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EPFL

Resampling estimation of discrete choice models

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Nicola Ortelli[1,2], Matthieu de Lapparent[1], Michel Bierlaire[2]

[1] IIDE, HEIG-VD
[2] TRANSP-OR, EPFL

Motivation

DCMs in the era of big data

- Specifying DCMs is time-consuming.
- Ever-larger datasets:
  - "Wider" data — more variables;
  - "Taller" data — more observations.
- Two distinct problems: specification and estimation.

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- Two distinct problems: specification and estimation.

Speeding up model estimation

- Obtain estimation results faster.
- Spend more time looking for good specifications.

Intuition

Maximum likelihood estimation

- Let $\mathcal{N} = \{(\boldsymbol{x}_n, i_n) : n = 1, \ldots, N\}$ be a choice dataset.
- Log likelihood function:

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{n=1}^{N} \log P\left(i_n \mid \boldsymbol{x}_n; \boldsymbol{\theta}\right).$$

- Computational time is linear in N.

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Factoring-out redundancy

- Suppose that some observations in $\mathcal{N}$ are identical.
- For $U < N$ unique observations:

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{u=1}^{U} N_u \cdot \log P\left(i_u \mid \boldsymbol{x}_u; \boldsymbol{\theta}\right).$$

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$\Rightarrow$ **Extend factorization to "nearly identical" observations!**

Resampling estimation of DCMs

Procedure

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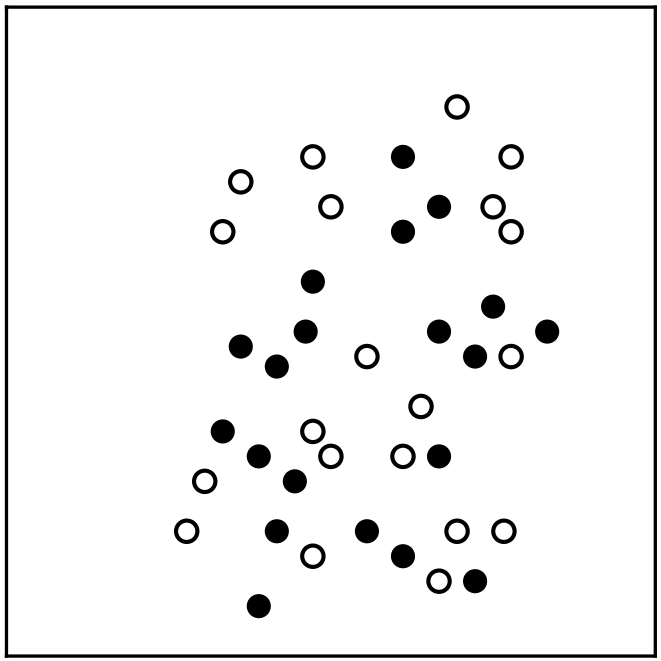
$\Rightarrow$ **Use locality-sensitive hashing (LSH).**

LSH-based dataset reduction (LSH-DR)

Toy data

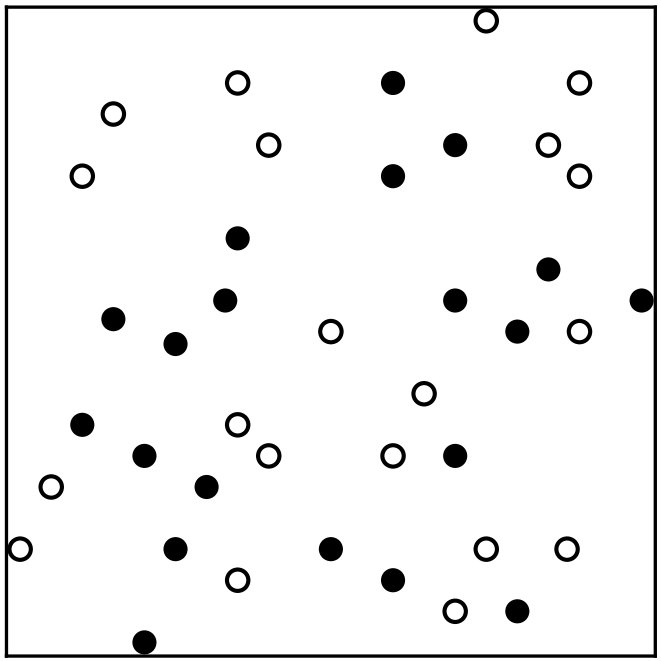
- 2 alternatives.
- 2 explanatory variables.



LSH-based dataset reduction (LSH-DR)

Min-max normalization

$$\frac{\boldsymbol{x} - \min(\boldsymbol{x})}{\max(\boldsymbol{x}) - \min(\boldsymbol{x})}$$

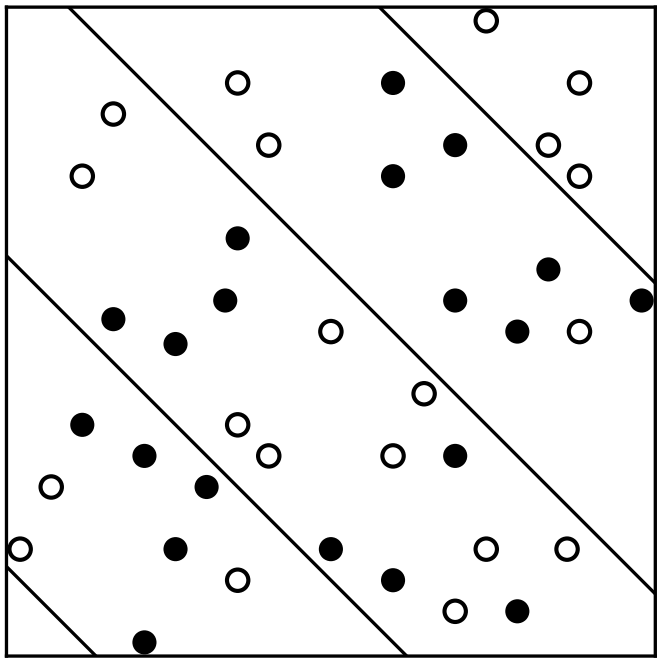


LSH-based dataset reduction (LSH-DR)

A single LSH function

$$h_{\boldsymbol{a},b}(\boldsymbol{x}_n) = \left\lfloor \frac{\boldsymbol{a} \cdot \boldsymbol{x}_n + b}{w} \right\rfloor$$

- $\boldsymbol{a} \overset{iid}{\sim} \mathcal{N}(0,1)$.
- $b \sim \mathcal{U}(0,w)$.
- w is the bucket width.

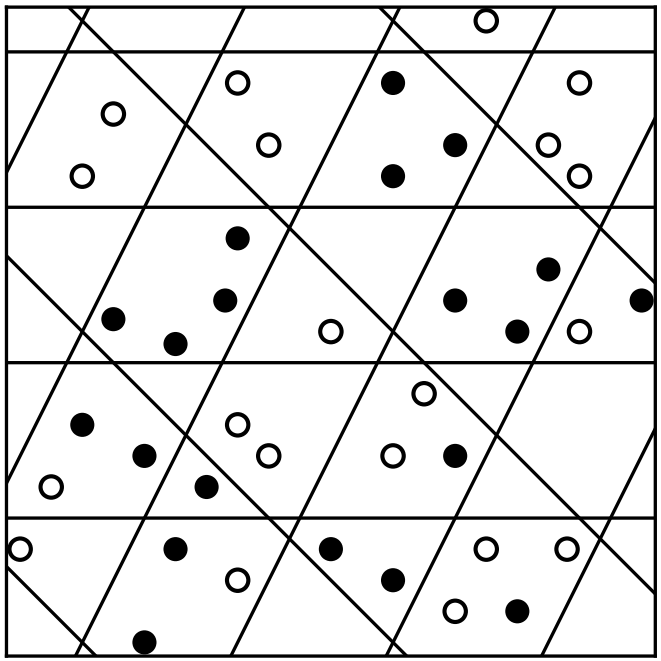


LSH-based dataset reduction (LSH-DR)

Combining LSH functions

$$H_{\boldsymbol{A},B}(\boldsymbol{x}_n) = H_{\boldsymbol{A},B}(\boldsymbol{x}_p) \Leftrightarrow$$
$$h_{\boldsymbol{a}_r,b_r}(\boldsymbol{x}_n) = h_{\boldsymbol{a}_r,b_r}(\boldsymbol{x}_p)\ \forall r.$$

