A Lagrangian decomposition scheme for the choice-based optimization framework

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Outline

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Mismatch between supply and demand

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- reduced profitability

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- decrease in consumer goodwill

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- decrease in consumer goodwill
- spillover effects

Supply-demand interplay

Take into account the interactions between supply and demand

Demand model

- Behavioral realism
- Disaggregate representation

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- Heterogeneity of tastes and preferences in high detail

Supply-related decisions

- Optimization models
- Tractability

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Mixed Integer Linear Problems (MILP)

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- Modeling flexibility (integer and continuous variables)
- Commercial MILP solvers to find the global optima
- Variety of strategies and solution techniques

DCM and MILP: an illustrative example

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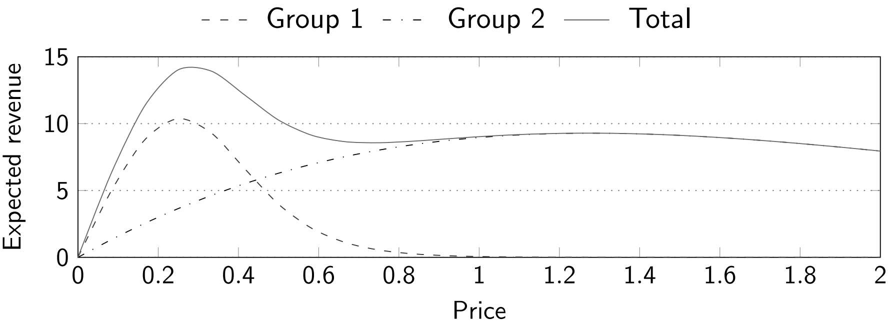

Choice-based optimization framework

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- Decision variables of MILP as explanatory variables of DCM
- Simulation-based linearization of the preference structure of DCM

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- Expected demand: $D_i = \sum_n P_n(i)$

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- Integration in MILP: $V_{in}(x^d_{in}, x^e_{in})$ linear in x^e

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- **Choice:** only one alternative can be chosen for each n and r

$$w_{inr} = \begin{cases} 1 & \text{if } z_{inr} = U_{nr} := \max_{j \in \mathcal{C}} z_{jnr} \\ 0 & \text{otherwise} \end{cases}$$

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- Sequence of optimal sols. of $\mathcal{P}_R$ converges to an optimal sol. of $\mathcal{P}$

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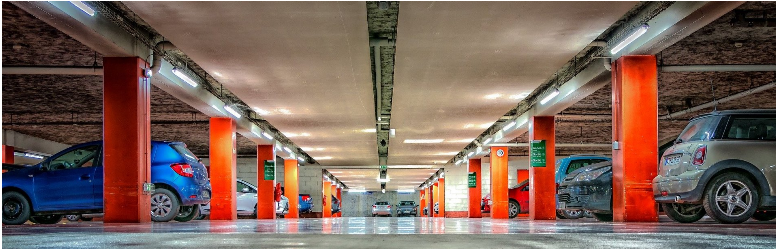
obj. fun.	max profit from all services but the opt-out
DCM	availability, discounted utility, choice
capacity	fixed or variable (discretized)
price	p_{in} endogenous (continuous or discrete)

Parking case study



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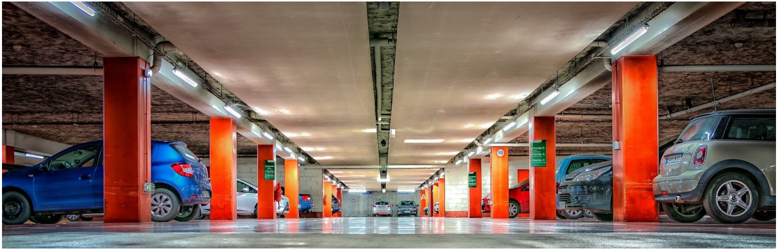

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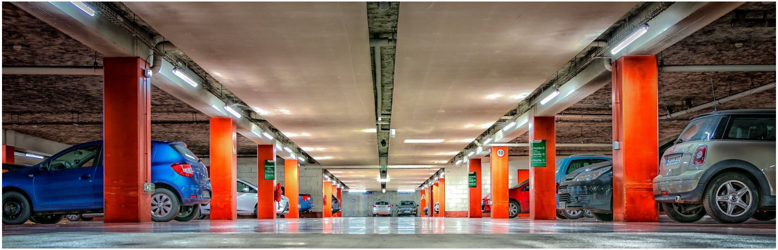

