Pedestrian-oriented flow characterization

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Pedestrian and Evacuation Dynamics conference (PED 2014)

October 24, 2014

Introduction

Objective

- Definitions of flow characteristics by adapting Edie's definitions
  - Stream-based approach
  - Data-driven discretization framework

Motivation

- Definitions and measurement methods currently available in the literature
  - Mostly fail to account for the multidirectional nature of pedestrian flows
  - Rely on arbitrarily chosen space and time discretization
- Realistic flow characterization important to many areas

Edie's definitions

- Density

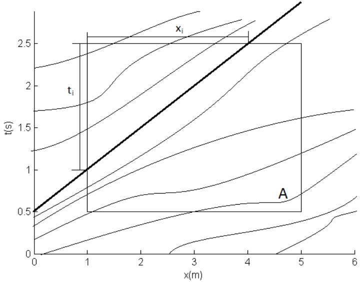
$$k(A) = \frac{\sum_i t_i}{|A|}$$

- Flow

$$q(A) = \frac{\sum_i x_i}{|A|}$$

- Speed

$$v(A) = \frac{\sum_i x_i}{\sum_i t_i}$$



L.C. Edie, Discussion of Traffic Stream Measurements and Definitions, *Proceedings of the Second International Symposium on the Theory of Traffic Flow, Paris, OECD, 1965*

Edie's definitions in 3D

- Density

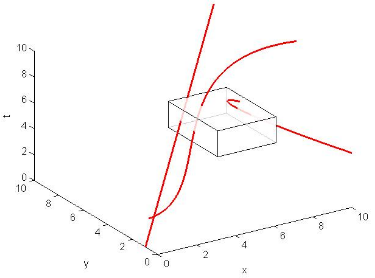
$$k(V) = \frac{\sum_i t_i}{Vol(V)}$$

- Flow

$$q(V) = \frac{\sum_i d_i}{Vol(V)}$$

- Speed:

$$v(V) = \frac{\sum_i d_i}{\sum_i t_i}$$



Saberi, M., and Mahmassani, H. (2014) Exploring Area-Wide Dynamics of Pedestrian Crowds Using a Three-Dimensional Approach, *Transportation Research Record: Journal of the Transportation Research Board*.

Sample of points

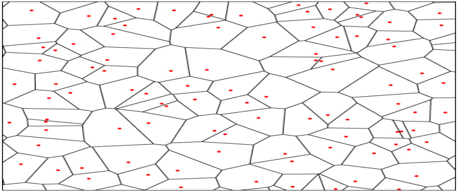
- $p_i^1, \ldots, p_i^{n(i)}$, where $n(i)$ is the number of data for pedestrian i
- For each observed point $p_i^k = (x_i^k, y_i^k, t_i^k)$ the trajectory is $p(t_i^k) = p_i^k$
- Many trajectories can interpolate the same set of points
  - Interpolation is not necessary if a discretization is data-driven
- Voronoi based space-time discretization

Data-driven discretization framework

Two-dimensional Voronoi diagrams

- $p_1, p_2, ..., p_N$ is a finite set of points
- Voronoi space decomposition assigns a region to each point

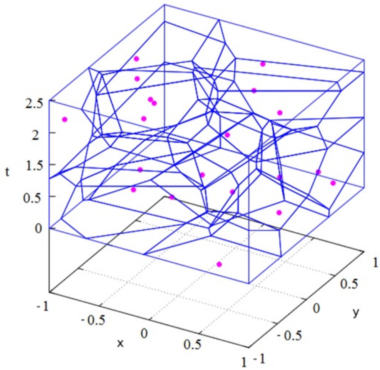
$$V(p_i) = \{p | \, \|p - p_i\| \leq \|p - p_j\|, i \neq j\}$$



Steffen, B., and Seyfried, A., Methods for measuring pedestrian density, flow, speed and direction with minimal scatter, *Physica A: Statistical mechanics and its applications, 389(9), 1902-1910.*

Data-driven discretization framework

Three-dimensional Voronoi diagrams



Data-driven discretization framework

Three-dimensional Voronoi diagrams

- Pedestrian i represented by $p_i = (x_i, y_i, t_i)$ and point $p = (x, y, t)$
- Space-time distance

$$d_\alpha = \sqrt{(x_i - x)^2 + (y_i - y)^2 + \alpha^2 (t_i - t)^2}$$

  α - 1 second of time is equivalent to α meters of distance

- Three-dimensional Voronoi diagram

$$V(p_i) = V_i = \{p | d_\alpha(p, p_i) \leq d_\alpha(p, p_j), j \neq i\}$$

- $Vol(V_i)$ - the volume of a Voronoi cell V_i associated with the point p_i with the unit square meters times seconds

