Route choice models: Introduction and recent developments

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Route choice model

Given

- a mono- or multi-modal transportation network (nodes, links, origin, destination)
- an origin-destination pair
- link and path attributes

identify the route that a traveler would select.

Choice model

Assumptions about

1. the decision-maker: n
2. the alternatives
   - Choice set $\mathcal{C}_n$
   - $p \in \mathcal{C}_n$ is composed of a list of links (i, j)
3. the attributes
   - link-additive: length, travel time, etc.

$$x_{kp} = \sum_{(i,j) \in P} x_{k(i,j)}$$

   - non link-additive: scenic path, usual path, etc.
4. the decision-rules: $\Pr(p|\mathcal{C}_n)$

Shortest path

Decision-makers all identical

Alternatives

- all paths between O and D
- $\mathcal{C}_n = \mathcal{U} \quad \forall n$
- $\mathcal{U}$ can be unbounded when loops are present

Attributes one link additive generalized cost

$$c_p = \sum_{(i,j) \in P} c_{(i,j)}$$

- traveler independent
- link cost may be negative
- no loop with negative cost must be present so that $c_p > -\infty$ for all p

Shortest path

Decision-rules path with the minimum cost is selected

$$\Pr(p) = \begin{cases} K & \text{if } c_p \leq c_q \quad \forall c_q \in \mathcal{U} \\ 0 & \text{otherwise} \end{cases}$$

- K is a normalizing constant so that $\sum_{p \in \mathcal{U}} \Pr(p) = 1$.
- $K = 1/S$, where S is the number of shortest paths between O and D.
- Some methods select one shortest path p^*

$$\Pr(p) = \begin{cases} 1 & \text{if } p = p^* \\ 0 & \text{otherwise} \end{cases}$$

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Shortest path

Advantages:

- well defined
- no need for behavioral data
- efficient algorithms (Dijkstra)

Disadvantages

- behaviorally irrealistic
- instability with respect to variations in cost
- calibration on real data is very difficult
  - inverse shortest path problem is NP complete
  - Burton, Pulleyblank and Toint (1997) The Inverse Shortest Paths Problem With Upper Bounds on Shortest Paths Costs *Network Optimization* , Series: Lecture Notes in Economics and Mathematical Systems , Vol. 450, P. M. Pardalos, D. W. Hearn and W. W. Hager (Eds.), pp. 156-171, Springer

Dial's approach

Dial R. B. (1971) A probabilistic multipath traffic assignment model which obviates path enumeration *Transportation Research* Vol. 5, pp. 83-111.

Decision-makers all identical

Alternatives efficient paths between O and D

Attributes link-additive generalized cost

Decision-rules multinomial logit model

Dial's approach

- Def 1: A path is efficient if every link in it has
  - its initial node closer to the origin than its final node, and
  - its final node closer to the destination than its initial node.
- Def 2: A path is efficient if every link in it has its initial node closer to the origin than its final node.

Efficient path: a path that does not backtrack.

Dial's approach

- Choice set $\mathcal{C}_n$ = set of efficient paths (finite, no loop)
- No explicit enumeration
- Every efficient path has a non zero probability to be selected
- Probability to select a path

$$\Pr(p) = \frac{e^{\theta(\sum_{(i,j)\in p^*} c(i,j) - \sum_{(i,j)\in p} c(i,j))}}{\sum_{q\in\mathcal{C}_n} e^{\theta(\sum_{(i,j)\in p^*} c(i,j) - \sum_{(i,j)\in p} q(i,j))}}$$

where p^* is the shortest path and θ is a parameter

Dial's approach

Note: the length of the shortest path is constant across $\mathcal{C}_n$

$$\Pr(p) = \frac{e^{-\theta \sum_{(i,j)\in p} c(i,j))}}{\sum_{q\in\mathcal{C}_n} e^{-\theta \sum_{(i,j)\in q} q(i,j))}} = \frac{e^{-\theta c_p}}{\sum_{q\in\mathcal{C}_n} e^{-\theta c_q}}$$

Multinomial logit model with

$$V_p = -\theta c_p$$

Dial's approach

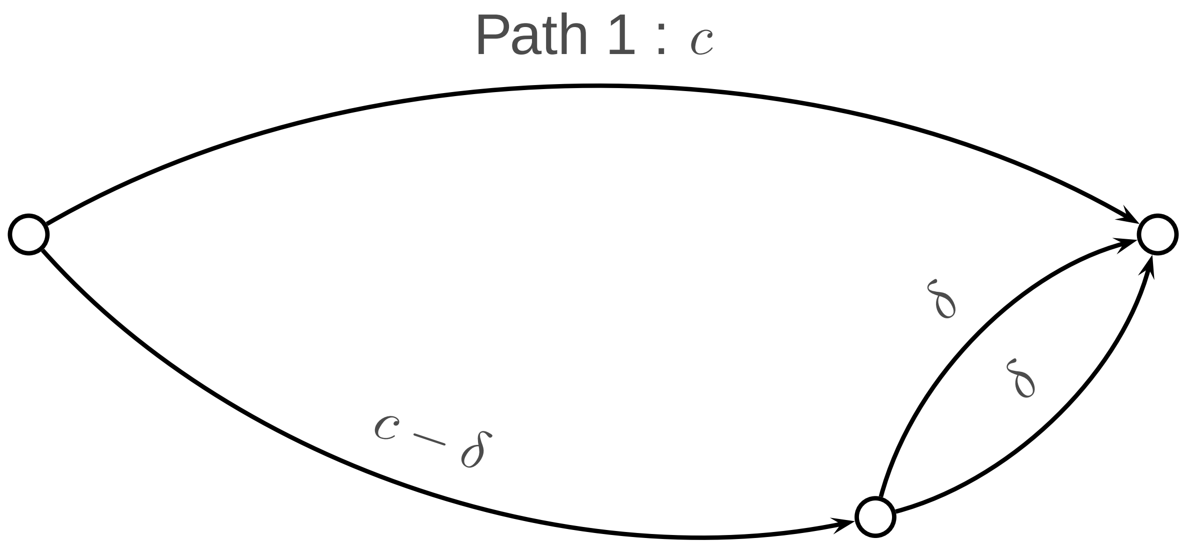
Advantages:

