Integrating demand uncertainty in inventory routing for recyclable waste collection

Iliya Markov[a], Michel Bierlaire[a], Jean-François Cordeau[b], Yousef Maknoon[a], Sacha Varone[c]

[a]Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
École Polytechnique Fédérale de Lausanne

[b]HEC Montréal and CIRRELT

[c]Haute École de Gestion de Genève
University of Applied Sciences Western Switzerland (HES-SO)

14th Joint OR Days

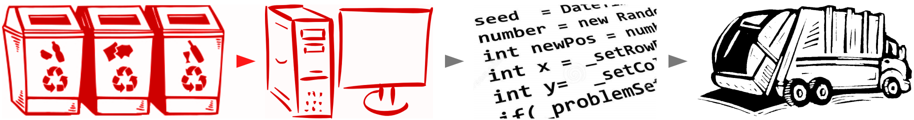
Lugano, September 8–9, 2016

Outline

Outline

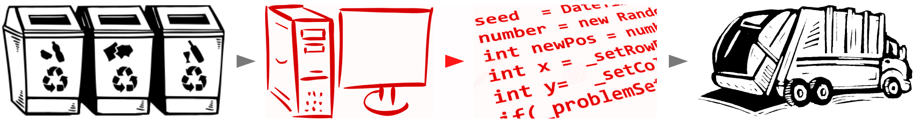
Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.



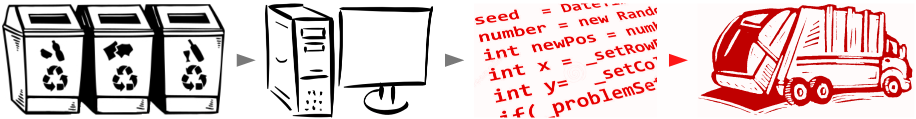
Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and route planning.



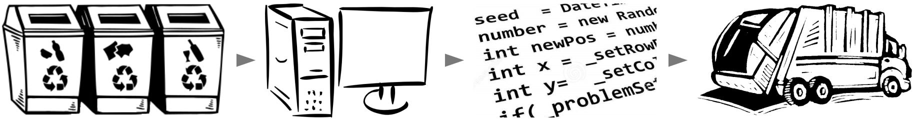
Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and route planning.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.



Setup

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and route planning.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.
- Efficient waste collection thus depends on the ability to:
  - forecast container levels,
  - select the containers to collect each day,
  - and route the vehicles in an (near-)optimal way.



Problem Definition

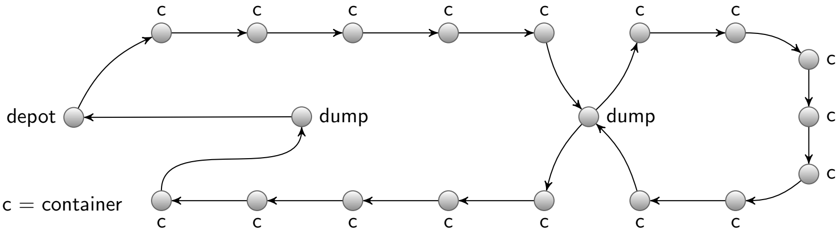
- The setup falls within the framework of the Stochastic Inventory Routing Problem (SIRP) with:
  - stochastic demands,
  - Order-Up-to level (OU) policy,
  - no allowed expected overflows,
  - single-day backorder limit (i.e. if a container overflows on a given day, it must be collected on that day).

Problem Definition

- The setup falls within the framework of the Stochastic Inventory Routing Problem (SIRP) with:
  - stochastic demands,
  - Order-Up-to level (OU) policy,
  - no allowed expected overflows,
  - single-day backorder limit (i.e. if a container overflows on a given day, it must be collected on that day).
- The routing component includes:
  - intermediate facility visits (recycling plants),
  - heterogeneous capacitated vehicles,
  - site dependencies,
  - vehicle-to-period availabilities,
  - time windows,
  - maximum tour duration.

