Faster estimation of discrete choice models via dataset reduction

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Outline

1. Background
2. LSH-based dataset reduction
3. Case study
4. Conclusion

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Discrete choice models (DCMs)

What are DCMs?

- Suppose N observations, each containing:
  - a vector of explanatory variables x_n;
  - the observed choice i_n.
- A DCM calculates the choice probabilities as a function of x_n and θ:

$$P\left(i \mid x_n; \theta\right),$$

- where θ is a vector of model parameters.

Estimating DCMs

Maximum likelihood estimation (MLE)

- Find θ so as to maximize the joint probability of the observed choices:

$$\max_{\theta} \mathcal{L}(\theta) = \sum_{n=1}^{N} P\left(i_n \mid x_n; \theta\right).$$

  - Solved using iterative methods — BFGS, Newton, etc.
  - Each iteration is $\mathcal{O}(N)$.
- MLE is burdensome for large datasets!

Intuition

Factoring out redundancy

- If the data contains groups of **identical observations**:

$$\mathcal{L}(\theta) = \sum_{u=1}^{U} N_u \cdot P\left(i_u \mid x_u; \theta\right),$$

  - U unique observations.
  - Each appears N_u times in the original data.
- Can we extend this "factorization trick" to **nearly identical observations**?

Faster estimation of DCMs

Assumption

- If $i_p = i_q$ and $x_p \approx x_q$, then $P(i_p \mid x_p; \theta) \approx P(i_q \mid x_q; \theta)$.

Approach

- Aggregate similar observations together.
- Associate weights.
- MLE on the reduced, weighted sample.

Challenges

- Clustering must be fast $\Rightarrow$ Use locality-sensitive hashing (LSH).
- Minimize degradation of estimation results.

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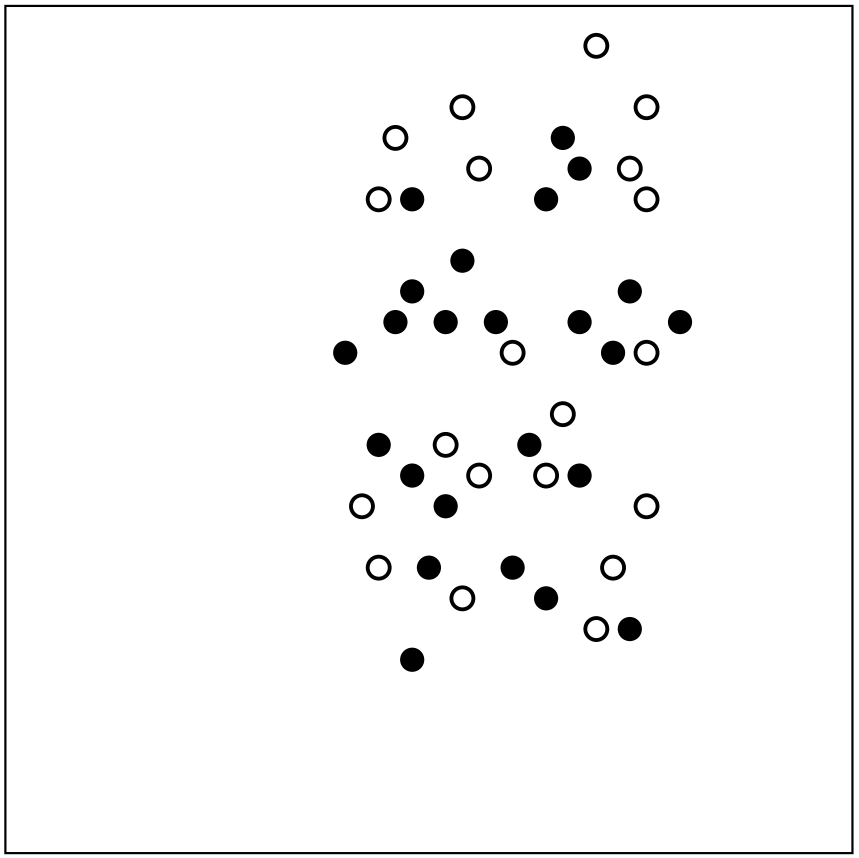
Procedure

Example

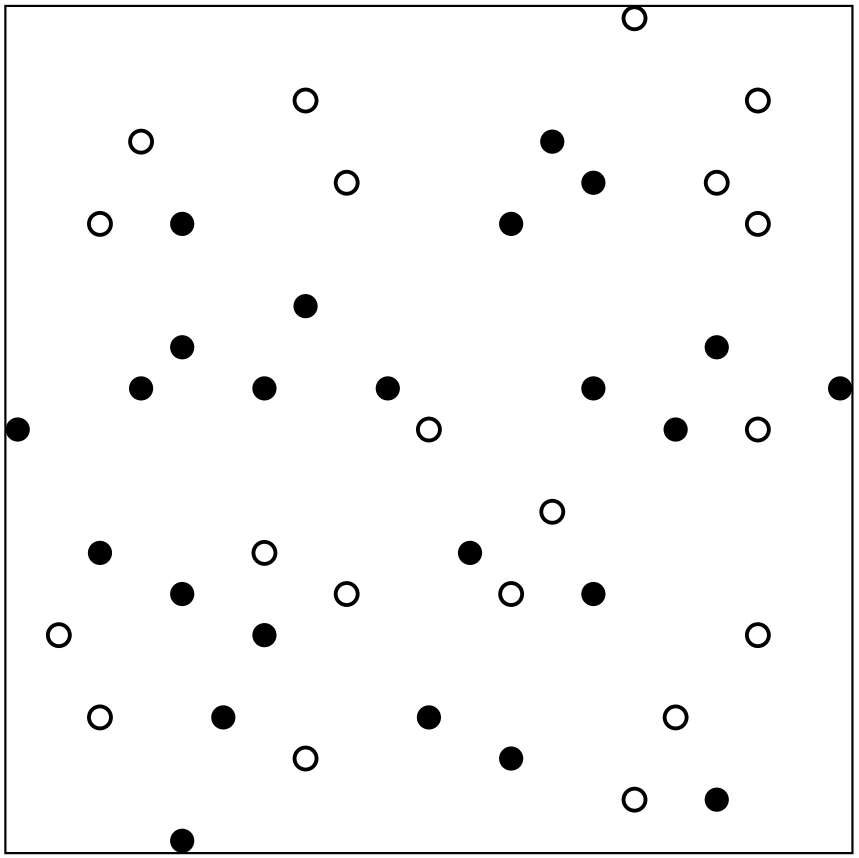
- 2 alternatives.
- 2 explanatory variables.