- probabilistic model, more stable
- calibration parameter θ
- avoid path enumeration
- designed for traffic assignment

Disadvantages:

- MNL assumes independence among alternatives
- efficient paths are mathematically convenient but not behaviorally motivated

Dial's approach



$$\Pr(1) = \frac{e^{-\theta c_1}}{\sum_{q \in \mathcal{C}} e^{-\theta c_q}} = \frac{e^{-\theta c}}{3e^{-\theta c}} = \frac{1}{3} \text{ for any } c, \delta, \theta$$

Path Size Logit

- With MNL, the utility of overlapping paths is overestimated
- When δ is large, there is some sort of “double counting”
- Idea: include a correction

$$V_p = -\theta c_p + \beta \ln \mathsf{PS}_p$$

where

$$\mathsf{PS}_p = \sum_{(i,j)\in p} \frac{c_{(i,j)}}{c_p} \frac{1}{\sum_{q\in\mathcal{C}} \delta^q_{i,j}}$$

and

$$\delta^q_{i,j} = \begin{cases} 1 & \text{if link } (i,j) \text{ belongs to path } q \\ 0 & \text{otherwise} \end{cases}$$

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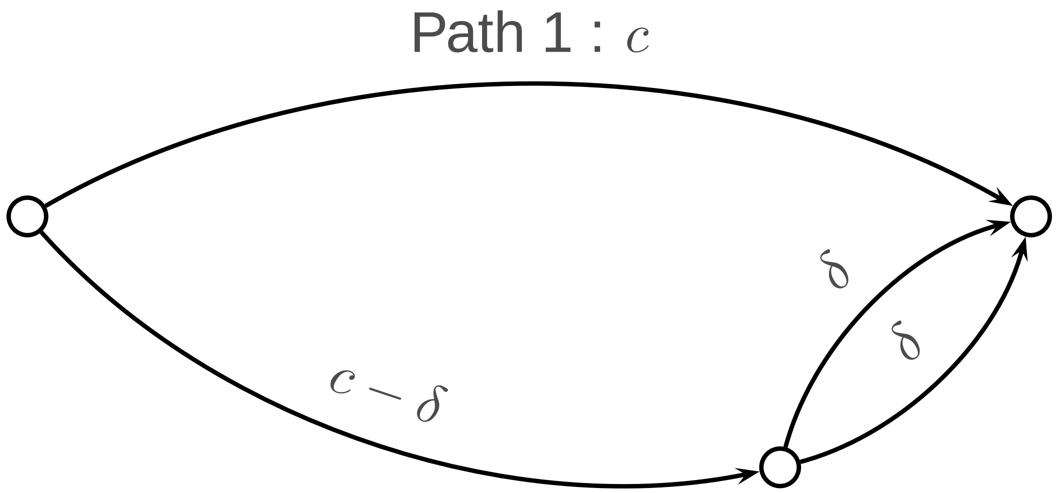
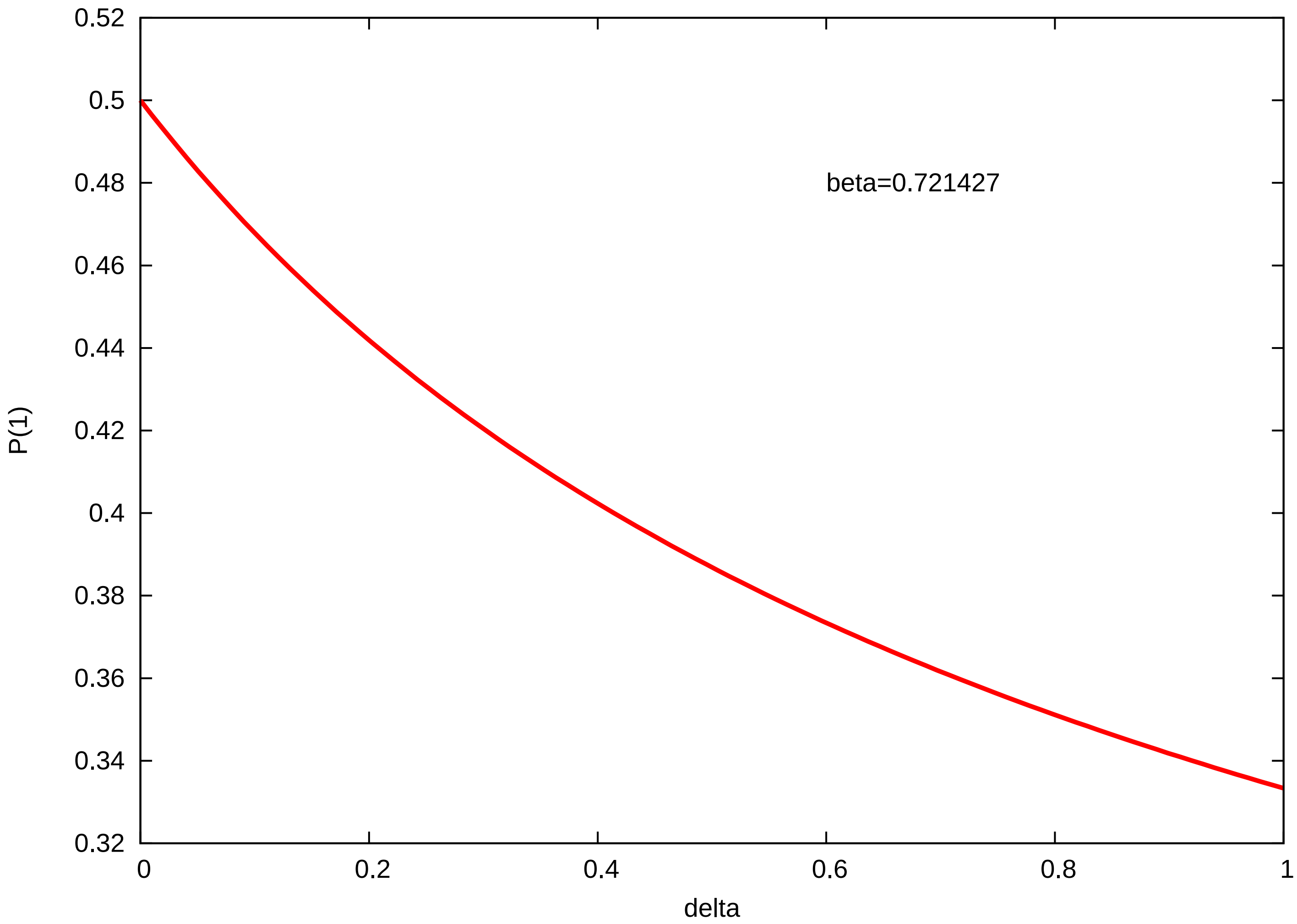
Path Size Logit