Voronoi-based Edie's definitions

Density indicator

- The set of all points in V_i corresponding to a given location (x_i, y_i) is a set of dimension 1 - a time interval

$$V_i(x_i, y_i) = \{(x_i, y_i, t) \in V_i\}$$

- $V_i(x_i, y_i)$ - the time interval that the pedestrian i occupies the location (x_i, y_i)

$$k(p_i) = \frac{V_i(x_i, y_i)}{Vol(V_i)}$$

Voronoi-based Edie's definitions

Flow indicator

- The set of all points in V_i corresponding to a specific time t_i is a set of dimension 2 - a physical area on the floor

$$V_i(t_i) = \{(x, y, t_i) \in V_i\}$$

- Distance d_i - a maximum distance in $V_i(t_i)$ in the movement direction of pedestrian i

$$q(p_i) = \frac{d_i}{Vol(V_i)}$$

Speed indicator

$$v(p_i) = \frac{d_i}{V_i(x_i, y_i)}$$

Voronoi-based Edie's definitions

Disaggregated

$$k(p_i) = \frac{V_i(x_i,y_i)}{Vol(V_i)}$$

$$q(p_i) = \frac{d_i}{Vol(V_i)}$$

$$v(p_i) = \frac{d_i}{V_i(x_i,y_i)}$$

Aggregated

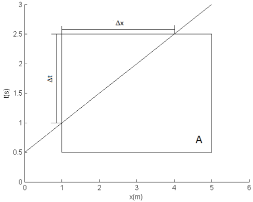
$$k(V) = \frac{\sum_i k(p_i) \cdot Vol(V_i)}{\sum_i Vol(V_i)}$$

$$q(V) = \frac{\sum_i q(p_i) \cdot Vol(V_i)}{\sum_i Vol(V_i)}$$

$$v(V) = \frac{\sum_i q(p_i) \cdot Vol(V_i)}{\sum_i k(p_i) \cdot Vol(V_i)}$$

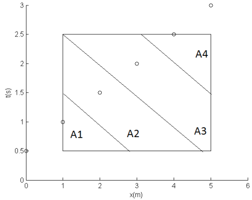
Asymptotic analysis

Edie



$$k(A) = \frac{\Delta t}{|A|}$$
$$q(A) = \frac{\Delta x}{|A|}$$
$$v(A) = \frac{\Delta x}{\Delta t}$$

Voronoi



$$k(A) = \frac{\sum_i \Delta t_i}{|A|}$$
$$q(A) = \frac{\sum_i \Delta x_i}{|A|}$$
$$v(A) = \frac{\sum_i \Delta x_i}{\sum_i \Delta t_i}$$

Asymptotic analysis

Two-dimensional case

- Sampling interval $\Delta t_s \rightarrow 0$
- Path L specified in parametric form: $x = x(t), t \in [\alpha, \beta]$

$$\lim_{\Delta t_i \rightarrow 0} \sum_i \Delta t_i = \int_L dt = \Delta t$$

$$\lim_{\Delta x_i \rightarrow 0} \sum_i \Delta x_i = \int_L dx = \int_L \dot{x} dt = \Delta x$$

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Asymptotic analysis

Three-dimensional case

- Sampling interval $\Delta t_s \to 0$
- Pedestrian identifier n
- Path L_n specified in parametric form:

$$x_n = x_n(t), y_n = y_n(t), t_n \in [\alpha_n, \beta_n]$$

$$\lim_{\Delta t_i \to 0} \sum_n \sum_i \Delta t_i = \sum_n \int_{L_n} dt = \sum_n \Delta t_n$$

$$\lim_{\Delta x_i \to 0} \sum_n \sum_i \Delta d_i = \sum_n \int_{L_n} \sqrt{\dot{x}^2 + \dot{y}^2} dt = \sum_n \Delta d_n$$

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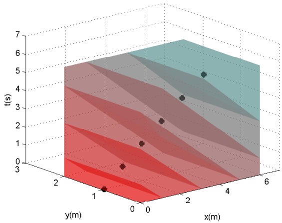
Simulation experiment

Single pedestrian

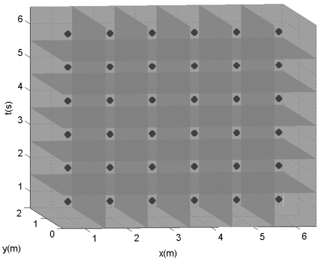
$$p(t) = (x(t), y(t), t) = (0.02t^2 + 0.9t + 0.1, 1, t)$$

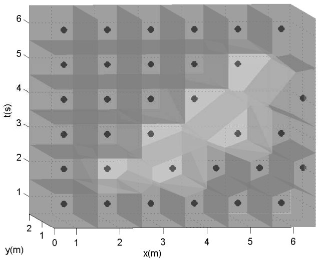
Inst. speed	Voronoi	Analytical trajectory
v_1	1.196	0.902
v_2	1.031	0.940
v_3	1.020	0.980
v_4	0.980	1.020
v_5	0.943	1.060
v_6	0.913	1.082



