22nd International Symposium on Transportation and Traffic Theory
Northwestern University, Evanston, USA

Data-driven spatio-temporal discretization for pedestrian flow characterization

Marija Nikolić, Michel Bierlaire

July, 24

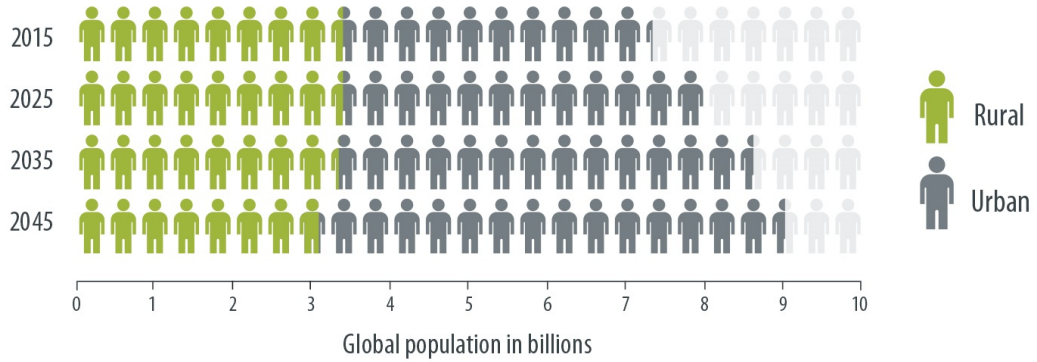
Outline

Urbanization

1950: **30%** of the population lives in cities
2014: **54%** of the population lives in cities

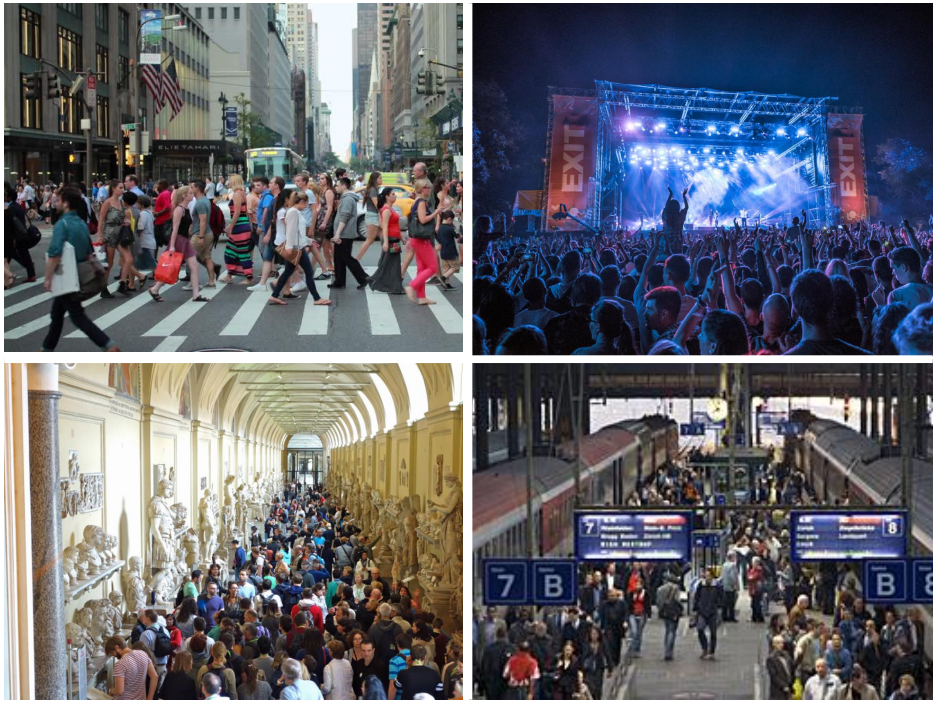
Challenges

Energy consumption, pollution, climate change
Increased traffic and congestion



Source: UN World Urbanization Prospects: 2011 Revision

Congestion: Pedestrian movements



Research challenges

Understand, describe and predict

Optimization of current infrastructure and operations

Efficient planning and management of future pedestrian facilities

Characterization

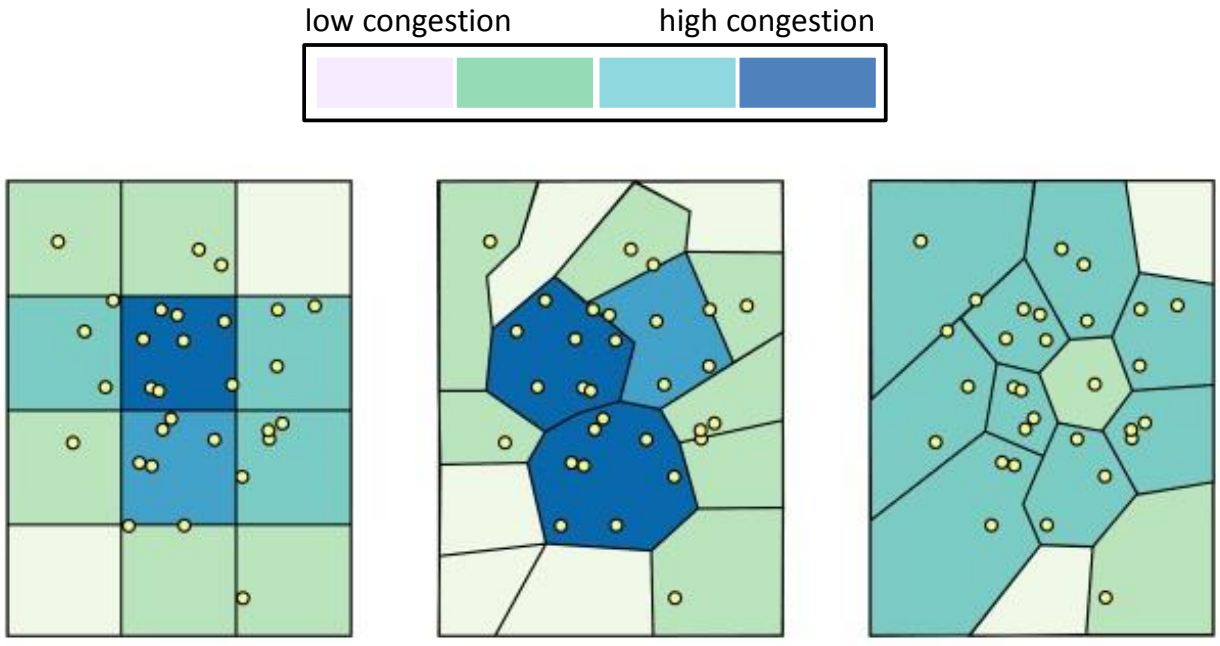
Quantities

- Density **k** (ped/m^2)
- Speed **v** (m/s)
- Flow **q** (ped/m·s)

Limitations

- Highly inspired by vehicular traffic
- Arbitrary spatial and temporal discretization

Discretization



Research challenges

Results sensitive to minor changes

Arbitrary discretization may introduce noise in data

How to define the discretization...

...independent of arbitrary chosen values?

How to define the discretization...

...independent of arbitrary chosen values?

Data-driven approach: Voronoi diagrams

Outline

Context

Model

Space-time representation: $\Omega \subset \mathbb{R}^3$

Units: meters and seconds

$p = (x, y, t) \in \Omega$: physical position (x, y) in space at a specific time t

Assumption: Ω is convex (obstacle-free and bounded)

Data: trajectories

Continuous: $\Gamma_i : \{p_i(t) | p_i(t) = (x_i(t), y_i(t), t)\}$

Discrete (sample):
$\Gamma_i : \{p_{is} | p_{is} = (x_{is}, y_{is}, t_s)\}, t_s = [t_0, t_1, ..., t_f]$

3D Voronoi diagrams: 3DVoro

Definition

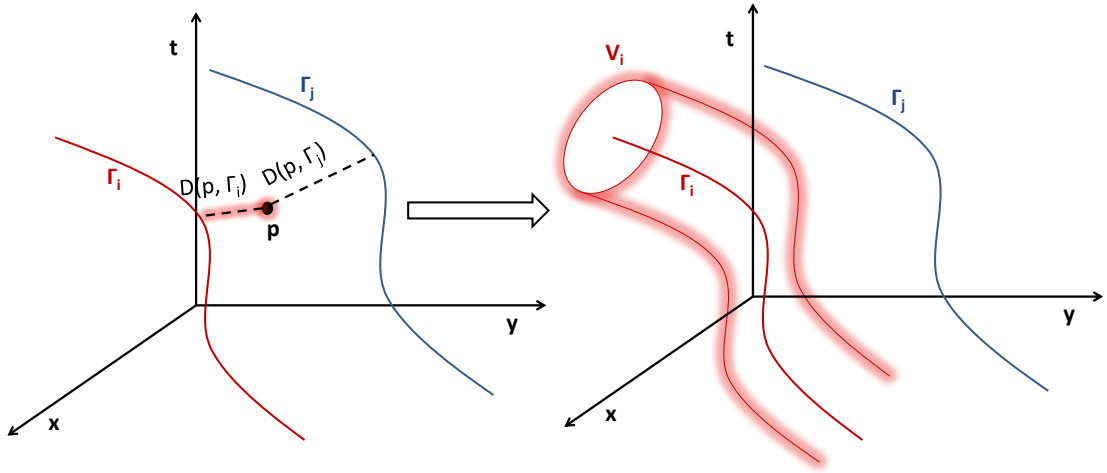
Associate $p \in \Omega$ with the closest Γ_i:

$$\delta_\Gamma(p, \Gamma_i) = \begin{cases} 1, & D(p, \Gamma_i) \leq D(p, \Gamma_j), \forall j \\ 0, & \text{otherwise} \end{cases}$$

$$D(p, \Gamma_i) = \min_{p_i} \{d(p, p_i)\}$$

Voronoi cell for Γ_i:

$$V_i = \{p \in |\delta_\Gamma(p, \Gamma_i) = 1\}$$



3DVoro: Distances

Spatial Euclidean distance

$$d_E(p, p_i) = \begin{cases} \sqrt{(x - x_i)^2 + (y - y_i)^2}, & t = t_i \\ \infty, & \text{otherwise} \end{cases}$$

Each point in time is independent

Motivated by the availability of snapshots of the floor area

All pedestrians must be observed at the exact same time

3DVoro: Distances

Time-Transform distances

$$d_{TT_1}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + v^2(t - t_i)^2}$$

$$d_{TT_2}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + \hat{v}_i(t_i)^2(t - t_i)^2}$$

$$d_{TT_3}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2} + \hat{v}_i(t_i)|t - t_i|$$