Routing Component

Figure 1: Example of a Collection Tour



Outline

Related VRP Literature

- VRP with Intermediate Facilities (VRP-IF):
  - Bard et al. (1998a), Kim et al. (2006), Crevier et al. (2007).
- Electric and alternative fuel VRP:
  - Conrad and Figliozzi (2011), Erdoğan and Miller-Hooks (2012), Schneider et al. (2014), Schneider et al. (2015).
- Heterogeneous fixed fleet VRP:
  - Taillard (1999), Baldacci and Mingozzi (2009), Subramanian et al. (2012), Penna et al. (2013).
  - Hiermann et al. (2014) and Goeke and Schneider (2015) use some form of vehicle heterogeneity in the electric VRP.

Related SIRP Literature

- Early research on optimal replenishment policies in a stochastic setting:
  - Trudeau and Dror (1992), Jaillet et al. (2002), Bard et al. (1998b).
- Robust optimization:
  - Solyalı et al. (2012).
- Chance constraints:
  - Soysal et al. (2015), Abdollahi et al. (2014), Yu et al. (2012).
- Scenario based:
  - roll-out/branch-and-cut: Bertazzi et al. (2013), Bertazzi et al. (2015).
  - stochastic optimization: Hemmelmayr et al. (2010), Nolz et al. (2014), Adulyasak et al. (2015).

Motivation and Contribution

- We use an approach with dynamic probabilistic information on container overflows and route failures:
  - scenario-based approaches are computationally expensive,
  - we can frequently revisit the states of random variables unlike in robust optimization,
  - we have a monetary value associated with the realization of undesirable events.

Motivation and Contribution

- We use an approach with dynamic probabilistic information on container overflows and route failures:
  - scenario-based approaches are computationally expensive,
  - we can frequently revisit the states of random variables unlike in robust optimization,
  - we have a monetary value associated with the realization of undesirable events.
- Rich routing features rarely considered in the IRP literature.

Motivation and Contribution

- We use an approach with dynamic probabilistic information on container overflows and route failures:
  - scenario-based approaches are computationally expensive,
  - we can frequently revisit the states of random variables unlike in robust optimization,
  - we have a monetary value associated with the realization of undesirable events.
- Rich routing features rarely considered in the IRP literature.
- Methodology has excellent performance on benchmark instances.

Motivation and Contribution

- We use an approach with dynamic probabilistic information on container overflows and route failures:
  - scenario-based approaches are computationally expensive,
  - we can frequently revisit the states of random variables unlike in robust optimization,
  - we have a monetary value associated with the realization of undesirable events.
- Rich routing features rarely considered in the IRP literature.
- Methodology has excellent performance on benchmark instances.
- Probabilistic approach very competitive wrt alternative practical policies.

Motivation and Contribution

- We use an approach with dynamic probabilistic information on container overflows and route failures:
  - scenario-based approaches are computationally expensive,
  - we can frequently revisit the states of random variables unlike in robust optimization,
  - we have a monetary value associated with the realization of undesirable events.
- Rich routing features rarely considered in the IRP literature.
- Methodology has excellent performance on benchmark instances.
- Probabilistic approach very competitive wrt alternative practical policies.
- We derive empirical lower and upper bounds for the solution cost of a rolling horizon approach.

Outline

1. Introduction
2. Related Literature
3. Formulation and Solution
4. Numerical Experiments
5. Conclusion

Nomenclature

Sets

o	origin	d	destination
$\mathcal{D}$	set of dumps	$\mathcal{P}$	set of containers
$\mathcal{N}$	$= \{o\} \cup \{d\} \cup \mathcal{D} \cup \mathcal{P}$	$\mathcal{K}$	set of vehicles
$\mathcal{T}$	$= \{0, ..., u\}$	$\mathcal{T}^+$	$= \{1, ..., u+1\}$

Parameters

ρ_{it}	demand of container i on day t (random variable)
ς	forecasting model error (st. dev. of the fit's residuals)
π_{ij}	travel distance of arc (i, j)
τ_{ijk}	travel time of vehicle k on arc (i, j)
λ_i, μ_i	lower and upper time window bound at point i
δ_i	service duration at point i
ω_i	capacity of container i
χ	container overflow cost (monetary)
ζ	emergency collection cost (monetary)

