A unified modeling and solution framework for stochastic routing problems

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1 Introduction

2 Capturing demand uncertainty

3 Optimization model

4 Methodology

5 Numerical experiments

6 Conclusion
Outline

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Problem description

- Sensorized containers transmit level data to the server.
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- Level data is used for demand forecasting and tour planning over a finite planning horizon.
- Vehicles perform the resulting tours.
- Solving this inventory routing problem involves
  - deciding which containers to visit each day
  - and optimizing the collection tours.
Introduction

Daily tour structure

Figure 1: Basic vehicle tour
Information uncertainty

- Information-wise, the problem is:
  - stochastic due to uncertain demands with distributional information
  - dynamic due to their periodic revelation
Information uncertainty

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  - *stochastic* due to uncertain demands with distributional information
  - *dynamic* due to their periodic revelation

- Thus, we can apply a *rolling horizon* approach:
  1. solve the problem for the planning horizon
  2. implement the first day decisions
  3. roll over and solve for updated levels and forecasts
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- *Solving the problem day by day in isolation leads to myopic decisions.*
Main approaches in the literature:

- stochastic programming, MDP (Pillac et al., 2013)
- approximate dynamic programming (Powell, 2011)
- robust optimization (Bertsimas and Sim, 2003, 2004)
- chance constraints (Gendreau et al., 2014)
Related literature and contributions

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  - robust optimization (Bertsimas and Sim, 2003, 2004)
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- Characteristics of our approach:
  - unified approach with few distributional assumptions
  - explicit modeling of *undesirable events* and *recourse actions*
  - cost-oriented with priced risk
  - applicable to rich real-world problems
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Demand forecasting

Sets

- $\mathcal{K}$: set of vehicles
- $\mathcal{T}$: set of days in the planning horizon
- $\mathcal{P}$: set of containers
Demand forecasting

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Forecasting model

- stochastic non-stationary demand $\rho_{it}$ for container $i \in \mathcal{P}$ on day $t \in \mathcal{T}$:

$$\rho_{it} = \mathbb{E}(\rho_{it}) + \varepsilon_{it} \quad (1)$$

- combine $\varepsilon_{it}$ in a vector:

$$\varepsilon = (\varepsilon_{11}, \ldots, \varepsilon_{1|\mathcal{T}|}, \varepsilon_{21}, \ldots, \varepsilon_{|\mathcal{P}||\mathcal{T}|}) \quad (2)$$

- let $\varepsilon \sim \Phi$ with $\text{var}(\varepsilon) = K$ that can be simulated
- use any forecasting model that provides $\mathbb{E}(\rho_{it})$ and $\Phi$
Inventory policy

Context

- Order-Up-to (OU) level policy \cite{Bertazzi2002}
- Maximum Level (ML) policy \cite{Archetti2011}
Inventory policy

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- Order-Up-to (OU) level policy (Bertazzi et al., 2002)
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Discretized ML policy
- for tractable pre-processing of stochastic information
- $\Lambda_{it}$: inventory after collection of container $i$ on day $t$

Figure 2: Discretized ML policy example
Capturing demand uncertainty

Undesirable events

Container overflows

- $\sigma_{it} = 1$ for overflow of container $i$ on day $t$, 0 otherwise
- entails an overflow cost
- *recourse*: emergency collection with a cost
## Undesirable events

### Container overflows
- $\sigma_{it} = 1$ for overflow of container $i$ on day $t$, 0 otherwise
- entails an overflow cost
- *recourse*: emergency collection with a cost

### Route failures
- inability to complete a depot-to-dump or dump-to-dump trip $S$
- due to insufficient vehicle capacity
- *recourse*: detour to the nearest dump with a cost
Overflow probabilities

- Overflow probability of container $i$ on day $t$:
  \[
  p_{it}^{DP} = P(\sigma_{it} = 1 \mid \Lambda_{im} : m = \max(0, g < t : \exists k \in K : y_{ikg} = 1))
  \]  
  \( (3) \)

  where:
  - $y_{ikg}$ 1 if vehicle $k$ visits container $i$ on day $g$, 0 otherwise
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- For a discretized ML policy, expression (3) can be pre-computed for $\varepsilon \sim \Phi$ with $\text{var}(\varepsilon) = K$ using simulation.
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- For a discretized ML policy, expression (3) can be **pre-computed** for $\varepsilon \sim \Phi$ with $\text{var}(\varepsilon) = K$ using **simulation**.