Simulation experiment

Density map





- Reproduced movement with uniform and non-uniform density
- Smooth transitions in flow characteristics over space and time

Stream-based approach

- Pedestrian traffic composed of different streams
- A stream definition: direction-based and exogenous

$$(\varphi_j)_{j=1}^{S}, S \geq 2$$

- Trajectories are assumed to contribute to the streams to some extent
- The contribution is related to the angle between a movement direction of a pedestrian and the corresponding stream

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Stream-based approach

- Pedestrian trajectory: $p(t) = (x(t), y(t), t)$
- Tangential direction associated with each point $p(t)$ of a trajectory

$$\nabla p(t) = (\frac{dx(t)}{dt}, \frac{dy(t)}{dt}, 1)$$

- Pedestrian movement direction - normalized vector e composed of the first two components of $\nabla p(t)$
- The individual contribution to the stream

$$c_i^{\varphi_j} = \begin{cases} \|e\| \, \|\varphi_j\| \, cos\theta & : 0^\circ < \theta \leq 90^\circ \\ 0 & : 90^\circ < \theta \leq 180^\circ. \end{cases}$$

θ - the angle between the vectors e and φ_j

Stream-based Voronoi definitions

Disaggregated

$$k(p_i) = \frac{V_i(x_i, y_i)}{Vol(V_i)}$$

$$q_{\varphi_j}(p_i) = \frac{d_i^{\varphi j}}{Vol(V_i)} \varphi j$$

$$v_{\varphi_j}(p_i) = \frac{d_i^{\varphi j}}{V_i(x_i, y_i)} \varphi j$$

Aggregation

$$k(V) = \frac{\sum_i k(p_i) \cdot Vol(V_i)}{\sum_i Vol(V_i)}$$

$$q_{\varphi_j}(V) = \frac{\sum_i q_{\varphi_j}(p_i) \cdot Vol(V_i)}{\sum_i Vol(V_i)} \varphi j$$

$$v_{\varphi_j}(V) = \frac{\sum_i q_{\varphi_j}(p_i) \cdot Vol(V_i)}{\sum_i k(p_i) \cdot Vol(V_i)} \varphi j$$

Conclusions

- The framework for pedestrian-oriented flow characterization
- Definitions based on data-driven discretization
  - Asymptotically consistent with Edie's definitions
  - Smooth transition in measured characteristics from point to point in 3D
- Stream-based approach to account for the multidirectional nature of pedestrian flows

Future research

- More numerical analysis needed
- Investigation of the role of conversion constant α
- Stream-based fundamental relationships for pedestrians
- Case study: Gare de Lausanne

Thank you

Measurement methods

Method A

- A reference location in space (x) is considered
- The mean value of q and v are calculated over time (Δt)

$$q = \frac{n}{\Delta t}, \; v = \frac{1}{n}\sum_i v_i(t)$$

n - number of pedestrians passing the location x during Δt
$v_i(t)$ - instantaneous speed of pedestrian i

Zhang, J., 2012. Pedestrian fundamental diagrams: Comparative analysis of experiments in different geometries. volume 14. Forschungszentrum Jülich

Method B

- The measures of k and v are averaged over time (Δt) and space

$$k = \frac{1}{\Delta t} \int_t \frac{n}{b \Delta x} dt, \; v = \frac{\sum_i v_i}{n}$$

$b, \Delta x$ - width and length of the measurement area

$v_i = \frac{\Delta x}{\Delta t_i}$ - individual space-mean speed

Zhang, J., 2012. Pedestrian fundamental diagrams: Comparative analysis of experiments in different geometries. volume 14. Forschungszentrum Jülich

Measurement methods

Method C

- The measures of k and v are specified per space unit

$$k = \frac{n}{b\Delta x}, \; v = \frac{\sum\limits_i v_i}{n}$$

$b, \Delta x$ - width and length of the measurement area

$v_i = \frac{\Delta x}{\Delta t_i}$ - individual space-mean speed

Zhang, J., 2012. Pedestrian fundamental diagrams: Comparative analysis of experiments in different geometries. volume 14. Forschungszentrum Jülich

Measurement methods

Method D

- The measures of k and v are specified via Voronoi diagrams

$$k = \frac{\int\int \rho_{xy} dxdy}{\Delta x \Delta y}, \quad v = \frac{\int\int v_{xy} dxdy}{\Delta x \Delta y}$$

$\rho_{xy} = \frac{1}{A_i}$, A_i - area of Voronoi cell associated to pedestrian i

v_{xy} - instantaneous speed of pedestrian i

Zhang, J., 2012. Pedestrian fundamental diagrams: Comparative analysis of experiments in different geometries. volume 14. Forschungszentrum Jülich