Convert seconds into meters using speed

$d_{TT_1}(p, p_i)$, $d_{TT_2}(p, p_i)$: combine components based on the Euclidean norm

d_{TT_3}: weighted sum of two norms

3DVoro: Distances

Predictive distance

$$d_P(p, p_i) = \begin{cases} \sqrt{(x_i^a - x)^2 + (y_i^a - y)^2}, & t - t_i \geq 0 \\ \infty, & \text{otherwise} \end{cases}$$

$$x_i^a = x_i^a(t) = x_i + (t - t_i) v_i^x(t_i)$$

$$y_i^a = y_i^a(t) = y_i + (t - t_i) v_i^y(t_i)$$

Accounts for the pedestrian dynamics

Anticipates future position when performing the assignment

Anticipation time: from zero to $t - t_i$

Points backward in time: infinitely distant

3DVoro: Distances

Mahalanobis distance

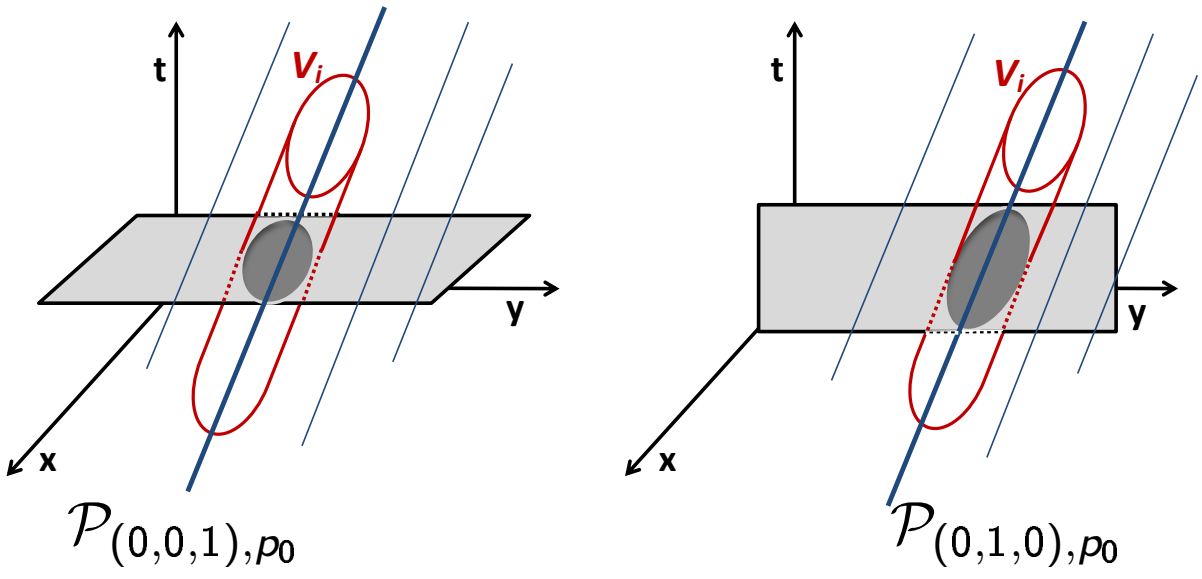
$$d_M(p, p_i) = \sqrt{(p - p_i)^T M_i (p - p_i)}$$

M_i: a change of variable matrix

TPoints in the movement direction of a pedestrian are "closer" than the points from other directions

Intersection with a plane

$\mathcal{P}_{(a,b,c),p_0}$: plane through p_0 with normal vector (a, b, c)



Voronoi-based traffic quantities

Consider $(x, y, t) \in \Omega$, and i such that $(x, y, t) \in V_i$

Density: $k(x, y, t) = \frac{1}{|V_i \cap \mathcal{P}_{(0,0,1),(x,y,t)}|}$

Flow: $\vec{q}_{(a,b,0)}(x, y, t) = \frac{1}{|V_i \cap \mathcal{P}_{(a,b,0),(x,y,t)}|}$

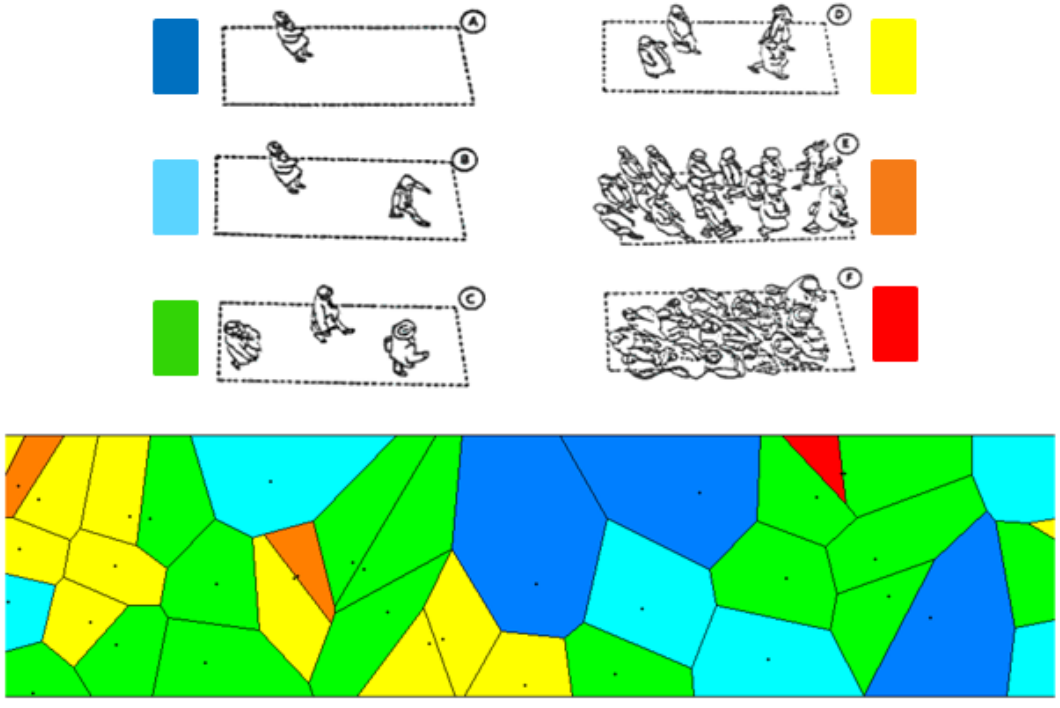
Velocity: $\vec{v}_{(a,b,0)}(x, y, t) = \frac{|V_i \cap \mathcal{P}_{(0,0,1),(x,y,t)}|}{|V_i \cap \mathcal{P}_{(a,b,0),(x,y,t)}|}$

Outline

Data

- Lausanne train station: Illustration
- Synthetic data
  - Nature of results
  - Robustness to simulation noise
  - Robustness to sampling frequency

3DVoro illustration: Lausanne train station



- Data-driven discretization
- General framework
- Microscopic characterization
- Applicable to continuous and discrete data

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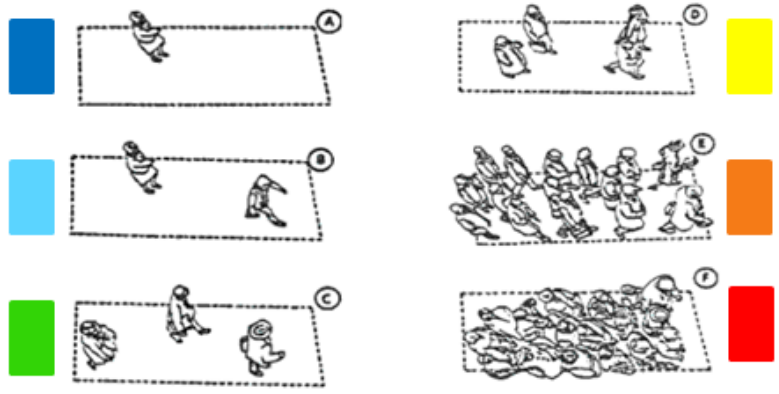

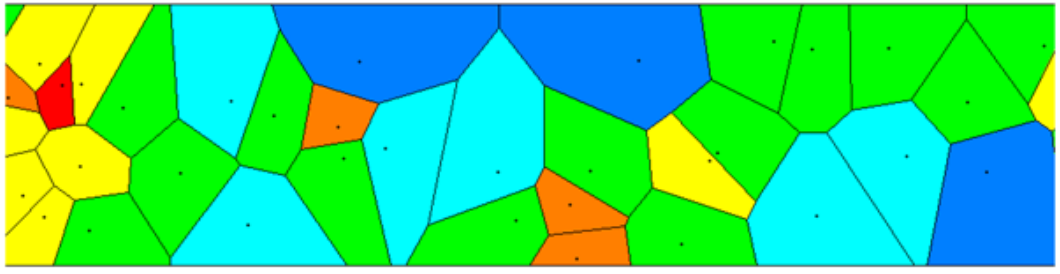

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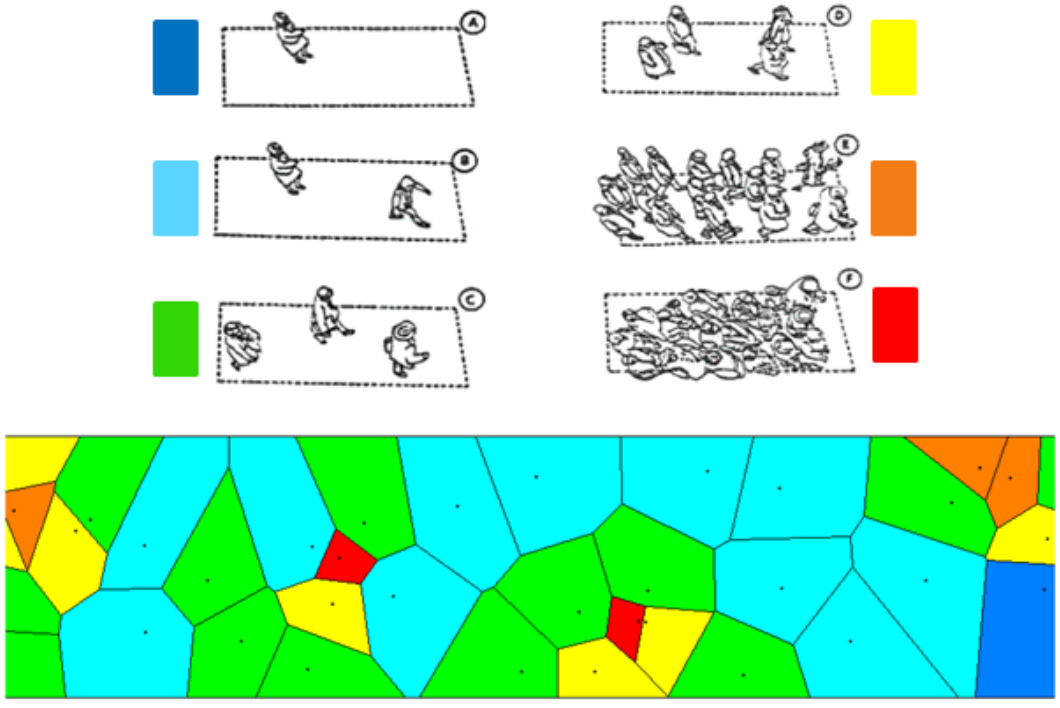

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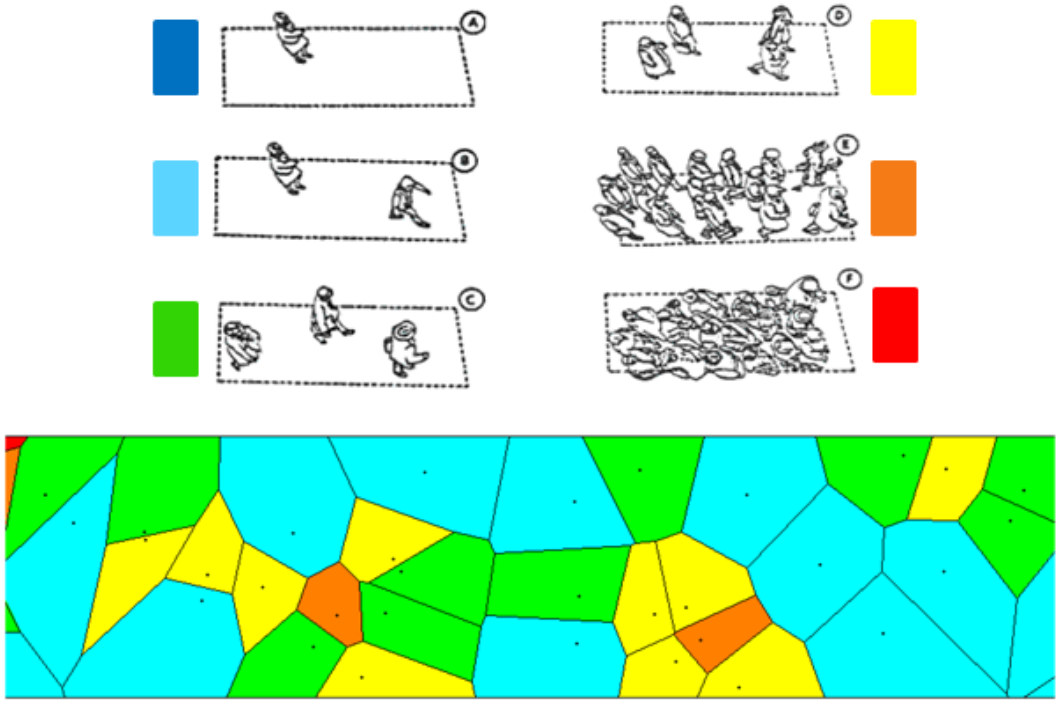