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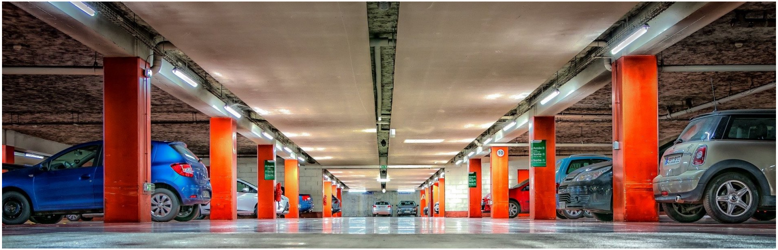

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- Common price (same price proposed to everyone): p_i

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- Exploit the decomposable structure of the framework!

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- Simulation-based linearization

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- Framework built on two dimensions that can be addressed separately:
  - **Individuals:** most fundamental unit of demand
  - **Draws:** independent behavioral scenario

Revenue maximization problem

obj. fun. (Z)	max revenue from all services but the opt-out
DCM	availability, discounted utility, choice
price	p_{in} continuous, $\eta_{inr} = p_{in} w_{inr}$ $+$ linearizing constraints (revenue calculation)
capacity	uncapacitated or fixed capacity

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$$
\begin{aligned}
Z^{LR}(\rho, \psi) = \max \quad & \underbrace{\sum_{i \in \mathcal{C} \setminus \{0\}} \sum_{n} \sum_{r} \frac{1}{R} \eta_{inr}}_{\text{revenue}} \\
& + \underbrace{\sum_{i \in \mathcal{C}} \sum_{n} \sum_{r} \rho_{inr} (U_{inr} - d_{inr} - \beta_{in} p_{in})}_{\text{relaxation utility function}} \\
& + \underbrace{\sum_{i \in \mathcal{C}} \sum_{n} \sum_{r} \gamma_{inr} (v_{inr} - w_{inr})}_{\text{relaxation duplicate choice}}
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- Preserve supply-demand interplay: Lagrangian decomposition

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$$p_{in1} = p_{in2} = \cdots = p_{inR}$$

$$Z^{LD}(\alpha) = \max \underbrace{\sum_{i \in \mathcal{C}\setminus\{0\}} \sum_{n} \sum_{r} \frac{1}{R} \eta_{inr}}_{\text{revenue}} + \underbrace{\sum_{i \in \mathcal{C}\setminus\{0\}} \sum_{n} \sum_{r} \alpha_{inr} (p_{inr} - p_{in(r+1)})}_{\text{relaxation copy constraints}}$$

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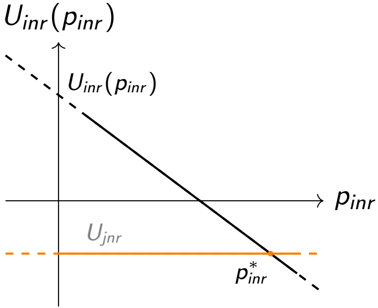
$$p_{inr} = \begin{cases} \max\{a_{in}, p^*_{inr}\}, & \text{if } \alpha_{inr} - \alpha_{in(r-1)} \leq 0, \\ b_{in}, & \text{otherwise}, \end{cases} \quad \forall i \in \mathcal{C}_n \setminus \{0\} \mid w_{inr} = 0,$$



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Key idea

1. S groups of R/S draws each
2. $p_{in}^1 = p_{in}^2 = \cdots = p_{in}^S$

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- Motivated by scenario grouping in stochastic programming
- Grouping scenarios within each subproblem
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  - Solving larger subproblems

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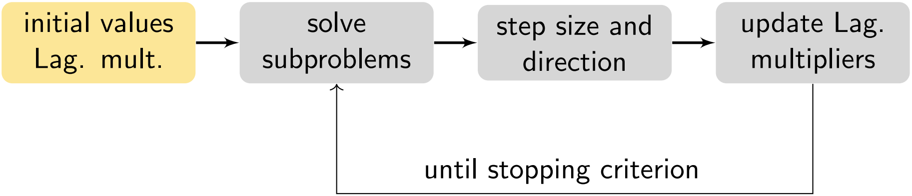
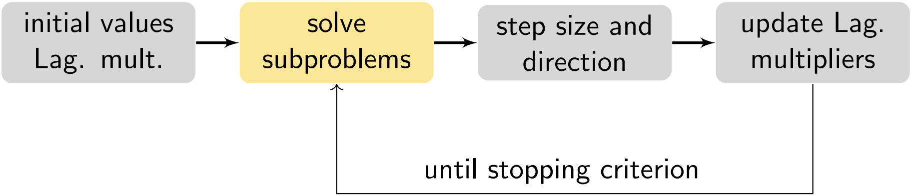
- Best upper bound (Lagrangian dual): subgradient method

