A general framework for routing problems with stochastic demands

Iliya Markov[a], Michel Bierlaire[a], Jean-François Cordeau[b]
Yousef Maknoon[c], Sacha Varone[d]

[a]Transport and Mobility Laboratory
École Polytechnique Fédérale de Lausanne

[b]HEC Montréal and CIRRELT

[c]Faculty of Technology, Policy, and Management
Delft University of Technology

[d]Haute École de Gestion de Genève
University of Applied Sciences Western Switzerland (HES-SO)

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Outline

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Setup and concepts

- Logistic setting:
  - depots, supply points, demand points
  - non-stationary stochastic demands over a planning horizon
  - distribution or collection context

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  - depots, supply points, demand points
  - non-stationary stochastic demands over a planning horizon
  - distribution or collection context
- Decisions:
  - visits
  - routing
  - inventory management

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- Undesirable events:
  - stock-outs
  - overflows
  - breakdowns
  - route failures

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- Undesirable events:
  - stock-outs
  - overflows
  - breakdowns
  - route failures
- The objective:
  - minimize cost
  - satisfying all constraints
  - avoiding the occurrence of undesirable events

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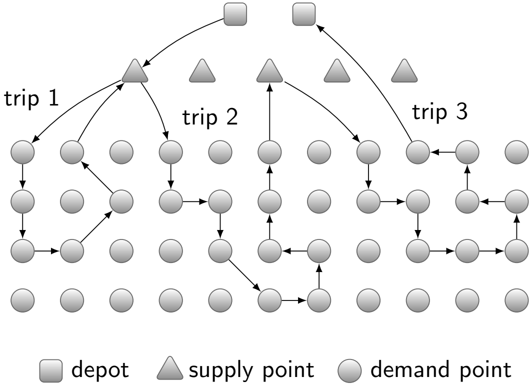
Routing

Figure 1: Tour example



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- Relies on dynamic probabilistic information to integrate the cost of undesirable events
- Uses recourse actions to recover from undesirable events
- Integrates demand forecasting
- Modeling framework corroborated by practical application
- High quality meta-heuristic solution approach
- Intuitive evaluation of various solution aspects by simulation

Outline

Forecasting

- The demand of point $i \in \mathcal{P}$ in period $t \in \mathcal{T}$ decomposes trivially as:

$$\rho_{it} = \mathbb{E}\left(\rho_{it}\right) + \varepsilon_{it} \tag{1}$$

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Definition 2

A forecasting model provides the expected demands $\mathbb{E}\left(\rho_{it}\right)$ for all $i \in \mathcal{P}, t \in \mathcal{T}$ and the error distribution $D(\boldsymbol{\varpi})$.

Demand point states and probabilities

- Notation:
  - Λ_{i0}: inventory after delivery of demand point i in period 0
  - ω_i: inventory capacity of demand point i
  - σ_{it}: state of demand point i in period t
    - $\sigma_{it} = 0$: normal
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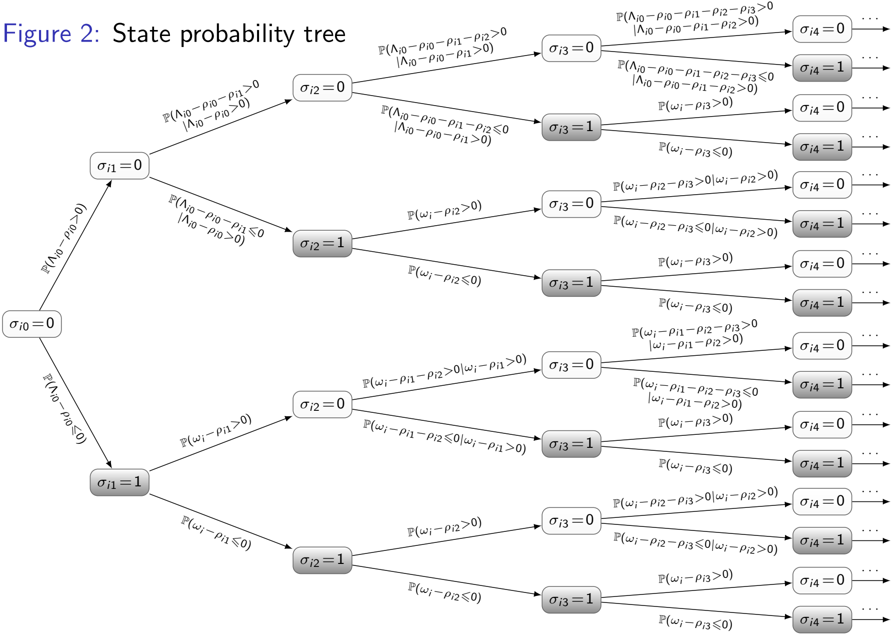
- Relevant costs:
  - **stock-out cost** χ: paid in a state of stock-out
  - **emergency delivery cost** ζ: paid in a state of stock-out when no vehicle visits the point