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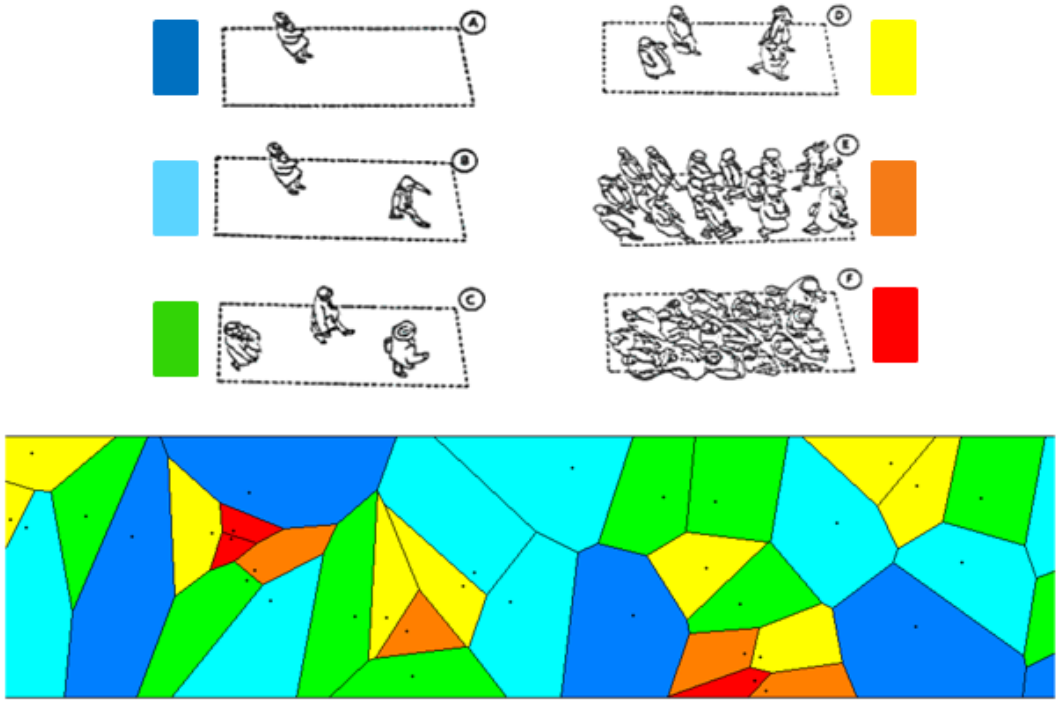

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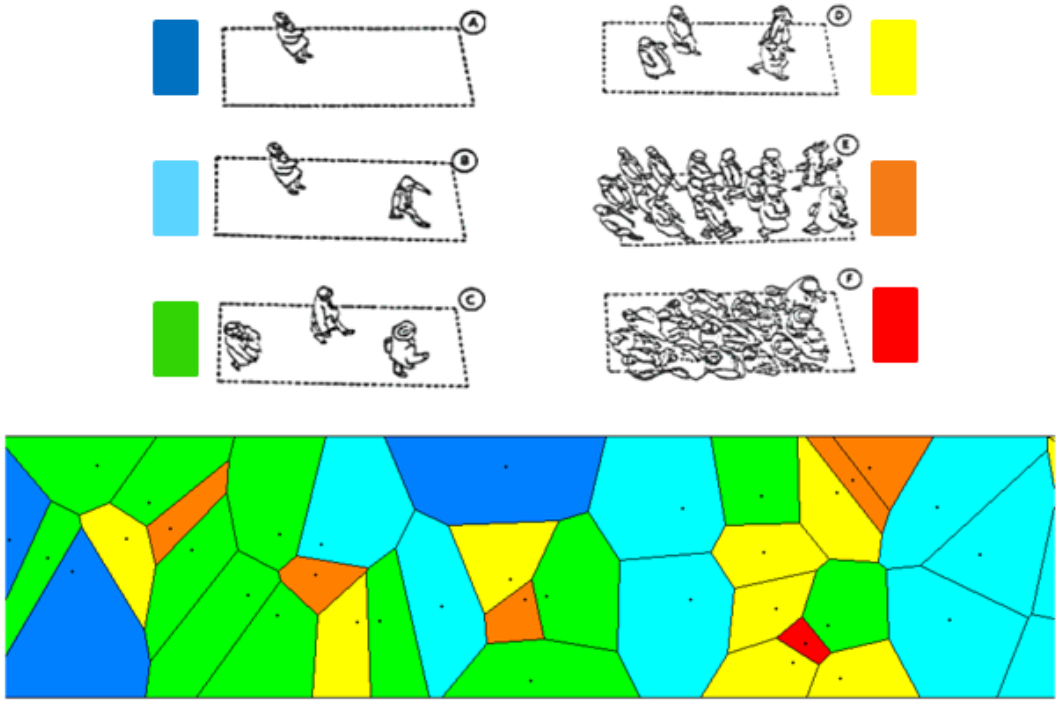

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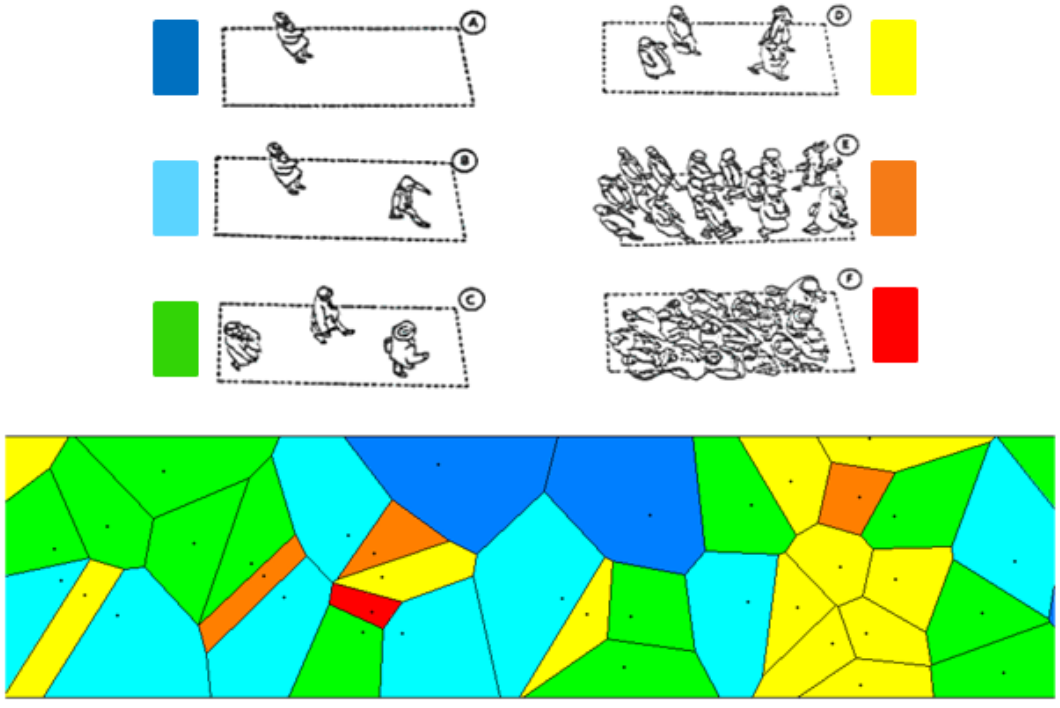

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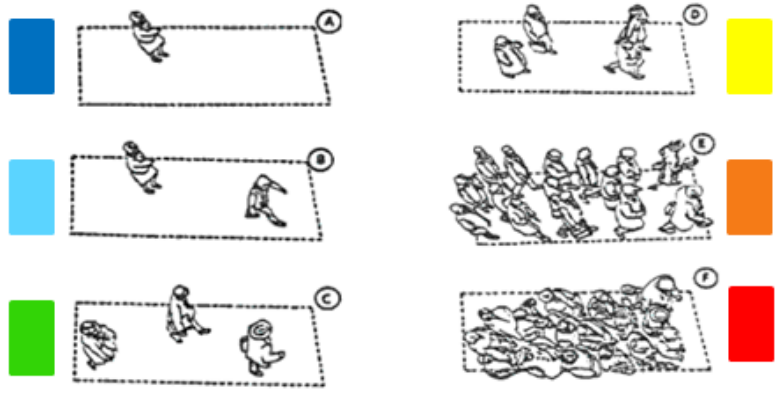

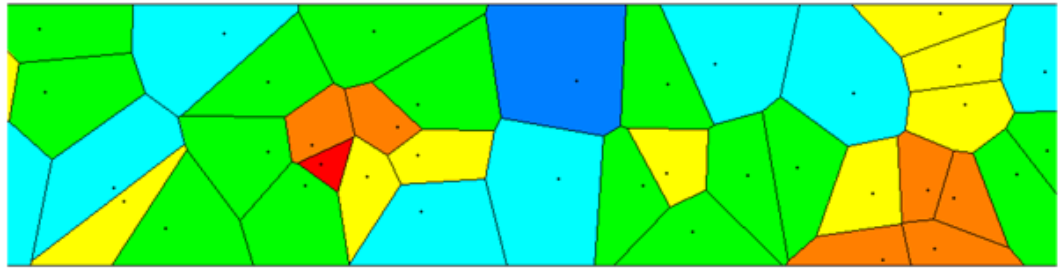

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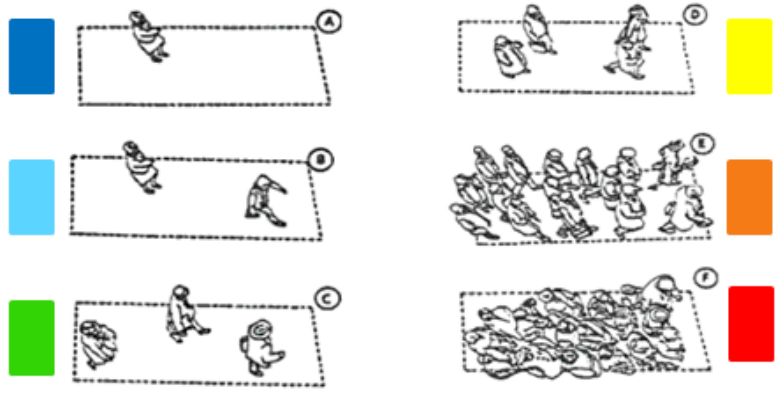

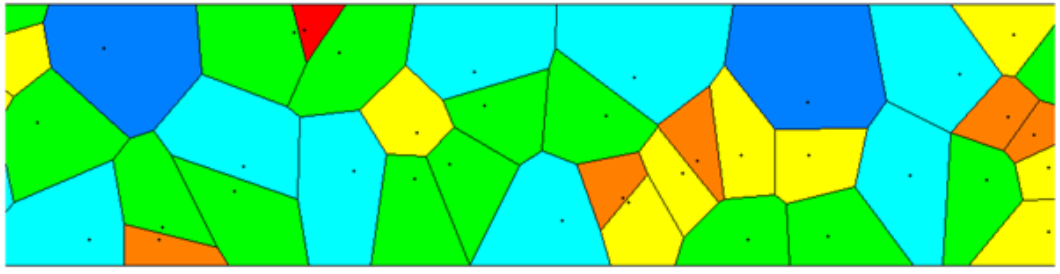

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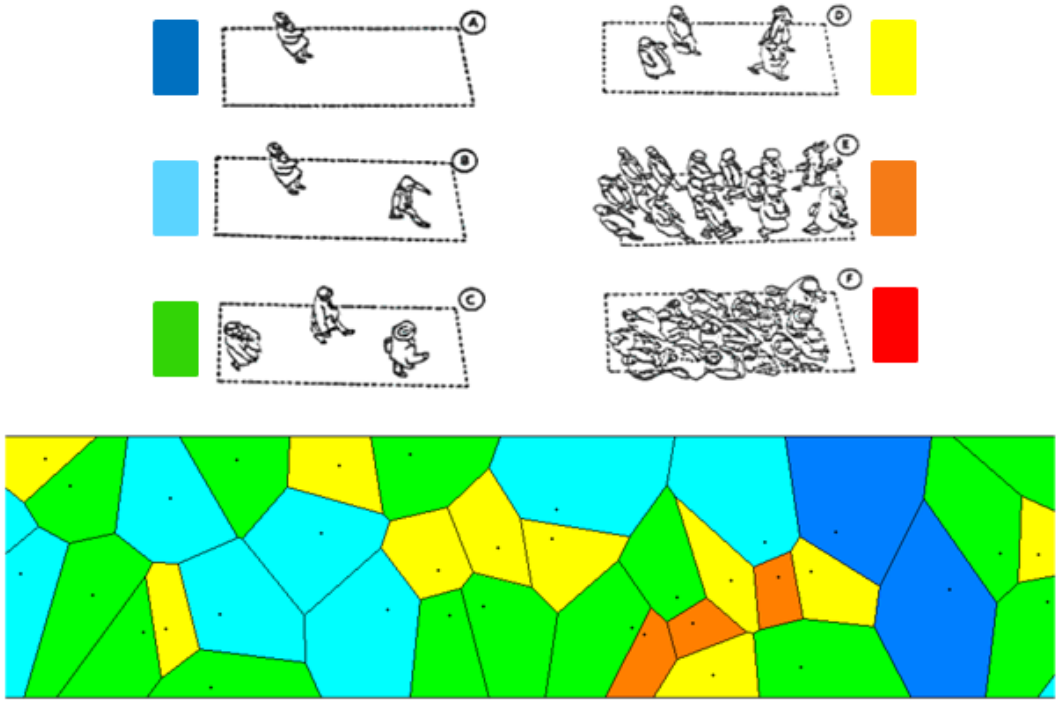