Procedure

Data rescaling

$$\frac{x - \min(x)}{\max(x) - \min(x)}$$



Procedure

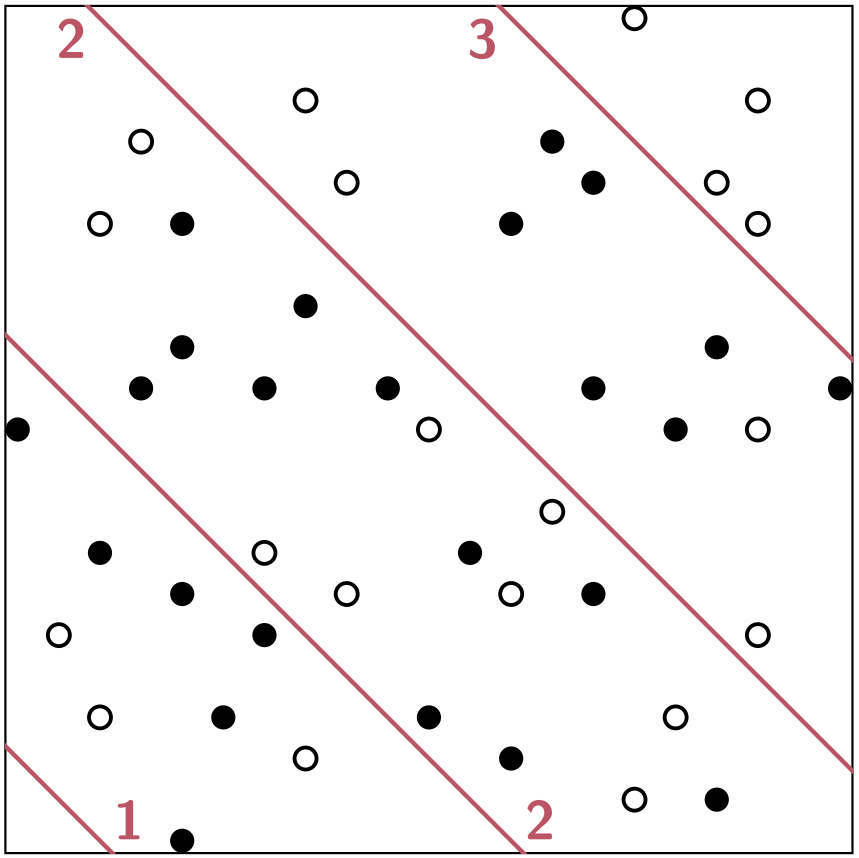
LSH function

$$h(x) = \left\lfloor \frac{a \cdot x + b}{w} \right\rfloor$$

- $a \overset{iid}{\sim} \mathcal{N}(0,1)$.[◇]
- $b \sim \mathcal{U}(0, w)$.
- w is the **bucket width**.*

Example

$$a = \left(\tfrac{1}{2}, \tfrac{1}{2}\right),\ b = \tfrac{1}{5},\ w = \tfrac{1}{4}.$$



Procedure

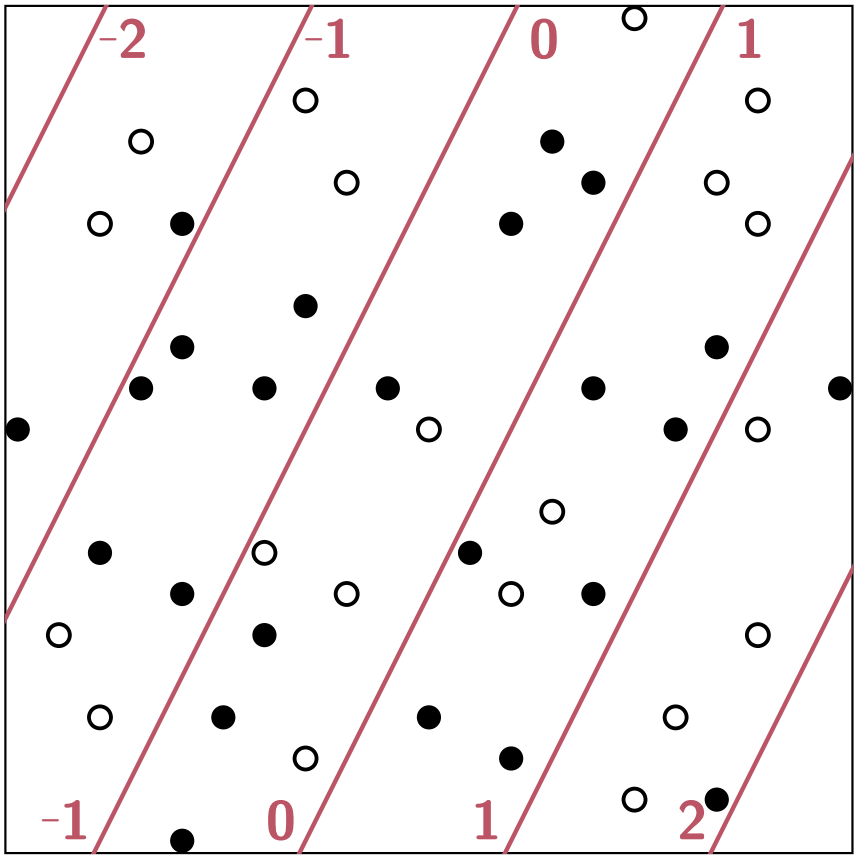
LSH function

$$h(x) = \left\lfloor \frac{a \cdot x + b}{w} \right\rfloor$$

- $a \overset{iid}{\sim} \mathcal{N}(0,1)$.◇
- $b \sim \mathcal{U}(0, w)$.
- w is the **bucket width**.*

