A Lagrangian relaxation technique for the demand-based benefit maximization problem

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Outline

1. Introduction
2. Demand-based benefit maximization problem
3. Lagrangian relaxation
4. Preliminary results
5. Conclusions and future work

Demand vs. supply

- Discrete choice models
- Disaggregate demand modeling
- Behavioral realism
- Complex formulations

Demand vs. supply

- MILP models
- Supply decisions
- Tractability of the formulations
- Linearity and/or convexity

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Bridging the gap

- Linear characterization of a discrete choice model
  - Simulation to address stochasticity
  - Direct usage of the utility variables (instead of the choice probabilities)
- Demand-based benefit maximization problem (MILP example)
- General framework that can be applied with an existing choice model
- Computationally expensive: need for decomposition techniques

Linearization of the discrete choice model

Choice set $\mathscr{C}$ (i)

Population N (n)

$U_{in} = V_{in} + \varepsilon_{in}$ $\xrightarrow{\text{draw distribution } (R)}$ $U_{inr} = V_{in} + \xi_{inr}$

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Demand representation

Demand is not obtained from the choice probabilities!

Choice w_{inr}

$$w_{inr} = \begin{cases} 1 & \text{if } U_{inr} \geq U_{jnr}, \forall j \in \mathscr{C}_n, j \neq i \\ 0 & \text{otherwise} \end{cases} \xrightarrow{\text{demand}} D_i = \frac{1}{R} \sum_r \sum_n w_{inr}$$

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demand

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Benefit maximization problem (1)

- Set of alternatives $\mathscr{C}$ $(i > 0)$
- Opt-out option $i = 0$
- Population N $(n \geq 1)$
- Price $a_{in} \leq p_{in} \leq b_{in}$
- Capacity levels c_{iq} (Q levels, each with a certain cost)

Benefit maximization problem (2)

obj. fun.	$\sum_{i>0} \text{Revenue}_i - \text{Cost}_i$
availability	operator level and scenario level
disc. utility	variable capturing availability and utility
choice	linearization of the highest utility
price	linearization of the product $w_{inr} p_{in}$ (revenue)
capacity	relation with the availability at scenario level

Computational results

- Parking choices: mixtures of logit model
- Distributed parameters (and correlated)
- $R = 50$ draws and $N = 50$ customers
- $|\mathscr{C}| = 3$: PSP, PUP and FSP (opt-out)

- Several experiments
  - Price calibration (discrete and continuous prices)
  - Price differentiation by population segmentation
- Computational times up to 34 hours!

Motivation: why decomposition techniques?

Customer (n)

- Utility maximization
- Objective function and capacity constraints

Draw (r)

- Behavioral scenario
- Objective function

Lagrangian relaxation

obj. fun.	obj. fun.	$+\alpha$	hard constraints
easy constraints	easy constraints		
hard constraints			

Lagrangian decomposition

obj. fun.		obj. fun. (x)		obj. fun. (y)
	$x = y$		$+$	
constraints		constraints (x)		constraints (y)

Lagrangian relaxation

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Lagrangian decomposition

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	$x = y$		$+$	
constraints		constraints (x)		constraints (y)

The problem now: revenue maximization & infinite capacity

obj. fun.	$\sum_{i>0} \text{Revenue}_i$
availability	
utility	no need for discounted utility (no availability)
choice	linearization of the highest utility
price	linearization of the product $w_{inr} p_{in}$ (revenue)
capacity	

Lagrangian decomposition

Idea: decompose the problem in subproblems per customer and draw

⚠ Price p_{in} is the same across draws ⇒ **no** decomposition by n and r

$$p_{in1} = p_{in2} = \cdots = p_{inR} = p_{in1}$$

$p_{inr} - p_{in(r+1)} = 0$ ⇒ Lagrangian multipliers α_{inr} ⇒ decomposition by n and r

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Lagrangian subproblems

- One subproblem per customer and draw
- Objective function:
  - Revenue term: $p_{inr} w_{inr}$ (linearized)
  - Lagrangian term: $(\alpha_{inr} - \alpha_{in(r-1)}) p_{inr}$
- Constraints:
  - Utility maximization
  - One alternative is chosen
  - Linearization revenue term

Subgradient method

What are the values of the Lagrangian multipliers (α_{inr})?

initial values Lag. mult. → solve subproblems → calculate step and direction → update Lag. multipliers

until stopping criterion

At every iteration:

- UB: from the subgradient method
- LB: from a feasible solution (based on the subgradient method)

Subgradient method

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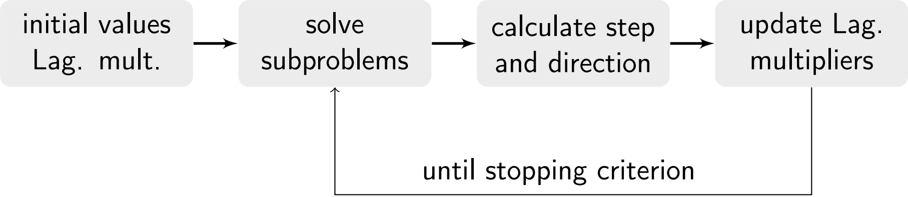