Figure 2: State probability tree



Order-Up-to (OU) inventory policy

Proposition 1

Under an OU policy in a distribution context, the stock-out probabilities can be pre-computed for any distribution $D(\varpi)$ using simulation.

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Corollary 1

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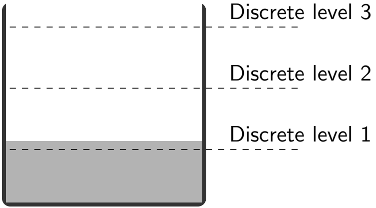
Maximum Level (ML) inventory policy

- Discretized ML policy:

Figure 3: Level discretization for a demand point



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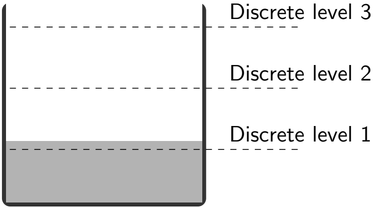


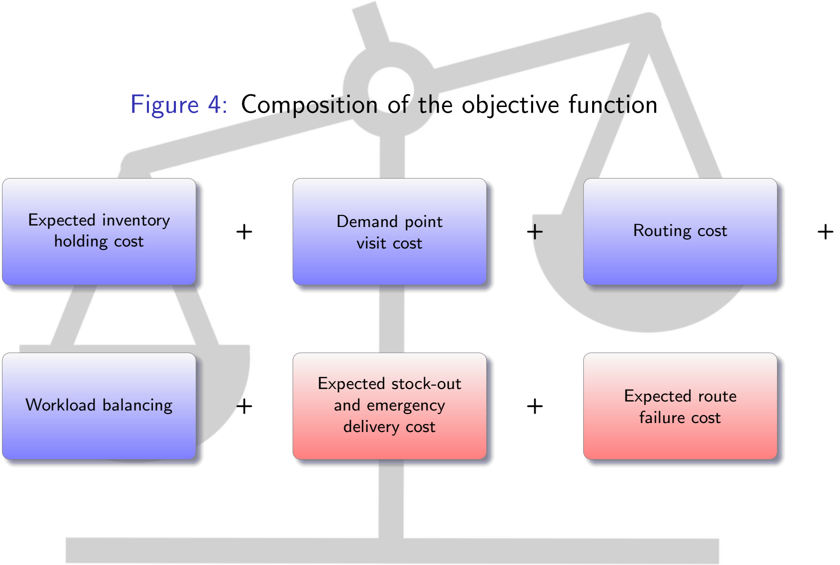
Proposition 3

Under a discretized ML policy, the relevant probabilities can be pre-computed, and the complexity is linear with the number of discrete levels.