Example

$$a = \left(1, -\tfrac{1}{2}\right),\ b = -\tfrac{1}{10},\ w = \tfrac{1}{4}.$$



Procedure

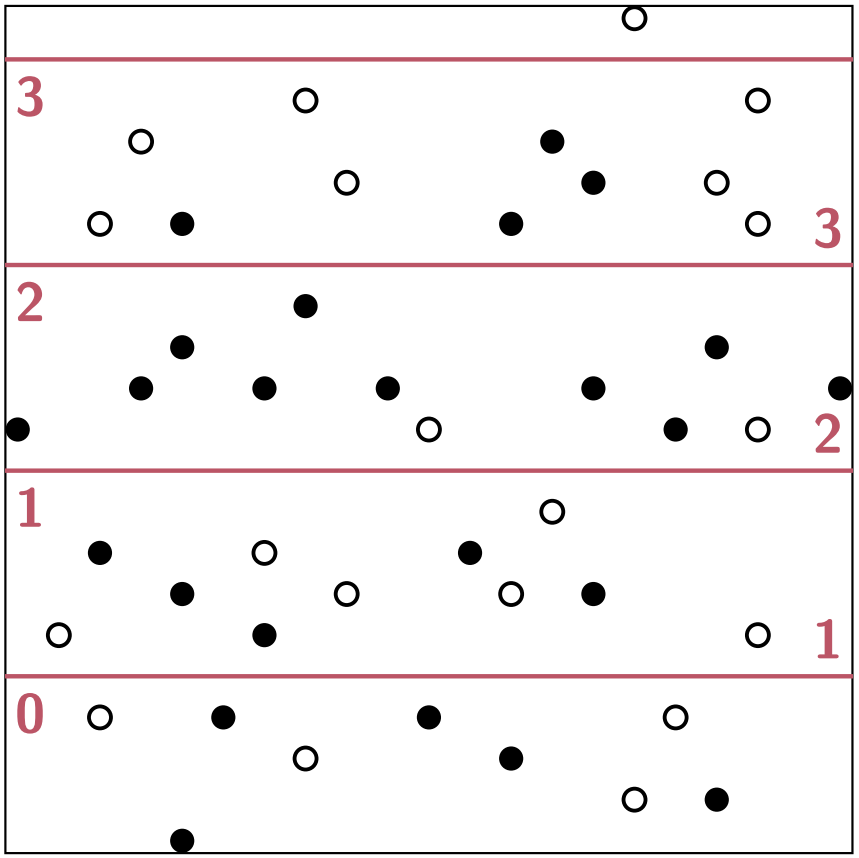
LSH function

$$h(x) = \left\lfloor \frac{a \cdot x + b}{w} \right\rfloor$$

- $a \overset{iid}{\sim} \mathcal{N}(0,1)$.[◇]
- $b \sim \mathcal{U}(0, w)$.
- w is the **bucket width**.*

Example

$$a = (0,1),\ b = \tfrac{1}{20},\ w = \tfrac{1}{4}.$$

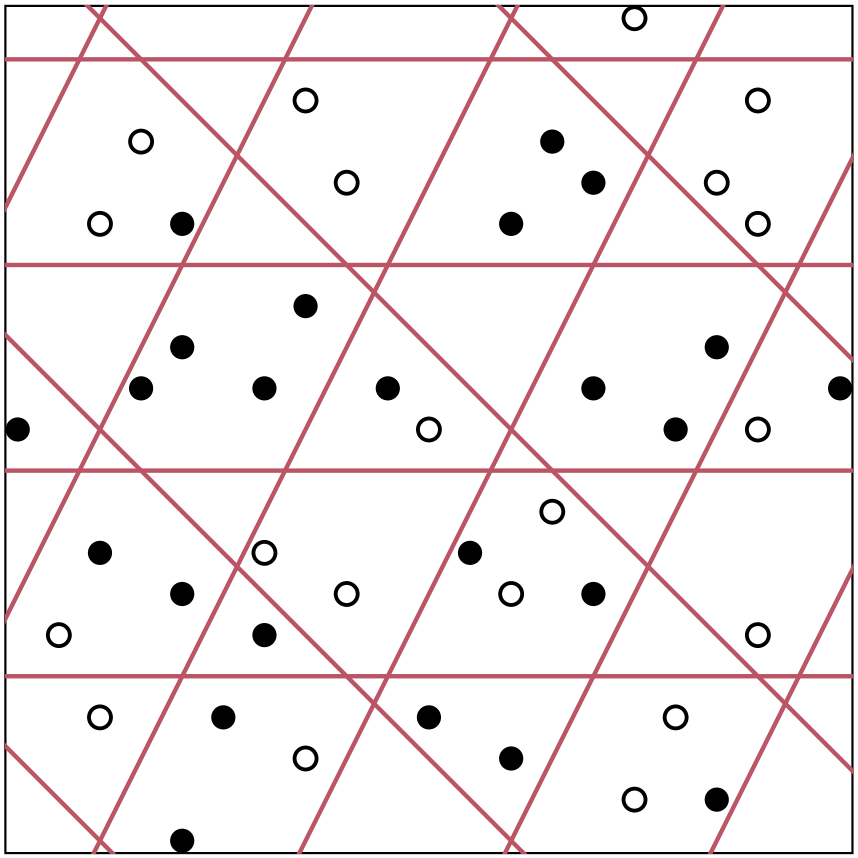


Procedure

AND-construction

- Combine r LSH functions.*
- For any pair of points (x_p, x_q):

$$h'(x_p) = h'(x_q) \Leftrightarrow$$
$$h_k(x_p) = h_k(x_q) \;\; \forall k = 1, \ldots, r.$$



