A Unified Framework for Rich Routing Problems with Stochastic Demands

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Outline

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Introduction

- Rich routing features (Lahyani et al., 2015)

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- Distribution, collection, or other context
- Stochastic demands that can be non-stationary
- Real-world demand forecasting
- Bridging the gap between theory and practice (Gendreau et al., 2016)

Contributions

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- Probabilities and costs of undesirable events, resp. recourse actions:
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- Computational tractability for a general inventory policy
- Generality and practical relevance of the approach
- Real case study showing superiority wrt deterministic approaches

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Key Notations and Terms

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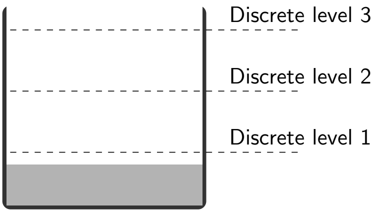
- Demand stochasticity leads to stock-outs:
  - $\sigma_{it} = 1$ for stock-out of point $i \in \mathcal{P}$ in period $t \in \mathcal{T}$, 0 otherwise
- And route failures:
  - Tours vs. trips: depot-delimited vs. supply point-delimited
  - $\mathscr{S} \in \mathfrak{S}_k$: a trip in the set of trips performed by vehicle $k \in \mathcal{K}$

Discretized Maximum Level (ML) Policy

- For tractable pre-processing of the stochastic information
- I_{it}: inventory of point $i \in \mathcal{P}$ at the start of period $t \in \mathcal{T}$
- Λ_{it}: inventory of point $i \in \mathcal{P}$ after delivery in period $t \in \mathcal{T}$
- ω_i: inventory capacity of point $i \in \mathcal{P}$

Figure 1: Discretization example for Λ_{it}



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Demand Forecasting

- Stochastic non-stationary demand ρ_{it} for point $i \in \mathcal{P}$ in period $t \in \mathcal{T}$:

$$\rho_{it} = \mathbb{E}\left(\rho_{it}\right) + \varepsilon_{it} \tag{1}$$

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- Combine $\varepsilon_{it}, \forall t \in \mathcal{T}, i \in \mathcal{P}$ in a vector:

$$\boldsymbol{\varepsilon} = \left(\varepsilon_{11}, \ldots, \varepsilon_{1|\mathcal{T}|}, \varepsilon_{21}, \ldots, \varepsilon_{|\mathcal{P}||\mathcal{T}|}\right) \tag{2}$$

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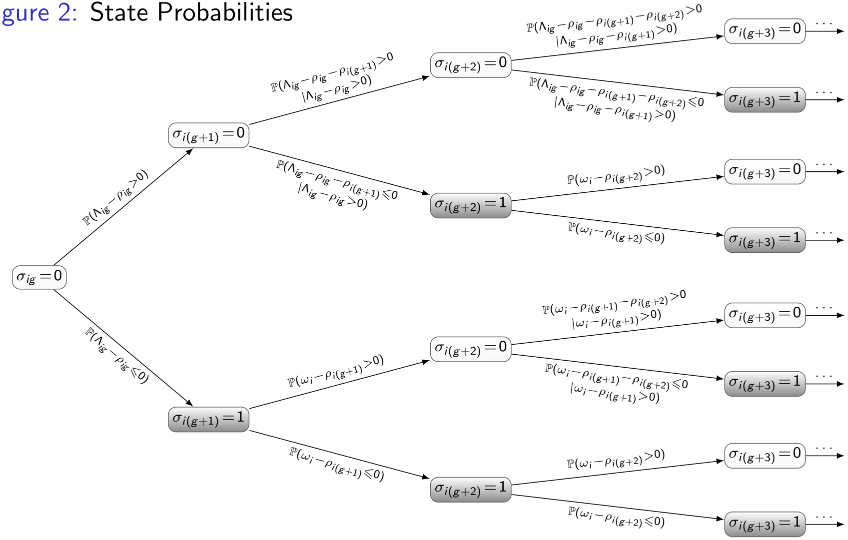
- Use any model that provides $\mathbb{E}(\rho_{it}), \forall t \in \mathcal{T}, i \in \mathcal{P}$ and Φ

Stock-out Probabilities: Branching

Figure 2: State Probabilities



Stock-out Probabilities: Formulation and pre-computing

- DVar: $y_{ikt} = 1$ if vehicle $k \in \mathcal{K}$ visits point $i \in \mathcal{P}$ in period $t \in \mathcal{T}$

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- Stock-out probability at point $i \in \mathcal{P}$ in period $t \in \mathcal{T}$:

$$\mathrm{p}_{it}^{\mathrm{DP}} = \mathbb{P}\left(\sigma_{it} = 1 \mid \Lambda_{im} \colon m = \max\left(0, g < t \colon \exists k \in \mathcal{K} \colon y_{ikg} = 1\right)\right) \quad (3)$$

