Integrating discrete choice models in mixed integer linear programming to capture the interactions between supply and demand

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Outline

Discrete choice models and optimization

- Disaggregate demand modeling
- Behavioral realism
- Complex formulations

- Supply decisions
- Linearity and/or convexity
- MILP models

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Bridging the gap

- Linear characterization of a discrete choice model
- Simulation to address stochasticity
- Demand-based benefit maximization problem

Linearization of the choice model (1)

$$U_{in} = V_{in} + \varepsilon_{in} \xrightarrow{\text{draw distribution } (R)} U_{inr} = V_{in} + \xi_{inr}$$

Linearization of the choice model (2)

$$w_{inr} = \begin{cases} 1 & \text{if } U_{inr} \geq U_{jnr}, \forall j \in \mathscr{C}_n, j \neq i \\ 0 & \text{otherwise} \end{cases}$$

$$D_i = \frac{1}{R} \sum_r \sum_n w_{inr}$$

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Benefit maximization problem (1)

- Set of services $\mathscr{C}$ $(i > 0)$
- Opt-out option $i = 0$
- Population N $(n \geq 1)$

- Price $a_i \leq p_{in} \leq b_{in}$
- Capacity levels c_{iq} (fixed f_{iq} and variable v_{iq} costs)

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Benefit maximization problem (2)

obj. fun.	$\sum_{i>0} \text{Revenue}_i - \text{Cost}_i$
availability	operator level and scenario level
disc. utility	variable capturing availability and utility
choice	linearization of the highest utility
price	linearization of the variable η_{iqnr} (revenue)
capacity	relation with the availability at scenario level

Computational results

- Parking choices: mixtures of logit model
- Distributed parameters (and correlated)
- $R = 50$ draws and $N = 50$ customers
- $|\mathscr{C}| = 3$: PSP, PUP and FSP (opt-out)

Computational times up to 34 hours!

Motivation

- Maximization of own utility
- Objective function and capacity constraints

- Behavioral scenario
- Objective function

Uncapacitated case and revenue maximization

obj. fun.	$\sum_{i>0} \text{Revenue}_i$
availability	
utility	no need for discounted utility (no availability)
choice	linearization of the highest utility
price	linearization of the variable η_{iqnr} (revenue)
capacity	

Lagrangian decomposition

Relax complicating constraints ⇒ subproblems easier to solve

⚠ Price p_{in} is the same across draws ⇒ **no** decomposition by n and r

$$p_{in1} = p_{in2} = \cdots = p_{inR} = p_{in1}$$

$p_{inr} - p_{in(r-1)} = 0 \Rightarrow$ Lagrangian multipliers $\alpha_{inr} \Rightarrow$ decomposition by n and r

Subgradient method

Input: UB: $Z(\overline{\alpha})$ with $\overline{\alpha}$ starting values, LB: Z^* (from a feasible solution)

while *$k < K$ or $Z(\alpha(k))$ has not improved in ω_3 iterations* **do**

  for $r = 1 \dots R$ **do**

    for $n = 1 \dots N$ **do**

      Lagrangian subproblem $Z_{nr}(\alpha(k))$ (MILP);

      Obtain p_{inr}, w_{inr} and $Z_{nr}(\alpha(k))$;

    end

  end

  Compute $Z(\alpha(k)) = \sum_r \sum_n Z_{nr}(\alpha(k))$;

  $k \leftarrow k + 1$;

  Obtain $\omega(k)$ (**step**) and $d_{inr}(k)$ (**direction**);

  Update the **Lagrangian multipliers**: $\alpha_{inr}(k) = \alpha_{inr}(k-1) - \omega(k) d_{inr}(k)$

end

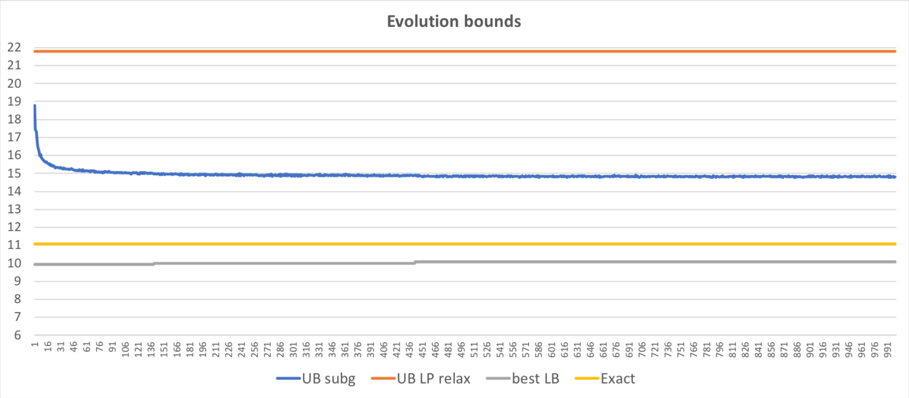
Preliminary results (1)

- $N = 20$ and $R = 100$
- $\lambda(0) = 0.5$, $\theta = 0.5$, $\omega_3 = 30$
- Number of iterations: $K = 1000$

Computational time:

- Exact method: 33.7 min
- Subgradient method: 21.4 min

Preliminary results (2)



Conclusions and future work

- Efficient method to obtain lower and upper bounds
- Calibrate the parameters of the subgradient method
- Changes in the formulation (tighter LP relaxation)
- Gradually include the complexity back (capacity, availability...)

Questions?

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Subgradient method: step size and direction

$$\alpha_{inr}(k) = \alpha_{inr}(k-1) - \omega(k) d_{inr}(k)$$

Step:

- $\omega(k) = \lambda(k) \frac{Z(\alpha(k-1)) - Z^*}{\|\gamma(k)\|^2}$
- $\lambda(0) \in [0,2)$
- $\gamma_{inr}(k) = p_{inr}(k) - p_{in(r-1)}(k)$ (subgradients)
- $\lambda(k)$ divided by ω_1 if $Z(\alpha(k))$ has not improved in ω_2 iterations

Direction:

- $d(k) = \gamma(k) + \theta d(k-1)$
- $\theta \in [0,1)$