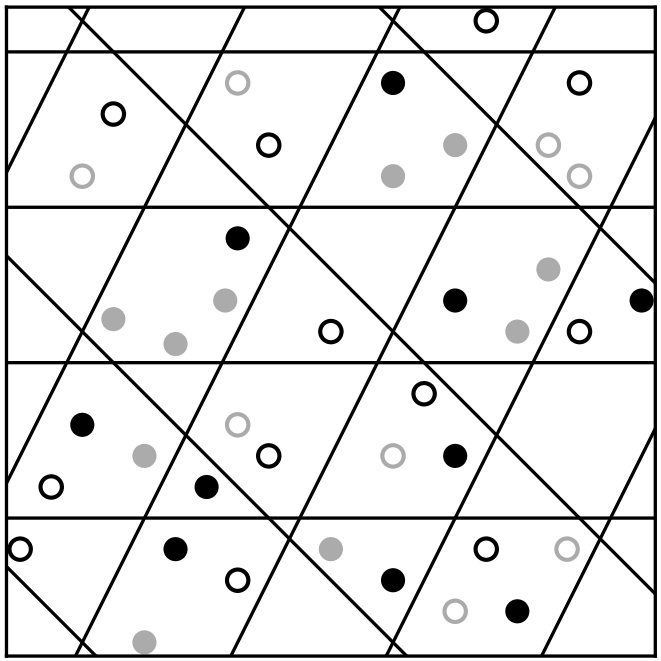
- $\boldsymbol{A} = (\boldsymbol{a}_1, \ldots, \boldsymbol{a}_R)$.
- $B = (b_1, \ldots, b_R)$.



LSH-based dataset reduction (LSH-DR)

Sampling

- In each bucket, sample one observation per alternative.

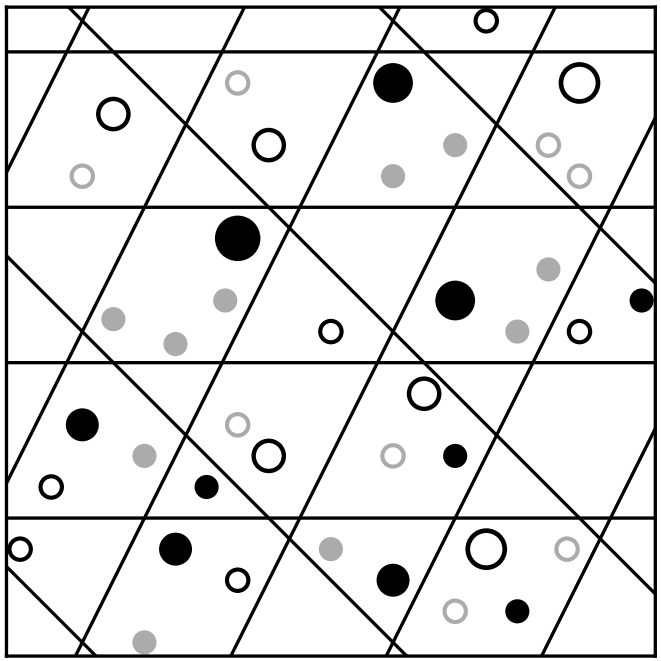


LSH-based dataset reduction (LSH-DR)

Weighting

- Each selected observation $(\boldsymbol{x}_g, i_g)$ is given a weight N_g:

$$N_g = |\{(\boldsymbol{x}_n, i_n) : i_g = i_n, \\ H_{\boldsymbol{A},B}(\boldsymbol{x}_g) = H_{\boldsymbol{A},B}(\boldsymbol{x}_n), \}|$$



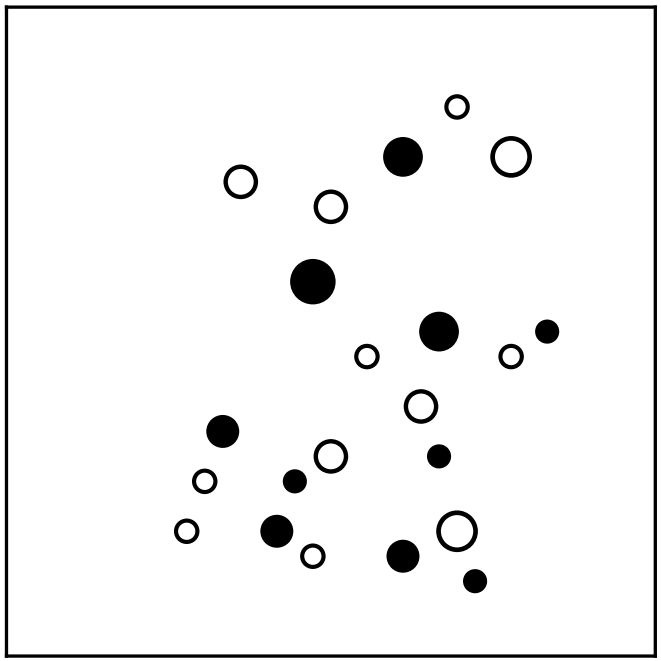
LSH-based dataset reduction (LSH-DR)

Weighted subsample

$$\{(\boldsymbol{x}_g, i_g, N_g) : g = 1, \ldots, G\}$$

Weighted likelihood

$$\mathcal{L}^w(\boldsymbol{\theta}) = \sum_{g=1}^{G} N_g \cdot \log P\left(i_g \mid \boldsymbol{x}_g; \boldsymbol{\theta}\right)$$



Experimental design

LPMC data [Hillel *et al.*, 2018]

- Mode choice, 4 alternatives.
- 81k observations (55k/26k).

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Models [Hillel, 2019; Ortelli *et al.*, 2023]

Logit-S

- 10 cont. variables.
- 0 dummies.
- **13 parameters**.

Nested-S

- 10 cont. variables.
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- **14 parameters**.