Outline

Objective function

Figure 4: Composition of the objective function



Objective Function: Stochastic components

- Expected Stock-Out and Emergency Delivery Cost (ESOEDC) component:

$$\text{ESOEDC} = \sum_{t \in \mathcal{T}^+} \sum_{i \in \mathcal{P}} \left(\mathbb{P}\left(\sigma_{it}{=}1 \mid \Lambda_{im}\right) \left(\chi + \zeta - \zeta \sum_{k \in \mathcal{K}} y_{ikt} \right) \right), \tag{2}$$

where

- $\mathcal{T}^+$: planning horizon plus following day
- $\mathcal{P}$: set of demand points
- $\mathcal{K}$: set of vehicles
- $\sigma_{it} = 1$: state of stock-out of point i in period t
- Λ_{im}: inventory after delivery of point i in period m
- m: period of the previous delivery to point i
- χ: stock-out cost
- ζ: emergency delivery cost
- $y_{ikt} = 1$ if point i is visited by vehicle k in period t, 0 otherwise

Objective Function: Stochastic components

- Expected Route Failure Cost (ERFC) component:

$$\text{ERFC} = \sum_{k \in \mathcal{K}} \sum_{\mathscr{S} \in \mathfrak{S}_k} \sum_{n=1}^{N_{\mathscr{S}}-1} C_{\mathscr{S}} \mathbb{P}\left(n\Omega_k < \Xi_{\mathscr{S}} \leqslant (n+1)\Omega_k\right), \qquad (3)$$

where

- $\mathcal{K}$: set of vehicles
- $\mathfrak{S}_k$: set of supply point delimited trips for vehicle k
- $N_{\mathscr{S}}$: number of demand points in trip $\mathscr{S}$
- $C_{\mathscr{S}}$: route failure cost for trip $\mathscr{S}$
- $\Xi_{\mathscr{S}}$: volume delivered in trip $\mathscr{S}$
- Ω_k: capacity of vehicle k

Objective function: Tractability

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The route failure probabilities cannot be pre-computed.

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Assumption 1

Restrict the error terms as $\varepsilon_{it} \overset{\text{iid}}{\sim} D(\varpi)$, where $D(\varpi)$ may be any theoretical or empirical distribution.

- While we cannot pre-compute the probabilities themselves, we can derive their ECDFs
- The number of ECDFs to derive is bounded by the number of demand points times the number of periods in the planning horizon

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Proposition 5

In the absence of inventory holding costs, the objective function always overestimates the real cost.

Constraints

- Routing aspect:
  - multiple depots
  - supply point visits
  - open tours
  - multi-period tours
  - periodicities and service frequency
  - etc...

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- Vehicle capacity related
- Duration and time window related
- Etc...

Applications

- Stochastic demand problems:
  - vehicle routing problem
  - waste collection inventory routing
  - supermarket delivery routing
  - fuel delivery routing
  - home health care routing
  - maritime inventory routing
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- Probability-based routing problems:
  - facility maintenance
  - epidemic prevention
  - etc...

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Adaptive Large Neighborhood Search (ALNS)

- State-of-the-art meta-heuristic framework
- Operators compete in modifying the current solution
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- The weights ω_i are periodically updated by an adaptive layer that tracks operator performance
- **Rich operator pools reflecting the problem structure**
- **Simulated annealing solution guiding principle**

Outline

Benchmarking: Archetti et al. (2007) Instances

- First classical IRP testbed
- 160 instances in total
- 5 to 50 customers
- 3 or 6 periods in the planning horizon
- Single vehicle
- Low and high inventory holding costs
- Optimal solutions (branch-and-cut) by Archetti et al. (2007)