Subgradient method



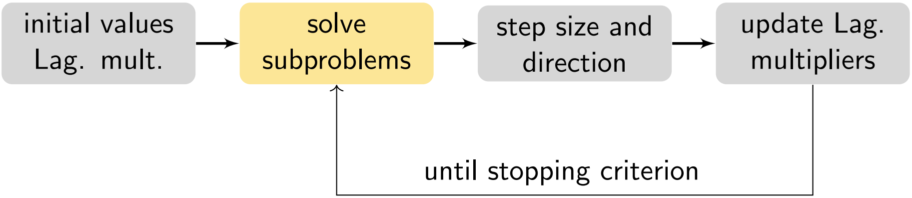
- **Initialize Lagrangian multipliers**: α^0 (e.g., $\alpha^0 = 0$)

Subgradient method



- **Upper bound**: Solve $Z_s^{\mathrm{UB}}(\alpha^0), \forall s$ (CPLEX solver)

Subgradient method



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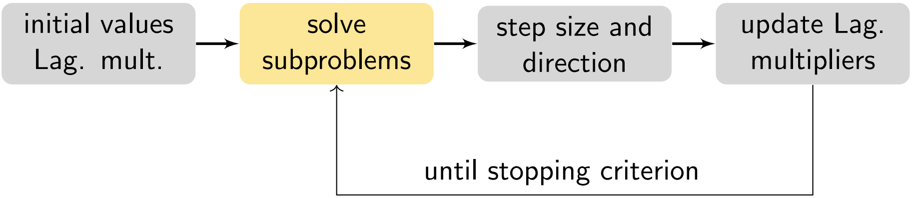

- **Upper bound**: Solve $Z_s^{\text{UB}}(\alpha^0), \forall s$ (CPLEX solver)
- **Lower bound**: Obtain Z^{LB} by generating feasible solutions for Z
- Keep track of the best bounds found so far: $Z^{\text{UB,best}}$ and $Z^{\text{LB,best}}$

Feasible solutions

- Sequence of prices: $\{\bar{p}_{in}^s\}_s$

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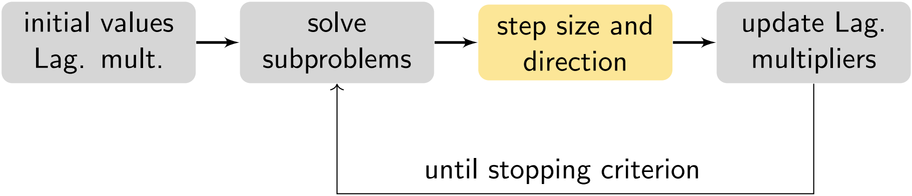
Input: Fixed prices $\bar{p}_{in}$;
Output: Values for y_{inr}, w_{inr}, U_{inr}, U_{nr} and Z;
Initialize $Z = 0$;
for $r = 1 \dots R$ **do**
  Initialize occupancy level $o_{ir} = 0$ and $y_{inr} = 1$;
  for $n = 1 \dots N$ **do**
    for $i \in \mathcal{C}_n \setminus \{0\}$ **do**
      if $o_{ir} < c_i$ **then**
        Calculate $U_{inr} = \beta_{in}\bar{p}_{in} + d_{inr}$;
      else
        Set $y_{inr} = 0$ and $U_{inr} = \ell_{nr}$;
    Determine w_{inr}, U_{inr}, U_{nr};
    Update $Z = Z + \sum_{i \in \mathcal{C}_n \setminus \{0\}} \frac{1}{R} w_{inr} \bar{p}_{in}$ and $o_{jr} = o_{jr} + 1$;

Subgradient method



- **Step size:** $\gamma^k = \lambda^k \frac{Z^{\mathrm{UB}}(\alpha^k) - Z^{\mathrm{LB,best}}}{\|v^k\|^2}$ (λ^k step decreasing parameter)

