Characterization of multidirectional pedestrian flows based on three-dimensional Voronoi tessellations

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Traffic characteristics

- **Density**: the number of pedestrians present in an area at a certain time instance [#ped/m^2]
- **Flow**:the number of pedestrians passing a line segment in a unit of time [#ped/ms]
- **Velocity**: the average of the velocities of pedestrians present in an area at a certain time instance / passing a line segment in a unit of time [m/s]

- LOS indicators
- Fundamental diagram specification
- Models of pedestrian dynamics

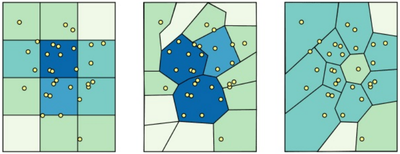
Pedestrian flow characterization

- Several approaches proposed in the literature [Duives, 2012; Zhang, 2012]
- Arbitrary discretization
- Inconsistent results
- Multi-directional flow composition neglected

Pedestrian flow characterization - Issues

Arbitrary discretization

- It may generate noise in the data [Openshaw, 1983]
- Results may be highly sensitive to minor changes of discretization



Pedestrian flow characterization - Issues

Inconsistent results in observations and modeling

- Averaging over different degrees of freedom may lead to incomparable results [Seyfried et al., 2005]

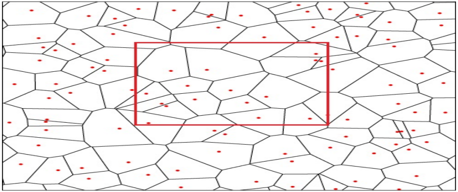
Multi-directional nature of pedestrian flows

- Definitions may not results in the desired outcome if pedestrians do not walk in the same direction [van Wageningen-Kessels et al., 2014]

Voronoi-based spatial discretization

- Assigns a personal region A_i to each pedestrian i: each point in the personal region is closer to i than to any other pedestrian, with respect of the Euclidean distance

$$A_i = \{p | d_E(p, p_i) \leq d_E(p, p_j), \forall j\}$$



Voronoi-based characterization

Steffen and Seyfried, 2010

- Density and speed are defined per unit of space via Voronoi diagrams

$$k = \frac{\int\int \rho_{xy} dxdy}{\Delta x \Delta y}, \; v = \frac{\int\int v_{xy} dxdy}{\Delta x \Delta y}$$

$\rho_{xy} = \frac{1}{A_i}$, ρ_{xy} - density distribution, A_i - area of Voronoi cell associated to pedestrian i

v_{xy} - instantaneous speed of pedestrian i

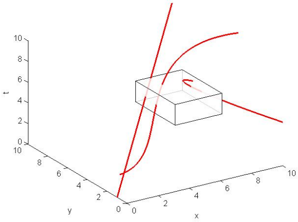
Characterization based on Edie's definitions

van Wageningen-Kessels et al., 2014

Density: $k(V) = \frac{\sum_i^N t_i}{dx \times dy \times dt}$

Flow: $\vec{q}(V) = \begin{pmatrix} \frac{\sum_i^N x_i}{dx \times dy \times dt} \\ \frac{\sum_i^N y_i}{dx \times dy \times dt} \end{pmatrix}$

Velocity: $\vec{v}(V) = \frac{\vec{q}(V)}{k(V)} = \begin{pmatrix} \frac{\sum_i^N x_i}{\sum_i t_i} \\ \frac{\sum_i^N y_i}{\sum_i t_i} \end{pmatrix}$



Data-driven discretization

Pedestrian trajectories

- The trajectory of pedestrian i is a curve in space and time

$$p_i(t) = (x_i(t), y_i(t), t)$$

- Voronoi diagram associated with trajectories
- A point $p(t)$ belongs to the set $V_i(t)$ if

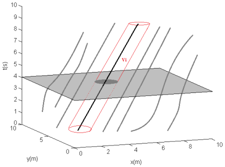
$$d(p(t), p_i(t)) \leq d(p(t), p_j(t)), \forall j$$

- Each pedestrian i is associated with a Voronoi tube V_i

Voronoi-based traffic indicators

The set of all points in V_i corresponding to a specific time t

$$V_i(t) = \{(x, y, t) \in V_i\} \sim [m^2]$$



Density indicator

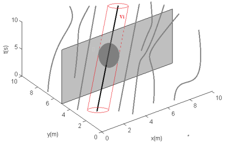
$$k_i(x, y, t) = \frac{1}{V_i(t)}$$

Voronoi-based traffic indicators

The set of all points in V_i corresponding to a given location x and y

$$V_i(x) = \{(x, y, t) \in V_i\} \sim [ms]$$

$$V_i(y) = \{(x, y, t) \in V_i\} \sim [ms]$$



Flow indicator

$$\vec{q}_i(x, y, t) = \begin{pmatrix} \frac{1}{V_i(x)} \\ \frac{1}{V_i(y)} \end{pmatrix}$$