Benchmarking: Archetti et al. (2007) instances

Table 1: Results on Archetti et al. (2007) Instances

		High Inventory Holding Cost				Low Inventory Holding Cost			
$\|\mathcal{T}\|$	n	Runtime(s.)	Best Gap(%)	Avg Gap(%)	Worst Gap(%)	Runtime(s.)	Best Gap(%)	Avg Gap(%)	Worst Gap(%)
3	5	69.08	0.00	0.00	0.00	85.69	0.00	0.00	0.00
3	10	183.94	0.00	0.00	0.00	156.36	0.00	0.00	0.00
3	15	317.93	0.00	0.00	0.00	274.05	0.00	0.00	0.00
3	20	440.02	0.00	0.00	0.01	444.68	0.00	0.00	0.02
3	25	523.42	0.00	0.08	0.25	501.78	0.01	0.20	0.66
3	30	835.21	0.01	0.15	0.32	649.09	0.00	0.41	0.98
3	35	866.06	0.00	0.15	0.36	731.21	0.00	0.46	1.68
3	40	896.91	0.02	0.18	0.44	976.83	0.16	0.47	0.97
3	45	1124.57	0.05	0.42	0.91	1074.19	0.00	1.05	2.53
3	50	1424.27	0.06	0.35	0.79	1223.56	0.13	1.19	2.15
6	5	105.86	0.00	0.00	0.00	73.28	0.00	0.00	0.00
6	10	184.48	0.00	0.01	0.08	181.93	0.00	0.00	0.00
6	15	333.82	0.01	0.09	0.15	272.03	0.00	0.03	0.16
6	20	394.39	0.00	0.17	0.41	420.28	0.05	0.34	0.82
6	25	636.27	0.12	0.34	0.82	546.85	0.09	0.67	1.60
6	30	725.63	0.10	0.47	0.93	733.12	0.44	1.43	2.63
Average		566.37	0.02	0.15	0.34	521.56	**0.05**	**0.39**	**0.89**

Waste collection inventory routing problem

- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016
- Planning horizon of 7 days
- Up to 2 heterogeneous vehicles
- Up to 53 containers (41 on average)
- 2 dumps located far apart from each other

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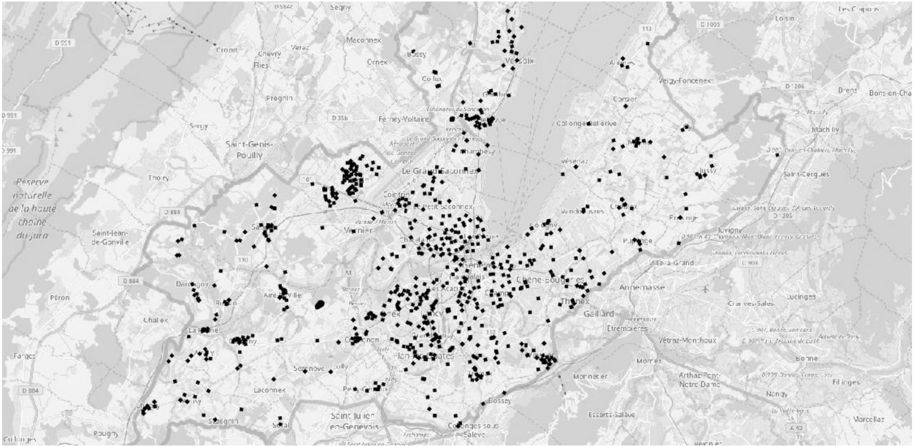
- **Simulation of undesirable events on the final solution**

Waste collection IRP: Geography

Figure 5: Geneva service area



Waste collection IRP: Policies

- **Probabilistic objective:**
  - *routing cost*
  - *expected overflow and emergency collection cost*
  - *expected route failure cost*
  - we vary the emergency collection cost (100 CHF, 50 CHF, 25 CHF) and the route failure cost multiplier (1.00, 0.50, 0.25, 0.00)

