UFO

Uncertainty **F**eature **O**ptimization, an Implicit Paradigm for Problems with Noisy Data

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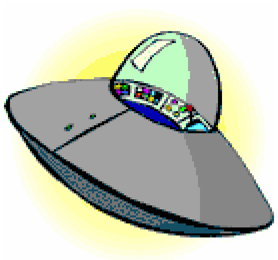

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Schedule

- Motivations
- The UFO framework
- Existing Methods seen as UFO
- Example: Problem with **M**ultiple **K**napsack **C**onstraints (MKC)
- Preliminary Simulation Results for MKC
- Future Work and Conclusions

Motivation

- Uncertainty should not be neglected
- Uncertainty is hard to characterize exactly
- Problems under uncertainty are hard to solve in general
- Few guarantees on real solution

Four Approaches

1. Neglect and solve deterministic problem

   ➢ Not realistic (Herroelen 2005, Sahinidis 2004)

Four Approaches

1. Neglect and solve deterministic problem
2. On-line Optimization
   - Data-driven
   - Not feasible for some problems (e.g. airline schedules)

Four Approaches

1. Neglect and solve deterministic problem
2. On-line Optimization
3. Characterize the Uncertainty and solve robust or stochastic problems
   - Need explicit Uncertainty characterization
   - Hard to characterize/model in general
   - Leads to difficult problems
   - Solutions tend to "simple" properties

Examples from Airline Scheduling

- Increase plane's idle time (Al-Fawzana & Haouari 2005)
- Decrease plane rotation length (Rosenberger et al. 2004)
- Departure de-peaking (Jiang 2006, Frank et al. 2005)
- More plane crossings (Bian et al. 2004, Klabjan et al. 2002)
- …

Four Approaches

1. Neglect and solve deterministic problem
2. On-line Scheduling
3. Characterize the Uncertainty
4. Model Uncertainty Implicitly => Uncertainty Features

Uncertainty **F**eature **O**ptimization

I. Increase robustness/stability (e.g. idle time)

II. Increase recoverability (e.g. plane crossings)

UF: Definition

Given a problem with Decision Variables ***x***

UF: a function $\mu(\boldsymbol{x})$ measuring the "*quality*" of a solution ***x***

OBJECTIVE: MAX $\mu(\boldsymbol{x})$

s.t. ***x*** feasible solution to initial problem

How to Derive an **UF** ?

Know WHAT changes, not HOW

- UF is problem dependent
- use practitioner's experience/intuition

If recovery strategy is known

- seek UF improving recovery's performance

$\Rightarrow$ RECOVERABILITY

General Optimization Problem

$$MIN \quad f(x)$$

$$s.t. \qquad a(x) \leq b$$

$$x \in X$$

UFO: Multi-Objective Problem

$$
\begin{aligned}
&OPT \quad [f(x), \mu(x)] \\
&s.t. \qquad a(x) \leq b \\
&\qquad\qquad x \in X
\end{aligned}
$$

UF and Optimality Budget

$\mu(\boldsymbol{x})$	Uncertainty Feature
f^*	Original Optimum
ρ	Maximal Optimality Gap

UFO with Budget Relaxation

$$MAX\ \mu(x)$$

$$s.t. \quad a(x) \leq b$$

$$f(x) \leq (1+\rho)f^*$$

$$x \in X$$

UFO Properties

I. Complexity not changed if $\mu(x)$ similar to $f(x)$
II. Implicit modeling of uncertainty
III. Differentiate solutions on optimal facet
IV. “Plug” tool for any existing method
V. Can use UF based on explicit uncertainty set
VI. Generalizes existing methods

Stochastic Problem as an **UFO**

Given an Uncertainty Set ***U*** with a probability measure on it

$$\begin{aligned} \min \quad & E_U\{f(\boldsymbol{x})\} \\ s.t. \quad & a(x) \leq b \\ & x \in X \end{aligned}$$

Stochastic problem as an **UFO**

$$MAX\ \mu(x) = -E_U\{f(x)\}$$

$$s.t. \quad a(x) \leq b$$

$$f(x) \leq (1 + \infty) f^*$$

$$x \in X$$

Robust problem as an **UFO**

Original LP Problem

$$MAX\ \boldsymbol{c}^T\boldsymbol{x}$$

$$s.t.\quad A\boldsymbol{x} \leq \boldsymbol{b}$$
$$\boldsymbol{x} \in X$$

Robust problem as an **UFO**

Formulation of Bertsimas and Sim (2004)

$$MAX\ \mathbf{c}^T\mathbf{x}$$

$$s.t. \quad A\mathbf{x} + \boldsymbol{\beta}(\mathbf{x},\boldsymbol{\Gamma}) \leq \mathbf{b}$$

$$\mathbf{x} \in X$$

$$\tilde{a}_{ij} \in [a_{ij} - \hat{a}_{ij}; a_{ij} + \hat{a}_{ij}] \quad \forall\ j \in J_i$$

$$\beta_i(x, \Gamma_i) = \max_{\{S_i \cup \{t_i\} | S_i \in J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}}$$

$$\left\{ \sum_{j \in S_i} \hat{a}_{ij} \mid x_j \mid + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} \mid x_{t_i} \mid \right\}$$

