Integrating advanced discrete choice models in mixed integer linear optimization

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Outline

1. Introduction
2. Choice model
3. Optimization model
4. Demand-based benefit maximization
5. Case study
6. Conclusions and ongoing work

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2 Choice model

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Discrete choice models

- Demand modeling
- Disaggregate level
- Heterogeneity of the population
- Predict the choice

Why are they not in OR?

- Tractability
- Linearity and/or convexity
- **MILP models**

Why are they not in OR?

- Tractability
- Linearity and/or convexity
- **MILP models**

- Behavioral realism
- Unrealistic assumptions
- Complex formulations

Bridging the gap

- General framework integrating
  - choice model (*demand*)
  - MILP model (*supply*)
- Simulation to linearize the choice model

General framework

- **Exogenous** variables explaining the **choice**: $x^d \in \mathbb{R}^D$

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General framework

- **Exogenous** variables explaining the **choice**: $x^d \in \mathbb{R}^D$
- **Exogenous** variables involved in the **optimization** model: $x^s \in \mathbb{R}^S$
- **Endogenous** variables appearing in **both models**: $x^e \in \mathbb{R}^E$
  - Characterize the interactions (e.g.: price)
  - $\ell^e \leq x^e \leq m^e$

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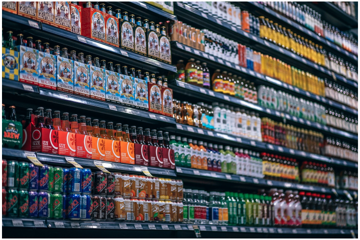
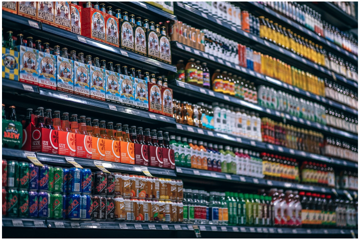
Choice set and population



- Set of alternatives $\mathcal{C}$ (i)
- Capacity: $c_i \geq 1$

- N individuals (n)
- Individual choice set $\mathcal{C}_n \subseteq \mathcal{C}$

Choice set and population



- Set of alternatives $\mathcal{C}$ (i)
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$$y_{in} = \begin{cases} 1 & \text{if alternative } i \text{ is offered to } n \\ 0 & \text{otherwise} \end{cases}$$

Utility function and behavioral assumption

$$U_{in}(x^d, x^e; \varepsilon_{in}) = V_{in}(x^d, x^e) + \varepsilon_{in}$$

- Utility function
  - Deterministic part: $V_{in}(x^d, x^e) = \sum_k \beta_k x^e_{ink} + g^d(x^d)$
  - Random term: ε_{in}

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  - Random term: ε_{in}
- Behavioral assumption
  - i chosen by n if $U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n$
  - $P_n(i|x^d, x^e) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n)$

Simulation

- R draws from the distribution of ε_{in}
- $\xi_{in1}, \ldots, \xi_{inR}$
- Behavioral scenario

$$U_{inr} = \sum_k \beta_k x^e_{ink} + g^d(x^d) + \xi_{inr}$$

Availability

- Not considered
- $y_{in} = 0 \; \forall i \notin \mathcal{C}_n$

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- Reached capacity
- $\boldsymbol{y_{inr}} \leq y_{in}$

Discounted utility

$$z_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{nr} & \text{if } y_{inr} = 0 \end{cases}$$

$$\begin{aligned} \ell_{nr} &\leq z_{inr} \\ z_{inr} &\leq \ell_{nr} + M_{inr} y_{inr} \\ U_{inr} - M_{inr}(1 - y_{inr}) &\leq z_{inr} \\ z_{inr} &\leq U_{inr} \end{aligned}$$

Choice

$$U_{nr} = \max_{i \in \mathcal{C}} z_{inr}$$

$$\begin{aligned} z_{inr} &\leq U_{nr} \\ U_{nr} &\leq z_{inr} + M_{nr}(1 - w_{inr}) \\ w_{inr} &\leq y_{inr} \\ \sum_{i \in \mathcal{C}} w_{inr} &= 1 \end{aligned}$$

Expected demand

$$D_i = \frac{1}{R}\sum_{r=1}^{R}\sum_{n=1}^{N} w_{inr}$$

Capacity allocation: priority list

- Exogenous to the model
- Relationship with individuals
- Numbering of individuals
- Example: random arrival

Capacity allocation: capacity cannot be exceeded

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n-1)(1 - y_{inr})$$

Capacity allocation: capacity has been reached

$$c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr}$$

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General optimization model (MILP)

$$g^s(x^s, x^e)$$

- Objective function (linear)
- Relates decisions at an aggregate level

$$\begin{aligned} h^s(x^s, x^e) &= 0 \\ \ell^e \leq x^e &\leq m^e \\ x^e_z &\in \mathbb{Z} \end{aligned}$$

- Set of constraints (linear)
- Feasible configuration of the variables

Applications

- Design of a train timetable
- Objective: maximize profit
- Constraints: passenger satisfaction

- Schedule, what movie...
- Objective: total benefit
- Constraints: one movie per theater, capacity

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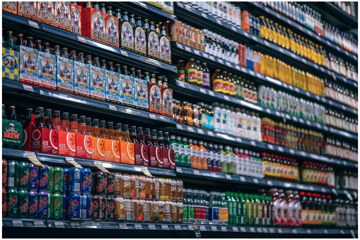
General setting



- Set of services $\mathcal{C}$ (i)
- Capacity: $c_i \geq 1$
- Opt-out option $(i = 0)$
- Maximize benefit