Velocity indicator

$$\vec{v}_i(x, y, t) = \frac{\vec{q}_i(x,y,t)}{k_i(x,y,t)}$$

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Pedestrian trajectory data

- In practice, collected through an appropriate tracking technology [Daamen and Hoogendoorn, 2003, Alahi et al., 2011]
- Time is discretized: $t_s = [t_0, t_1, ..., t_f]$
- The trajectory is described as a finite collection of triplets

$$p_{is} = (x_{is}, y_{is}, t_s)$$

Characterization based on the sample of points

- Interpolation
  - Introduces errors
- Voronoi diagrams at t_s
  - Needs data that are synchronized
  - Otherwise, the density is underestimated
- 3D Voronoi diagrams for the sample of points
  - The points at t_s are the only available data
  - Spatio-temporal distance (assignment rule)

Data-driven discretization

- Voronoi diagram associated with the points p_{is}
- Each point p_{is} is associated with a Voronoi cell V_{is}
- A point p belongs to the set V_{is} if

$$d_*(p, p_{is}) \leq d_*(p, p_{js}), \forall j$$

- $d_*(p, p_{is})$ - spatio-temporal distance

Voronoi-based traffic indicators

- The set of all points in V_{is} corresponding to a given location (x, y)

$$V_{is}(x, y) = \{(x, y, t) \in V_{is}\} \sim [s]$$

Density indicator

$$k_i(x, y, t) = \frac{V_{is}(x_{is}, y_{is})}{Vol(V_{is})}$$

Voronoi-based traffic indicators

- The set of all points in V_{is} corresponding to a specific time t

$$V_{is}(t) = \{(x, y, t) \in V_{is}\} \sim [m^2]$$

Flow indicator

$$\vec{q}_i(x, y, t) = \begin{pmatrix} \frac{x_i}{V_{is}} \\ \frac{y_i}{V_{is}} \end{pmatrix}$$

x_i - a maximum distance in x direction in $V_{is}(t_{is})$

y_i - a maximum distance in y direction in $V_{is}(t_{is})$

Velocity indicator

$$\vec{v}_i(x, y, t) = \frac{\vec{q}_i(x,y,t)}{k_i(x,y,t)}$$

Spatio-temporal distances

Euclidean distance

$$d_E(p, p_{is}) = \sqrt{(p - p_{is})^T (p - p_{is})}$$

Mahalanobis distance

$$d_M(p, p_{is}) = \sqrt{(p - p_{is})^T M_{is} (p - p_{is})}$$

- M_{is} - symmetric, positive-definite matrix
- M_{is} - defines how distances are measured from the perspective of pedestrian i

Spatio-temporal distances

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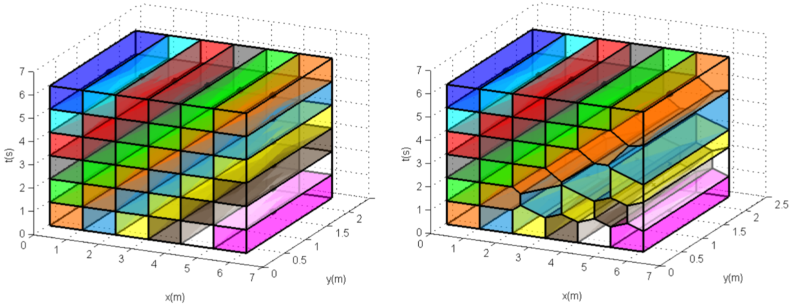
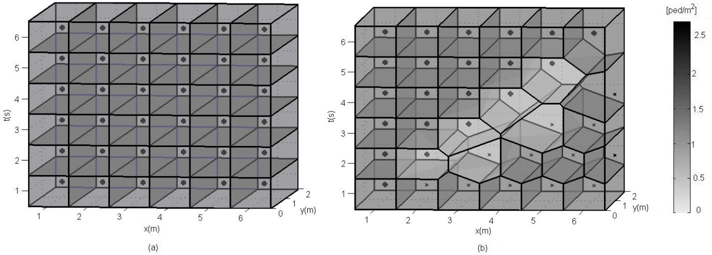
3D Voronoi discretization