Logit-L

- 11 cont. variables.
- 15 dummies.
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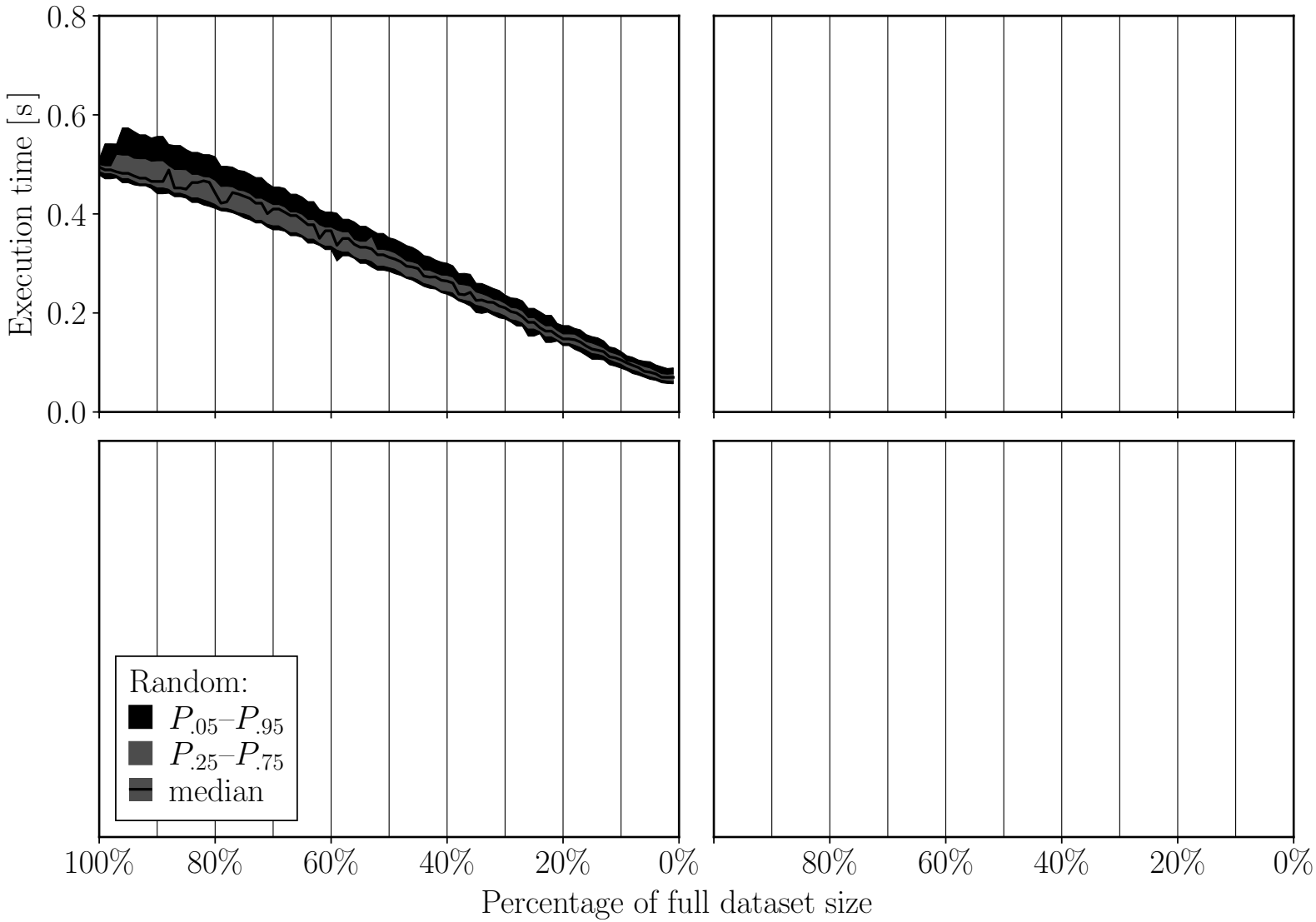
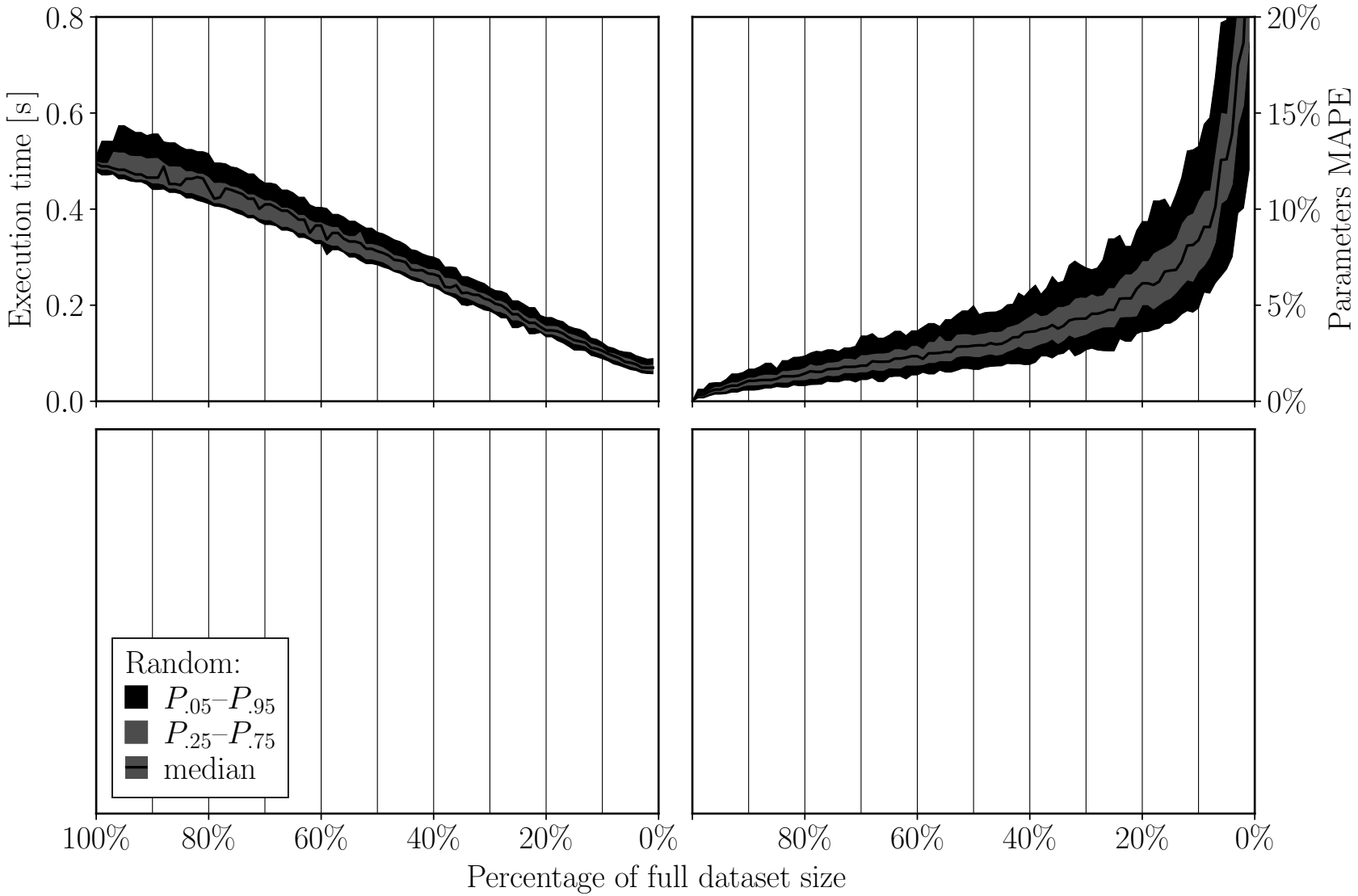
Estimation

- Biogeme. [Bierlaire, 2023]
- 36-core processor @2.4 GHz, 256 GB of RAM.

Results — Logit-S



Results — Logit-S



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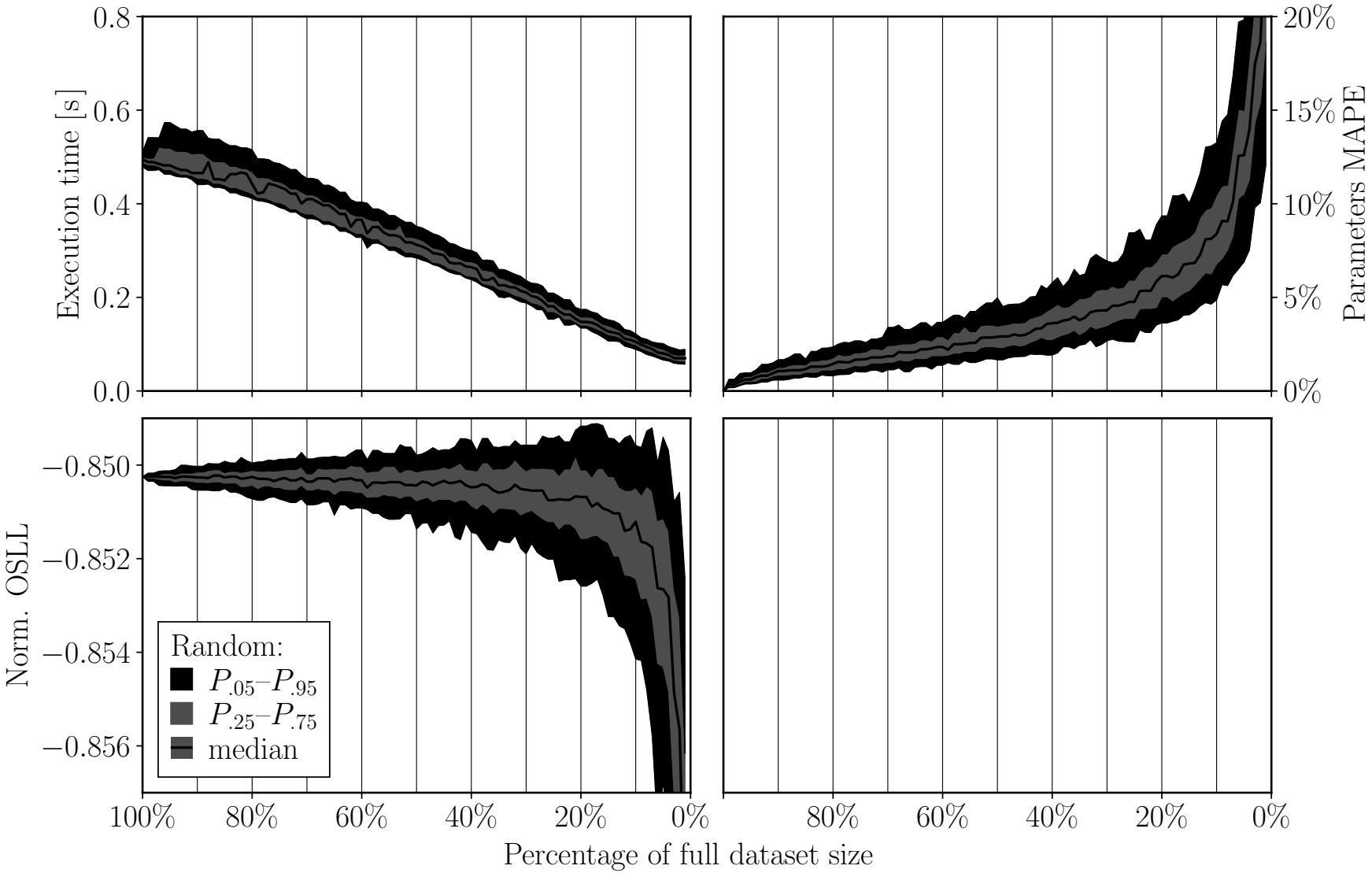