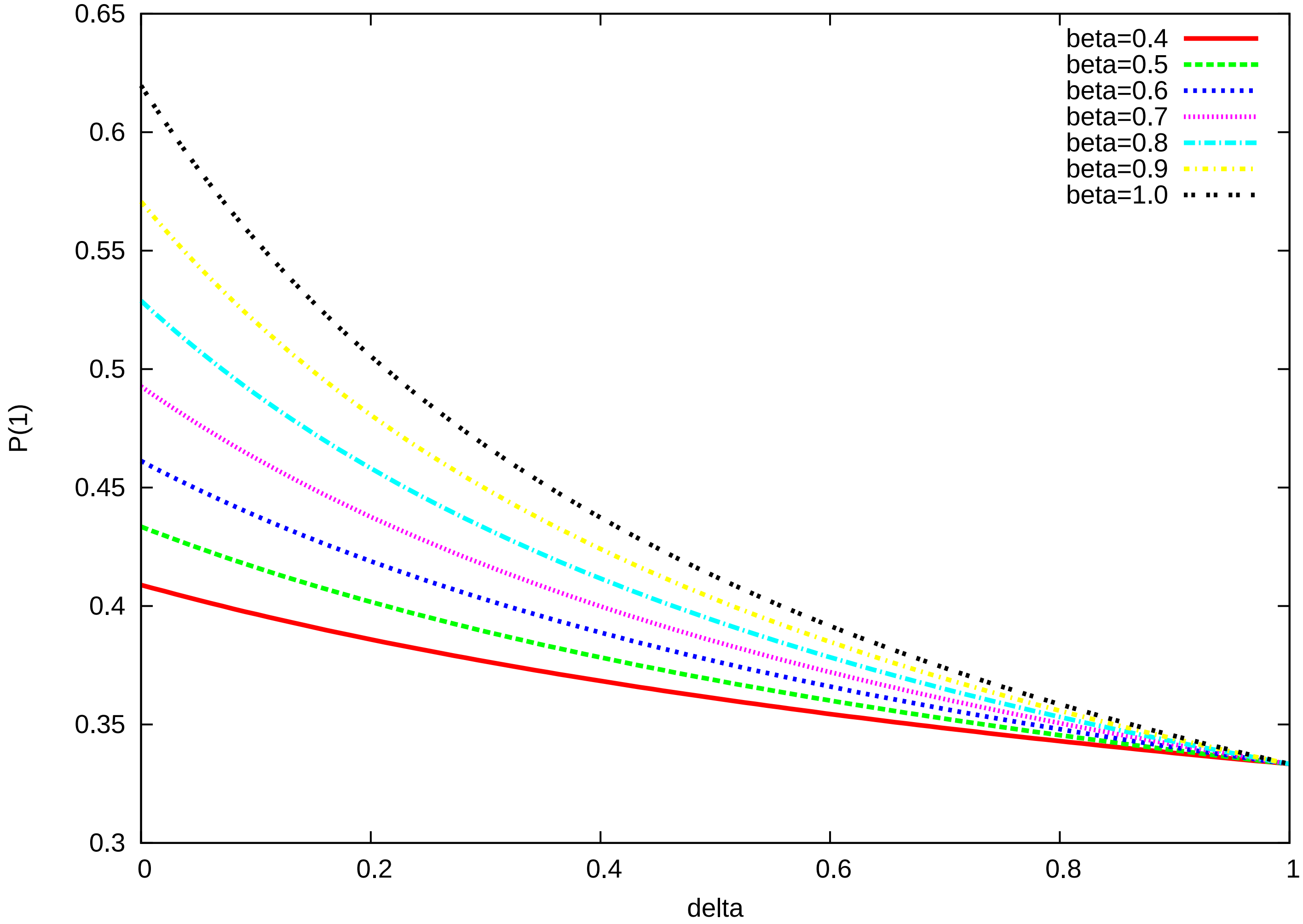
$$\text{PS}_1 = \frac{c}{c}\frac{1}{1} = 1$$