Euclidean distance



Voronoi-based density maps

Euclidean distance

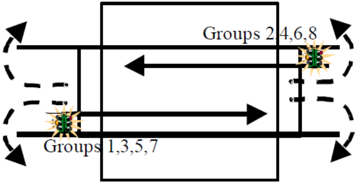


Reproduces settings with uniform and non-uniform movement

Delft case study

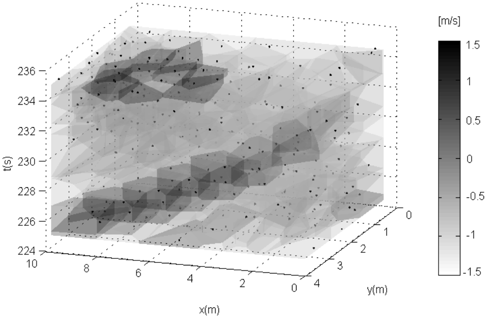
Bidirectional flow [Daamen and Hoogendoorn, 2003]

- Trajectories extracted from the digital video sequences
- The position of each individual is available every 0.1s
- Total number of trajectories: 1,123
- The average length of the trajectories: 10 meters
- The average time of the trajectories: 10 seconds.



Voronoi-based velocity maps

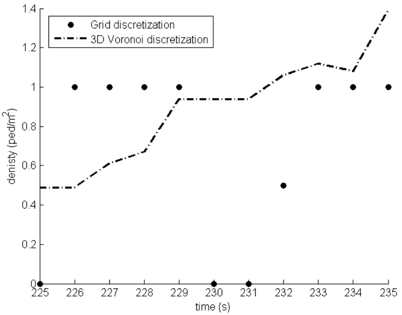
Lane formation



Allows to correlate the momentary speed of an individual pedestrian (or a group of pedestrians) with the availability of space

3D Voronoi vs. grid-based method

Density sequences



Voronoi-based approach leads to smooth transitions in measured characteristics

Spatio-temporal distances

Euclidean distance

$$d_E(p, p_{is}) = \sqrt{(p - p_{is})^T (p - p_{is})}$$

Mahalanobis distance

$$d_M(p, p_{is}) = \sqrt{(p - p_{is})^T M_{is} (p - p_{is})}$$

- M_{is} - symmetric, positive-definite matrix
- M_{is} - defines how distances are measured from the perspective of pedestrian i

Mahalanobis distance

Directions of interest

$$p_{is} = (x_{is}, y_{is}, t_s),\ v_i(t_s) = \frac{1}{t_{(s+1)} - t_s} \begin{pmatrix} x_{i(s+1)} - x_{is} \\ y_{i(s+1)} - y_{is} \\ 1 \end{pmatrix}$$

$$d^1(t_s) = \frac{v_i(t_s)}{||v_i(t_s)||},\ ||d^1(t_s)|| = 1$$

$$d^2(t_s) = \begin{pmatrix} d_x^1(t_s) \\ d_y^2(t_s) \\ 0 \end{pmatrix},\ d^1(t_s)^T d^2(t_s) = 0,\ ||d^2(t_s)|| = 1$$

$$d^3(t_s) = \begin{pmatrix} 0 \\ 0 \\ t_{(s+1)} - t_s \end{pmatrix},\ ||d^3(t_s)|| = t_{(s+1)} - t_s$$

Mahalanobis distance

Change of coordinates

$$S_1(t_s,\delta) = p_{is} + (t_{(s+1)} - t_s)v_i(t_s) + \delta d^1(t_s)$$

$$S_2(t_s,\delta) = p_{is} - (t_{(s+1)} - t_s)v_i(t_s) - \delta d^1(t_s)$$

$$S_3(t_s,\delta) = p_{is} + \delta d^2(t_s)$$

$$S_4(t_s,\delta) = p_{is} - \delta d^2(t_s)$$

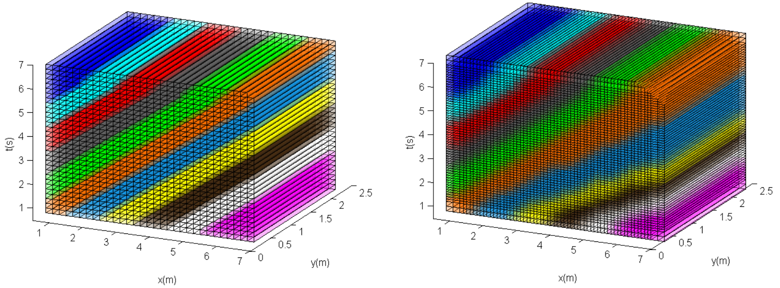
$$S_5(t_s,\delta) = p_{is} + \delta d^3(t_s)$$

$$S_6(t_s,\delta) = p_{is} - \delta d^3(t_s)$$

$$d_M = \sqrt{(S_j(t_s,\delta) - p_{is})^T M_{is} (S_j(t_s,\delta) - p_{is})} = \delta, j = 1,..,6$$