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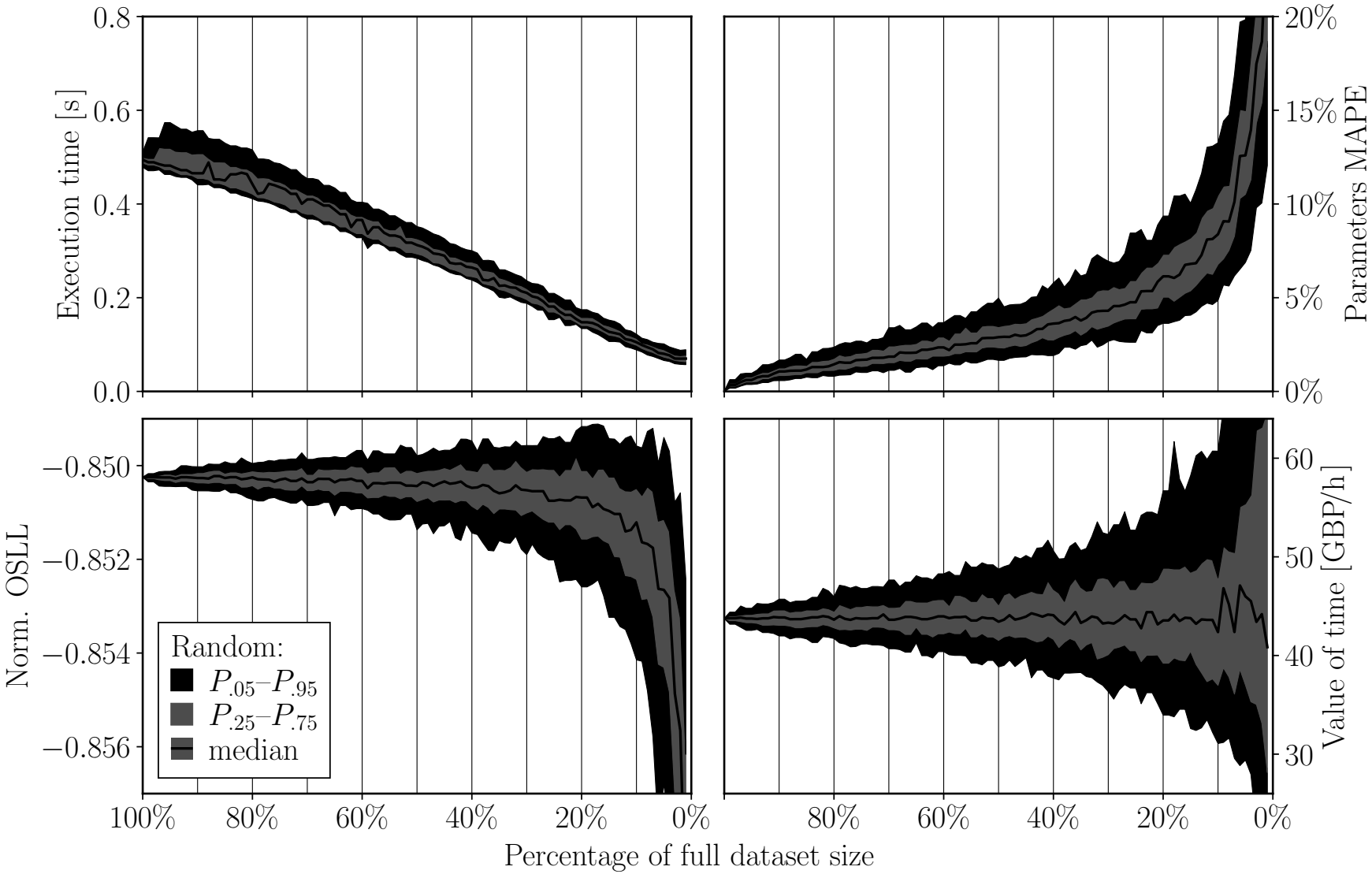