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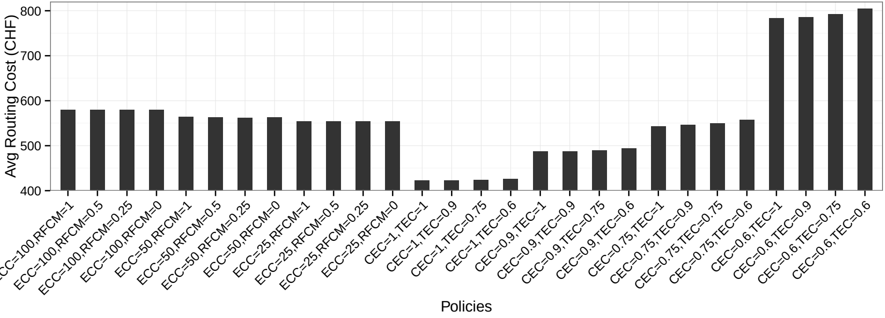
  - we vary the emergency collection cost (100 CHF, 50 CHF, 25 CHF) and the route failure cost multiplier (1.00, 0.50, 0.25, 0.00)
- **Deterministic objective:**
  - *routing cost only*
  - reduced container effective capacity
  - reduced truck effective capacity
  - we vary the container and truck effective capacities (1.00, 0.90, 0.75, 0.60)

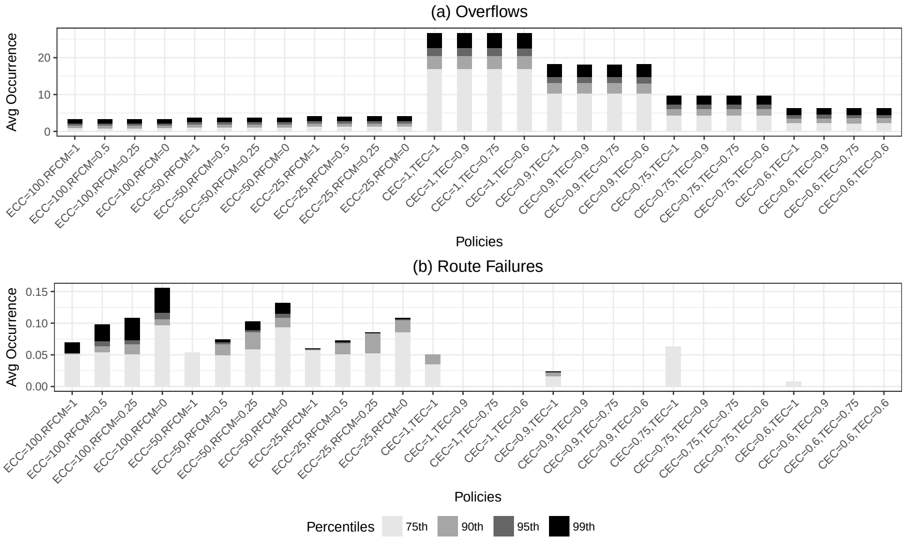
Waste collection IRP: Routing costs

Figure 6: Comparison of routing costs for probabilistic and deterministic policies



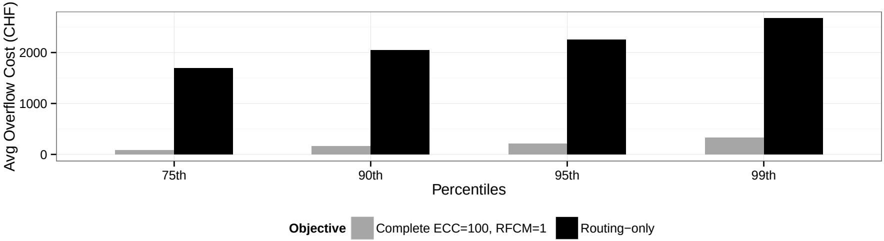
Waste collection IRP: Overflows and route failures