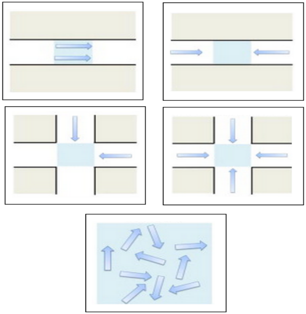
3D Voronoi discretization

Mahalanobis distance



Numerical analysis

Scenarios



Benchmark

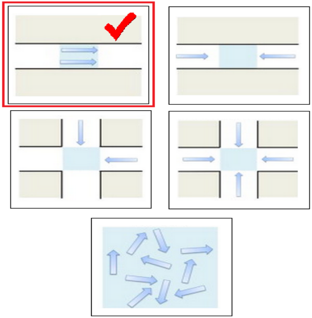
- Synthetic pedestrian trajectories
- Voronoi-based method for trajectories

Sample of points from trajectories

- Different sampling frequency
- Method
  - Voronoi diagrams with d_E
  - Voronoi diagrams with d_M

Numerical analysis

Scenarios



Benchmark

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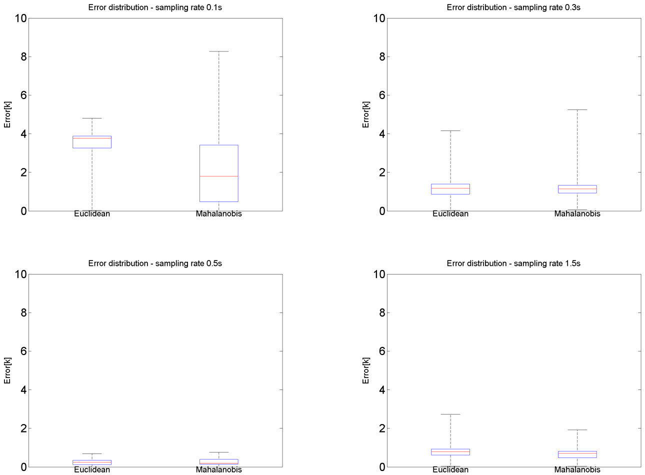
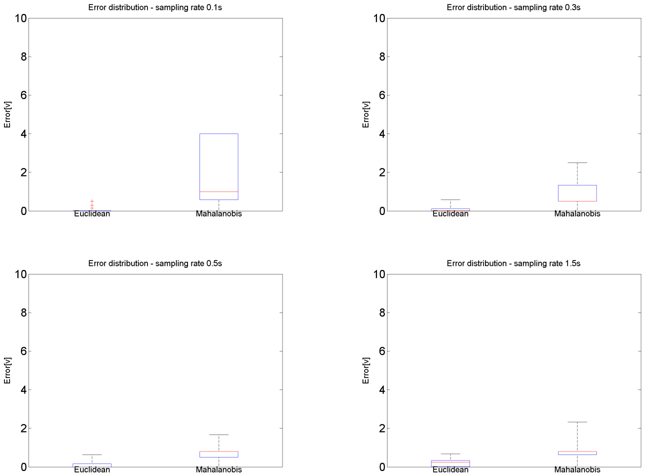
Density indicator



Speed indicator



Next...

More numerical analysis

- Different scenarios
- Importance sampling
- Comparison with interpolation
- Different assignment rules - anticipation of the forward movement of pedestrians

Conclusions

- The framework for pedestrian-oriented flow characterization
- Edie's definitions adapted through a data-driven discretization

- Reproduces the settings with uniform and non-uniform movement
- Reflects the self-organization phenomena
- Leads to smooth transitions in measured traffic characteristics
- Sampling frequency affects the accuracy of 3D Voronoi results

Future research

- More numerical analysis
- Real case study: train stations in Lausanne and Basel [Alahi et al., 2014]
- Stream-based definitions of indicators and their interaction [Nikolić and Bierlaire, 2014]
- Stream-based fundamental relationships for pedestrians

Thank you for your attention