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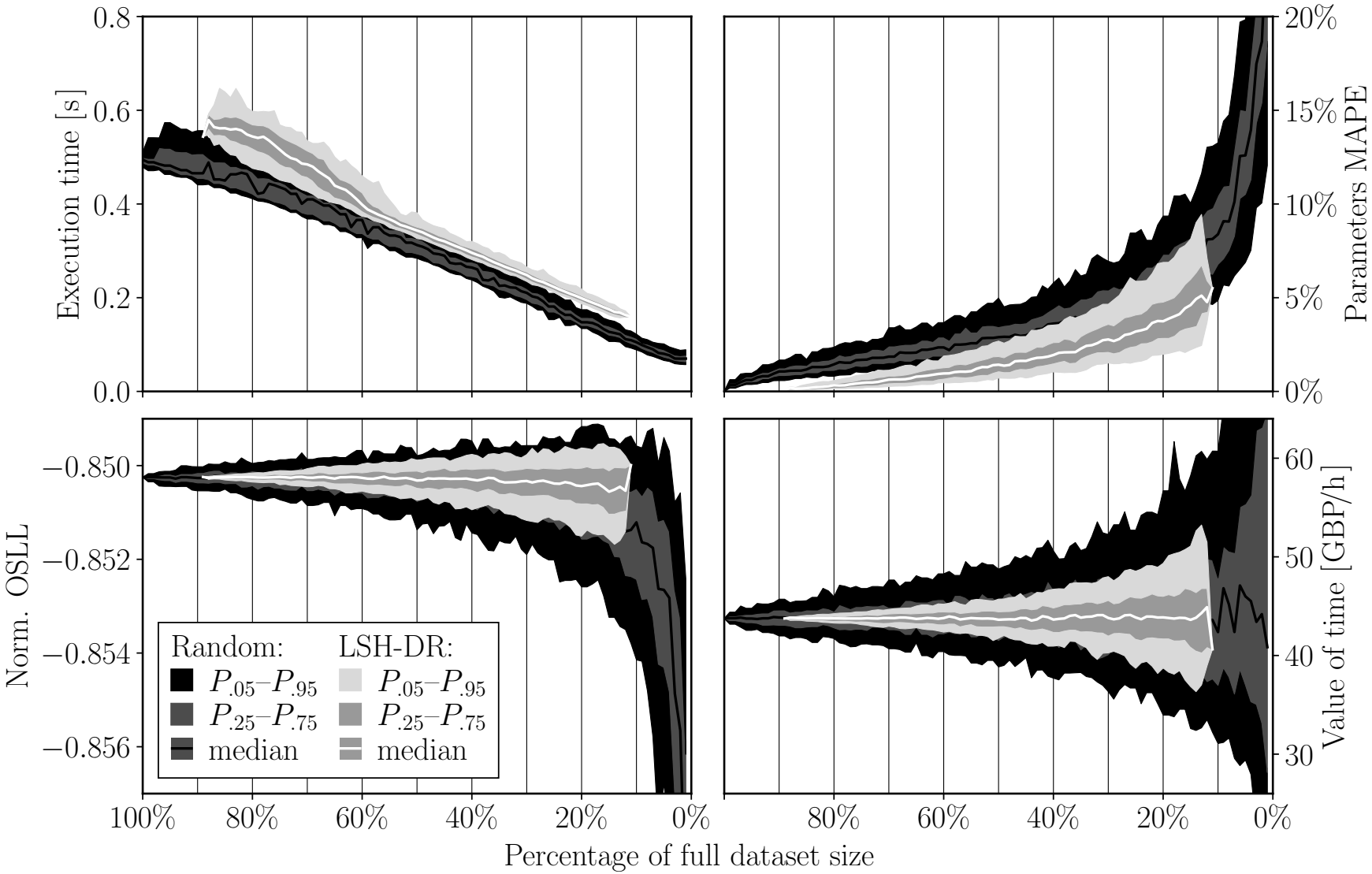
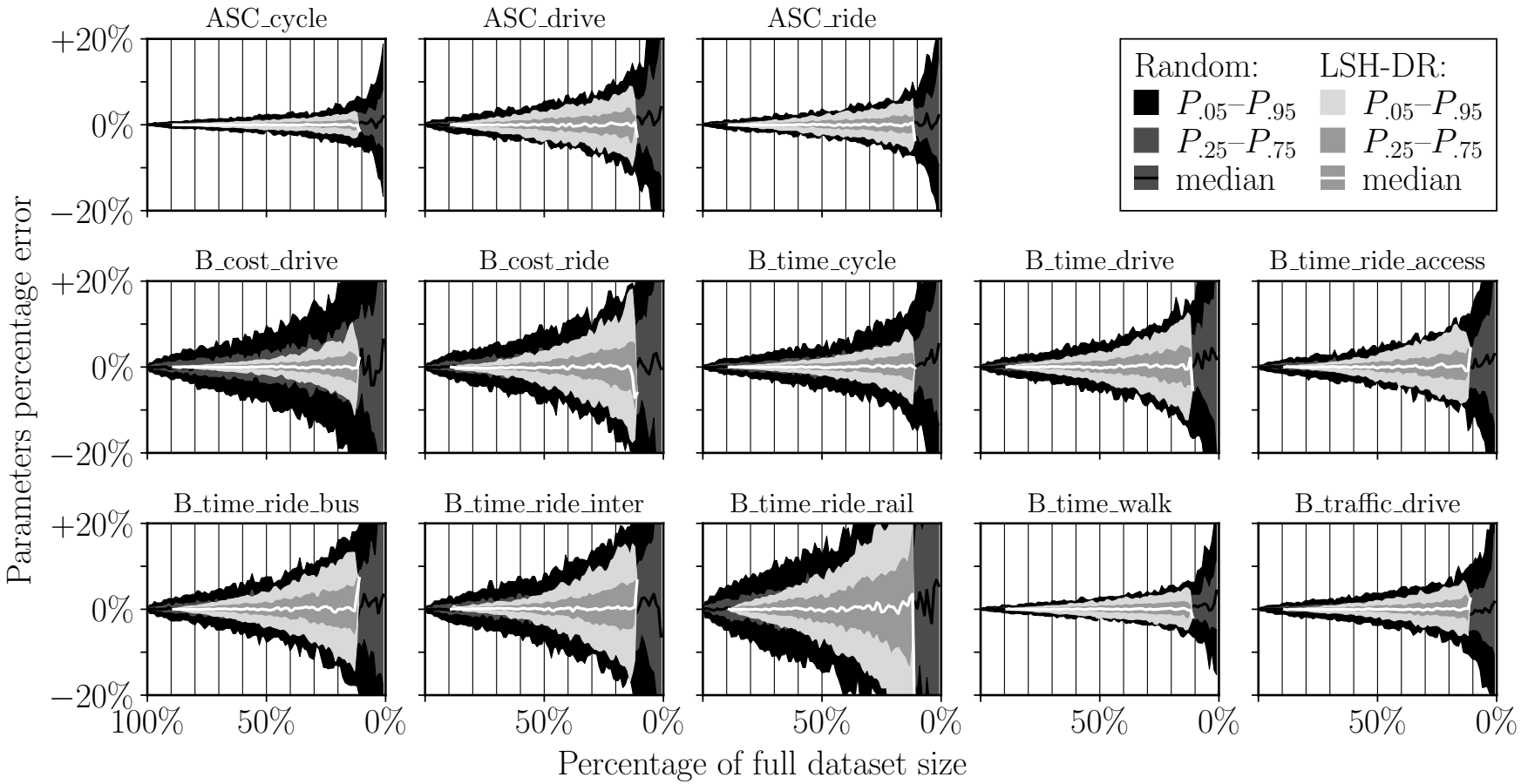
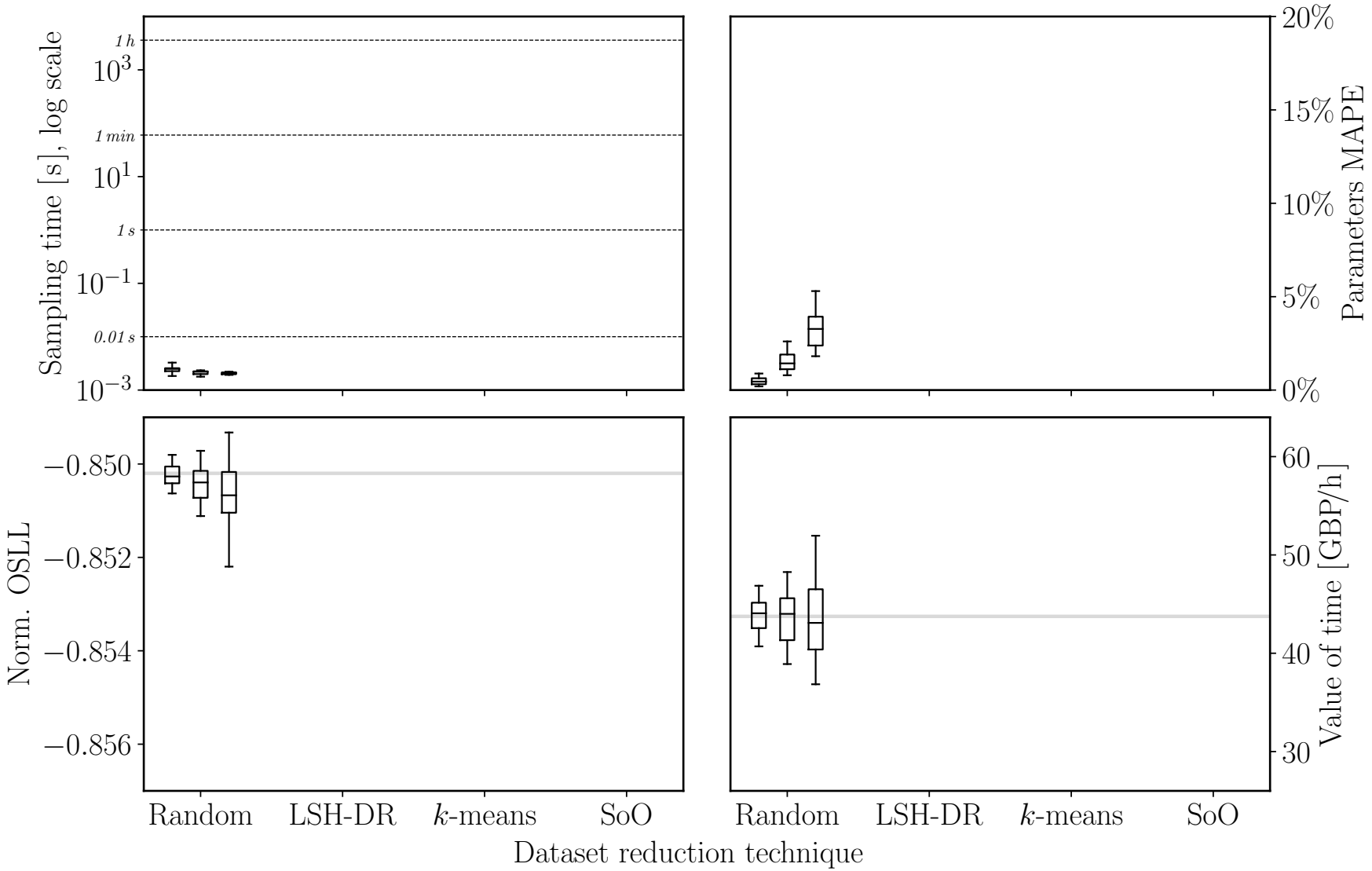