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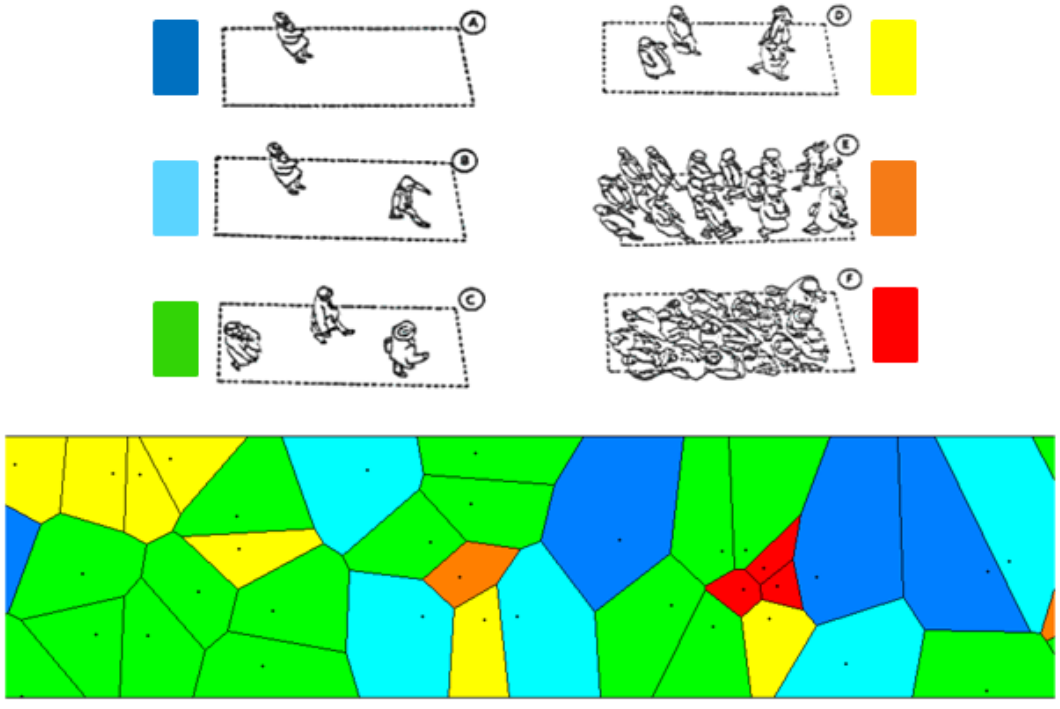

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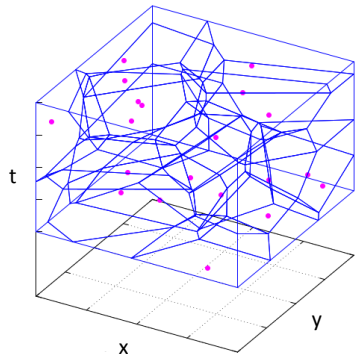
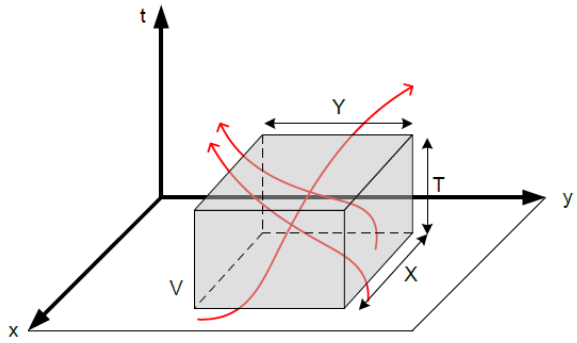
3DVoro performance: Synthetic data

NOMAD simulation tool (Campanella et al.; 2014)
Flow composition: uni-directional and bi-directional
Scenarios: low/high demand, homogenous/heterogeneous population

