Probabilistic speed-density relationship for heterogeneous pedestrian traffic

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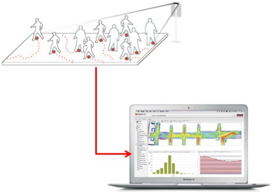
EPFL ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Data

Visiosafe technology

- Spin-off of EPFL
- Anonymous tracking of pedestrians
- Large-scale data collection
- Thermal and range sensors



Visiosafe data

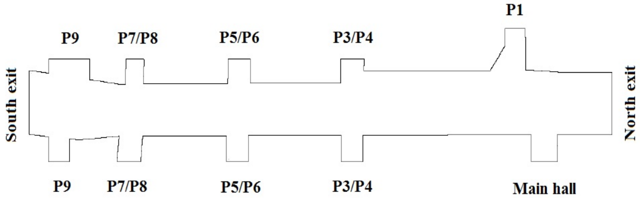
- Position of every single individual over time

$$(t, x(t), y(t), pedestrian_{id})$$

[Alahi et al., 2011]

Gare de Lausanne

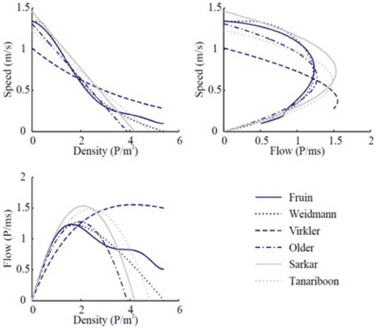
Pedestrian underpass West



- The busiest walking area in the station
- Area $\approx 685m^2$
- Area covered by 32 sensors

Related research

Deterministic speed-density models



[Daamen et al., 2005]

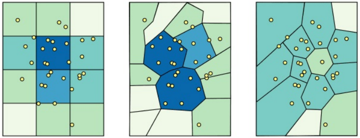
Pedestrian traffic

Density

- Number of pedestrians per square meter at a given moment

Issues

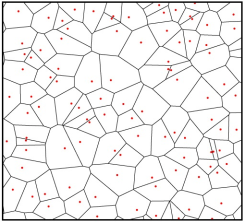
- Spatial discretization is arbitrary
- Results may be highly sensitive
- Idea: data driven spatial discretization



Voronoi tessellations

- $p_1, p_2, ..., p_N$ is a finite set of points
- Voronoi space decomposition assigns a region to each point p_i

$$V(p_i) = \{p | \|p - p_i\| \leq \|p - p_j\|, i \neq j\}$$



[Steffen and Seyfried, 2010]

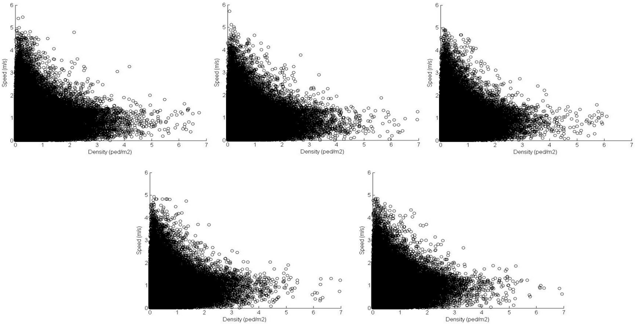
Methodology

Voronoi tessellations

- Consider pedestrian p
- V_p is the Voronoi cell associated with p
- $|V_p|$ is the area of cell V_p (in m^2)
- Density associated with the cell: $k_p = 1/|V_p|$
- Speed associated with the cell: the speed of a pedestrian occupying the cell

Empirical speed-density relationship

Speed-density profiles



February, 2013.: morning peak hour

Probabilistic speed-density model

Theoretical foundation

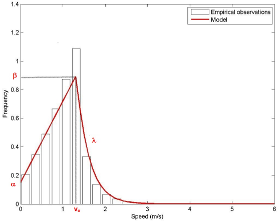
- Speed is affected by different factors
  - congestion level, trip purpose, age, health condition, etc.
- Congestion level: speed decreases with increasing density
- Pedestrian heterogeneity
  - Slower walkers: elderly people, people unfamiliar with environment, people influenced by static and dynamic objects from the scene, etc.
  - Faster walkers (less sensitive to congestion): business travelers, people in a hurry to catch a train, etc.
- Characterization of the observed phenomena: probabilistic approach

