A new mathematical formulation to integrate supply and demand within a choice-based optimization framework

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Outline

1 Introduction

2 Demand modeling
   A probabilistic formulation
   A linear formulation

3 Supply side: demand-based revenues maximization

4 Case study

5 Conclusions and future work
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Motivation
Supply and demand

Supply

- Decision variables to design and configure the supply
- Maximize revenues
- **Here:** MILP

Demand

- Formalization of preferences for demand forecasting
- Maximize satisfaction
- **Here:** discrete choice models
State of the art: Integration paradigms

**Linear choice-based optimization models**
- Decision variables are not in the utility function
- Exogeneous utility

**Nonlinear choice-based optimization models**
- Endogeneous utility
- Nonlinearity and nonconvexity to the optimization model

**General observations**
- The assumption of exogeneously given demand is in most of the cases unrealistic
- **Motivation:** consider utility as endogeneous to the optimization model (better representation of the demand)
- Complexity increases
  - Mathematical model
  - Resolution approach
Integration of supply and demand

Integration of discrete choice models in MILP
- Probabilistic
- Nonlinearity and nonconvexity

Linear approach addressing
- Nonconvex representation of probabilities
- Wide class of discrete choice models
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Demand modeling 

Utility

Supply and demand

- Population of $N$ individuals
- Set of alternatives $\mathcal{C}$
  - artificial opt-out alternative
- $\mathcal{C}_n \subseteq \mathcal{C}$ subset of available alternatives to individual $n$

Utility

$U_{in} = V_{in} + \varepsilon_{in}$: associated score with alternative $i$ by individual $n$

- $V_{in}$: deterministic part
- $\varepsilon_{in}$: error term

Behavioral assumption: $n$ chooses $i$ if $U_{in}$ is the highest in $\mathcal{C}_n$
Probabilistic model

Availability

\[ y_{in} = \begin{cases} 
1 & \text{if } i \in C_n \\
0 & \text{otherwise} 
\end{cases} \]

Choice

\[ w_{in} = \begin{cases} 
1 & \text{if } n \text{ chooses } i \\
0 & \text{otherwise} 
\end{cases} \]

Probabilistic model

- \( \Pr(w_{in} = 1) = \Pr(U_{in} \geq U_{jn}, \forall j \in C_n) \) and \( i \) available \( (y_{in} = 1) \)
- \( D_i = \sum_{n=1}^{N} \Pr(w_{in} = 1) \), in general non linear
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Demand modeling

A linear formulation

Simulation

Behavioral scenarios

- Assume a distribution for $\varepsilon_{in}$
- Generate $R$ draws $\xi_{in1} \ldots \xi_{inR}$
- The choice problem becomes deterministic

Demand model

$$U_{inr} = V_{in} + \xi_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr} \quad (1)$$

Endogeneous part of $V_{in}$
- Decision variables $x_{ink}$
- Assumption: linear

Exogeneous part of $V_{in}$
- Other variables $z_{in}$
- $f$ not necessarily linear
Availability of alternatives

**Operator level**

\( y_{in} \) decision of the operator

\[
 y_{in} = 0 \ \forall i \notin C_n 
\]  

(2)

**Scenario level**

\( y_{inr} \) availability at scenario level (e.g. demand exceeding capacity)

\[
 y_{inr} \leq y_{in} 
\]  

(3)
Choice of alternatives

Choice at scenario level

\[ w_{inr} = \begin{cases} 
1 & \text{if } i = \arg \max_j \{ U_{jnr} \} \\
0 & \text{otherwise}
\end{cases} \]

Choice and availability

\[ w_{inr} \leq y_{inr} \]
Linearization of the choice \((I)\)

**Auxiliary variables**

\[ \nu_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ l_{inr} & \text{if } y_{inr} = 0 \end{cases} \]

**Linearizing constraints**

\[ l_{inr} \leq \nu_{inr} \]  \hspace{1cm} (5)
\[ \nu_{inr} \leq l_{inr} + M_{inr} y_{inr} \]  \hspace{1cm} (6)
\[ U_{inr} - M_{inr}(1 - y_{inr}) \leq \nu_{inr} \]  \hspace{1cm} (7)
\[ \nu_{inr} \leq U_{inr} \]  \hspace{1cm} (8)

where \( l_{inr} \leq U_{inr} \leq m_{inr}, \ M_{inr} = m_{inr} - l_{inr} \)
Linearization of the choice (II)

Highest utility

\[ U_{nr} = \max_{i \in C_n} \nu_{inr} \]

Linearizing constraints

\[ \nu_{inr} \leq U_{nr} \quad (9) \]
\[ U_{nr} \leq \nu_{inr} + M'_{inr}(1 - w_{inr}) \quad (10) \]
\[ \sum_{i \in C} w_{inr} = 1 \quad (11) \]

where \( M'_{inr} = \max_{j \in C} \{ m_{jnr} \} - l_{inr} \)
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Supply side: demand-based revenues maximization