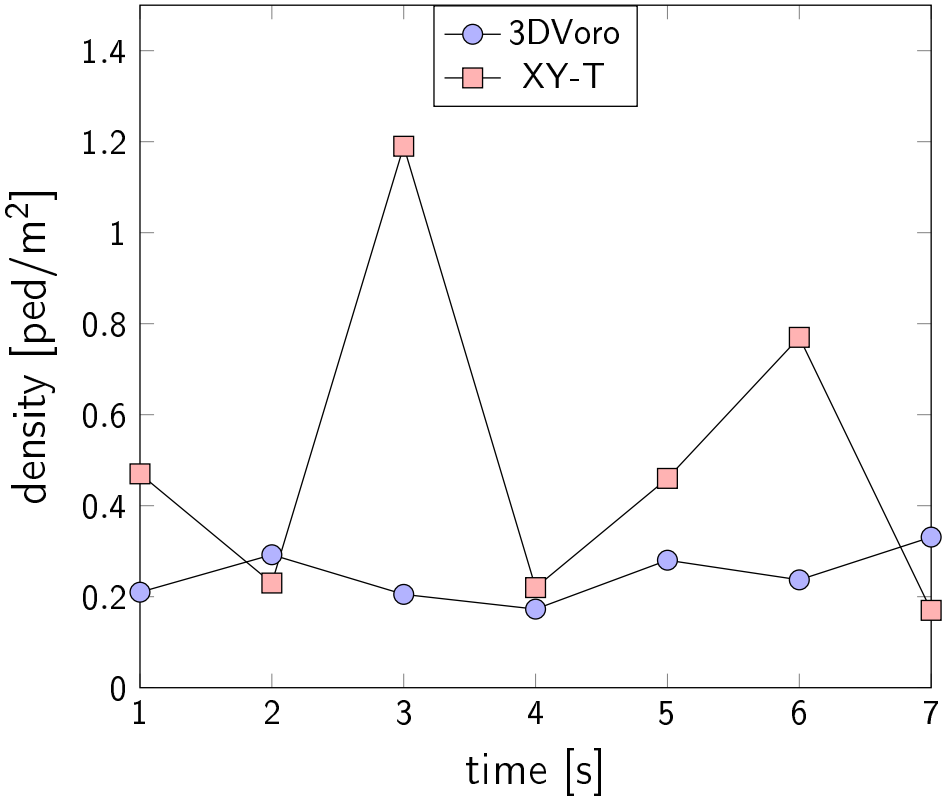
Analysis

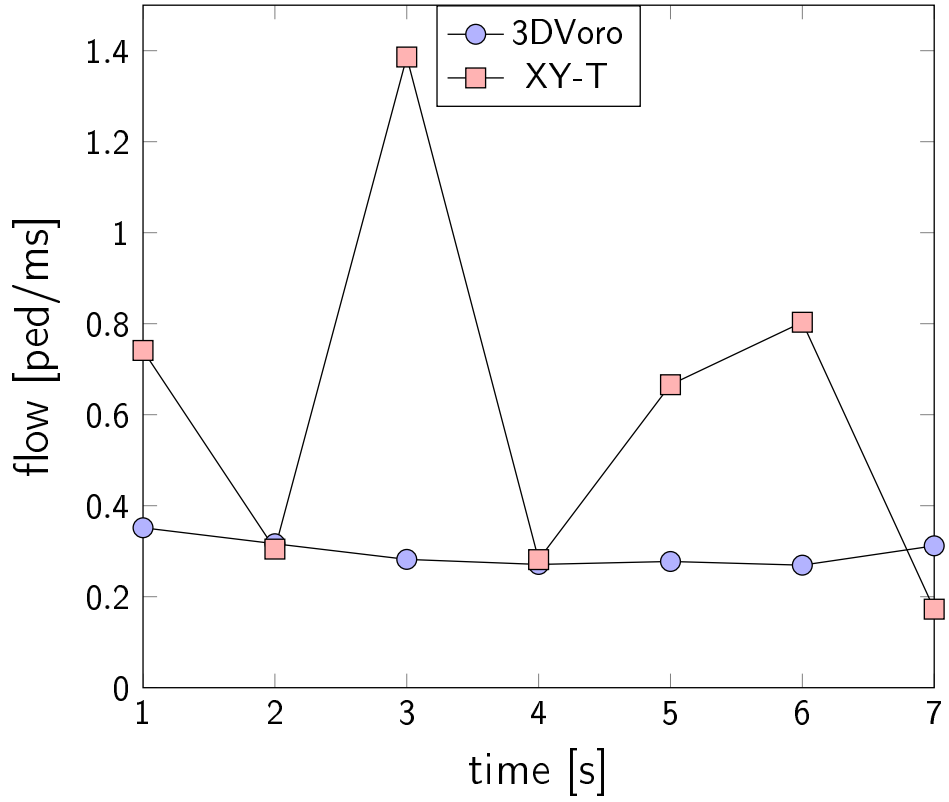
3DVoro and XY-T methods





Nature of the results





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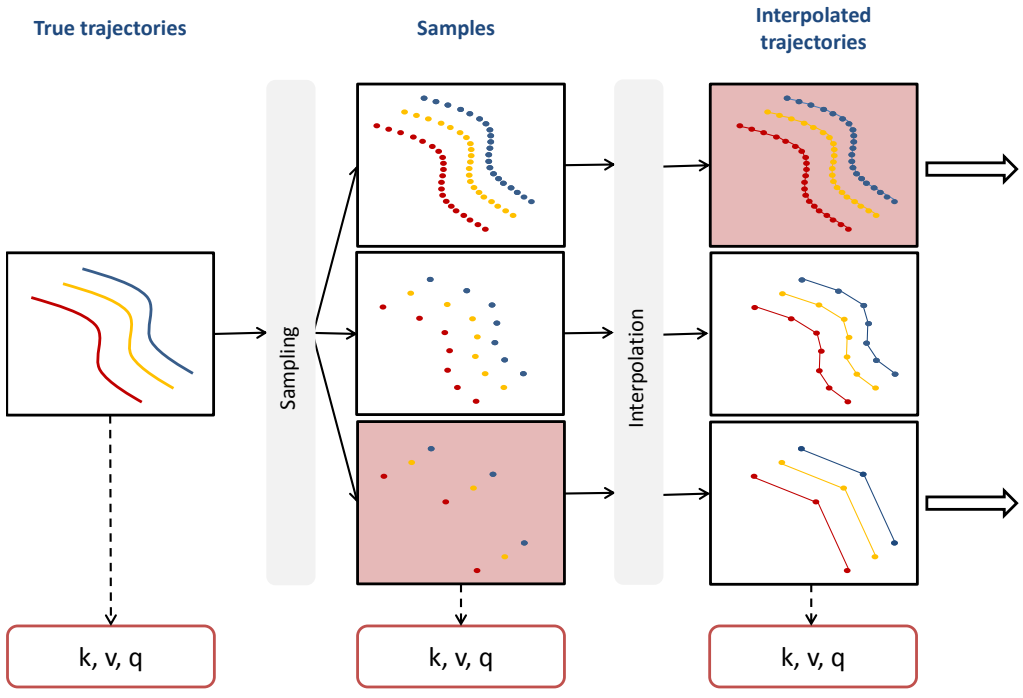
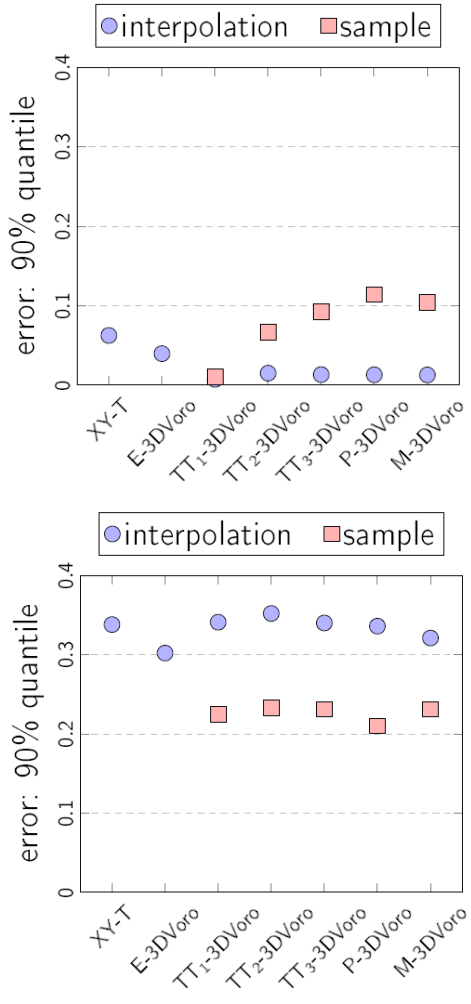
Robustness to sampling of trajectories





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Outline

Conclusion

Main findings

- Data-driven and microscopic discretization
- Well defined, flexible and general framework
- Smooth transitions in measured characteristics
- Robust to noise in the data
- Robust to sampling of trajectories

Future directions

- Anisotropy and presence of obstacles

Thank you

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Characteristics of methods

Method	Scale	Spatial aggregation		Temporal aggregation		Data type
		Unit	Assumptions	Unit	Assumptions	
XY-T	Macroscopic	Area	Shape Size Location	Interval	Duration	Trajectories
Grid-based (GB)	Macroscopic	Cell	Size Location	Interval	Duration	Trajectories Sync. sample
Range-based (RB)	Macroscopic	Circle	Radius Location	Interval	Duration	Trajectories Sync. sample
Exponentially-weighted (EW)	Macroscopic	Range	Influence function Range of influence	Interval	Duration	Trajectories Sync. sample
Voronoi-based (VB)	Microscopic	Voronoi cell	Boundary conditions	Interval	Duration	Trajectories Sync. sample

3DVoro: Distances

Mahalanobis distance

$$d^1(t_i) = \frac{v_i(t_i)}{||v_i(t_i)||}, ||d^1(t_i)|| = 1$$

$$d^2(t_i) = \begin{pmatrix} d_x^1(t_i) \\ d_y^2(t_i) \\ 0 \end{pmatrix}$$

$$d^3(t_i) = \begin{pmatrix} 0 \\ 0 \\ \Delta t \end{pmatrix}$$

$$d_M(S_j, p_i) = \alpha, j = 1, ..., 6$$

$$S_1(t_i, \alpha) = p_i + \Delta t v_i(t_i) + \alpha d^1(t_i)$$

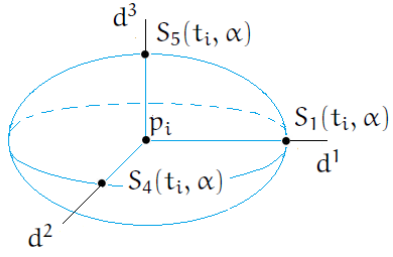
$$S_2(t_i, \alpha) = p_i - \Delta t v_i(t_i) - \alpha d^1(t_i)$$

$$S_3(t_i, \alpha) = p_i + \alpha d^2(t_i)$$

$$S_4(t_i, \alpha) = p_i - \alpha d^2(t_i)$$

$$S_5(t_i, \alpha) = p_i + \alpha d^3(t_i)$$

$$S_6(t_i, \alpha) = p_i - \alpha d^3(t_i)$$

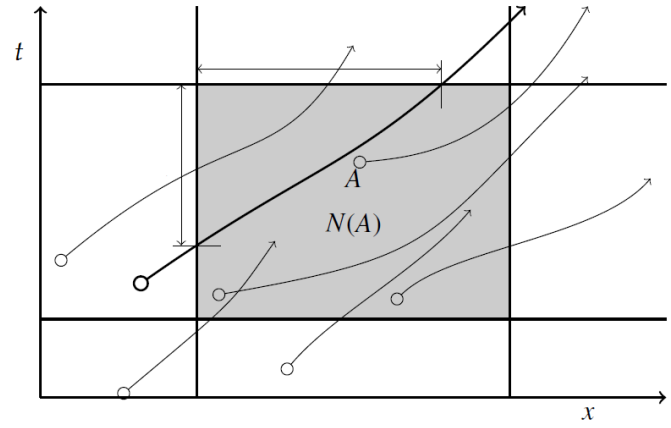


Edie (1963)

$$k(A) = \frac{\sum_{i=1}^{N} t_i}{dxdt}$$

$$q(A) = \frac{\sum_{i=1}^{N} x_i}{dxdt}$$

$$v(A) = \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} t_i}$$

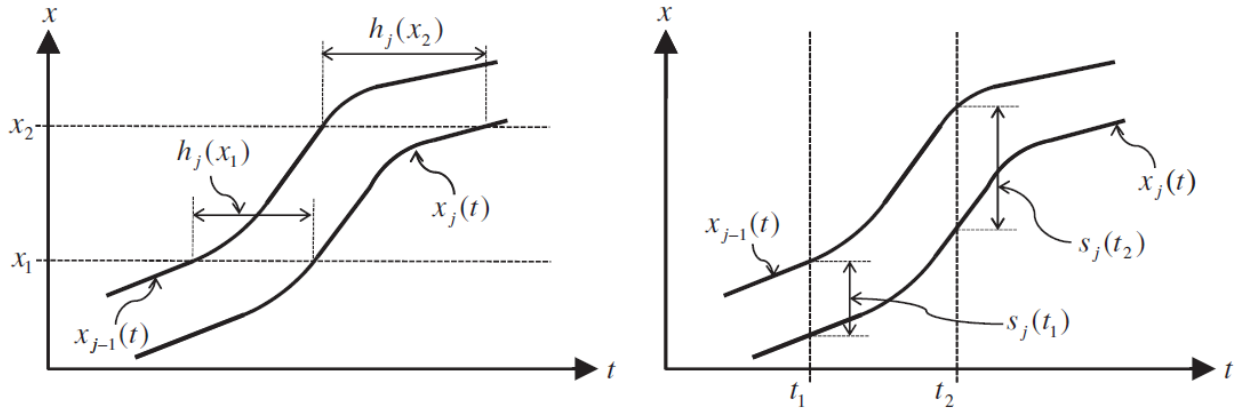


Jabari et al. (2014)

$$k(x,t) = \frac{1}{s_i(t)}, \text{ for } x \in [x_i(t), x_{i-1}(t))$$

$$q(x,t) = \frac{1}{h_i(x)}, \text{ for } t \in (t_{i-1}(x), t_i(x)]$$

$$v(x,t) = \frac{s_i(t)}{h_i(x)}, \text{ for } x \in [x_i(t), x_{i-1}(t)), t \in (t_{i-1}(x), t_i(x)]$$

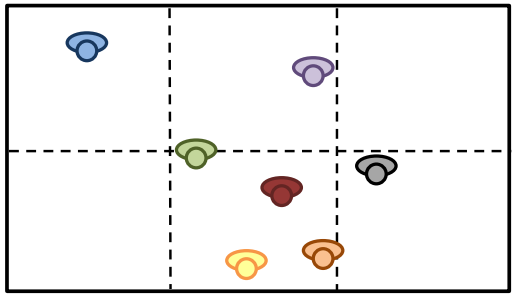


Fruin (1971)

$$k(x, y, t) = \frac{N_A(t)}{|A|}, \text{ for } (x, y) \in A$$

$$\vec{q}(x, y, t) = k(x, y, t)\vec{v}(x, y, t)$$

$$\vec{v}_i(t) = \frac{\begin{pmatrix} x_i(t_2) \\ y_i(t_2) \end{pmatrix} - \begin{pmatrix} x_i(t_1) \\ y_i(t_1) \end{pmatrix}}{t_2 - t_1}$$

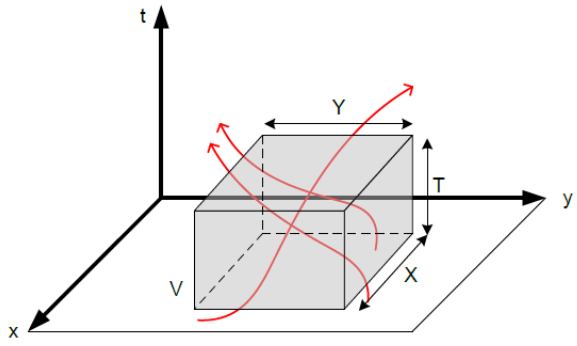


$$\vec{v}(x, y, t) = \frac{\sum_{i=1}^{N_A} \vec{v}_i(t)}{N_A}, \text{ for } (x, y) \in A$$

van Wageningen-Kessels et al. (2014)
Saberi and Mahmassani (2014)

$$k(A) = \frac{\sum_{i=1}^{N} t_i}{dxdydt}$$

$$\vec{q}(A) = \begin{pmatrix} q_x(A) \\ q_y(A) \end{pmatrix} = \begin{pmatrix} \frac{\sum_{i=1}^{N} x_i}{dxdydt} \\ \frac{\sum_{i=1}^{N} y_i}{dxdydt} \end{pmatrix}$$



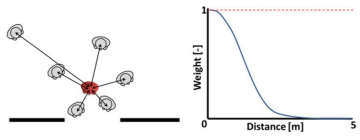
$$\vec{v}(A) = \begin{pmatrix} v_x(A) \\ v_y(A) \end{pmatrix} = \begin{pmatrix} \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} t_i} \\ \frac{\sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} t_i} \end{pmatrix}$$

Helbing et al. (2007)

$$f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{1}{\pi R^2} \exp\left(-\frac{\left\|\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right\|^2}{R^2}\right)$$

$$k(x, y, t) = \sum_i f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right)$$

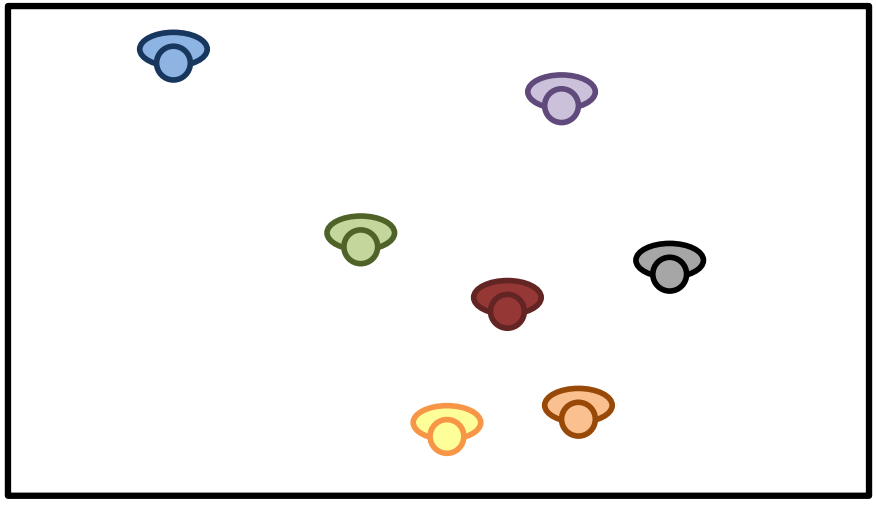
$$\vec{q}(x, y, t) = k(x, y, t)\vec{v}(x, y, t)$$