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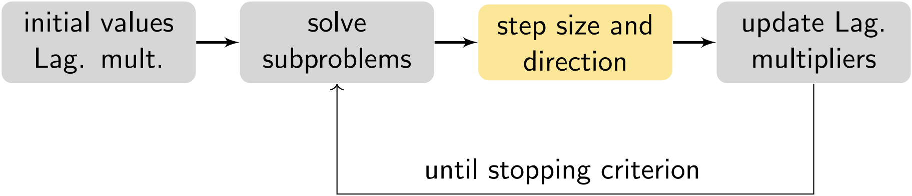

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- **Step direction:** $v^k = -(g^k + \zeta^k v^{k-1})$
  - subgradient: $g^k_{ins} = p^k_{ins} - p^k_{in(s+1)}$
  - deflection parameter: ζ^k

Deflected subgradient method: zigzagging of kind I

- Angle between current subgradient and previous one might be obtuse

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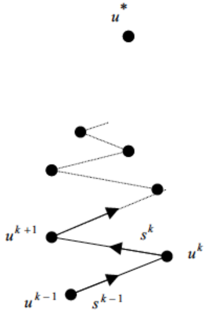

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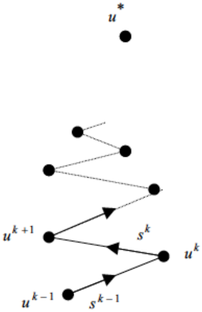

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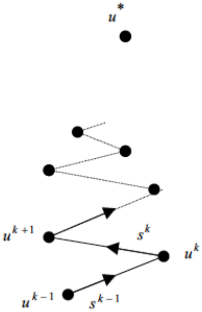
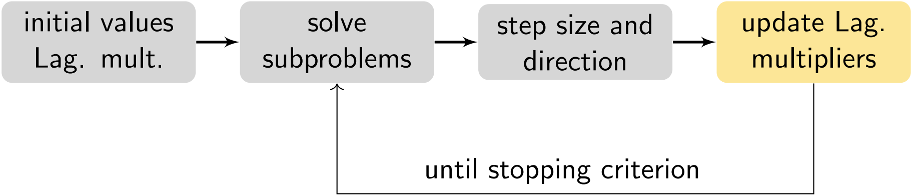


- Deflect the step direction to decrease the angle
- Only when g^k forms an obtuse angle with the previous direction

Subgradient method



- **Update Lagrangian multipliers:** $\alpha^{k+1} = \alpha^k + \gamma^k v^k$

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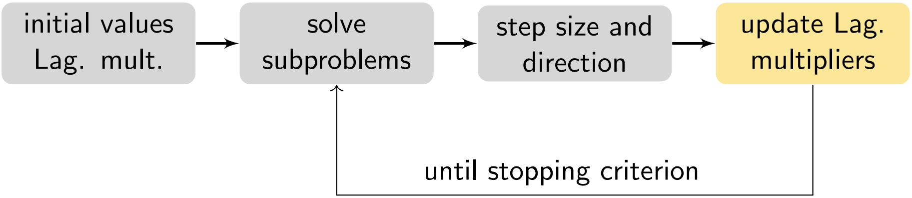
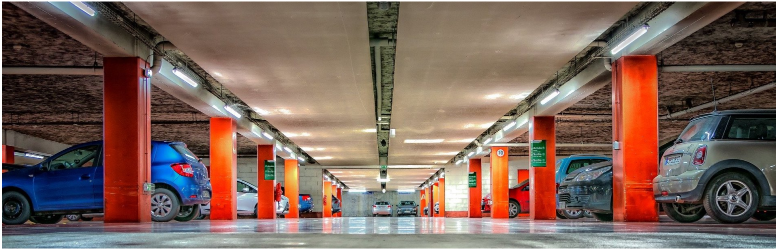


- **Update Lagrangian multipliers:** $\alpha^{k+1} = \alpha^k + \gamma^k v^k$
- Stopping criterion: computational time

Outline

Case study



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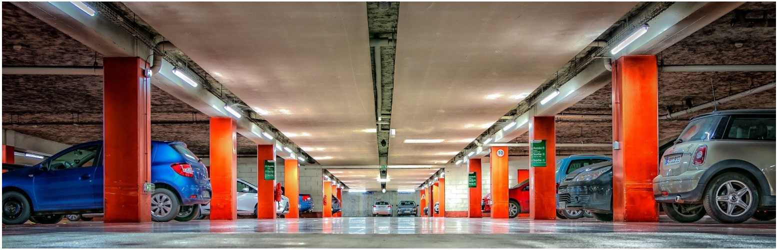

