Lagrangian relaxation for the demand-based benefit maximization problem

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Outline

Discrete choice models and optimization

- Disaggregate demand modeling
- Behavioral realism
- Complex formulations

- Tractability
- Linearity and/or convexity
- MILP models

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Bridging the gap

- Linear characterization of a discrete choice model
- Simulation to address stochasticity
- Demand-based benefit maximization problem

Linearization of the choice model (1)

$$U_{in} = V_{in} + \varepsilon_{in} \xrightarrow{\text{draw distribution } (R)} U_{inr} = V_{in} + \xi_{inr}$$

Linearization of the choice model (2)

$$w_{inr} = \begin{cases} 1 & \text{if } U_{inr} \geq U_{jnr} \forall j \in \mathcal{C}_n, j \neq i \\ 0 & \text{otherwise} \end{cases}$$

$$D_i = \frac{1}{R} \sum_r \sum_n w_{inr}$$

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Benefit maximization problem (1)

- Set of services $\mathscr{C}$ $(i > 0)$
- Opt-out option $i = 0$
- Population N $(n \geq 1)$
- Price $a_i \leq p_{in} \leq b_{in}$
- Capacity levels c_{iq}
- Fixed f_{iq} and variable v_{iq} costs

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Benefit maximization problem (2)

obj. fun.	$\sum_{i>0} \text{Revenue}_i - \text{Cost}_i$
availability	operator level and scenario level
disc. utility	variable capturing availability and utility
choice	linearization of the highest utility
price	linearization of the variable η_{iqnr} (revenue)
capacity	relation with the availability at scenario level

Computational results

- Parking choices: mixtures of logit model
- $R = 50$ draws and $N = 50$ customers
- $|\mathscr{C}| = 3$: PSP, PUP and FSP (opt-out)
- Computational times up to 34 hours!

Motivation

- Maximization of own utility
- Objective function and capacity constraints

- Behavioral scenario
- Objective function

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Uncapacitated case and revenue maximization

obj. fun.	$\sum_{i>0} \text{Revenue}_i$
availability	
utility	no need for discounted utility (no availability)
choice	linearization of the highest utility
price	linearization of the variable η_{iqnr} (revenue)
capacity	

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Lagrangian decomposition

⚠ Price p_{in} is the same across draws ⇒ **no** decomposition by n and r

$$p_{in1} = p_{in2} = \cdots = p_{inR} = p_{in1}$$

Lagrangian multipliers λ_{inr} ⇒ decomposition by n and r

The Lagrangian is not stable...

- Subgradient method to update the Lagrangian multipliers
- Lagrangian is not stable (dual not equivalent to the primal)
- Augmented Lagrangian to overcome this limitation

$$\begin{aligned} \min \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & x_1 + x_2 = 1.5 \\ & 0 \leq x_1 \leq 1 \\ & 0 \leq x_2 \leq 1 \end{aligned}$$

Optimal solution

$x_1 = 1,\ x_2 = 0.5$

- Relax $x_1 + x_2 = 1.5$
- True value $\lambda = 2$
- λ little lower than 2: $x_1 = 1,\ x_2 = 0$
- λ little greater than 2: $x_1 = 1,\ x_2 = 1$
- Even at $\lambda = 2$, $x_1 = 1,\ x_2 = 0.5$ will not be reached!

Augmented Lagrangian relaxation

Lagrangian relaxation

$$\max \sum_{i>0} \sum_{n} \sum_{r} \left[\frac{1}{R} \eta_{inr} + \lambda_{inr} (p_{inr} - p_{in(r+1)}) \right]$$

Augmented Lagrangian (additional penalty)

$$\max \sum_{i>0} \sum_{n} \sum_{r} \left[\frac{1}{R} \eta_{inr} + \lambda_{inr} (p_{inr} - p_{in(r+1)}) - \frac{c}{2} \underbrace{\left\| (p_{inr} - p_{in(r+1)}) \right\|^2}_{p_{inr}^2 + p_{in(r+1)}^2 - 2 p_{inr} p_{in(r+1)}} \right]$$

⚠ Nonseparable quadratic term ⇒ **no** decomposition by n and r

Auxiliary Problem Principle (1)

- Partial linearization of the quadratic term at the current iteration (k)
- Addition of a quadratic separable term (if appropriately chosen)

$$\max \sum_{i>0} \sum_{n} \sum_{r} \left[\frac{1}{R} \eta_{inr} + \lambda_{inr} (p_{inr} - p_{in(r+1)}) - \frac{c}{2} \left\| (p_{inr} - p^{k-1}_{in(r+1)}) \right\|^2 - \frac{b-c}{2} \left\| (p_{inr} - p^{k-1}_{inr}) \right\|^2 \right]$$

Auxiliary Problem Principle (2)

Update Lagrangian multipliers

$$\lambda_{inr}^{k+1} = \lambda_{inr}^{k} - \theta\big(p_{inr} - p_{in(r+1)}\big)$$

Update penalty term

$$\sum_{i>0}\sum_{n}\sum_{r}\left\|p_{inr}^{k} - p_{in(r+1)}^{k}\right\| > \alpha \sum_{i>0}\sum_{n}\sum_{r}\left\|p_{inr}^{k-1} - p_{in(r+1)}^{k-1}\right\| \Longrightarrow \begin{cases} c \leftarrow \beta c \\ b \leftarrow \gamma c \end{cases}$$

⚠ Best convergence performance when $b \rightarrow 2c$ and $\theta = c$

Preliminary results

- Multipliers: $c = 1,\ b = 2c,\ \theta = c$
- Auxiliary multipliers: $\alpha = 1.1,\ \beta = 2,\ \gamma = 2$
- Number of iterations: $K = 25$

		Exact method				Augmented Lagrangian			
N	R	**Revenue**	Dem PSP	Dem PUP	Dem FSP	**Obj fun**	Dem PSP	Dem PUP	Dem FSP
5	5	2.785	1.2	2.4	1.4	2.784	1.2	2.4	1.4
10	10	5.215	5.2	2.7	2.1	4.953	5	2.7	2.3
25	10	15.383	7.8	11.6	5.6	15.365	7.3	12.2	5.5
25	25	14.580	10.52	10.16	4.32	14.438	7.88	11.88	5.24
25	50	14.383	11.6	9.1	4.3	13.507	5.96	11.58	7.46

Conclusions

- Augmented Lagrangian to overcome Lagrangian relaxation's limitation
- Close aggregate quantities for low values of N and R
- Higher differences for larger values of N and R

Future work

- Other strategies for the augmented Lagrangian (absolute value)
- Assess computational time with respect to the exact method
- Characterize a solution to the original problem
- Gradually include the complexity back (capacity, availability...)

Questions?

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