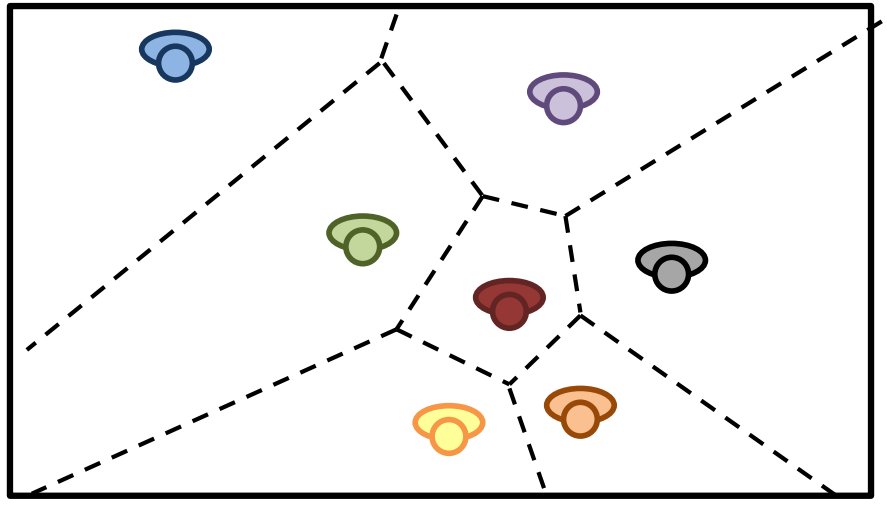
$$\vec{v}_i(t) = \frac{\begin{pmatrix} x_i(t_2) \\ y_i(t_2) \end{pmatrix} - \begin{pmatrix} x_i(t_1) \\ y_i(t_1) \end{pmatrix}}{t_2 - t_1}$$

$$\vec{v}(x, y, t) = \frac{\sum_i \vec{v}_i(t) f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right)}{\sum_i f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right)}$$

Voronoi diagrams: 2D



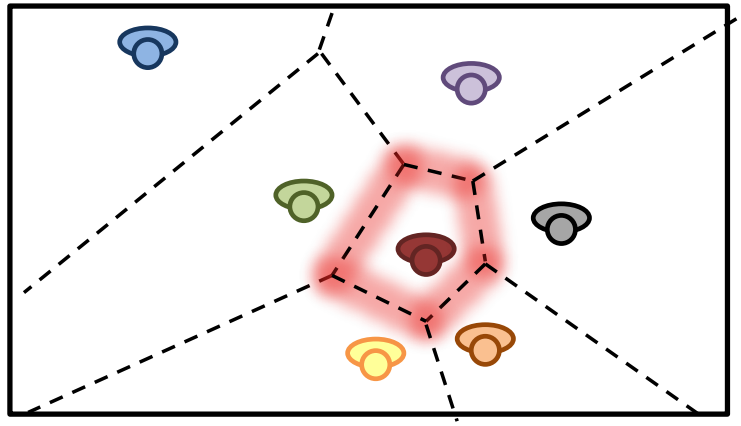
Voronoi diagrams: 2D



Steffen and Seyfried (2010)

$$k(x, y, t) = \frac{1}{|A_i|}, \text{ for } (x, y) \in A_i$$

$$\vec{v}(x, y, t) = \frac{\begin{pmatrix} x_i(t_2) \\ y_i(t_2) \end{pmatrix} - \begin{pmatrix} x_i(t_1) \\ y_i(t_1) \end{pmatrix}}{t_2 - t_1}$$



q: half a person has passed a segment if half of the Voronoi cell has passed it

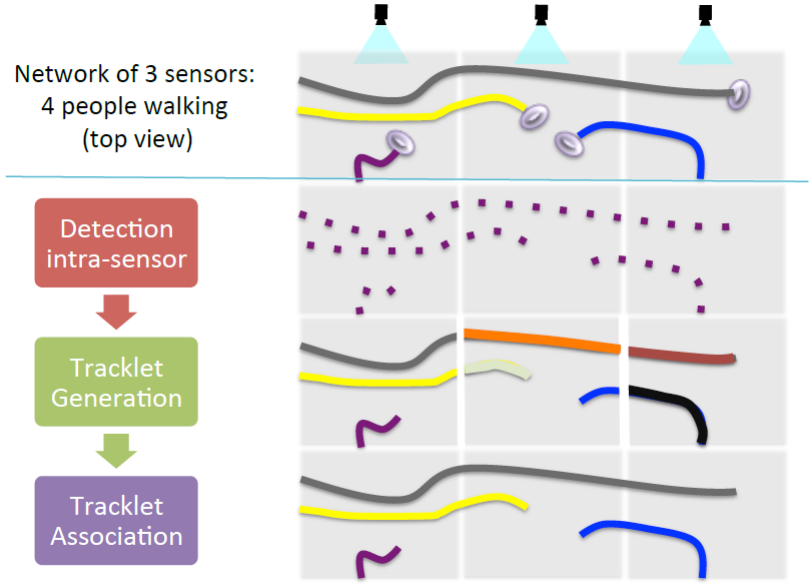
Lausanne data

Tracklet generation

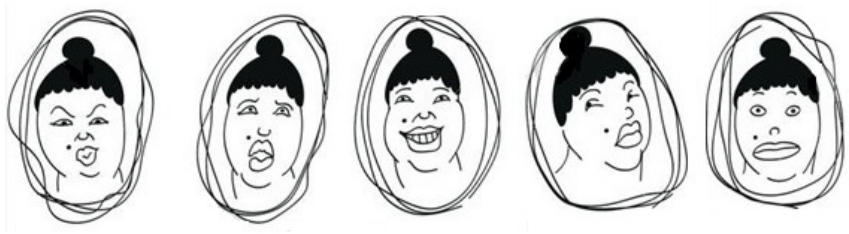
- A graph-based tracking algorithm is implemented to link the detected points
- A directed graph: vertices representing the 3D coordinates of detected pedestrians, edges defining the connectivity between vertices
- The connectivity prevents too long or unrealistic connections

Tracklet association

- Task: find the set of trajectories Θ that best explains the extracted tracklets
- Formally:maximizing the a-posterior probability of Θ given the set of tracklets



3DVoro: Robustness to noise in the data



100 sets of pedestrian trajectories synthesized per scenario

$\theta_r^M(p) = (k_r^M(p), v_r^M(p), q_r^M(p))$: a vector of indicators at point p obtained by applying the method M to the r^{th} set of trajectories

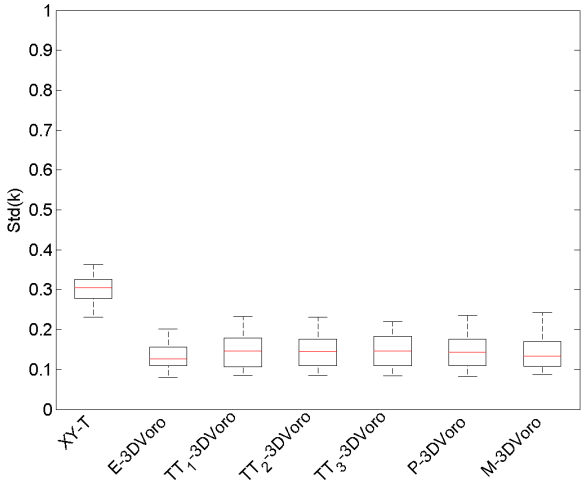
The standard deviation of the indicators at p as

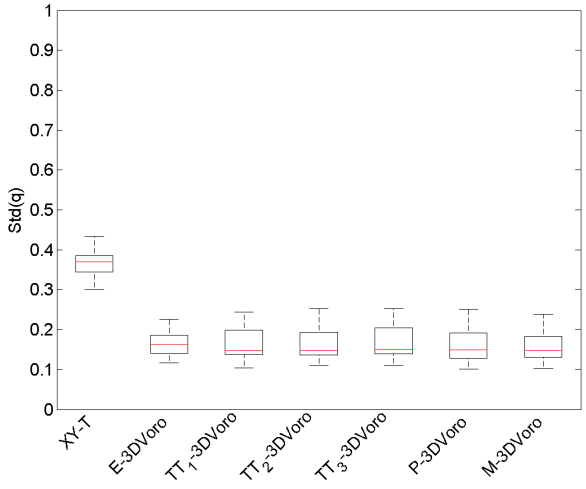
$$\sigma_R^M(p) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}(\theta_r^M(p) - \mu_R^M(p))^2}$$

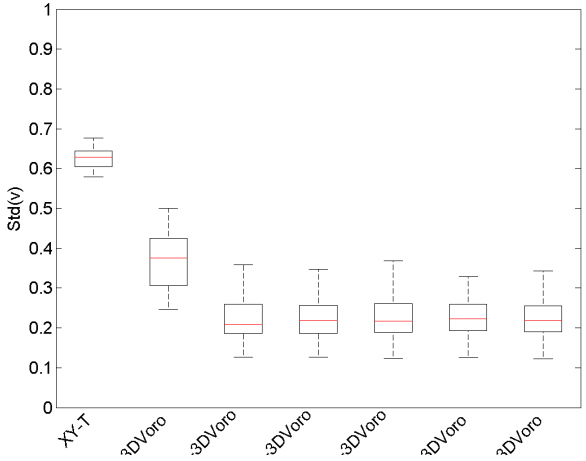
$\mu_R^M(p) = \frac{1}{R}\sum_{r=1}^{R}\theta_r^M(p)$, $R = 100$

The procedure is repeated for 1000 randomly selected points p

3DVoro: Robustness to noise in the data - Sc.I



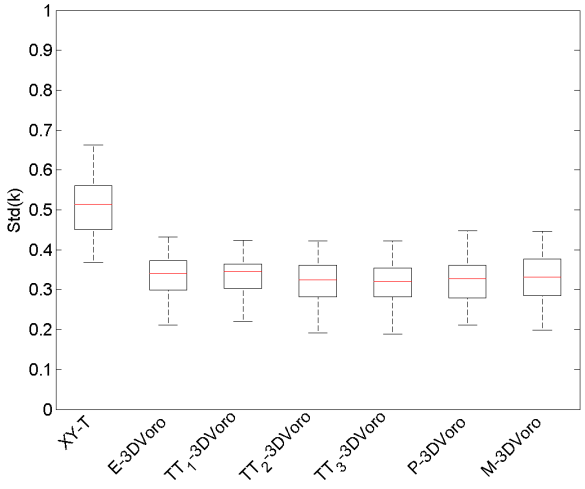


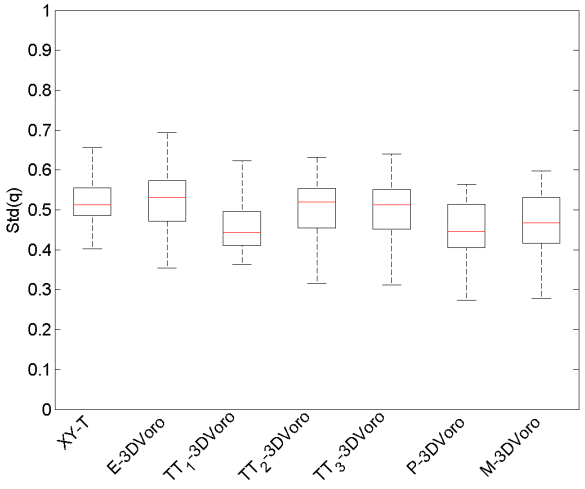


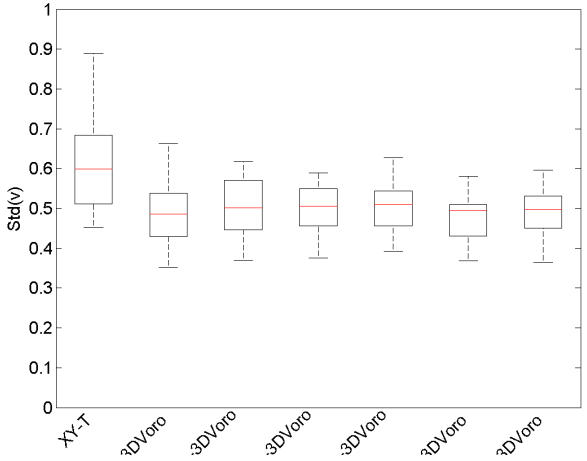
TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