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- The complexity is linear in the number of discrete levels

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$$\Gamma_{\mathscr{S}} = \sum_{\mathcal{S}_0 \in \mathscr{S}} \sum_{s \in \mathcal{S}_0} (\Lambda_{s0} - I_{s0}) + \sum_{t \in \mathcal{T} \setminus 0} \sum_{\mathcal{S}_t \in \mathscr{S}} \sum_{s \in \mathcal{S}_t} \left(\Lambda_{st} - \Lambda_{sm} + \sum_{h=m}^{t-1} \rho_{sh} \right), \quad (4)$$

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- Route failure probability:

$$\mathrm{p}^{\mathrm{RF}}_{\mathscr{S},k} = \mathbb{P}\left(\Gamma_{\mathscr{S}} > \Omega_k\right) \quad (5)$$

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- Use simulation to pre-process empirical distribution functions to be used at runtime (limited number)

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Objective

- Expected Stock-Out and Emergency Delivery Cost (ESOEDC), using stock-out cost χ and emergency delivery cost ζ:

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{P}} \left(\chi + \zeta - \zeta \sum_{k \in \mathcal{K}} y_{ikt} \right) \mathrm{p}_{it}^{\mathrm{DP}} \tag{7}$$

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- Expected Route Failure Cost (ERFC), using supply point detour cost $C_{\mathscr{S}}$ and weight multiplier ψ:

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- Deterministic cost components (routing, work balancing, visits, etc.)
- Overestimates the real cost due to modeling simplifications
  - Do-nothing vs. optimal reaction policy

Deterministic Constraints

- Open and multi-period tours
- Periodicities, service choice
- Accessibility restrictions
- Time windows, max tour duration, equity
- Inventory management (inventory policy)
- Vehicle capacity management
- etc...

Probabilistic Constraints

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- Maximum stock-out probability, for a constant $\gamma^{\mathrm{DP}} \in (0, 1]$:

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- Performance on real-world stochastic waste collection IRP instances:
  - Stability: on average 1-2% between best and worst over 10 runs
  - Speed: 10-15 min per problem, suitable for operational purposes

Waste Collection IRP: Instances

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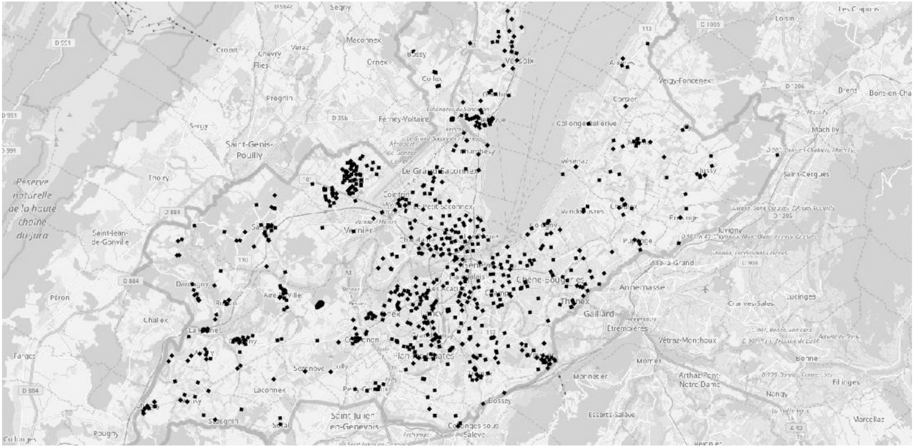
- Simulate undesirable events on final solution for original capacities