Figure 7: Comparison of undesirable events at different simulated percentiles



Waste collection IRP: Realized costs

Figure 8: Comparison of realized costs at different simulated percentiles



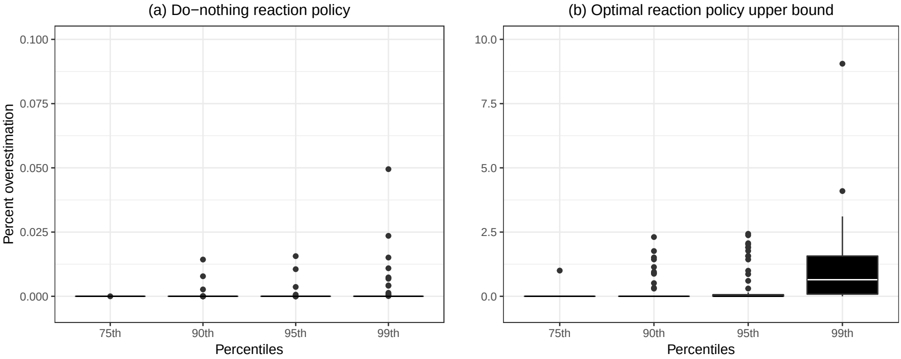
Waste collection IRP: Simulation and Tractability

Table 2: Impact of ECDFs on computation time

ALNS version	Bins	ECC	RFCM	Cost (CHF) Best	Cost (CHF) Avg	Cost (CHF) Worst	Runtime (s.) Best	Runtime (s.) Avg	Runtime (s.) Worst	ECDF calls (millions) Best	ECDF calls (millions) Avg	ECDF calls (millions) Worst
Original	-	100	1	662.65	666.64	672.87	870.65	906.84	936.40	-	-	-
Original	1000	100	1	662.82	666.97	673.43	1028.87	1096.86	1153.05	84.91	94.93	105.52
Original	100	100	1	662.29	666.61	673.40	912.54	955.96	990.57	84.11	94.54	103.84
Efficient	1000	100	1	662.63	666.74	673.35	909.06	948.77	982.68	52.95	58.90	65.00
Efficient	100	100	1	662.49	666.46	672.73	869.52	903.81	932.79	52.94	58.44	63.90

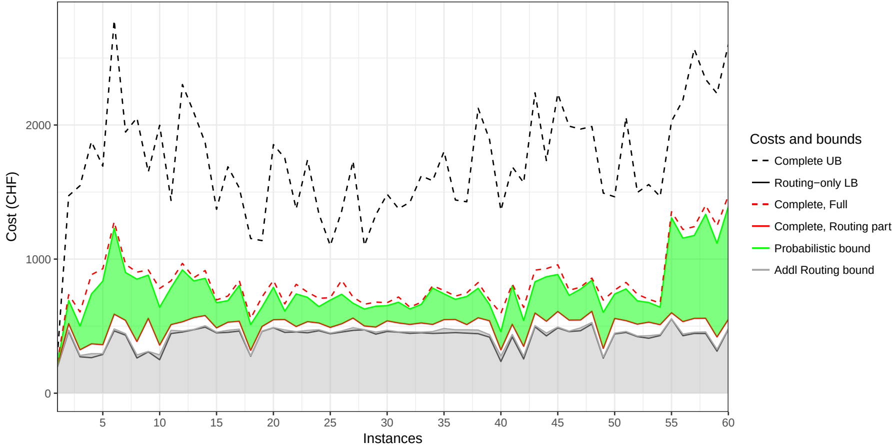
Waste collection IRP: Objective Overestimation