$$\text{PS}_2 = \text{PS}_3 = \frac{c-\delta}{c}\frac{1}{2} + \frac{\delta}{c}\frac{1}{1} = \frac{1}{2} + \frac{\delta}{2c}$$

Path Size Logit



Path Size Logit



Path Size Logit

Advantages:

- MNL formulation: simple
- Easy to compute
- Exploits the network topology
- Practical

Disadvantages:

- Derived from the theory on nested logit
- Several formulations have been proposed
- Correlated with observed and unobserved attributes
- May give biased estimates

Path Size Logit: readings

- Cascetta, E., Nuzzolo, A., Russo, F., Vitetta, A. 1996. A modified logit route choice model overcoming path overlapping problems. Specification and some calibration results for interurban networks. In Lesort, J.B. (Ed.), Proceedings of the 13th International Symposium on Transportation and Traffic Theory, Lyon, France.
- Ramming, M., 2001. Network Knowledge and Route Choice, PhD thesis, Massachusetts Institute of Technology.
- Ben-Akiva, M., and Bierlaire, M. (2003). Discrete choice models with applications to departure time and route choice. In Hall, R. (ed) *Handbook of Transportation Science, 2nd edition* pp.7-38. Kluwer.

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Path Size Logit: readings

- Hoogendoorn-Lanser, S., van Nes, R. and Bovy, P. (2005) Path Size Modeling in Multimodal Route Choice Analysis. *Transportation Research Record* vol. 1921 pp. 27-34
- Frejinger, E., and Bierlaire, M. (2007). Capturing correlation with subnetworks in route choice models, *Transportation Research Part B: Methodological* 41(3):363-378. doi:10.1016/j.trb.2006.06.003

Path enumeration

- Dial's approach avoids path enumeration
- Computationally convenient but behaviorally incorrect
- MNL inappropriate due to significant path overlap
- Generalized cost must be link-additive
- Heterogeneity in terms of behavior, equipments, etc. cannot be accounted for.
- With other DCM models, choice sets must be explicitly defined
- Path enumeration heuristics have been proposed:
    - Deterministic approaches: link elimination (Azevedo et al., 1993), labeled paths (Ben-Akiva et al., 1984)
    - Stochastic approaches: simulation (Ramming, 2001) and doubly stochastic (Bovy and Fiorenzo-Catalano, 2006)

Path enumeration

- Underlying assumption in existing approaches: the actual choice set is generated
- Empirical results suggest that this is not always true
- Our approach:
  - Choice set contains all paths
  - Too large for computation
  - Solution: sampling of alternatives

Sampling of Alternatives

- Multinomial Logit model (e.g. Ben-Akiva and Lerman, 1985):

$$P(i|\mathcal{C}_n) = \frac{q(\mathcal{C}_n|i)P(i)}{\sum_{j\in\mathcal{C}_n} q(\mathcal{C}_n|j)P(j)} = \frac{e^{V_{in}+\ln q(\mathcal{C}_n|i)}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}+\ln q(\mathcal{C}_n|j)}}$$

  $\mathcal{C}_n$: set of sampled alternatives
  $q(\mathcal{C}_n|j)$: probability of sampling $\mathcal{C}_n$ given that j is the chosen alternative

- If purely random sampling, $q(\mathcal{C}_n|i) = q(\mathcal{C}_n|j)$ and

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}+\ln q(\mathcal{C}_n|i)}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}+\ln q(\mathcal{C}_n|j)}} = \frac{e^{V_{in}}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}}}$$

Importance Sampling of Alternatives

- Attractive paths have higher probability of being sampled than unattractive paths
- In this case, $q(\mathcal{C}_n|i) \neq q(\mathcal{C}_n|j)$

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in} + \ln q(\mathcal{C}_n|i)}}{\sum_{j \in \mathcal{C}_n} e^{V_{jn} + \ln q(\mathcal{C}_n|j)}} \neq \frac{e^{V_{in}}}{\sum_{j \in \mathcal{C}_n} e^{V_{jn}}}$$

