A new mathematical formulation to integrate supply and demand within a choice-based optimization framework

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1. Introduction

2. Demand modeling
   - A probabilistic formulation
   - A linear formulation

3. Supply side: demand-based revenues maximization

4. Case study

5. Conclusions and future work
Outline

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Supply and demand

Supply

- Decision variables to design and configure the supply
- Maximize revenues
- Here: MILP

Demand

- Formalization of preferences for demand forecasting
- Maximize satisfaction
- Here: discrete choice models
State of the art: Integration paradigms

Linear choice-based optimization models
- Decision variables are not in the utility function
- Exogeneous utility

Nonlinear choice-based optimization models
- Endogeneous utility
- Nonlinearity and nonconvexity to the optimization model

General observations
- The assumption of exogeneously given demand is in most of the cases unrealistic
- **Motivation:** consider utility as endogeneous to the optimization model (better representation of the demand)
- Complexity increases
  - Mathematical model
  - Resolution approach
Integration of supply and demand

- Integration of discrete choice models in MILP
  - Probabilistic
  - Nonlinearity and nonconvexity
- Linear approach addressing
  - Nonconvex representation of probabilities
  - Wide class of discrete choice models
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Introduction

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Utility

\[ U_{in} = V_{in} + \varepsilon_{in} \]: associated score with alternative \( i \) by individual \( n \)

- \( V_{in} \): deterministic part
- \( \varepsilon_{in} \): error term

**Behavioral assumption:** \( n \) chooses \( i \) if \( U_{in} \) is the highest in \( C_n \)

Supply and demand

- Population of \( N \) individuals
- Set of alternatives \( C \)
  - artificial *opt-out* alternative
- \( C_n \subseteq C \) subset of available alternatives to individual \( n \)
Probabilistic model

Availability

\[ y_{in} = \begin{cases} 
1 & \text{if } i \in C_n \\
0 & \text{otherwise} 
\end{cases} \]

Choice

\[ w_{in} = \begin{cases} 
1 & \text{if } n \text{ chooses } i \\
0 & \text{otherwise} 
\end{cases} \]

Probabilistic model

- \( \Pr(w_{in} = 1) = \Pr(U_{in} \geq U_{jn}, \forall j \in C_n) \) and \( i \) available (\( y_{in} = 1 \))
- \( D_i = \sum_{n=1}^{N} \Pr(w_{in} = 1) \), in general non linear
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Demand modeling

A linear formulation

Simulation

Behavioral scenarios

- Assume a distribution for $\varepsilon_{in}$
- Generate $R$ draws $\xi_{in1} \ldots \xi_{inR}$
- The choice problem becomes deterministic

Demand model

$$U_{inr} = V_{in} + \xi_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr} \quad (1)$$

Endogeneous part of $V_{in}$

- Decision variables $x_{ink}$
- Assumption: linear

Exogeneous part of $V_{in}$

- Other variables $z_{in}$
- $f$ not necessarily linear
Availability of alternatives

Operator level

\( y_{in} \) decision of the operator

\[
    y_{in} = 0 \quad \forall i \notin C_n
\]  

(2)

Scenario level

\( y_{inr} \) availability at scenario level (e.g. demand exceeding capacity)

\[
    y_{inr} \leq y_{in}
\]  

(3)
Choice of alternatives

Choice at scenario level

\[ w_{inr} = \begin{cases} 
1 & \text{if } i = \arg \max_{j} \{ U_{jnr} \} \\
0 & \text{otherwise} 
\end{cases} \]

Choice and availability

\[ w_{inr} \leq y_{inr} \quad (4) \]
Linearization of the choice (I)

Auxiliary variables

\[ \nu_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ l_{nr} & \text{if } y_{inr} = 0 \end{cases} \]

Linearizing constraints

\[ l_{nr} \leq \nu_{inr} \quad (5) \]
\[ \nu_{inr} \leq l_{nr} + M_{inr} y_{inr} \quad (6) \]
\[ U_{inr} - M_{inr}(1 - y_{inr}) \leq \nu_{inr} \quad (7) \]
\[ \nu_{inr} \leq U_{inr} \quad (8) \]

where \( l_{nr} = \min_{i \in C} \{ U_{inr} \} \), \( m_{inr} \geq U_{inr} \), \( M_{inr} = m_{inr} - l_{nr} \)
Linearization of the choice (II)

Highest utility

\[ U_{nr} = \max_{i \in C_n} \nu_{inr} \]

Linearizing constraints

\[ \nu_{inr} \leq U_{nr} \]  \hspace{1cm} (9)
\[ U_{nr} \leq \nu_{inr} + M_{nr}(1 - w_{inr}) \]  \hspace{1cm} (10)
\[ \sum_{i \in C} w_{inr} = 1 \]  \hspace{1cm} (11)

where \( M_{nr} = \max_{i \in C} \{ U_{inr} \} - l_{nr} \)
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Maximization of revenues