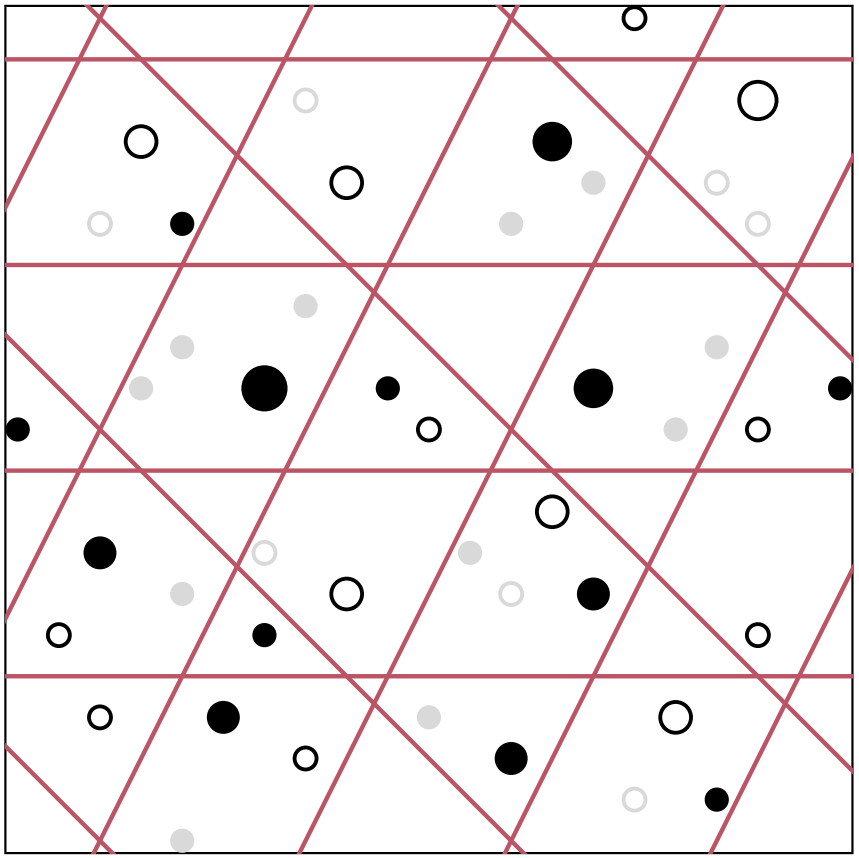
Procedure

Instance selection

- In each bucket, **for each class**, pick 1 instance x_s **at random**.$^{\diamond}$

Weight assignment

- The associated weight ω_s is equal to the number of instances of alternative i_s in the bucket.
- $\sum_s \omega_s = N$.

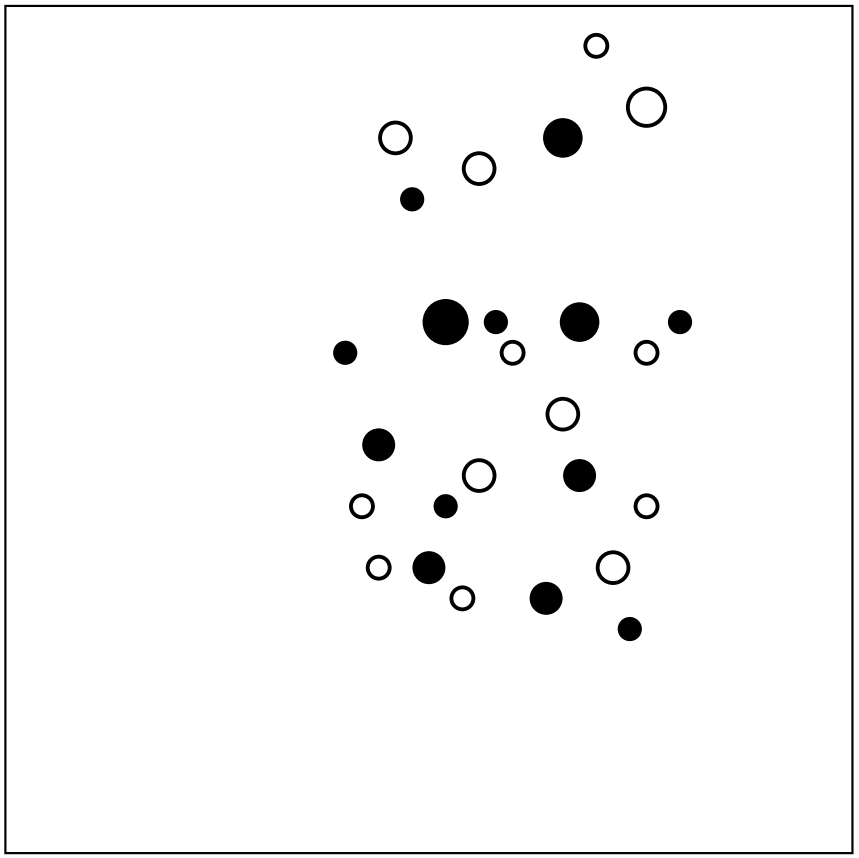


Procedure

MLE

- Data are scaled back.
- Log likelihood function:

$$\mathcal{L}(\theta) = \sum_s \omega_s P(i_s \mid x_s, \theta)$$



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Dataset and models

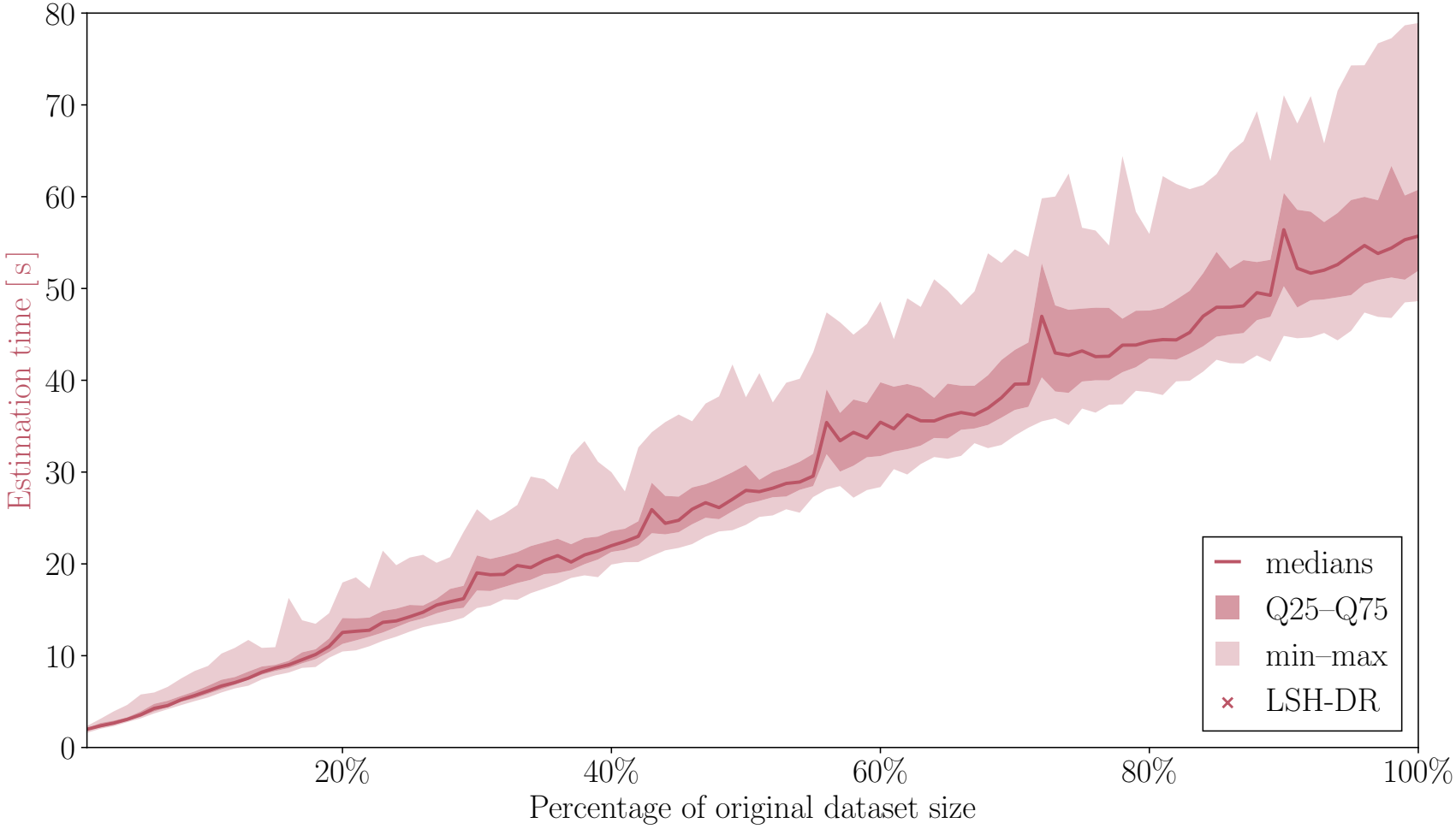
LPMC data

- Mode choice, 4 alternatives: walk, cycle, drive and public transport.
- 81k observations:
  - 55k for estimation;
  - 26k for out-of-sample validation.