- Path utilities must be corrected in order to obtain unbiased estimation results

Stochastic Path Enumeration

- Key feature: we must be able to compute $q(\mathcal{C}_n|i)$
- One possible idea: a biased random walk between s_o and s_d which selects the next link at each node v.
- Initialize: $v = s_o$
- Step 1: associate a weight with each outgoing link $\ell = (v, w)$:

$$\omega(\ell|b_1) = 1 - (1 - {x_\ell}^{b_1})$$

where

$$x_\ell = \frac{SP(v, s_d)}{C(\ell) + SP(w, s_d)},$$

is 1 if ℓ is on the shortest path, and decreases when ℓ is far from the shortest path

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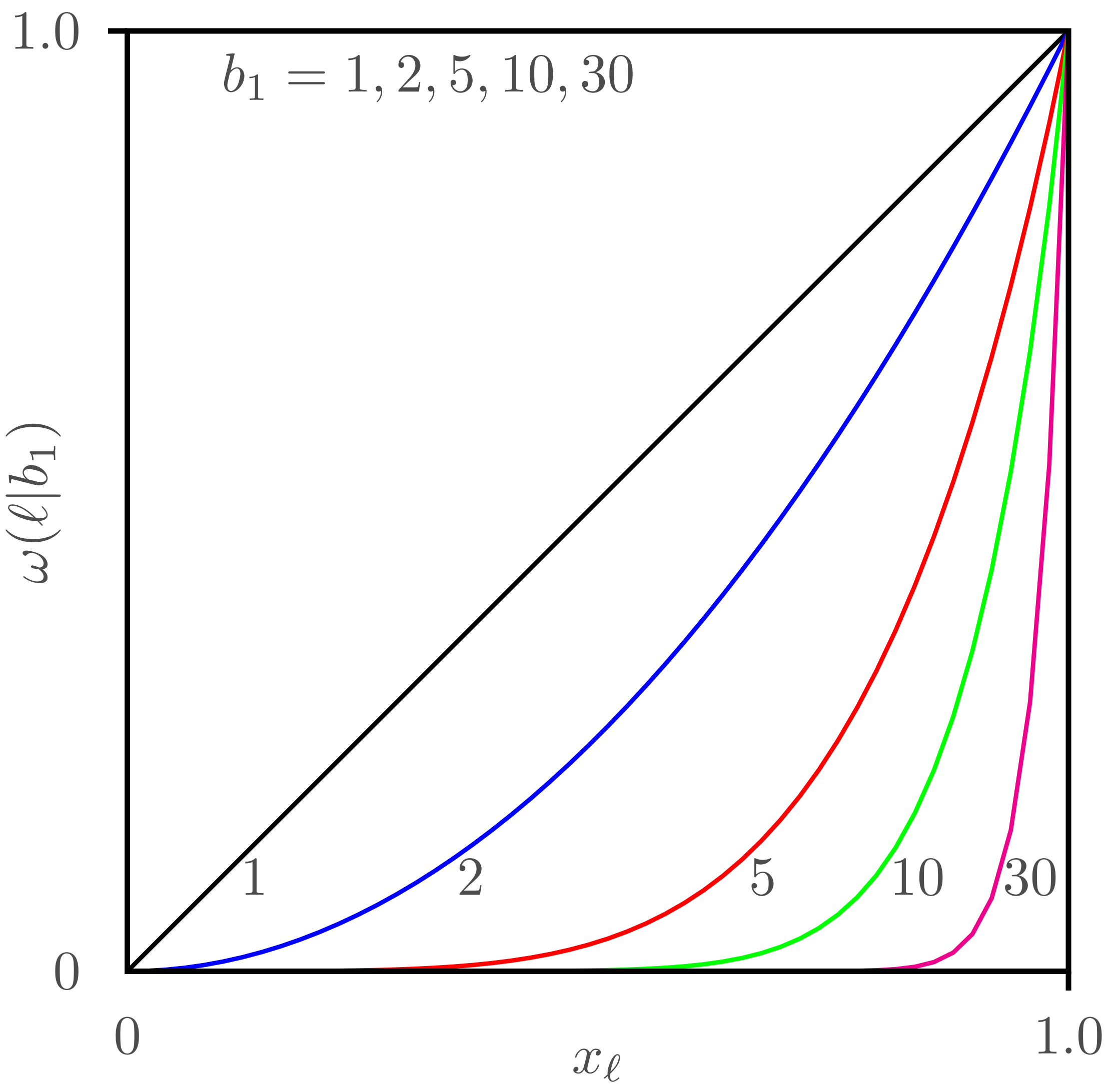
Stochastic Path Enumeration



Stochastic Path Enumeration

- Step 2: normalize the weights to obtain a probability distribution

$$q(\ell|\mathcal{E}_v, b_1) = \frac{\omega(\ell|b_1, b_2)}{\sum_{m\in\mathcal{E}_v} \omega(m|b_1)}$$

- Random draw a link (v, w^*) based on this distribution and add it to the current path
- If $w^* = s_d$, stop. Else, set $v = w^*$ and go to step 1.