3DVoro: Robustness to noise in the data - Sc.II







TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$1.47e^{-02}$	/	$1.25e^{-02}$	/	$1.25e^{-02}$	/	$6.25e^{-02}$	/
E-3DVoro	$1.17e^{-02}$	/	0	/	$4.48e^{-04}$	/	$3.96e^{-02}$	/
TT_1-3DVoro	$2.70e^{-03}$	$6.70e^{-03}$	0	0	$3.00e^{-04}$	$2.30e^{-03}$	$7.30e^{-03}$	$1.02e^{-02}$
TT_2-3DVoro	$5.80e^{-03}$	$3.50e^{-02}$	0	$2.80e^{-03}$	$6.00e^{-04}$	$2.08e^{-02}$	$1.50e^{-02}$	$6.69e^{-02}$
TT_3-3DVoro	$5.40e^{-03}$	$4.34e^{-02}$	0	$8.00e^{-03}$	$6.00e^{-04}$	$2.83e^{-02}$	$1.32e^{-02}$	$9.22e^{-02}$
P-3DVoro	$8.20e^{-03}$	$5.36e^{-02}$	0	$6.10e^{-03}$	$2.40e^{-03}$	$3.03e^{-02}$	$1.30e^{-02}$	$1.14e^{-01}$
M-3DVoro	$4.50e^{-03}$	$5.65e^{-02}$	0	$6.80e^{-03}$	$1.10e^{-03}$	$4.55e^{-02}$	$1.28e^{-02}$	$1.04e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$1.90e^{-01}$	/	$1.00e^{-01}$	/	$1.50e^{-01}$	/	$3.38e^{-01}$	/
E-3DVoro	$1.64e^{-01}$	/	$1.12e^{-02}$	/	$1.46e^{-01}$	/	$3.02e^{-01}$	/
TT_1-3DVoro	$2.54e^{-01}$	$1.27e^{-01}$	$1.35e^{-02}$	$9.00e^{-03}$	$1.16e^{-01}$	$8.97e^{-02}$	$3.41e^{-01}$	$2.25e^{-01}$
TT_2-3DVoro	$1.64e^{-01}$	$1.22e^{-01}$	$1.44e^{-02}$	$1.06e^{-02}$	$1.21e^{-01}$	$7.30e^{-02}$	$3.52e^{-01}$	$2.33e^{-01}$
TT_3-3DVoro	$1.89e^{-01}$	$1.24e^{-01}$	$1.84e^{-02}$	$1.09e^{-02}$	$1.24e^{-01}$	$7.88e^{-02}$	$3.40e^{-01}$	$2.31e^{-01}$
P-3DVoro	$3.19e^{-01}$	$1.21e^{-01}$	$3.26e^{-02}$	$6.20e^{-03}$	$1.43e^{-01}$	$7.43e^{-02}$	$3.36e^{-01}$	$2.10e^{-01}$
M-3DVoro	$1.97e^{-01}$	$1.24e^{-01}$	$3.48e^{-02}$	$9.90e^{-03}$	$1.41e^{-01}$	$7.72e^{-02}$	$3.21e^{-01}$	$2.31e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of density indicator - $Uni_{LD-HomoPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$2.05e^{-02}$	/	0	/	$1.25e^{-02}$	/	$5.00e^{-02}$	/
E-3DVoro	$1.43e^{-02}$	/	0	/	$2.67e^{-02}$	/	$2.64e^{-02}$	/
TT_1-3DVoro	$8.00e^{-03}$	$4.55e^{-02}$	0	0	$8.00e^{-04}$	$1.75e^{-02}$	$2.36e^{-02}$	$8.52e^{-02}$
TT_2-3DVoro	$1.49e^{-02}$	$1.07e^{-01}$	0	0	$3.20e^{-03}$	$5.72e^{-02}$	$3.33e^{-02}$	$2.21e^{-01}$
TT_3-3DVoro	$1.24e^{-02}$	$1.60e^{-01}$	0	0	$3.50e^{-03}$	$9.62e^{-02}$	$2.98e^{-02}$	$3.41e^{-01}$
P-3DVoro	$2.10e^{-02}$	$1.66e^{-01}$	0	0	$4.20e^{-03}$	$1.16e^{-01}$	$5.27e^{-02}$	$3.64e^{-01}$
M-3DVoro	$1.31e^{-02}$	$2.40e^{-01}$	0	0	$2.50e^{-03}$	$1.75e^{-01}$	$2.91e^{-02}$	$5.58e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$5.29e^{-01}$	/	$1.63e^{-01}$	/	$4.75e^{-01}$	/	$1.01e^{00}$	/
E-3DVoro	$4.02e^{-01}$	/	0	/	$2.49e^{-01}$	/	1.03E+00	/
TT_1-3DVoro	$4.06e^{-01}$	$2.90e^{-01}$	$3.10e^{-01}$	$2.48e^{-02}$	$2.64e^{-01}$	$1.65e^{-01}$	$9.21e^{-01}$	$7.12e^{-01}$
TT_2-3DVoro	$3.92e^{-01}$	$4.58e^{-01}$	$2.85e^{-01}$	$2.34e^{-01}$	$2.48e^{-01}$	$2.34e^{-01}$	$9.30e^{-01}$	1.11E+00
TT_3-3DVoro	$4.41e^{-01}$	$5.07e^{-01}$	$2.89e^{-01}$	$5.89e^{-02}$	$2.37e^{-01}$	$3.06e^{-01}$	$9.81e^{-01}$	1.17E+00
P-3DVoro	$4.31e^{-01}$	$3.71e^{-01}$	$1.40e^{-03}$	0	$2.58e^{-01}$	$1.80e^{-01}$	$9.43e^{-01}$	$7.29e^{-01}$
M-3DVoro	$4.34e^{-01}$	$5.01e^{-01}$	$3.16e^{-01}$	$1.36e^{-01}$	$2.75e^{-01}$	$3.52e^{-01}$	$9.96e^{-01}$	$9.80e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of density indicator - $Uni_{HD-HeteroPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$6.50e^{-02}$	/	0	/	0	/	$8.65e^{-03}$	/
E-3DVoro	$1.20e^{-02}$	/	0	/	0	/	$4.66e^{-03}$	/
TT_1-3DVoro	$3.58e^{-03}$	$1.08e^{-02}$	0	0	0	$1.02e^{-03}$	$4.16e^{-03}$	$6.15e^{-03}$
$TT_2$3DVoro	$8.13e^{-03}$	$1.18e^{-02}$	0	0	0	$2.35e^{-03}$	$8.09e^{-03}$	$1.29e^{-02}$
TT_3-3DVoro	$1.49e^{-02}$	$2.06e^{-02}$	0	$3.91e^{-03}$	0	$8.43e^{-03}$	$7.46e^{-03}$	$3.10e^{-02}$
P-3DVoro	$2.29e^{-02}$	$5.42e^{-02}$	0	$1.94e^{-03}$	0	$2.72e^{-02}$	$9.25e^{-03}$	$1.06e^{-01}$
M-3DVoro	$2.15e^{-02}$	$4.82e^{-02}$	0	$4.31e^{-02}$	0	$2.42e^{-02}$	$7.69e^{-03}$	$1.29e^{-01}$

(a) Sampling frequency: $3\ s^{-1}$

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$1.66e^{-01}$	/	0	/	$6.84e^{-02}$	/	$7.00e^{-01}$	/
E-3DVoro	$1.65e^{-01}$	/	0	/	$1.19e^{-01}$	/	$3.40e^{-01}$	/
TT_1-3DVoro	$1.68e^{-01}$	$1.29e^{-01}$	$3.50e^{-02}$	$5.02e^{-02}$	$8.50e^{-02}$	$5.70e^{-02}$	$3.85e^{-01}$	$2.62e^{-01}$
TT_2-3DVoro	$1.70e^{-01}$	$1.02e^{-01}$	$4.52e^{-02}$	$5.63e^{-02}$	$8.49e^{-02}$	$6.15e^{-02}$	$3.82e^{-01}$	$5.57e^{-01}$
TT_3-3DVoro	$1.80e^{-01}$	$1.18e^{-01}$	$4.82e^{-02}$	$6.06e^{-02}$	$8.80e^{-02}$	$6.55e^{-02}$	$3.83e^{-01}$	$2.65e^{-01}$
P-3DVoro	$2.02e^{-01}$	$1.60e^{-01}$	$3.69e^{-02}$	$4.84e^{-02}$	$9.36e^{-02}$	$6.73e^{-02}$	$4.14e^{-01}$	$3.01e^{-01}$
M-3DVoro	$1.80e^{-01}$	$1.55e^{-01}$	$4.80e^{-02}$	$3.36e^{-02}$	$1.01e^{-01}$	$9.27e^{-02}$	$4.38e^{-01}$	$3.08e^{-01}$