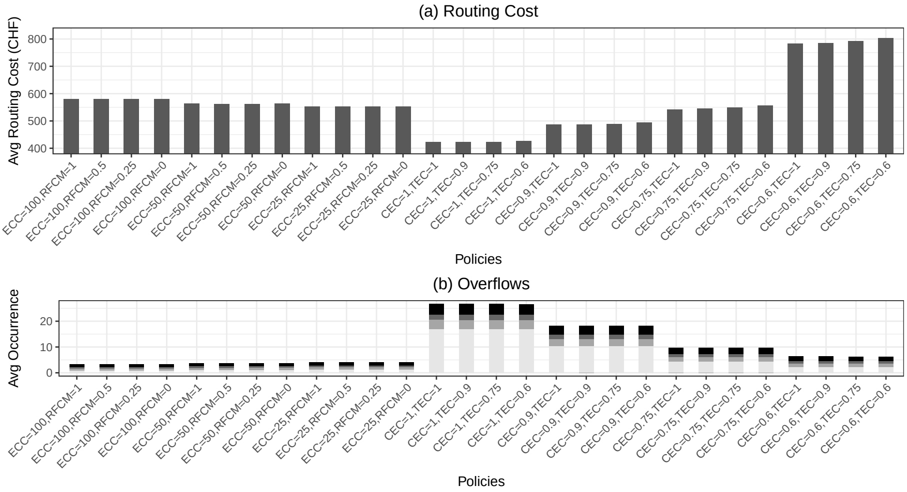
Waste Collection IRP: Instances

Figure 3: Geneva Service Area



Waste Collection IRP: Stochastic vs. Deterministic

Figure 4: Routing Cost and Number of Overflows



Waste Collection IRP: Calculating Route Failures

Table 1: Impact of ECDFs on Tractability

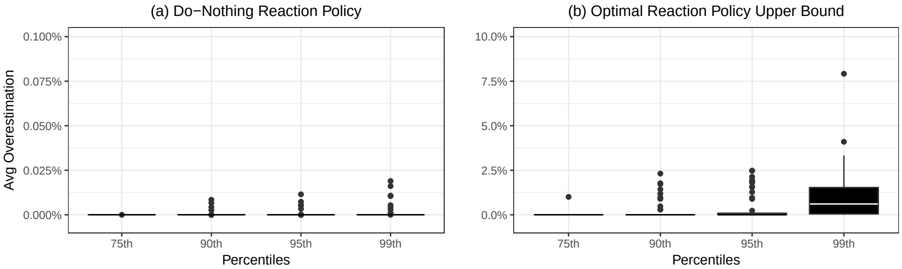
				Cost (CHF)			Runtime (s.)			ECDF calls (millions)		
ALNS version	Bins	ECC	RFCM	Best	Avg	Worst	Best	Avg	Worst	Best	Avg	Worst
Original	–	100.00	1.00	662.65	666.64	672.87	870.65	906.84	936.40	–	–	–
ECDFs	1000	100.00	1.00	662.63	666.74	673.35	909.06	948.77	982.68	52.95	58.90	65.00
ECDFs	100	100.00	1.00	662.49	666.46	672.73	869.52	903.81	932.79	52.94	58.44	63.90

Note. ECDF: Empirical Cumulative Distribution Function

Note. Bins: Number of bins in the ECDF binning implementation

Waste Collection IRP: Overestimation

Figure 5: Do-nothing vs. Optimal Reaction Policy



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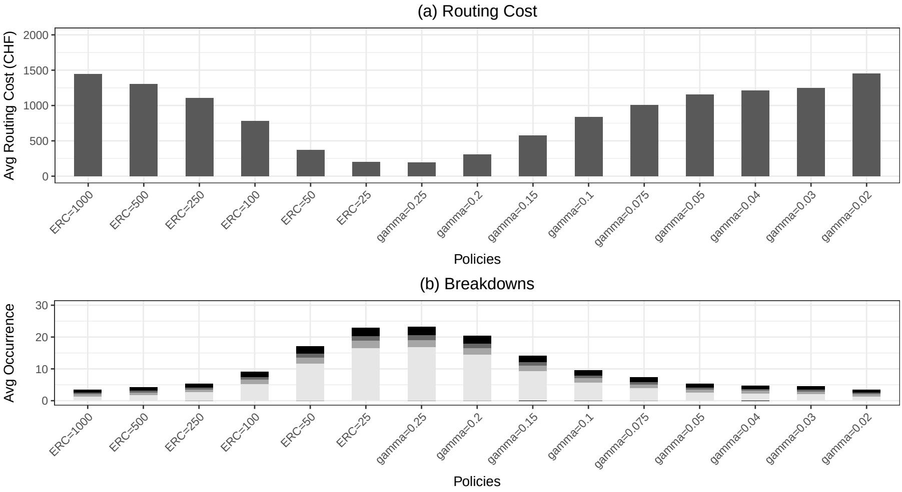
- Simulate undesirable events on final solution

Facility Maintenance Problem: Stochastic Approaches

Figure 6: Routing Cost and Breakdowns for Stochastic Approach



Facility Maintenance Problem: Stochastic vs. Deterministic

Table 2: Performance Indicators for Stochastic Approach

					Avg Num Breakdowns			
Model	ERC	γ^{DP}	Avg RC (CHF)	Avg EERC (CHF)	75th Perc.	90th Perc.	95th Perc.	99th Perc.
Prob. obj	250.00	–	1108.69	312.94	2.59	3.49	4.13	5.34
Prob. const	–	0.08	1010.44	0.00	3.91	5.06	5.84	7.29

Table 3: Performance Indicators for Deterministic Approach

					Avg Num Breakdowns			
Model	ERC	ν	Avg RC (CHF)	Avg EERC (CHF)	75th Perc.	90th Perc.	95th Perc.	99th Perc.
Deterministic	–	2	1945.96	0.00	3.16	4.10	4.56	5.71
Deterministic	–	1	1140.10	0.00	4.28	5.47	6.26	7.77

Note. Avg RC: Average routing cost

Note. Avg EERC: Average Expected Emergency Repair Cost

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- Future Research
  - More tests on real-world benchmarks
  - Lower bounds: column generation

Thank you

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