Probability of generating a path j:

$$q(j) = \prod_{\ell\in\Gamma_j} q(\ell|\mathcal{E}_v, b_1).$$

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Sampling of Alternatives

- Following Ben-Akiva (1993)
- Sampling protocol
  1. A set $\widetilde{\mathcal{C}}_n$ is generated by drawing R paths with replacement from the universal set of paths $\mathcal{U}$
  2. Add chosen path to $\widetilde{\mathcal{C}}_n$
- Outcome of sampling: $(\widetilde{k}_1, \widetilde{k}_2, \ldots, \widetilde{k}_J)$ and $\sum_{j=1}^{J} \widetilde{k}_j = R$

$$P(\widetilde{k}_1, \widetilde{k}_2, \ldots, \widetilde{k}_J) = \frac{R!}{\prod_{j \in \mathcal{U}} \widetilde{k}_j!} \prod_{j \in \mathcal{U}} q(j)^{\widetilde{k}_j}$$

- Alternative j appears $k_j = \widetilde{k}_j + \delta_{cj}$ in $\widetilde{\mathcal{C}}_n$

Sampling of Alternatives

- Let $\mathcal{C}_n = \{j \in \mathcal{U} \mid k_j > 0\}$

$$q(\mathcal{C}_n|i) = q(\widetilde{\mathcal{C}}_n|i) = \frac{R!}{(k_i-1)!\prod_{\substack{j\in\mathcal{C}_n\\ j\neq i}} k_j!} q(i)^{k_i-1} \prod_{\substack{j\in\mathcal{C}_n\\ j\neq i}} q(j)^{k_j} = K_{\mathcal{C}_n}\frac{k_i}{q(i)}$$

$$K_{\mathcal{C}_n} = \frac{R!}{\prod_{j\in\mathcal{C}_n} k_j!} \prod_{j\in\mathcal{C}_n} q(j)^{k_j}$$

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}+\ln\left(\frac{k_i}{q(i)}\right)}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}+\ln\left(\frac{k_j}{q(j)}\right)}}$$

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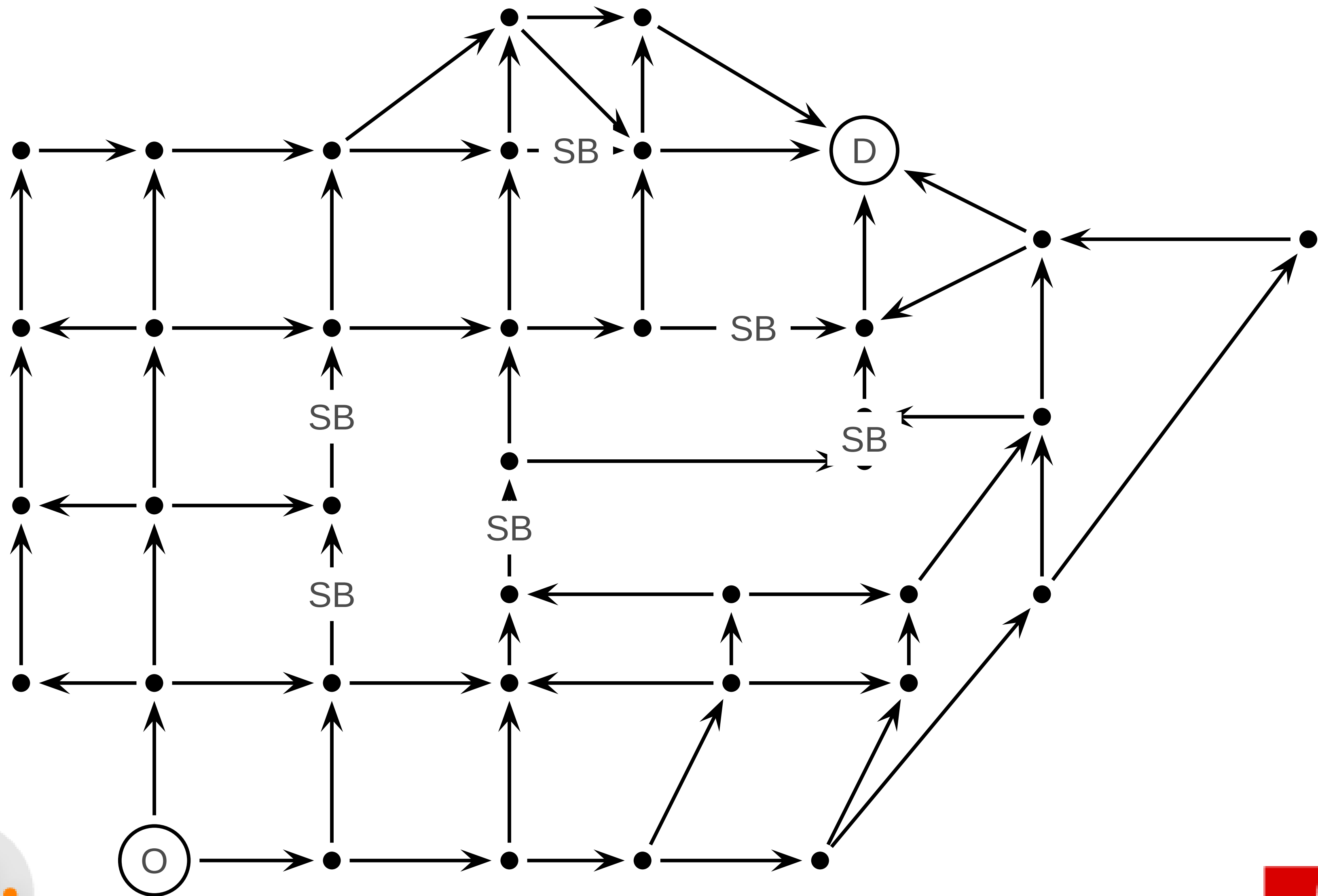
Numerical Results