- The complexity is linear in the number of discrete levels.
Route failure probabilities

Route failure probability of trip $S$ performed by vehicle $k$:

$$p_{S,k}^{RF} = \mathbb{P}(\Gamma_S > \Omega_k)$$  \hspace{1cm} (4)

where:
- $\Gamma_S$  \hspace{0.5cm} collection quantity in trip $S$
- $\Omega_k$  \hspace{0.5cm} capacity of vehicle $k$
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- Use simulation to derive an ECDF of the error of $\Gamma_S$, $\forall S$, the latter being sums of $\varepsilon_{it}$. 
Route failure probabilities

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  where:
  - \( \Gamma_S \)  collection quantity in trip \( S \)
  - \( \Omega_k \)  capacity of vehicle \( k \)

- Can be partially pre-processed for any iid error terms \( \varepsilon_{it} \).

- Use simulation to derive an ECDF of the error of \( \Gamma_S \), \( \forall S \), the latter being sums of \( \varepsilon_{it} \).

- Use the ECDFs at runtime to approximate route failure probabilities.
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Principal cost components, I

Routing cost

- daily deployment cost
- travel distance related cost
- travel, service and waiting time related cost
Expected overflow and emergency collection cost

\[ \sum_{t \in T \cup T^+} \sum_{i \in P} \left( \chi + \zeta - \zeta \sum_{k \in K} y_{ik} \right) p_{it}^{DP} \quad (5) \]

where:

- \( \chi \) overflow cost
- \( \zeta \) emergency collection cost
Expected route failure cost

\[
\sum_{t \in T \setminus 0} \sum_{k \in K} \sum_{S \in \mathcal{S}_{kt}} \psi C_S p_{S,k}^{RF}
\]  

(6)

where:
- \( \mathcal{S}_{kt} \) set of trips performed by vehicle \( k \) on day \( t \)
- \( C_S \) dump detour cost for trip \( S \)
- \( \psi \) route failure cost multiplier
Objective function

Components:
- routing cost
- expected overflow and emergency collection cost
- expected route failure cost
- various deterministic cost components (inventory holding, number of visits, workload balancing)
Objective function

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Overestimates the real cost:
- due to modeling simplifications
- for tractability reasons
- do-nothing vs. optimal reaction policy
Deterministic constraints

- accessibility restrictions
- vehicle capacity and dump visits
- time windows
- maximum tour duration
- periodicities and service choice
- inventory tracking and container capacity
- inventory policy definition
- etc...
Probabilistic constraints

- Capture stochasticity in the constraints instead of the objective.
Probabilistic constraints

- Capture stochasticity in the constraints instead of the objective.
- Maximum overflow probability, for a constant $\gamma^{DP} \in (0, 1]$:
  \[ p^{DP}_{it} \leq \gamma^{DP} \quad \forall t \in T, i \in P \]  
  (7)
- Maximum route failure probability, for a constant $\gamma^{RF} \in (0, 1]$:
  \[ p^{RF}_{S,k} \leq \gamma^{RF} \quad \forall t \in T, k \in K, S \in S_{kt} \]  
  (8)
Applications

Stochastic demand problems

- vehicle routing
- waste collection inventory routing
- supermarket delivery routing
- fuel delivery routing
- home health care routing
- maritime inventory routing
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Probability-based routing problems

- e.g. facility maintenance
- facility breakdown probability grows with number of days since last visit
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Adaptive large neighborhood search

- State-of-the-art meta-heuristic (Ropke and Pisinger, 2006a,b).
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- Rich operator pools:
  - diversification vs. intensification
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- Admits intermediate infeasibilities.
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- Rich operator pools:
  - diversification vs. intensification
- Admits intermediate infeasibilities.
- Performance:
  - competitive on benchmarks (Archetti et al., 2007)
  - stable: 0-3% between best and worst over 10 runs
  - fast: 10-15 min. per instance; operational speed
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Waste collection case study