Maximization of revenues

Application
- Operator selling services to a market, each service:
  - Price
  - Capacity (number of individuals)
- $i = 0$ denotes the opt-out option
- Demand is price elastic and heterogenous
- **Goal**: best strategy in terms of capacity allocation and pricing
Pricing (I)

Revenues per alternative

- $p_{in}$ price that individual $n$ has to pay to access to alternative $i$
- Endogeneous variable in the utility function (1)

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}$$

Linearization (I)

- Discretization of the price: $p_{in}^1, \ldots, p_{in}^{Lin}$
- Binary variables $\lambda_{inl}$ such that $p_{in} = \sum_{l=1}^{Lin} \lambda_{inl} p_{in}^l$ and

$$\sum_{\ell=1}^{Lin} \lambda_{in\ell} = 1, \forall i > 0$$ (12)
Supply side: demand-based revenues maximization

Pricing (II)

Linearization (II)

- Revenues from alternative \(i\):
  \[
  R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{l=1}^{L_{in}} \lambda_{inl} p_{in}^{l} \sum_{r=1}^{R} w_{inr}
  \]

- Still non-linear \(\Rightarrow\) \(\alpha_{inr} = \lambda_{inl} w_{inr}\) to linearize it

\[
\begin{align*}
\lambda_{inl} + w_{inr} &\leq 1 + \alpha_{inr} \ell \quad \forall i > 0 \quad (13) \\
\alpha_{inr} \ell &\leq \lambda_{inl} \quad \forall i > 0 \quad (14) \\
\alpha_{inr} \ell &\leq w_{inr} \quad \forall i > 0 \quad (15)
\end{align*}
\]
Supply side: demand-based revenues maximization

Capacity (I)

Overview

- $c_i$: capacity of service $i$
- Who has access if the capacity is reached?
- The model favors customers bringing higher revenues
- ... but generally customers arrive in a random order

Priority list

- An individual is served only if all individuals before her in the list have been served
- Can account for fidelity programs, VIP customers, etc.
- We assume it given

$$y_{inr} \geq y_{i(n+1)r} \quad \forall i > 0$$ (16)
Capacity (II)

Capacity must not be exceeded

\[
\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n - 1)(1 - y_{inr}) \quad \forall i > 0, n > c_i \tag{17}
\]

- \( y_{inr} = 1 \Rightarrow 1 + \sum_{m=1}^{n-1} w_{imr} \leq c_i \)
- \( y_{inr} = 0 \Rightarrow \sum_{m=1}^{n-1} w_{imr} \leq n - 1 \)

Capacity has been reached

\[
c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} \quad \forall i > 0 \tag{18}
\]

- \( y_{in} = 1, y_{inr} = 0 \Rightarrow \sum_{m=1}^{n-1} w_{imr} \leq c_i \)
Full model

Objective function

\[
\max \sum_{i > 0} \frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} p_{in}^\ell \sum_{r=1}^{R} \alpha_{inr\ell}
\]  

(19)

Constraints

- Utility: (1)
- Availability of alternatives: (2) and (3)
- Choice: (4), (5), (6), (7), (8), (9), (10) and (11)
- Pricing: (12), (13), (14) and (15)
- Capacity allocation: (16), (17) and (18)
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Parking choices

Original experiment

- [Ibeas, 2014] *Modelling parking choices considering user heterogeneity*
- Stated preferences survey
- Analyze viability of an underground car park
- Mixed logit model (random taste parameters)

Free on-Street Parking (FSP)
Free (opt-out)

Paid on-Street Parking (PSP)
0.6 and 0.8

Paid Underground Parking (PUP)
0.8 and 1.5
Choice model

Survey
- 197 respondents
- 8 scenarios based on
  - AT (access time to parking area)
  - TD (time to reach the destination)
  - FEE (price)

Mixed Logit model
- **Attributes:** TD
- **Random parameters:** AT, FEE
- **Socioeconomic characteristics:** residence, age of the vehicle
- **Interactions:** price and low income, price and residence
Assumptions

- Subset of 25 individuals
- Uncapacitated vs. capacitated case
- Capacity of 10 individuals for both PSP and PUP
- 10 price levels from 0 to 3

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Computational results: revenue and computational time

![Graph showing revenue and computational time](image)
Computational results: demand

Uncapacitated case

Capacitated case

Demand (uncapacitated case)

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<th>Demand PSP</th>
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Demand (capacitated case)

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Conclusions and future work

Conclusions

- General framework (*any* assumption can be made for $\varepsilon_{in}$)
- Linear formulation integrating demand and supply
- High dimensionality of the problem ($N$ and $R$)
- Need for speeding up the computational results

Future work

- Decomposition techniques
- Two interesting subproblems
  - Choice subproblem (user’s side)
  - Pricing subproblem (operator’s side)
Questions?