- Parking choices
- Common price: p_i (aggregate formulation)

Comparison with optimal solutions

- $N = 50$, 5 draws per group and $R \in \{100, 250, 500\}$

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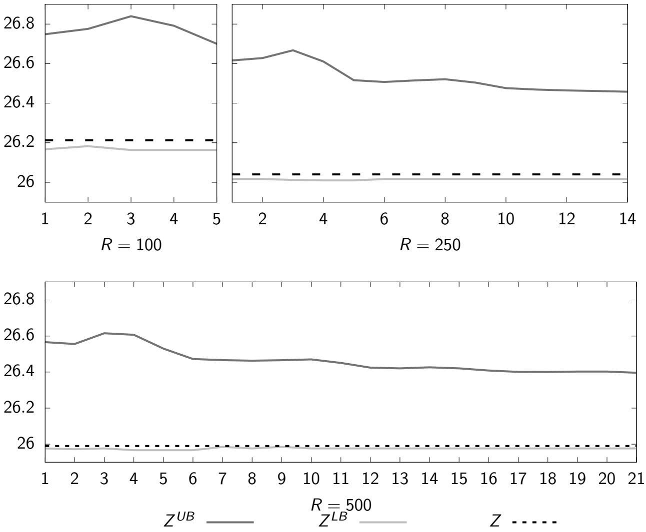
R	#Iter.	$Z^{\text{UB,best}}$ (it.)	$Z^{\text{LB,best}}$ (it.)	Avg. time it. (min)	gap_{dual}(%)	gap_{opt}(%)
100	5	26.70 (5)	26.18 (2)	5.16	1.98	0.11
250	14	26.46 (14)	26.02 (1)	16.81	1.70	0.09
500	21	26.40 (21)	25.99 (7)	35.82	1.58	0.02

Evolution of bounds



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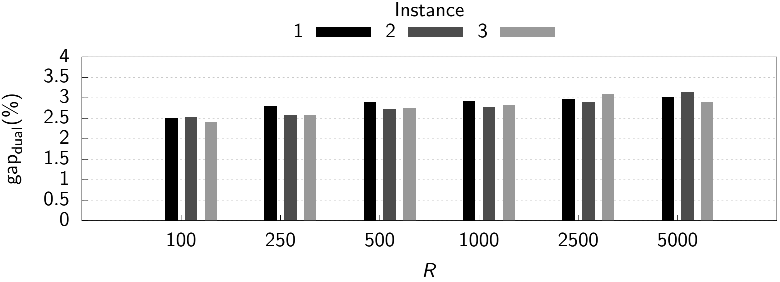

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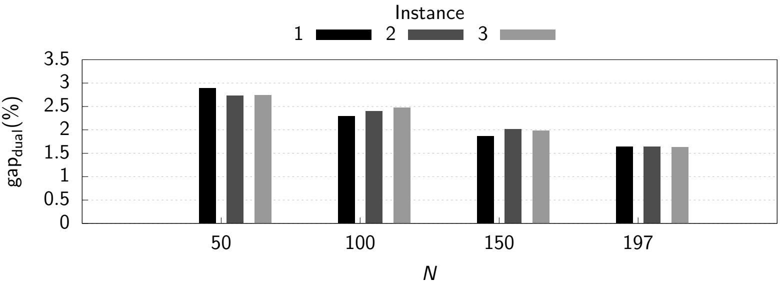

Trade-off with respect to the size of the draw groups

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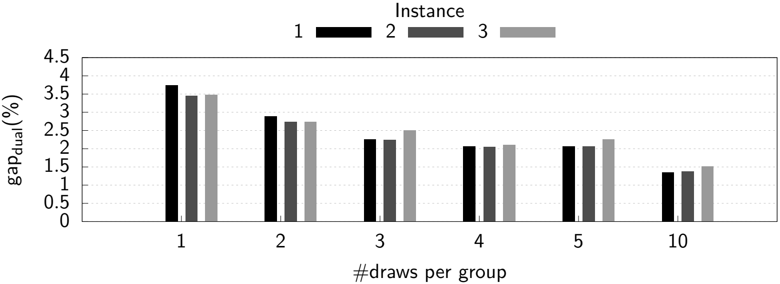

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Future research directions

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- Combination with other techniques (e.g., Benders decomposition in the presence of discrete design variables) and variance reduction methods (to decrease the number of draws)

Questions?