Figure 9: Objective function overestimation for two reaction policy extremes



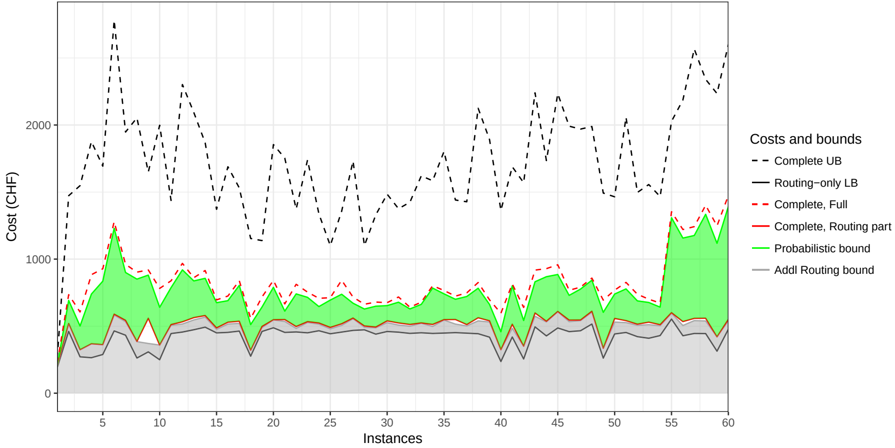
Waste collection IRP: Bounds

Figure 10: Heuristic bounds, single visit, gap = 17%



Waste collection IRP: Bounds

Figure 11: Heuristic bounds with re-optimization, single visit, gap = 7%



Facility maintenance

- 24 instances derived from the waste collection instances
- Planning horizon of 7 days
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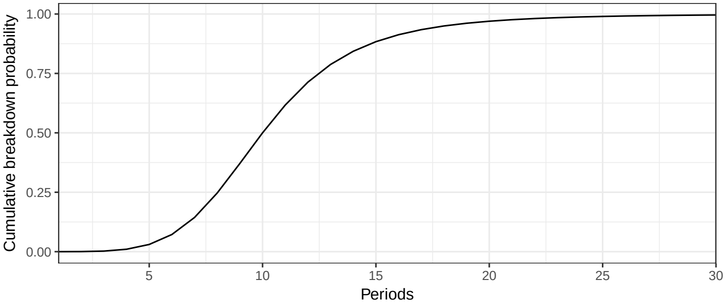
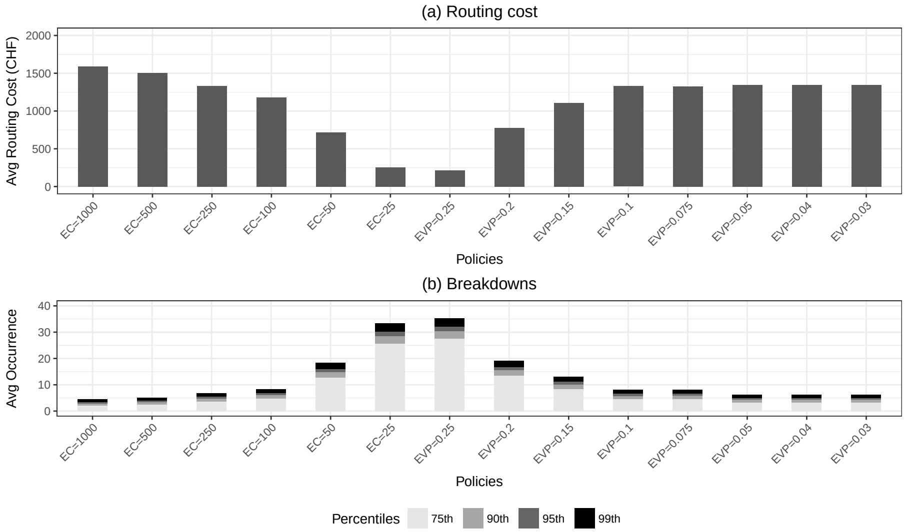
Facility maintenance: Breakdown probability

Figure 12: Facility cumulative breakdown probability



Facility maintenance: Routing cost and breakdowns

Figure 13: Verification of modeling approach



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- Computationally tractable
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- Much superior to classical deterministic approaches
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- Computationally tractable
- Corroborated by practical applications
- Much superior to classical deterministic approaches
- High quality efficient and stable solution methodology
- Next steps:
  - further work on the facility maintenance problem
  - further stability tests
- Future work:
  - further work on bounds
  - comparison to alternative approaches
  - generation of additional sets of realistic instances

Thank you.

Questions?

Archetti, C., Bertazzi, L., Laporte, G., and Speranza, M. G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41(3):382–391.