Start with Feasibilty Problem

$$f^* = MIN \quad f(\boldsymbol{x})$$

$$= MIN \; [A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \mathbf{J})] - \boldsymbol{b}$$

$$s.t. \qquad \boldsymbol{x} \in X$$

Define **UF** and budget

$$\mu(\boldsymbol{x}) = \boldsymbol{c}^T \boldsymbol{x} \qquad \rho = \max_i \left\{ \frac{\rho_i f_i(\boldsymbol{x}^*)}{f^*} - 1 \right\}$$

Where

$$\rho_i = \begin{cases} \dfrac{\bar{\beta}_i(\boldsymbol{x}, \Gamma_i)}{f_i(x^*)} \\ 0 \quad \text{if} \quad f_i(\boldsymbol{x}^*) = 0 \end{cases}$$

and

$$\boldsymbol{\beta}(\boldsymbol{x}, \mathbf{J}) = \boldsymbol{\beta}(\boldsymbol{x}, \boldsymbol{\Gamma}) + \bar{\boldsymbol{\beta}}(\boldsymbol{x}, \boldsymbol{\Gamma})$$

UFO formulation

$$MAX\ \mu(x) = \boldsymbol{c}^T \boldsymbol{x}$$

$$s.t. \quad [A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \mathbf{J})] - \boldsymbol{b} \leq (1+\rho)f^*$$

$$\boldsymbol{x} \in X$$

Replace Elements in Constraint

$$[Ax + \boldsymbol{\beta}(x, \mathrm{J})] - \boldsymbol{b} \leq (1+\rho) f^*$$

=

$$[Ax + \boldsymbol{\beta}(x, J)] - \boldsymbol{b} \leq \overline{\boldsymbol{\beta}}(x, \Gamma)$$

Which is equivalent to

$$Ax + \boldsymbol{\beta}(x, J) - \overline{\boldsymbol{\beta}}(x, \Gamma) \leq \boldsymbol{b}$$

=

$$Ax + \boldsymbol{\beta}(x, \Gamma) \leq \boldsymbol{b}$$

Retrieve Robust Formulation

$$MAX\ \mu(x) = \boldsymbol{c}^T \boldsymbol{x}$$

$$s.t. \quad A\boldsymbol{x} + \boldsymbol{\beta}(\boldsymbol{x}, \boldsymbol{\Gamma}) \leq \boldsymbol{b}$$

$$\boldsymbol{x} \in X$$

Q.E.D.

BONUS

Gives methodology to compute maximal values of Γ to ensure a robust solution exists.

Multiple **K**napsack **C**onstraints

$$\max \sum_{i=1}^{N} c_i x_i$$

$$s.t. \quad \sum_{j=1}^{N} a_{ij} x_i \leq b_i \quad \forall i \; = 1, \ldots, M$$

$$\boldsymbol{x} \in \mathbb{Z}_+^N$$

MKC with Max Taken Object **UFO**

$$\min \left\{ \mu(\boldsymbol{x}) = \max_{i=1,\dots,N} \{x_i\} \right\}$$

$$s.t. \quad A\boldsymbol{x} \leq \boldsymbol{b}$$

$$\boldsymbol{c}^T \boldsymbol{x} \geq (1-\rho)c^*$$

$$\boldsymbol{x} \in \mathbb{Z}_+^N$$

Other derived **UF**

- Max Taken (MTk): $\mu(\boldsymbol{x}) = \max_{i=1,\ldots,N} \{x_i\}$
- Diversification (Div): $\mu(\boldsymbol{x}) = \sum(\min \{1, x_i\})$
- Impact Ratio (IR): $\mu(\boldsymbol{x}) = -\max_{i} \frac{a_{ij} x_j}{b_i}$
- 2Sum: $\mu(\boldsymbol{x}) = -\max_{i,j \neq k} \frac{a_{ij} x_j + a_{ik} x_k}{b_i}$

MKC Simulator

- Generation of problems
- Solve Models inc. Robust (combining possible)
- Simulation with user-defined parameters

Simulation Results

- Partial results on 8 of 24 classes
- Classes according to
    i. Cost-correlated *A* matrix
    ii. Granularity
    iii. Number of Constraints
    iv. Number of varying coefficients

Simulation Results

- Simulations according to
  a. Exact variability matrix $\hat{A}$
  b. Variability matrix based on A
  c. Random Variability Matrix
  d. High or Low variances

MKC – Results III

	ROBUST Â	ROBUST 0.1*A	IR rho = 10%	_MTk + 2Sum rho = 10%	Div rho = 10%
Nbr Unfeasible	21470	25763	40680	40841	50212
% of unfeasible	25.56	30.67	48.43	48.62	59.78
Avg Optimality Gap (%)	18.07	11.66	9.25	9.23	7.94
Max Optimiality Gap (%)	98.13	96.89	96.4	96.4	96.89

MKC – Results IV

Used Model	ROBUST Â			_MTk rho = 10%		
Var. Matrix	Â	0.2*A	RANDOM	Â	0.2*A	RANDOM
Nbr Unfeasible	2	15	761	47	117	1186
% of unfeasible	<0.01	1.00	50.73	3.13	7.8	79.07
Avg Optimality Gap (%)	3.95	4.10	9.59	9.67	9.45	6.57
Max Optimiality Gap (%)	37.43	33.55	90.01	24.97	24.98	89.22

Future Work

- Extended Tests on MKC
- Application of UFO to Airline Transportation
- Find an UF generator ?

Conclusions

- UFO allows to cope with uncertainty IMPLICITLY
- Use explicit uncertainty model is still possible
- UFO can be combined with any already existing method
- It is NOT an alien method !

THANKS for your attention

Any Questions?