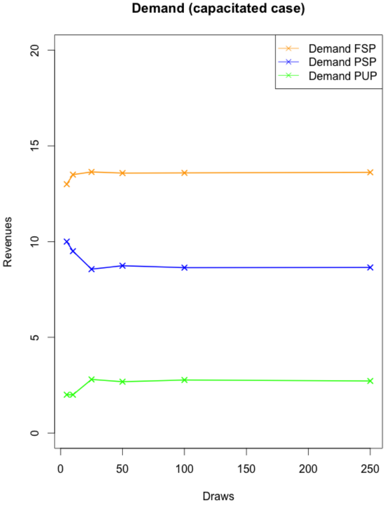


Computational results: demand

Uncapacitated case



Capacitated case



Outline

Conclusions and future work

Conclusions

- General framework (*any* assumption can be made for ε_{in})
- Linear formulation integrating demand and supply
- High dimensionality of the problem (N and R)
- Need for speeding up computational results

Future work

- Decomposable structure of the problem:
  - By simulation r
  - By individual n
- Lagrangian relaxation to decompose the problem:
  - Choice subproblem (user's side)
  - Pricing subproblem (operator's side)

Questions?

meritxell.pacheco@epfl.ch

Bibliography

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