Results — Logit-S: Parameters



Results — Logit-S: Comparison



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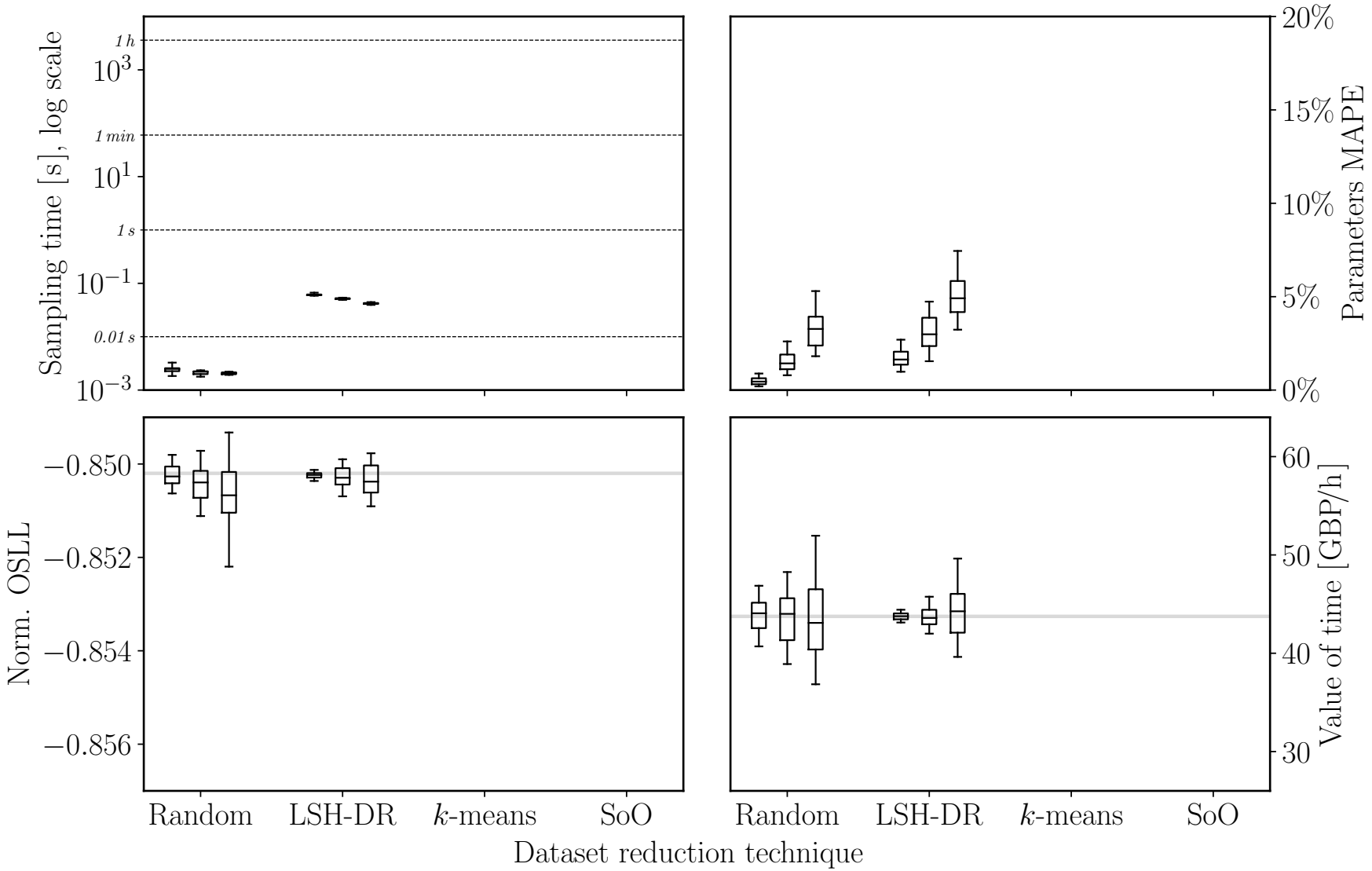

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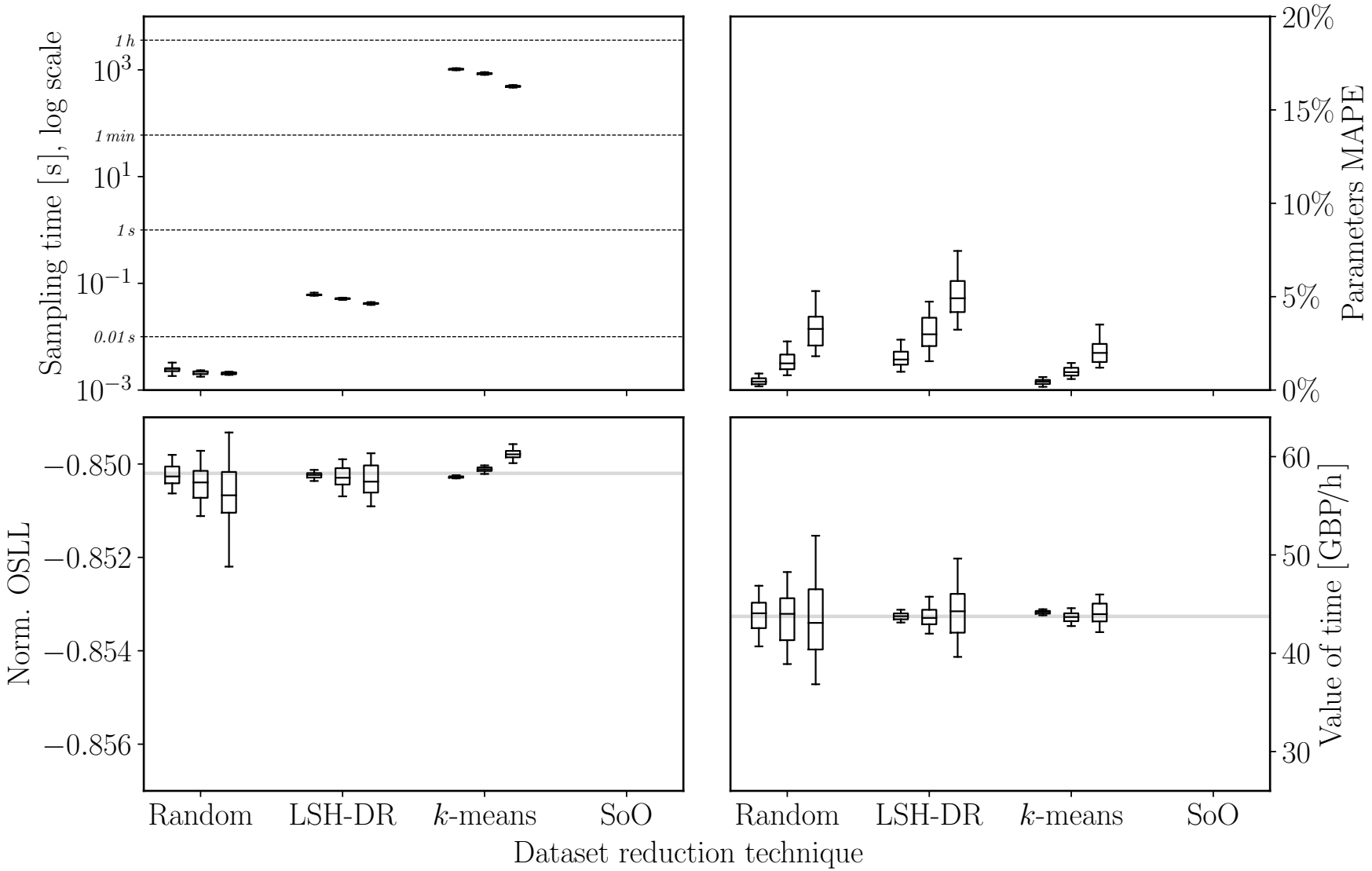