Probabilistic speed-density model

Piecewise specification

$$f(v,k) = \begin{cases} \frac{\beta(k)-\alpha(k)}{v_o(k)} \cdot v + \alpha(k), & v \leq v_o \\ exp(-\lambda \cdot v + log(\beta(k)) + \lambda \cdot v_o(k)), & v \geq v_o \end{cases}$$

Illustration - one density level



Model parameters

- v_o - mode of the distribution
- Assumed to follow symmetric triangular distribution

$$v_o(k) \sim f_{v_o}(\bar{v_o}(k), \sigma^2)$$

- The mean value corresponds to the Underwood's model

$$\bar{v_o}(k) = v_f \cdot exp(-\frac{k}{\gamma})$$

- α - frequency of occurrence of small speed values

$$\alpha(k) = a_\alpha \cdot k + b_\alpha$$

- β - frequency of occurrence of most frequent speed values

$$\beta(k) = a_\beta \cdot k + b_\beta$$

Model estimation

Notation

$$P_l(v_i, k_i) = \int_{v_o} f_l(v_i, k_i | v_o(k_i)) f_{v_o}(\bar{v_o}(k), \sigma^2) dv_o$$

$$P_e(v_i, k_i) = \int_{v_o} f_e(v_i, k_i | v_o(k_i)) f_{v_o}(\bar{v_o}(k), \sigma^2) dv_o$$

$$\omega_i = \begin{cases} 1, & P_l(v_i, k_i) \geq P_e(v_i, k_i) \\ 0, & otherwise \end{cases}$$

Maximum likelihood

$$\arg\max_{\alpha,\beta,\lambda,v_o} log \left\{ \prod_{i=1}^{n} \Big(\omega_i \cdot P_l(v_i, k_i) + (1 - \omega_i) \cdot P_e(v_i, k_i) \Big) \right\}$$

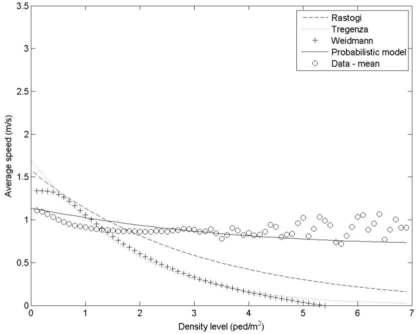
$$\begin{aligned} s.c. \quad & v_i \leq \bar{v}_o(k_i) + (1 - \omega_i) \cdot M \\ & v_i \geq \bar{v}_o(k_i) - \omega_i \cdot M \\ & \omega_i \in \{0, 1\} \end{aligned}$$

Estimation results

Parameter	Value	Std err
a_α	-0.026	$2.746e^{-06}$
b_α	0.264	$7.274e^{-06}$
a_β	0.130	$3.515e^{-06}$
b_β	0.851	$1.892e^{-06}$
λ	1.969	$1.432e^{-05}$
v_f	1.137	$6.555e^{-09}$
γ	4.743	$1.766e^{-07}$
σ	0.090	$1.168e^{-08}$
$log \mathcal{L}$	-509291.839	
$\#parameters$	8	
$\#observations$	756691	

Comparison with deterministic models

Exponential specifications



Goodness of Fit

Model	MSE
Tregenza	0.406
Weidmann	0.441
Rastogi	0.221
Probabilistic	0.015

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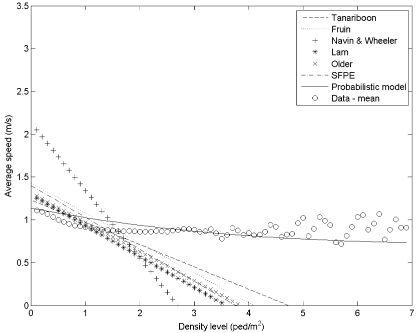
Comparison with deterministic models

Linear specifications



Goodness of Fit

Model	MSE
Tanariboon	0.591
Fruin	0.948
Navin and Wheeler	4.751
Lam	1.244
Older	1.044
SFPE	1.170
Probabilistic	0.015