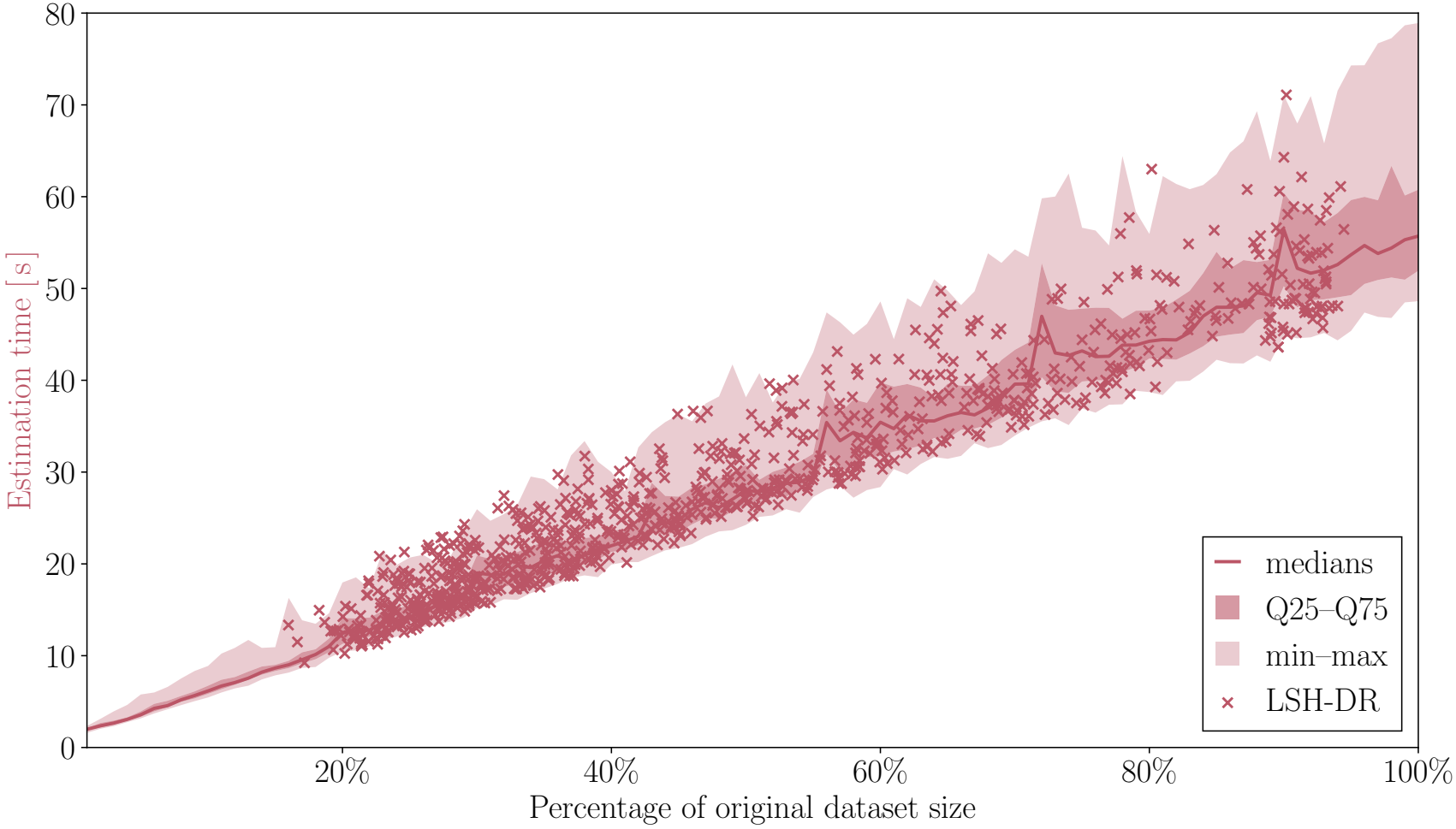
Model

- Multinomial logit.
- 11 continuous attributes.
- 15 binary variables.
- **53 parameters**.

Preliminary results — random sample



Preliminary results — $r = 5$, $w = 0.1, \ldots, 1.0$



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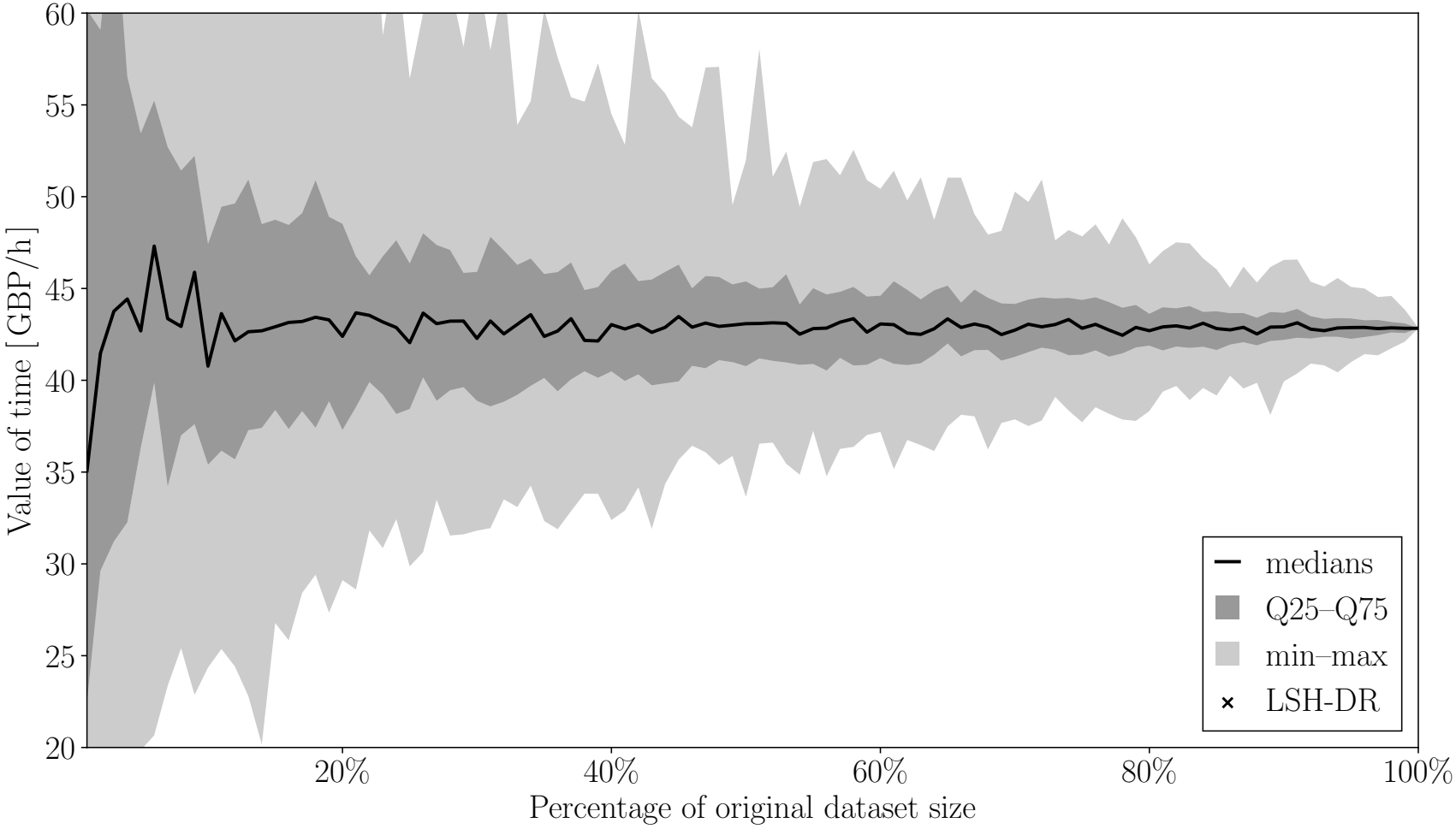