- Estimation of models based on synthetic data generated with a postulated model
- Evaluation of
  - Sampling correction
  - Path Size attribute
  - Biased random walk algorithm parameters

Numerical Results



Numerical Results

- True model: Path Size Logit

  $U_j = \beta_{\mathsf{PS}} \ln \mathsf{PS}_j^{\mathcal{U}} + \beta_{\mathsf{L}} \mathsf{Length}_j + \beta_{SB} \mathsf{SpeedBumps}_j + \varepsilon_j$

  $\beta_{\mathsf{PS}} = 1$, $\beta_{\mathsf{L}} = -0.3$, $\beta_{\mathsf{SB}} = -0.1$
  ε_j distributed Extreme Value with scale 1 and location 0

  $\mathsf{PS}_j^{\mathcal{U}} = \sum_{\ell \in \Gamma_j} \frac{L_\ell}{L_j} \frac{1}{\sum_{p \in \mathcal{U}} \delta_{\ell p}}$

- 3000 observations

Numerical Results

- Four model specifications

		Sampling Correction	
		Without	With
Path	$\mathcal{C}$	$M^{\text{NoCorr}}_{PS(\mathcal{C})}$	$M^{\text{Corr}}_{PS(\mathcal{C})}$
Size	$\mathcal{U}$	$M^{\text{NoCorr}}_{PS(\mathcal{U})}$	$M^{\text{Corr}}_{PS(\mathcal{U})}$

$$\text{PS}_i^{\mathcal{U}} = \sum_{\ell \in \Gamma_i} \frac{L_\ell}{L_i} \frac{1}{\sum_{j \in \mathcal{U}} \delta_{\ell j}}$$

$$\text{PS}_{in}^{\mathcal{C}} = \sum_{\ell \in \Gamma_i} \frac{L_\ell}{L_i} \frac{1}{\sum_{j \in \mathcal{C}_n} \delta_{\ell j}}$$

Numerical Results

- Model $M^{\text{NoCorr}}_{PS(\mathcal{C})}$:

$$V_{in} = \mu\left(\beta_{\text{PS}} \ln \text{PS}^{\mathcal{C}}_{in} - 0.3\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i\right)$$

- Model $M^{\text{Corr}}_{PS(\mathcal{C})}$:

$$V_{in} = \mu\left(\beta_{\text{PS}} \ln \text{PS}^{\mathcal{C}}_{in} - 0.3\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i\right) + \ln(\frac{k_i}{q(i)})$$

- Model $M^{\text{NoCorr}}_{PS(\mathcal{U})}$:

$$V_{in} = \mu\left(\beta_{\text{PS}} \ln \text{PS}^{\mathcal{U}}_{in} - 0.3\text{Length}_i + \beta_{SB}\text{SpeedBumps}_i\right)$$

- Model $M^{\text{Corr}}_{PS(\mathcal{U})}$:

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Numerical Results

	True PSL	$M^{\text{NoCorr}}_{PS(\mathcal{C})}$ PSL	$M^{\text{Corr}}_{PS(\mathcal{C})}$ PSL	$M^{\text{NoCorr}}_{PS(\mathcal{U})}$ PSL	$M^{\text{Corr}}_{PS(\mathcal{U})}$ PSL
β_{L} fixed	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**
$\widehat{\mu}$	**1**	**0.182**	**0.923**	**0.141**	**0.977**
standard error		0.0277	0.0246	0.0263	0.0254
t-test w.r.t. 1		-29.54	-3.13	-32.64	-0.91
$\widehat{\beta}_{\text{PS}}$	**1**	**1.94**	**0.308**	**-1.02**	**1.02**
standard error		0.428	0.0736	0.383	0.0539
t-test w.r.t. 1		2.20	-9.40	-5.27	0.37
$\widehat{\beta}_{\text{SB}}$	**-0.1**	**-1.91**	**-0.139**	**-2.82**	**-0.0951**
standard error		0.25	0.0232	0.428	0.024
t-test w.r.t. -0.1		-7.24	-1.68	-6.36	0.20

Numerical Results

	True	$M^{\text{NoCorr}}_{PS(\mathcal{C})}$	$M^{\text{Corr}}_{PS(\mathcal{C})}$	$M^{\text{NoCorr}}_{PS(\mathcal{U})}$	$M^{\text{Corr}}_{PS(\mathcal{U})}$
	PSL	PSL	PSL	PSL	PSL
Final log likelihood		-6660.45	-6147.79	-6666.82	-5933.62
Adj. rho-square		0.018	0.093	0.017	0.125