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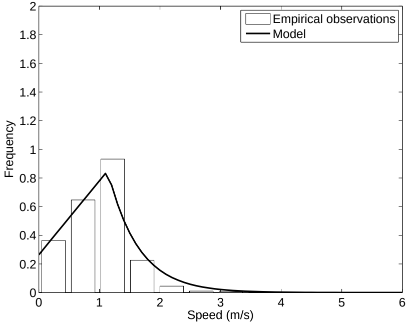
External validation - PDF

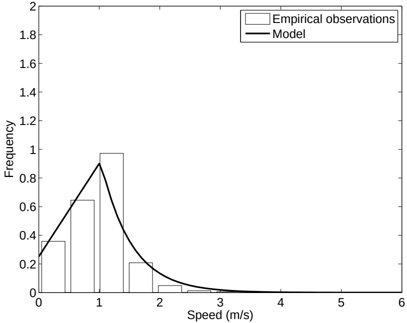
Density level: $< 0.1 ped/m^2$



$\#observations$: 21178

External validation - PDF

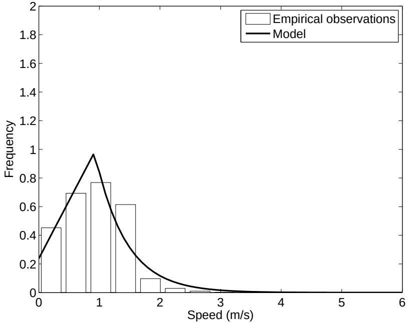
Density level: $0.5 ped/m^2$



$\#observations$: 40470

External validation - PDF

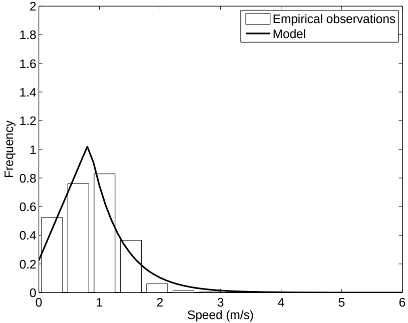
Density level: $1 ped/m^2$



$\#observations : 10705$

External validation - PDF

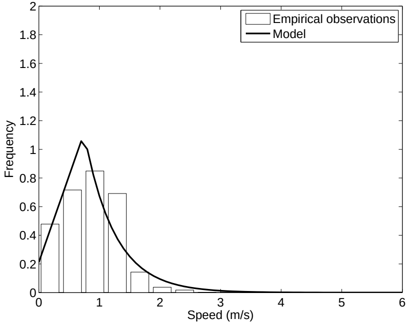
Density level: $1.5 ped/m^2$



$\#observations$: 6781

External validation - PDF

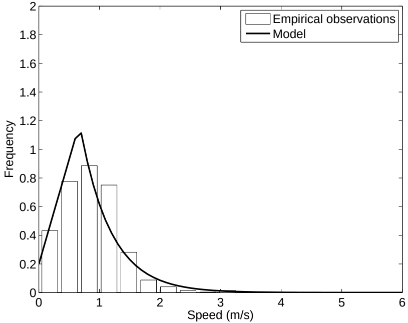
Density level: $2ped/m^2$



$\#observations$: 2509

External validation - PDF

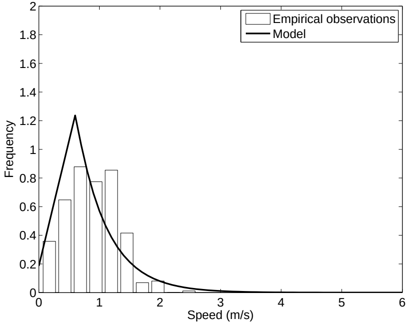
Density level: $2.5 ped/m^2$



$\#observations$: 898

External validation - PDF

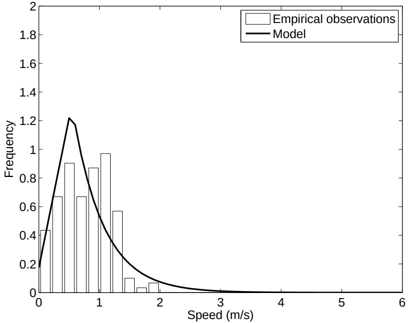
Density level: $3 ped/m^2$



$\#observations$: 354

External validation - PDF

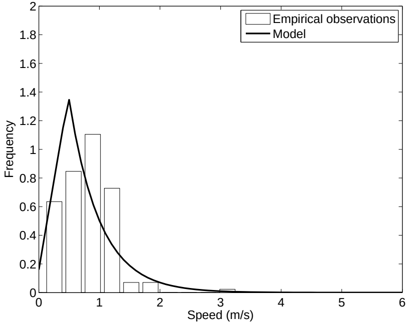
Density level: $3.5 ped/m^2$



$\#observations : 158$

External validation - PDF

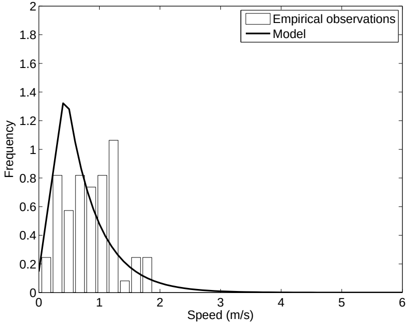
Density level: $4 ped/m^2$



$\#observations : 73$

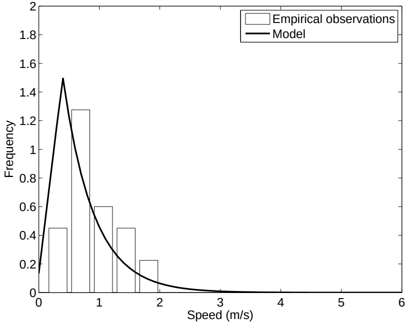
External validation - PDF

Density level: $4.5 ped/m^2$



$\# observations : 27$

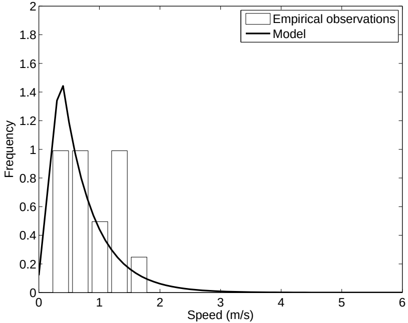