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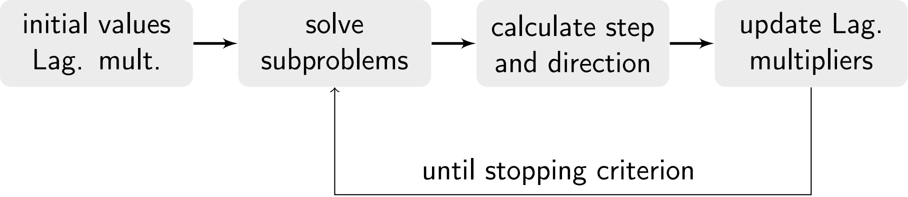

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- LB: from a feasible solution (based on the subgradient method)

Case study

- $N = 20$ and $R = 100$
- Price bounds PSP: $[0.5, 1.0]$
- Price bounds PUP: $[0.7, 1.2]$
- FSP is the opt-out
- Number of iterations: $K = 250$

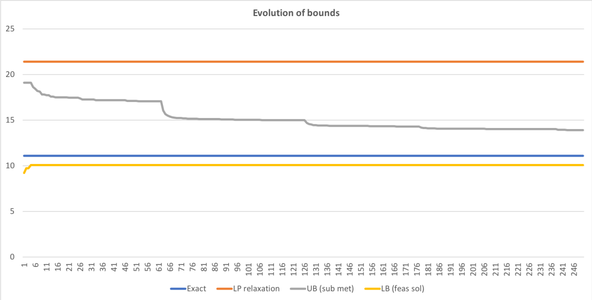
Evolution bounds

Computational time:

- Exact method: 32 min
- Subgradient method: 5.9 min (1.4 s/it)

Objective function:

- MILP: 11.08
- LP relaxation: 21.41



Evolution bounds (with valid inequality)

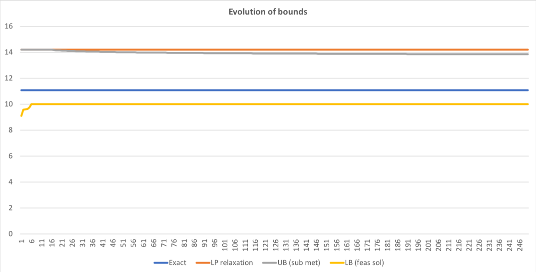
$$\sum_i U_{inr} w_{inr} \geq U_{jnr} \ \forall j, n, r$$

Computational time:

- Exact method: 11 min
- Subgradient method: 34 min (7 s/it)

Objective function:

- MILP: 11.08
- LP relaxation: 14.19



Conclusions

- Efficient method to obtain LB and UB + feasible solution
- Valid inequalities help to tighten the solution space
- Need for convergence: LB and UB sufficiently close

Future work

- Define other techniques to generate feasible solutions (improve LB)
- Try other valid inequalities: $(U_{nr} - U_{inr})w_{inr}$ in the objective function
- Evaluate other strategies (e.g., regularization term)
- Gradually include the complexity back (capacity, availability...)

Questions?

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Subgradient method: algorithm

Input: UB: $Z(\overline{\alpha})$ with $\overline{\alpha}$ starting values, LB: Z^* (from a feasible solution)

while $k < K$ *or* $Z(\alpha(k))$ *has not improved after some iterations* **do**

    for $r = 1 \dots R$ **do**

        for $n = 1 \dots N$ **do**

            Lagrangian subproblem $Z_{nr}(\alpha(k))$ (MILP);

            Obtain p_{inr} and $Z_{nr}(\alpha(k))$;

        end

    end

    Compute $Z(\alpha(k)) = \sum_r \sum_n Z_{nr}(\alpha(k))$;

    $k \leftarrow k + 1$;

    Obtain $\omega(k)$ (**step**) and $d_{inr}(k)$ (**direction**);

    Update the **Lagrangian multipliers**: $\alpha_{inr}(k) = \alpha_{inr}(k-1) - \omega(k) d_{inr}(k)$

end

Subgradient method: step size and direction

$$\alpha_{inr}(k) = \alpha_{inr}(k-1) - \omega(k) d_{inr}(k)$$

Step:

- $\omega(k) = \lambda(k) \frac{Z(\alpha(k-1)) - Z^*}{\|\gamma(k)\|^2}$
- $\lambda(0) \in [0,2)$
- $\gamma_{inr}(k) = p_{inr}(k) - p_{in(r-1)}(k)$ (subgradients)
- $\lambda(k)$ divided by ω_1 if $Z(\alpha(k))$ has not improved in ω_2 iterations

Direction:

- $d(k) = \gamma(k) + \theta d(k-1)$
- $\theta \in [0,1)$