Application

- Operator selling services to a market, each service:
  - Price
  - Capacity (number of individuals)
- \( i = 0 \) denotes the opt-out option
- Demand is price elastic and heterogenous
- **Goal:** best strategy in terms of capacity allocation and pricing
Supply side: demand-based revenues maximization

Pricing (I)

Revenues per alternative

- $p_{in}$ price that individual $n$ has to pay to access to alternative $i$
- Endogeneous variable in the utility function (1)

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}$$

Linearization (I)

- Discretization of the price: $p_{in}^1, \ldots, p_{in}^{Lin}$
- Binary variables $\lambda_{in\ell}$ such that $p_{in} = \sum_{l=1}^{Lin} \lambda_{inl} p_{in}^l$ and

$$\sum_{\ell=1}^{Lin} \lambda_{in\ell} = 1, \forall i > 0$$

(12)
Pricing (II)

Linearization (II)

- Revenues from alternative \( i \):
  \[
  R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{l=1}^{L_{in}} \lambda_{inl} p_{in}^l \sum_{r=1}^{R} w_{inr}
  \]

- Still non linear \( \Rightarrow \alpha_{inr\ell} = \lambda_{inl} w_{inr} \) to linearize it

\[
\begin{align*}
\lambda_{in\ell} + w_{inr} & \leq 1 + \alpha_{inr\ell} \quad \forall i > 0 \quad (13) \\
\alpha_{inr\ell} & \leq \lambda_{in\ell} \quad \forall i > 0 \quad (14) \\
\alpha_{inr\ell} & \leq w_{inr} \quad \forall i > 0 \quad (15)
\end{align*}
\]
Supply side: demand-based revenues maximization

Capacity (I)

Overview

- $c_i$: capacity of service $i$
- Who has access if the capacity is reached?
- The model favors customers bringing higher revenues
- ... but generally customers arrive in a random order

Priority list

- An individual is served only if all individuals before her in the list have been served
- Can account for fidelity programs, VIP customers, etc.
- We assume it given

\[ y_{inr} \geq y_{i(n+1)r} \quad \forall i > 0 \] (16)
Capacity (II)

Capacity must not be exceeded

\[ \sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n - 1)(1 - y_{inr}) \quad \forall i > 0, n > c_i \quad (17) \]

- \( y_{inr} = 1 \Rightarrow 1 + \sum_{m=1}^{n-1} w_{imr} \leq c_i \)
- \( y_{inr} = 0 \Rightarrow \sum_{m=1}^{n-1} w_{imr} \leq n - 1 \)

Capacity has been reached

\[ c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} \quad \forall i > 0 \quad (18) \]

- \( y_{in} = 1, y_{inr} = 0 \Rightarrow \sum_{m=1}^{n-1} w_{imr} \geq c_i \)
Supply side: demand-based revenues maximization

Full model

Objective function

\[ \max \sum_{i>0} \frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} p_{in}^{\ell} \sum_{r=1}^{R} \alpha_{inr\ell} \]  \hspace{1cm} (19)

Constraints

- Utility: (1)
- Availability of alternatives: (2) and (3)
- Choice: (4), (5), (6), (7), (8), (9), (10) and (11)
- Pricing: (12), (13), (14) and (15)
- Capacity allocation: (16), (17) and (18)
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Parking choices

Original experiment

- [Ibe, 2014] *Modelling parking choices considering user heterogeneity*
- Stated preferences survey
- Analyze viability of an underground car park
- Mixed logit model (random taste parameters)

Free on-Street Parking (FSP)
Free (opt-out)

Paid on-Street Parking (PSP)
0.6 and 0.8

Paid Underground Parking (PUP)
0.8 and 1.5
Case study

Choice model

Survey
- 197 respondents
- 8 scenarios based on
  - AT (access time to parking area)
  - TD (time to reach the destination)
  - FEE (price)

Mixed Logit model
- **Attributes:** TD
- **Random parameters:** AT, FEE
- **Socioeconomic characteristics:** residence, age of the vehicle
- **Interactions:** price and low income, price and residence
Computational results: overview

Assumptions

- Subset of 25 individuals
- Uncapacitated vs. capacitated case
- Capacity of 10 individuals for both PSP and PUP
- 10 price levels from 0 to 3

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Computational results: revenue and computational time
Computational results: demand

Uncapacitated case

Capacitated case
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Conclusions

- General framework (*any* assumption can be made for $\varepsilon_{in}$)
- Linear formulation integrating demand and supply
- High dimensionality of the problem ($N$ and $R$)
- Need for speeding up the computational results
Future work

Decomposition techniques

- Decomposition sources:
  - Individual (own optimization problem)
  - Draw (independent scenarios)
- Decomposable structures in practice:
  - Complicating constraints (Lagrangian relaxation techniques)
  - Complicating variables (Benders decomposition)

Lagrangian decomposition

- Two interesting subproblems with common variables
  - Choice subproblem (user’s side)
  - Pricing subproblem (operator’s side)
Questions?