- N customers (n)
- Individual choice set $\mathcal{C}_n \subseteq \mathcal{C}$
- $0 \in \mathcal{C}_n \; \forall n$
- Price to pay: p_{in}

Benefit maximization

$$\max \sum_{i>0} (G_i - C_i)$$

$$G_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} p_{in} w_{inr}$$

- Product of variables
- Binary representation of price
- $\frac{1}{10^k} a_{in} \leq p_{in} \leq \frac{1}{10^k} b_{in}$
- $L_{in} = \lceil \log_2(b_{in} - a_{in} + 1) \rceil$ variables

$$C_i = (f_i + v_i c_i) y_i$$

- f_i fixed cost
- v_i variable cost
- $y_i = 1$ if i is offered $\forall n$
- $y_{in} = y_i \; \forall n$

Price characterization

$$p_{in} = \frac{1}{10^k}\left(a_{in} + \sum_{\ell=0}^{L_{in}-1} 2^{\ell}\lambda_{in\ell}\right)$$

$$\begin{aligned}G_i =& \frac{1}{R}\frac{1}{10^k}\sum_{n=1}^{N}\sum_{r=1}^{R}\left(a_{in} + \sum_{\ell=0}^{L_{in}-1} 2^{\ell}\lambda_{in\ell}\right) w_{inr}\\ =& \frac{1}{R}\frac{1}{10^k}\left[\sum_{n=1}^{N}\sum_{r=1}^{R}\left(a_{in}w_{inr} + \sum_{\ell=0}^{L_{in}-1} 2^{\ell}\alpha_{inr\ell}\right)\right]\end{aligned}$$

where $\alpha_{inr\ell} = \lambda_{in\ell} w_{inr}$

Resulting MILP

max	benefit
subject to	availability
	utility definition
	discounted utility
	choice
	capacity allocation
	price selection

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Parking choices

PSP

PUP

FSP (opt-out)

- $N = 50$ customers
- $\mathcal{C} = \{\text{PSP}, \text{PUP}, \text{FSP}\}$
- $\mathcal{C}_n = \mathcal{C} \quad \forall n$
- Mixtures of a logit model
- $y_{in} = y_i \quad \forall n$
- $p_{in} = p_i \quad \forall n$

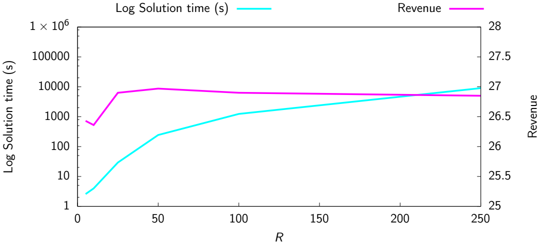
General experiments

- Maximization of revenue
- Unlimited capacity
- Capacity of 20 spots for PSP and PUP

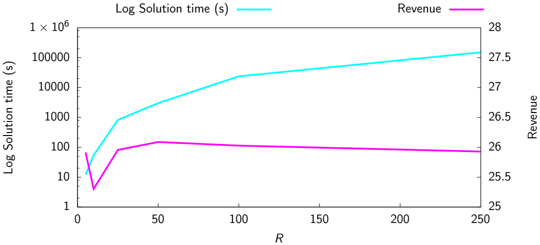
- Reduced price for residents
- Two scenarios
  1. Subsidy offered by the municipality
  2. Operator is obliged to offer a reduced price

Uncapacitated vs Capacitated case (1)

Uncapacitated

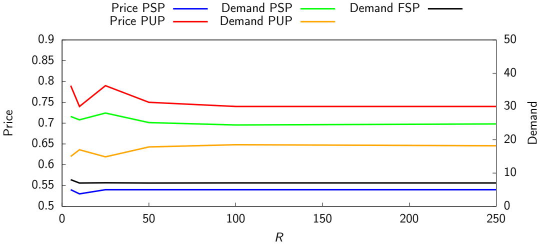


Capacitated

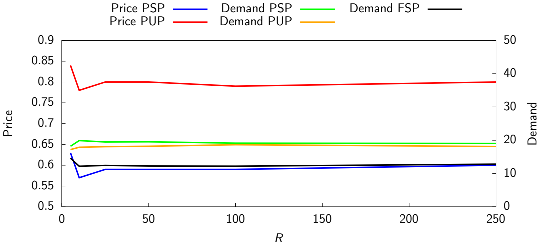


Uncapacitated vs Capacitated case (2)

Uncapacitated

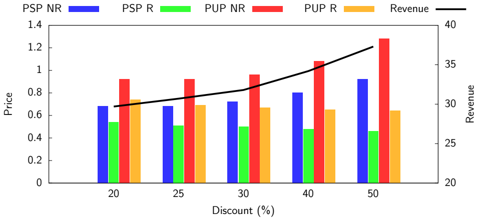


Capacitated

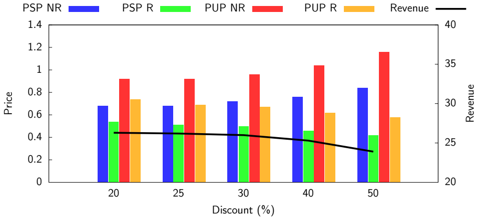


Price differentiation by population segmentation

Subsidy offered by the municipality



Operator is obliged to offer a reduced price



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Conclusions and ongoing work

- Powerful tool to configure systems based on **heterogenous** behavior
- Computationally **expensive**
- Decomposition techniques ⇒ **Lagrangian relaxation**
  - Operator subproblem: FLP
  - Customer subproblem: iterative method (customers)

Questions?

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