- Waste collection IRP instances:
  - 63 realistic instances from Geneva, Switzerland
  - rich routing features
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- Compare probabilistic policies varying the:
  - Emergency Collection Cost \((ECC)\)
  - Route Failure Cost Multiplier \((RFCM)\)

Simulate undesirable events on final solution.
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- Compare probabilistic policies varying the:
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- Against buffer capacity deterministic policies varying the:
  - Container Effective Capacity (CEC)
  - Truck Effective Capacity (TEC)
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Waste collection: Service area

Figure 3: Geneva service area
Waste collection: Policy comparison

Figure 4: Routing cost and overflows for probabilistic and deterministic policies

(a) Routing Cost

(b) Overflows
Waste collection: Rolling horizon

- **Static Deterministic IRP (SD-IRP):**
  - true demands; solve for planning horizon
Waste collection: Rolling horizon

- **Static Deterministic IRP (SD-IRP):**
  - true demands; solve for planning horizon

- **Static Stochastic IRP (SS-IRP):**
  - forecast demands; solve for planning horizon

Hypothesize:

\[ z_{\text{SS-IRP}} \geq z_{\text{DSIRP}} \geq z_{\text{SD-IRP}} \]
Waste collection: Rolling horizon

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- Dynamic and Stochastic IRP \((DSIRP)\):
  - forecast demands; rolling horizon approach over planning horizon
Numerical experiments

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Hypothesize:
- \( z(\text{SS-IRP}) \geq z(\text{DSIRP}) \geq z(\text{SD-IRP}) \)
Waste collection: Rolling horizon

Figure 5: Analysis of rolling horizon bounds

(a) $z(\text{DSIRP}) \geq z(\text{SD-IRP})$
(b) $z(\text{SS-IRP}) \geq z(\text{DSIRP})$
The rolling horizon approach is beneficial.
Waste collection: Impact of ECDFs

- Numerical approximation vs. ECDFs for route failure probabilities:
  - 100 bins: squared error of $10^{-6}$
  - 1000 bins: squared error of $10^{-7}$
Waste collection: Impact of ECDFs

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**Figure 6**: Runtimes of different configurations
Waste collection: Objective overestimation

Figure 7: Objective function’s overestimation of the real cost for ECC = 100 CHF, RFCM = 1

(a) Do-Nothing Reaction Policy

(b) Optimal Reaction Policy Upper Bound
Numerical experiments

Facility maintenance case study

Instances:

- 93 instances derived partially from real data
- rich routing features
Numerical experiments

Facility maintenance case study

- **Instances:**
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- **Compare probabilistic policies varying the:**
  - Emergency Repair Cost (*ERC*)
  - maximum breakdown probability (*gamma*)
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  - minimum number of required visits ($\nu$)
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  - minimum number of required visits \((\nu)\)

- **Simulate undesirable events on final solution.**
Figure 8: Routing cost and breakdowns for probabilistic objective vs. probabilistic constraints model

(a) Routing Cost

(b) Breakdowns

Policies

Percentiles 75th 90th 95th 99th
Facility maintenance: Policy comparison

Figure 9: Probabilistic vs. deterministic policies

(a) Breakdowns at 99th percentile

(b) Routing Cost

Policy
- Deterministic
- Probabilistic
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- Tractability through the ability to pre-process.
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- Explicit modeling of undesirable events, recourse actions, and costs.
- Few distributional assumptions.
- Negligible deviation of modeled from real cost.
- Efficient and competitive solution methodology.
- Tractability through the ability to pre-process.
- Clear-cut superiority of stochastic (rolling horizon) approach.
Future research

- More tests on real-world benchmarks.
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- Additional rich routing features.
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- Richer objective: modeling realism vs. tractability.
- Column generation for lower bounds.
Conclusion

Thank you

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Figure 10: Container state probability tree