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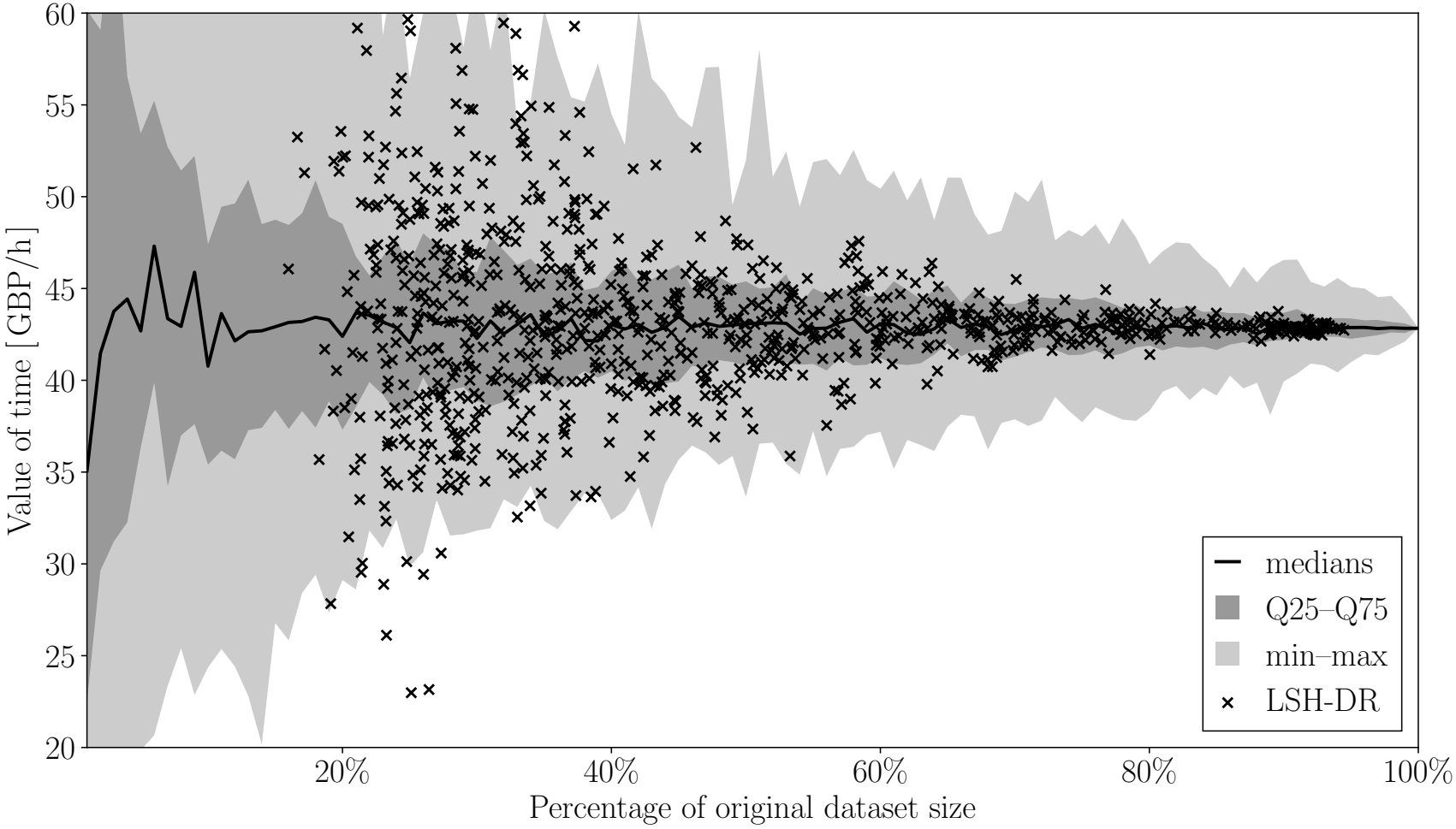

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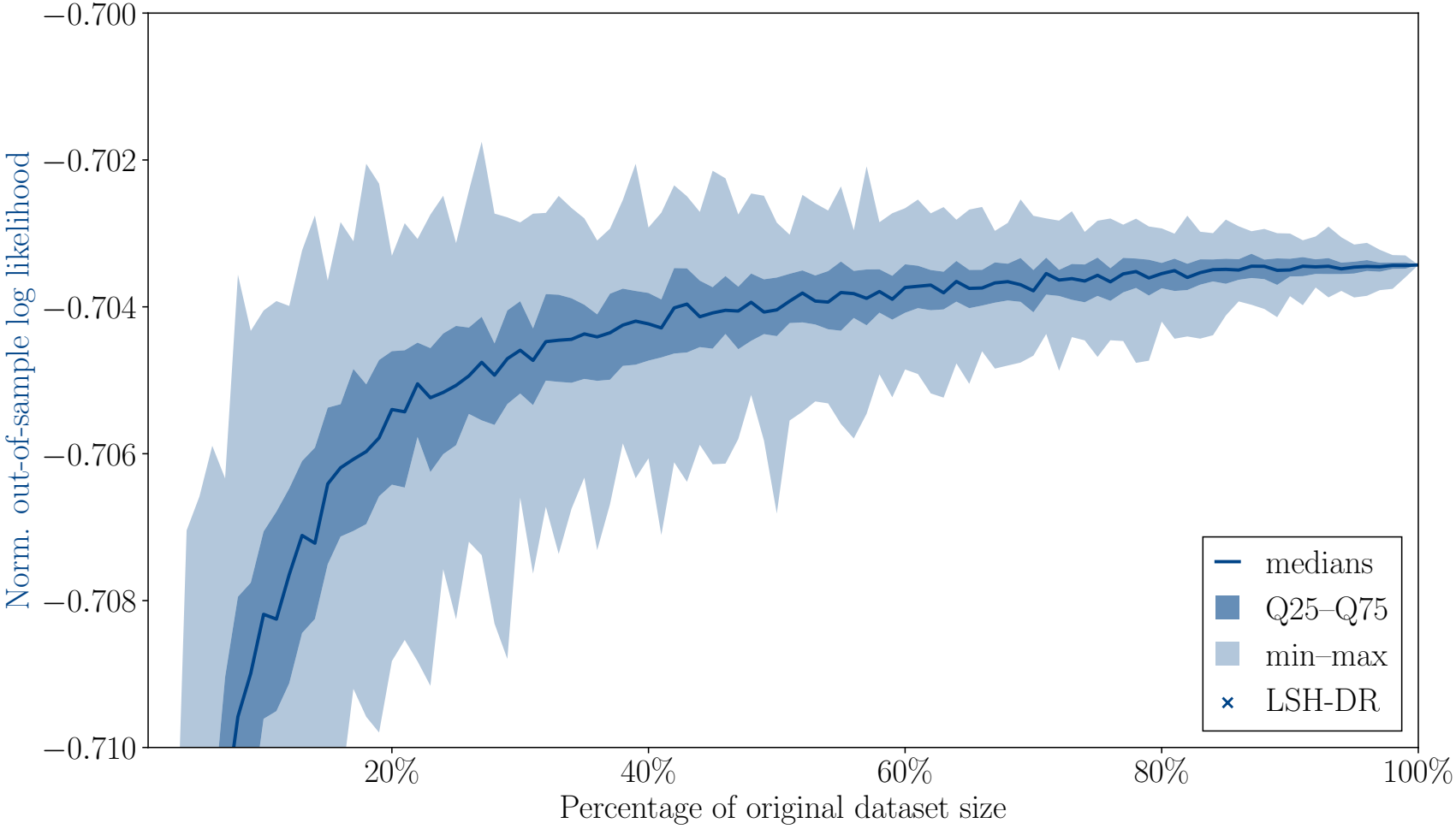