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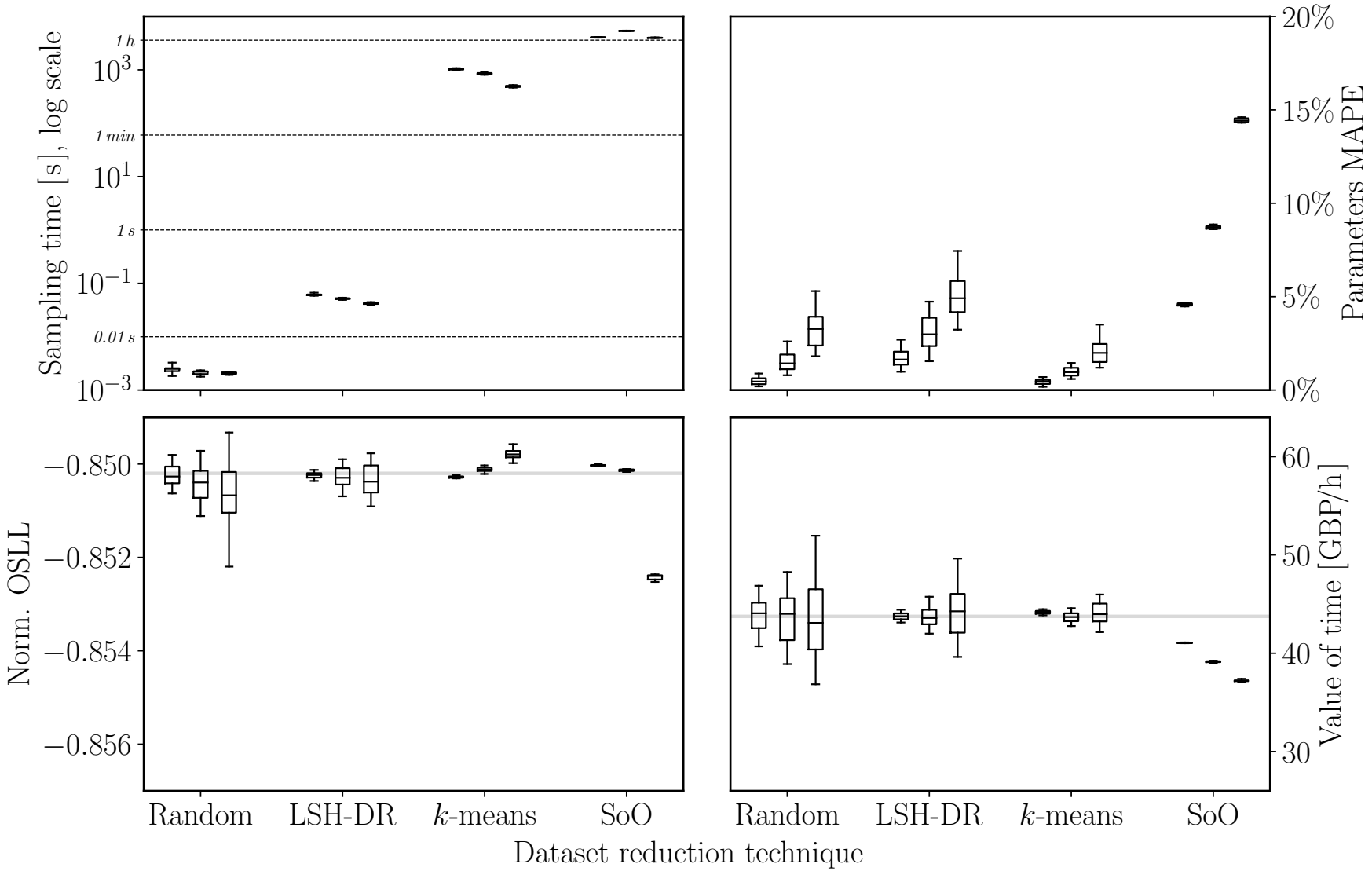
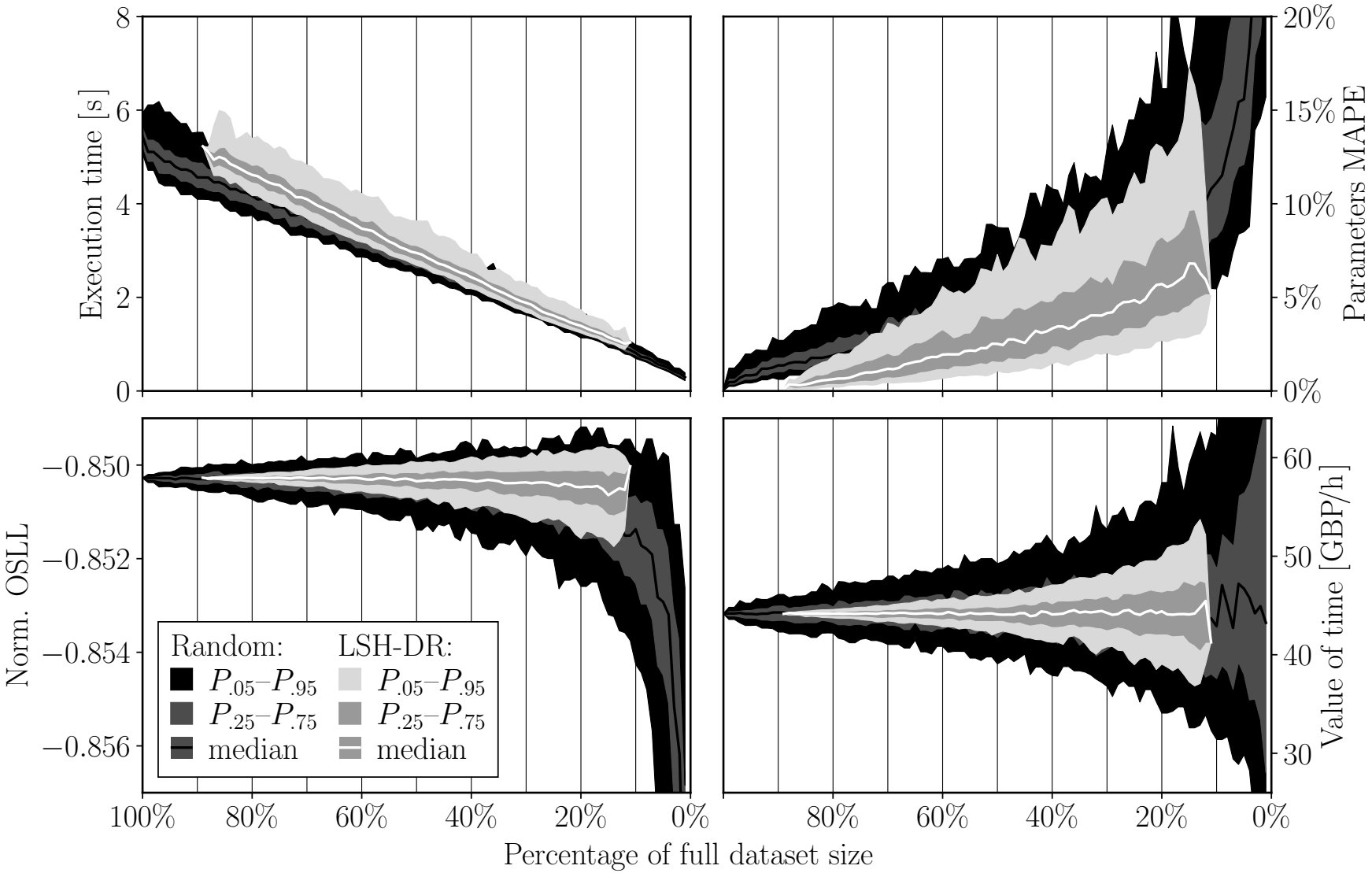
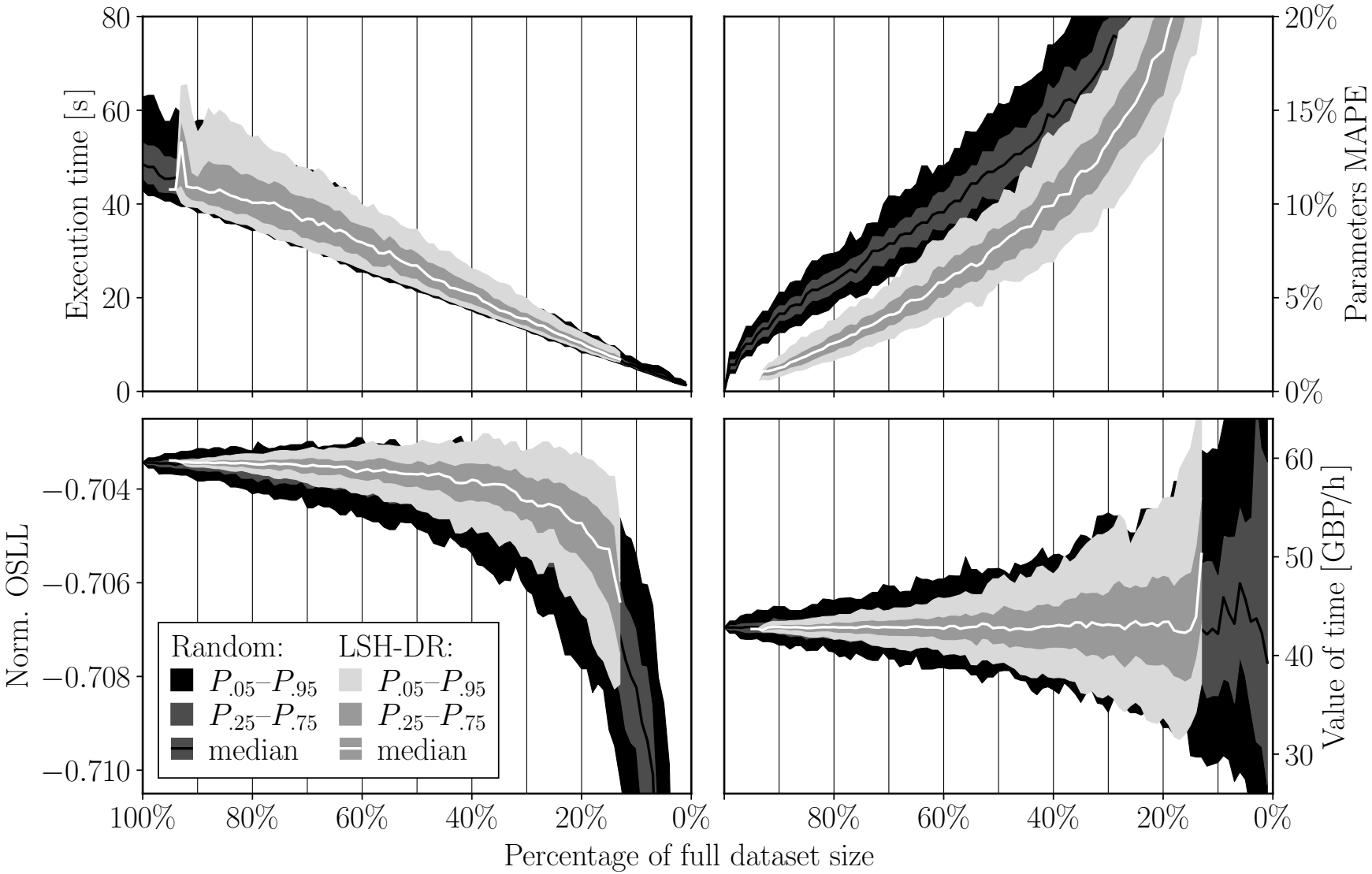