Null log likelihood: -6784.96, 3000 observations

Algorithm parameters: 10 draws, $b_1 = 5$, $b_2 = 1$, $C(\ell) = L_\ell$

Average size of sampled choice sets: 9.66

BIOGEME (Bierlaire, 2007 and Bierlaire, 2003) has been used for all model estimations

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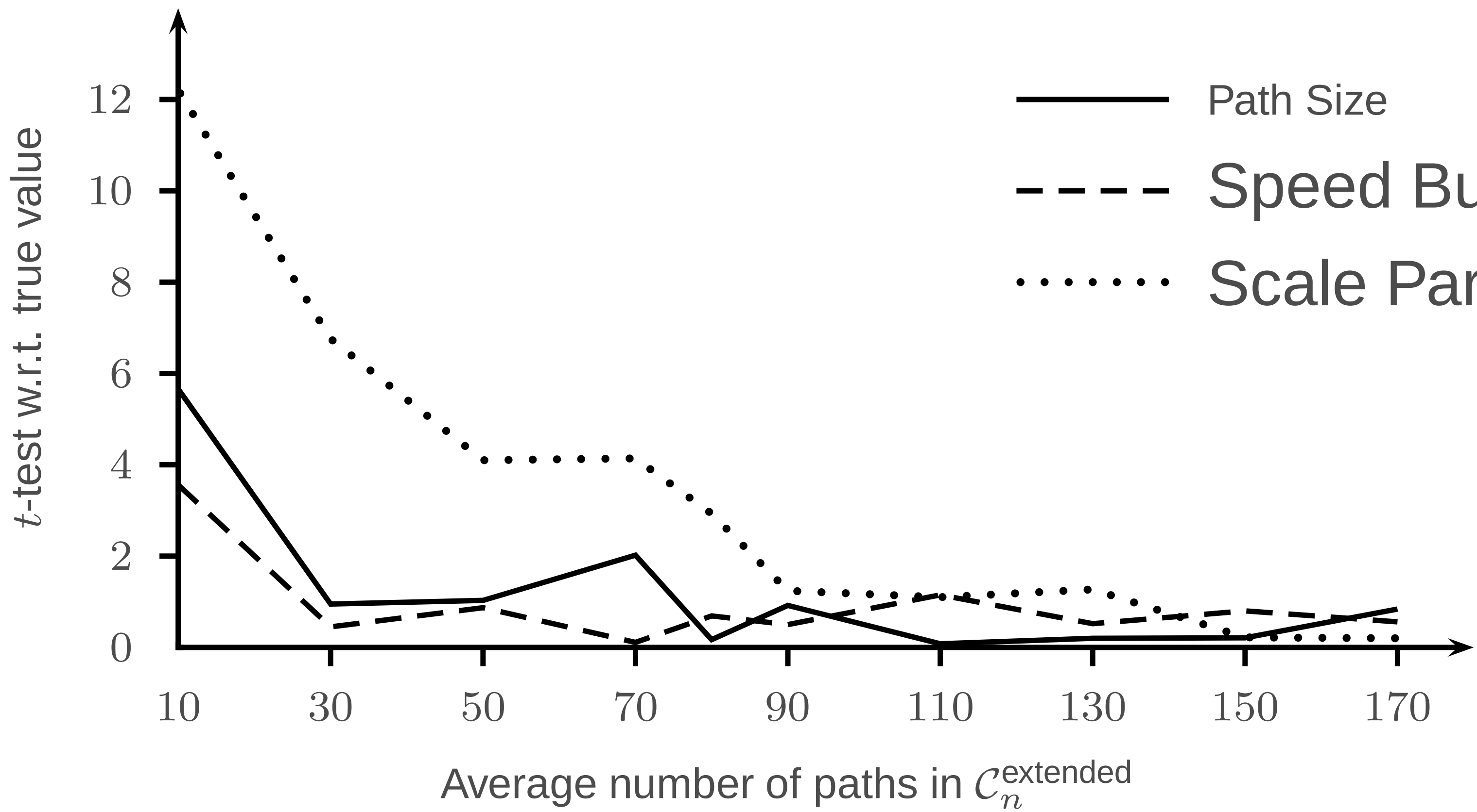
Extended Path Size

- Compute Path Size attribute based on an *extended choice set* $\mathcal{C}_n^{\text{extended}}$
- Simple random draws from $\mathcal{U} \backslash \mathcal{C}_n$ so that $|\mathcal{C}_n| \leq |\mathcal{C}_n^{\text{extended}}| \leq |\mathcal{U}|$

Extended Path Size



Extended Path Size

- Assume that the true choice set is the set of all paths
- Draw a subset for estitating the choice probability
- Draw a larger subset to compute the path size
- Various heuristics based on the same definition of the link weights can be used

Conclusions

- New point of view on choice set generation and route choice modeling
- Path generation is considered an importance sampling approach
- We present a path generation algorithm and derive the corresponding sampling correction
- Path Size should be computed on largest possible sets
- Numerical results are very promising

Readings

- Frejinger, Emma (2008) Route choice analysis : data, models, algorithms and applications. PhD thesis EPFL, no 4009 http://library.epfl.ch/theses/?nr=4009
- Frejinger, E., and Bierlaire, M. (2007). Sampling of Alternatives for Route Choice Modeling. Technical report TRANSP-OR 071121. Transport and Mobility Laboratory, ENAC, EPFL.