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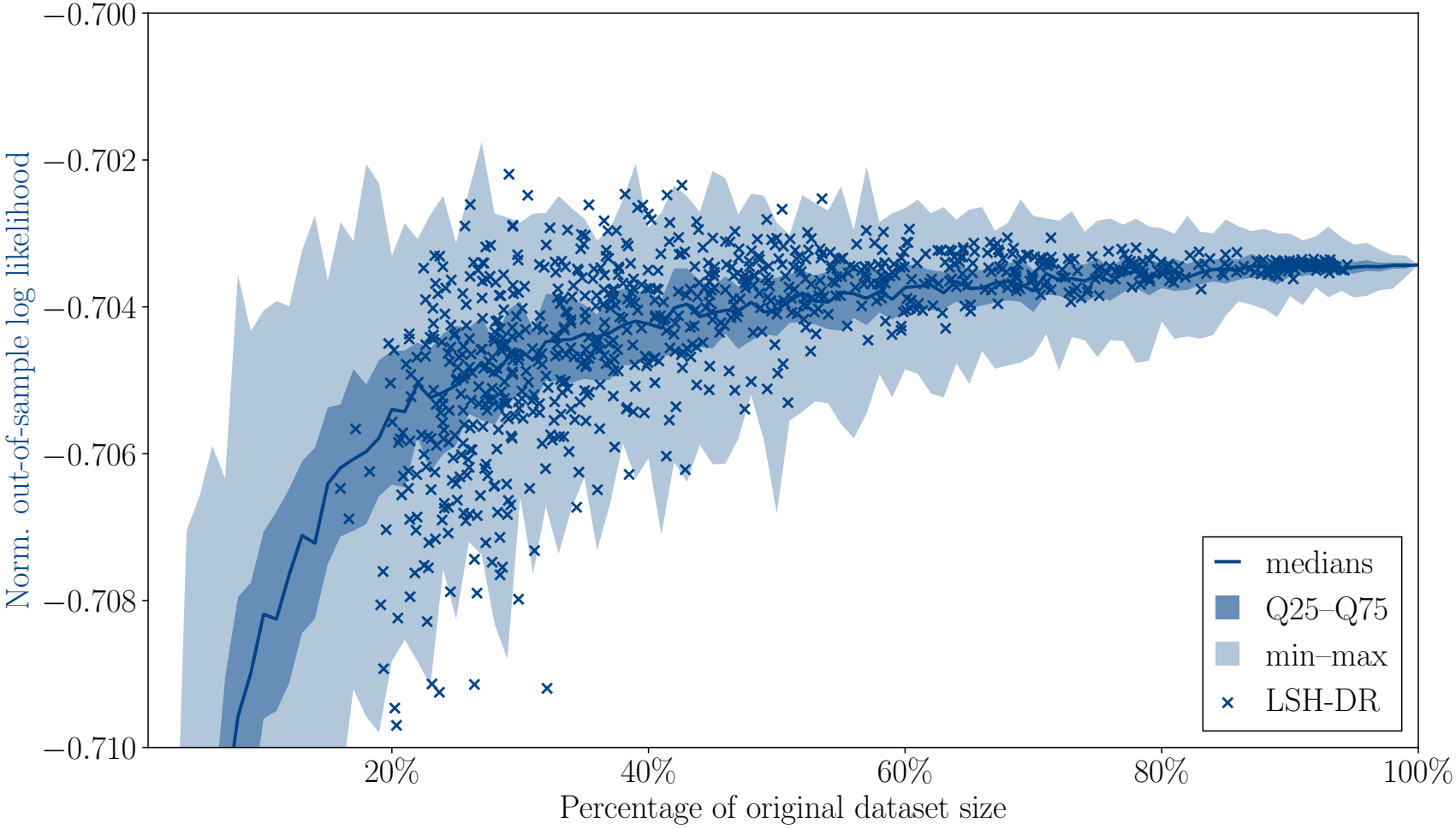

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Summary

- Reducing computational time comes at the cost of deteriorating results.
- This can be mitigated by carefully sampling observations.
- Factor out redundancy, but keep diversity!

Future work

- Knowledge-based LSH.
- Informed sampling from the buckets.
- Embed dataset reduction within MLE.

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