Results — Nested-S



Results — Logit-L



Conclusion

Summary

- Resampling technique designed to speed up DCM estimation.
- Time savings come at the cost of deteriorating the estimation results.
- Factor out redundancy, but keep diversity!

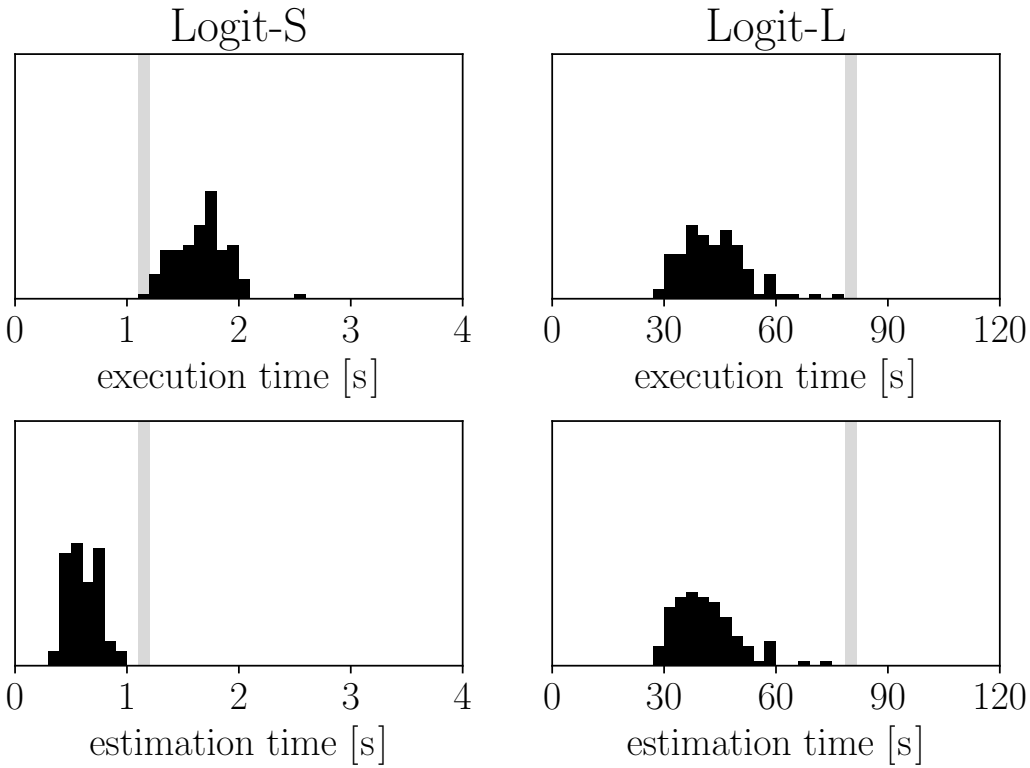
Next steps

- Synthetic prototypical observations.
- Knowledge-based LSH functions.
- Embed resampling within the model estimation process.

Spin-off

Adaptive resampling

- Generate weighted batches for stochastic optimization. [Lederrey et al., 2021]
- Start small and increase batch size dynamically.



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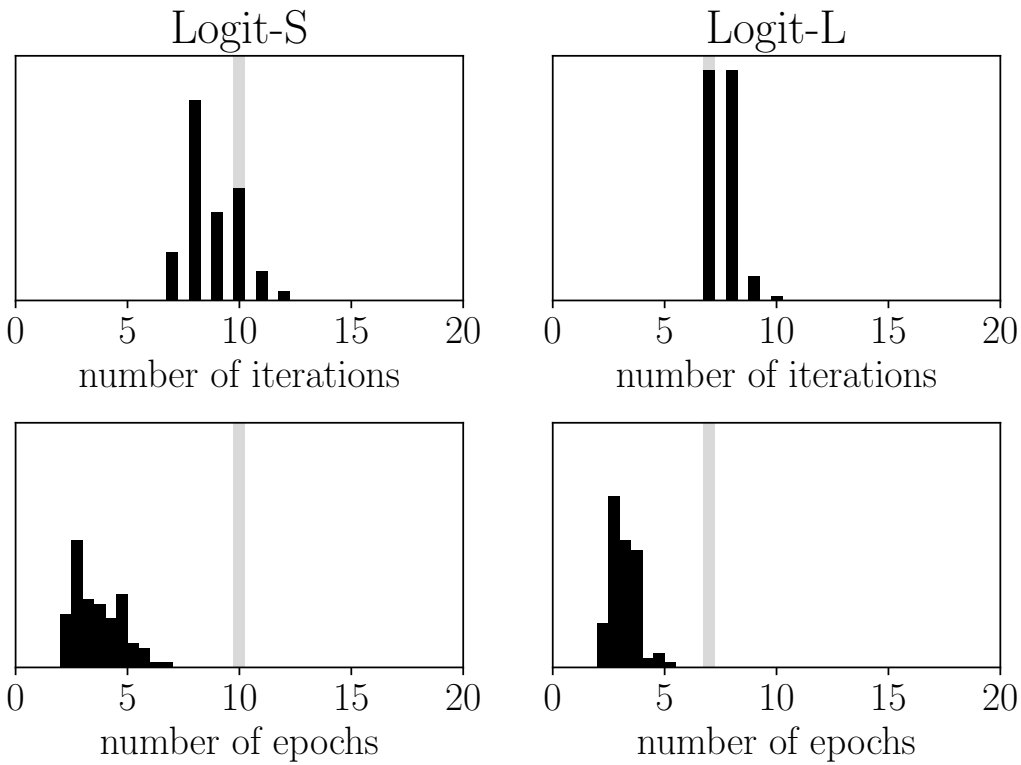

References

LSH-DR

- Ortelli, N., de Lapparent, M. and Bierlaire, M. (2023). Resampling estimation of discrete choice models, Technical Report, TRANSP-OR 230330. Transport and Mobility Laboratory, ENAC, EPFL.

Direct precedents

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- Schmid, B., Becker, F., Molloy, J., Axhausen, K. W., Lüdering, J., Hagen, J. and Blome, A. (2022). Modeling train route decisions during track works, Journal of Rail Transport Planning & Management 22: 100320.
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Dataset & models

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Results — Logit-S: Unweighted