Density level: $5 ped/m^2$



$\#observations : 22$

External validation - PDF

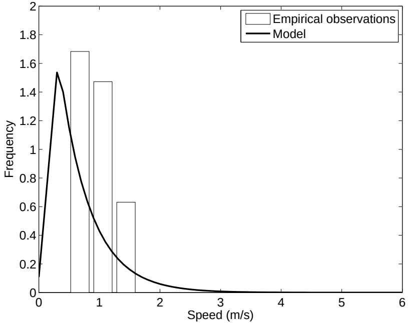
Density level: $5.5 ped/m^2$



$\#observations : 7$

External validation - PDF

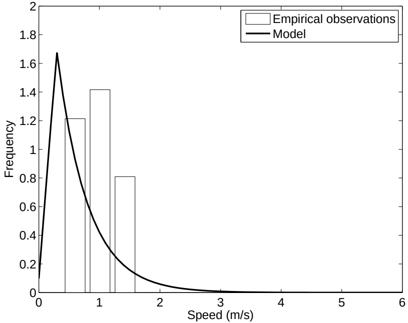
Density level: $6 ped/m^2$



$\#observations : 3$

External validation - PDF

Density level: $6.5 ped/m^2$



$\#observations : 3$

External validation - CDF

Kolmogorov-Smirnov distance

- The maximum value of the absolute difference between two cumulative distribution functions

$$D = \sup_v \left| F_{model}(v|k) - F_{data}(v|k) \right|$$

$k(ped/m^2)$	D	$k(ped/m^2)$	D	$k(ped/m^2)$	D
0	0.066	0.1	0.160	0.2	0.159
0.3	0.150	0.4	0.132	0.5	0.112
0.6	0.102	0.7	0.092	0.8	0.081
0.9	0.082	1	0.086	1.1	0.075
1.2	0.073	1.3	0.082	1.4	0.090
1.5	0.084	1.6	0.084	1.7	0.097
1.8	0.109	1.9	0.108	2	0.101
2.5	0.125	3	0.191	3.5	0.157

Conclusion and future directions

- Pedestrian-oriented flow characterization
- Data-driven discretization framework
- Probabilistic methodology to describe observed heterogeneity
- Case study: Gare de Lausanne
  - The results of internal and external validation indicate the good performance of the proposed approach
  - The model comparison with the predictions deterministic models prove the strength of the proposed methodology

- Stochastic conservation laws
- Multidirectional nature of pedestrians flows