(b) Sampling frequency: $0.5\ s^{-1}$

Robustness to the sampling frequency of density indicator - $Bi_{LD-HomoPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$2.85e^{-02}$	/	0	/	$3.28e^{-03}$	/	$1.00e^{-01}$	/
E-3DVoro	$3.00e^{-02}$	/	0	/	$9.64e^{-03}$	/	$6.50e^{-02}$	/
TT_1-3DVoro	$1.15e^{-01}$	$2.78e^{-02}$	0	0	$7.90e^{-04}$	$8.78e^{-03}$	$2.32e^{-02}$	$4.94e^{-02}$
TT_2-3DVoro	$9.72e^{-02}$	$9.34e^{-02}$	0	0	$3.21e^{-03}$	$5.16e^{-02}$	$3.50e^{-02}$	$2.15e^{-01}$
TT_3-3DVoro	$4.89e^{-02}$	$1.05e^{-01}$	0	0	$2.83e^{-03}$	$5.91e^{-02}$	$3.56e^{-02}$	$2.62e^{-01}$
P-3DVoro	$1.15e^{-01}$	$1.70e^{-01}$	0	$3.33e^{-02}$	$4.79e^{-03}$	$6.28e^{-02}$	$4.65e^{-02}$	$2.61e^{-01}$
M-3DVoro	$1.15e^{-01}$	$1.52e^{-01}$	0	$8.33e^{-02}$	$4.55e^{-03}$	$7.20e^{-02}$	$5.35e^{-02}$	$3.51e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T								
E-3DVoro	$2.79e^{-01}$	/	0	/	$1.29e^{-01}$	/	$7.14e^{-01}$	/
TT_1-3DVoro	$4.49e^{-01}$	$2.58e^{-01}$	$5.70e^{-03}$	$1.99e^{-03}$	$1.54e^{-01}$	$1.34e^{-01}$	$8.43e^{-01}$	$6.64e^{-01}$
TT_2-3DVoro	$3.71e^{-01}$	$2.98e^{-01}$	$4.28e^{-02}$	$9.34e^{-02}$	$1.61e^{-01}$	$1.40e^{-01}$	$8.07e^{-01}$	$7.90e^{-01}$
TT_3-3DVoro	$9.82e^{-01}$	$3.56e^{-01}$	$4.34e^{-02}$	$6.70e^{-03}$	$1.64e^{-01}$	$1.38e^{-01}$	$7.76e^{-01}$	$7.74e^{-01}$
P-3DVoro	$3.82e^{-01}$	$3.15e^{-01}$	$2.32e^{-03}$	$6.74e^{-03}$	$1.53e^{-01}$	$1.61e^{-01}$	$9.09e^{-01}$	$7.22e^{-01}$
M-3DVoro	$4.08e^{-01}$	$3.77e^{-01}$	$1.89e^{-02}$	$1.47e^{-02}$	$1.90e^{-01}$	$1.74e^{-01}$	$7.91e^{-01}$	$8.18e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of density indicator - $Bi_{HD-HeteroPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$4.30e^{-03}$	/	0	/	$3.40e^{-03}$	/	$1.16e^{-02}$	/
E-3DVoro	$1.55e^{-01}$	/	0	/	$3.56e^{-02}$	/	$4.99e^{-01}$	/
TT_1-3DVoro	$9.60e^{-03}$	$2.31e^{-02}$	0	0	$2.20e^{-03}$	$9.38e^{-03}$	$2.79e^{-02}$	$4.85e^{-02}$
TT_2-3DVoro	$2.04e^{-02}$	$7.66e^{-02}$	0	$4.10e^{-03}$	$5.80e^{-03}$	$4.48e^{-02}$	$6.48e^{-02}$	$1.68e^{-01}$
TT_3-3DVoro	$1.81e^{-02}$	$9.15e^{-02}$	0	$8.00e^{-04}$	$5.70e^{-03}$	$4.51e^{-02}$	$5.42e^{-02}$	$2.15e^{-01}$
P-3DVoro	$2.98e^{-02}$	$1.38e^{-01}$	0	$5.90e^{-03}$	$1.41e^{-02}$	$7.90e^{-02}$	$5.75e^{-02}$	$2.92e^{-01}$
M-3DVoro	$1.88e^{-02}$	$1.46e^{-01}$	0	$2.00e^{-04}$	$5.90e^{-03}$	$1.04e^{-01}$	$5.95e^{-02}$	$3.22e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$5.80e^{-01}$	/	1.02	/	$3.26e^{-01}$	/	1.42	/
E-3DVoro	1.77	/	$4.36e^{-02}$	/	$7.11e^{-01}$	/	1.27	/
TT_1-3DVoro	$5.42e^{-01}$	$5.40e^{-01}$	$2.28e^{-02}$	$2.10e^{-03}$	$3.43e^{-01}$	$3.02e^{-01}$	1.04	$9.66e^{-01}$
TT_2-3DVoro	$5.11e^{-01}$	$5.56e^{-01}$	$1.39e^{-01}$	$8.20e^{-03}$	$3.15e^{-01}$	$3.17e^{-01}$	1.07	1.04
TT_3-3DVoro	$6.08e^{-01}$	$5.52e^{-01}$	$3.72e^{-02}$	$7.50e^{-03}$	$3.29e^{-01}$	$3.18e^{-01}$	1.05	1.05
P-3DVoro	$5.60e^{-01}$	$5.41e^{-01}$	$8.75e^{-02}$	$1.30e^{-03}$	$3.32e^{-01}$	$3.04e^{-01}$	$9.76e^{-01}$	$9.82e^{-01}$
M-3DVoro	$5.03e^{-01}$	$5.43e^{-01}$	$3.93e^{-02}$	$6.91e^{-02}$	$3.76e^{-01}$	$3.15e^{-01}$	1.08	$9.52e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of velocity indicator - $Uni_{LD-HomoPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$1.92e^{-02}$	/	$9.60e^{-03}$	/	$6.20e^{-03}$	/	$3.42e^{-02}$	/
E-3DVoro	$3.17e^{-02}$	/	0	/	$6.30e^{-03}$	/	$3.86e^{-02}$	/
TT_1-3DVoro	$1.57e^{-02}$	$6.18e^{-02}$	0	0	$6.10e^{-03}$	$1.87e^{-02}$	$3.23e^{-02}$	$1.30e^{-01}$
TT_2-3DVoro	$1.83e^{-02}$	$1.38e^{-01}$	0	$1.73e^{-02}$	$7.90e^{-03}$	$4.27e^{-02}$	$3.82e^{-02}$	$3.88e^{-01}$
TT_3-3DVoro	$1.85e^{-02}$	$1.88e^{-01}$	0	$1.00e^{-01}$	$8.00e^{-03}$	$6.46e^{-02}$	$4.08e^{-02}$	$4.87e^{-01}$
P-3DVoro	$2.93e^{-02}$	$2.05e^{-01}$	0	$7.96e^{-02}$	$9.00e^{-03}$	$9.82e^{-02}$	$6.49e^{-02}$	$5.29e^{-01}$
M-3DVoro	$2.14e^{-02}$	$3.16e^{-01}$	0	$5.10e^{-03}$	$8.00e^{-03}$	$1.47e^{-01}$	$4.37e^{-02}$	$8.21e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$5.73e^{-01}$	/	1.15	/	$3.51e^{-01}$	/	1.58	/
E-3DVoro	1.01	/	$8.57e^{-01}$	/	$3.85e^{-01}$	/	1.67	/
TT_1-3DVoro	$5.82e^{-01}$	$5.80e^{-01}$	$8.69e^{-01}$	$5.85e^{-02}$	$4.51e^{-01}$	$3.13e^{-01}$	1.40	1.28
TT_2-3DVoro	$5.76e^{-01}$	$5.67e^{-01}$	$9.40e^{-01}$	$1.02e^{-01}$	$3.75e^{-01}$	$2.64e^{-01}$	1.54	1.16
TT_3-3DVoro	$5.79e^{-01}$	$5.94e^{-01}$	$8.50e^{-01}$	$5.73e^{-02}$	$3.70e^{-01}$	$2.77e^{-01}$	1.46	1.29
P-3DVoro	$5.66e^{-01}$	$5.62e^{-01}$	$8.92e^{-01}$	$4.61e^{-02}$	$3.83e^{-01}$	$2.95e^{-01}$	1.38	1.26
M-3DVoro	$6.27e^{-01}$	$7.11e^{-01}$	$9.13e^{-01}$	$1.43e^{-02}$	$5.05e^{-01}$	$2.86e^{-01}$	1.55	1.49

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of velocity indicator - $Uni_{HD-HeteroPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$1.93e^{-02}$	/	0	/	$1.77e^{-02}$	/	$7.73e^{-02}$	/
E-3DVoro	$1.65e^{-02}$	/	0	/	$5.60e^{-03}$	/	$3.75e^{-02}$	/
TT_1-3DVoro	$3.00e^{-04}$	$7.60e^{-03}$	0	0	0	$2.60e^{-03}$	$8.00e^{-04}$	$1.74e^{-02}$
TT_2-3DVoro	$1.40e^{-03}$	$4.16e^{-02}$	0	0	0	$3.17e^{-02}$	$3.60e^{-03}$	$8.99e^{-02}$
TT_3-3DVoro	$1.30e^{-03}$	$4.65e^{-02}$	0	$4.32e^{-02}$	0	$3.48e^{-02}$	$3.90e^{-03}$	$1.14e^{-01}$
P-3DVoro	$2.70e^{-03}$	$4.69e^{-02}$	0	$1.41e^{-02}$	$8.00e^{-04}$	$2.27e^{-02}$	$5.50e^{-03}$	$1.29e^{-01}$
M-3DVoro	$1.20e^{-03}$	$5.09e^{-02}$	0	$4.75e^{-02}$	0	$3.54e^{-02}$	$2.50e^{-03}$	$1.23e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$2.55e^{-01}$	/	$1.45e^{-01}$	/	$2.45e^{-01}$	/	$5.06e^{-01}$	/
E-3DVoro	$4.17e^{-01}$	/	$6.50e^{-02}$	/	$1.27e^{-01}$	/	$3.83e^{-01}$	/
3DVoro-δ_{TT_1}	$1.74e^{-01}$	$1.50e^{-01}$	$1.79e^{-01}$	$8.00e^{-04}$	$1.13e^{-01}$	$8.77e^{-02}$	$3.21e^{-01}$	$2.98e^{-01}$
TT_1-3DVoro	$2.07e^{-01}$	$1.53e^{-01}$	$1.92e^{-01}$	$1.00e^{-04}$	$1.39e^{-01}$	$8.52e^{-02}$	$3.71e^{-01}$	$3.29e^{-01}$
TT_2-3DVoro	$2.33e^{-01}$	$1.52e^{-01}$	$2.05e^{-01}$	$3.00e^{-04}$	$1.48e^{-01}$	$8.46e^{-02}$	$3.63e^{-01}$	$3.27e^{-01}$
TT_2-3DVoro	$2.17e^{-01}$	$1.43e^{-01}$	$1.53e^{-01}$	$1.40e^{-03}$	$1.34e^{-01}$	$8.49e^{-02}$	$3.01e^{-01}$	$2.98e^{-01}$
M-3DVoro	$1.75e^{-01}$	$1.48e^{-01}$	$1.83e^{-01}$	$1.00e^{-04}$	$1.36e^{-01}$	$9.11e^{-02}$	$3.43e^{-01}$	$3.22e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of flow indicator - $Uni_{LD-HomoPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$2.75e^{-02}$	/	$2.30e^{-03}$	/	$1.75e^{-02}$	/	$7.21e^{-02}$	/
E-3DVoro	$1.09e^{-02}$	/	0	/	$8.70e^{-04}$	/	$2.83e^{-02}$	/
TT_1-3DVoro	$7.80e^{-03}$	$6.06e^{-02}$	0	0	$7.00e^{-04}$	$1.21e^{-02}$	$2.22e^{-02}$	$1.58e^{-01}$
TT_2-3DVoro	$1.05e^{-02}$	$1.45e^{-01}$	0	0	$1.10e^{-03}$	$6.08e^{-02}$	$2.78e^{-02}$	$3.11e^{-01}$
TT_3-3DVoro	$1.06e^{-02}$	$2.03e^{-01}$	0	0	$1.00e^{-03}$	$8.27e^{-02}$	$2.19e^{-02}$	$4.64e^{-01}$
P-3DVoro	$1.62e^{-02}$	$1.95e^{-01}$	0	$4.86e^{-02}$	$1.80e^{-03}$	$8.54e^{-02}$	$3.70e^{-02}$	$4.90e^{-01}$
M-3DVoro	$1.29e^{-02}$	$3.06e^{-01}$	0	0	$1.60e^{-03}$	$1.48e^{-01}$	$2.92e^{-02}$	$8.95e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$5.18e^{-01}$	/	$3.50e^{-01}$	/	$4.48e^{-01}$	/	1.09	/
E-3DVoro	$6.54e^{-01}$	/	$3.69e^{-01}$	/	$2.03e^{-01}$	/	1.54	/
TT_1-3DVoro	$4.99e^{-01}$	$4.02e^{-01}$	$1.06e^{-01}$	$6.49e^{-02}$	$3.24e^{-01}$	$1.81e^{-01}$	1.35	$9.43e^{-01}$
TT_2-3DVoro	$5.66e^{-01}$	$4.16e^{-01}$	$1.47e^{-01}$	$5.55e^{-02}$	$2.73e^{-01}$	$1.73e^{-01}$	1.57	1.21
TT_3-3DVoro	$5.91e^{-01}$	$4.45e^{-01}$	$1.53e^{-01}$	$1.57e^{-01}$	$2.94e^{-01}$	$1.71e^{-01}$	1.68	1.31
P-3DVoro	$4.81e^{-01}$	$4.28e^{-01}$	$5.53e^{-02}$	$3.98e^{-02}$	$2.22e^{-01}$	$1.89e^{-01}$	1.34	1.12
M-3DVoro	$6.41e^{-01}$	$4.47e^{-01}$	$9.07e^{-02}$	$4.55e^{-02}$	$3.97e^{-01}$	$1.73e^{-01}$	1.66	1.24

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of flow indicator - $Uni_{HD-HeteroPop}$

3DVoro: Robustness to sampling frequency

Interpolation

Higher sampling frequency

Time-Transform distances lead to the best performance (TT1-3DVoro)

Samples

Lower sampling frequency

$Uni_{LD-HomoPop}$: the distances that take into account the speed and/or direction of pedestrians (TT_2-3DVoro, P-3DVoro and M-3DVoro)

$Uni_{HD-HeteroPop}$: Time-Transform distances (TT_1-3DVoro)

General

Time-Transform: more data available (the sampling frequency equal to 3 s^{-1} or the demand equal to 3.6 pedestrians per second)

Distances accounting for the dynamics: less data available (the sampling frequency equal to 0.5 s^{-1} and the demand equal to 1.2 pedestrians per second)