Nomenclature

Sets

o	origin	d	destination
$\mathcal{D}$	set of dumps	$\mathcal{P}$	set of containers
$\mathcal{N}$	$= \{o\} \cup \{d\} \cup \mathcal{D} \cup \mathcal{P}$	$\mathcal{K}$	set of vehicles
$\mathcal{T}$	$= \{0, ..., u\}$	$\mathcal{T}^+$	$= \{1, ..., u+1\}$

Parameters

σ_{it}	$= 1$ if container i is in a state of full and overflowing on day t, 0 otherwise
φ_k	daily deployment cost of vehicle k (monetary)
β_k	unit-distance running cost of vehicle k (monetary)
θ_k	unit-time running cost of vehicle k (monetary)
α_{kt}	$= 1$ if vehicle k is available on day t, 0 otherwise
α_{ik}	$= 1$ if point i is accessible by vehicle k, 0 otherwise
Ω_k	capacity of vehicle k
H	maximum tour duration
ψ	Route Failure Cost Multiplier (RFCM) $\in [0, 1]$

Nomenclature

Decision variables: binary

$$x_{ijkt} = \begin{cases} 1 & \text{if vehicle } k \text{ traverses arc } (i,j) \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ikt} = \begin{cases} 1 & \text{if vehicle } k \text{ visits point } i \text{ on day } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_{kt} = \begin{cases} 1 & \text{if vehicle } k \text{ is used on day } t \\ 0 & \text{otherwise} \end{cases}$$

Decision variables: continuous

q_{ikt}	expected pickup quantity by vehicle k from container i on day t
Q_{ikt}	expected cumulative quantity on vehicle k at point i on day t
I_{it}	expected inventory in container i at the start of day t
S_{ikt}	start-of-service time of vehicle k at point i on day t

Forecasting Model

- Demand is the amount deposited in a container on each day, and is random and non-stationary.

Forecasting Model

- Demand is the amount deposited in a container on each day, and is random and non-stationary.
- We can use any forecasting model that gives us:
  - the expected container demands $\mathbb{E}(\rho_{it})$ on each day,
  - a consistent estimate of the forecasting error ς.

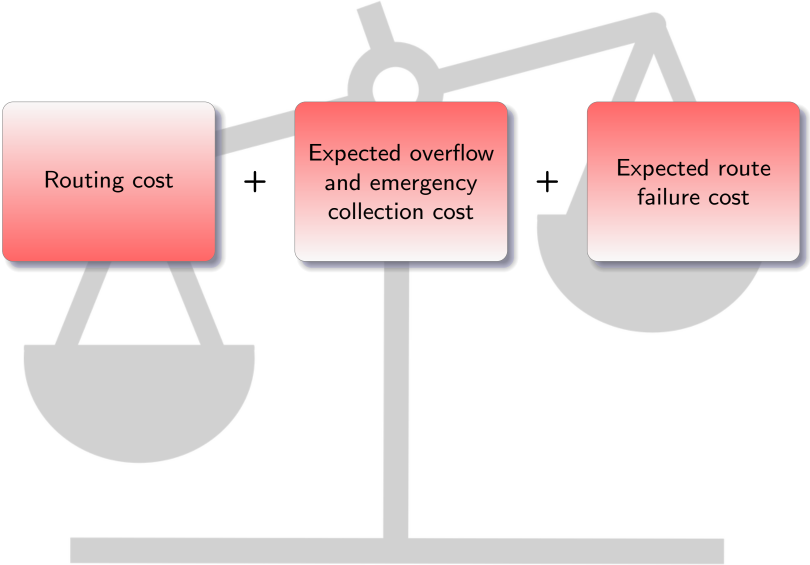
Forecasting Model

- Demand is the amount deposited in a container on each day, and is random and non-stationary.
- We can use any forecasting model that gives us:
  - the expected container demands $\mathbb{E}(\rho_{it})$ on each day,
  - a consistent estimate of the forecasting error ς.
- The forecasting error is the standard deviation of the residuals based on a historical fit.
- Its distribution can be approximated as a normal, and is used to calculate probabilities of container overflows and route failures.

Forecasting Model

- Demand is the amount deposited in a container on each day, and is random and non-stationary.
- We can use any forecasting model that gives us:
  - the expected container demands $\mathbb{E}(\rho_{it})$ on each day,
  - a consistent estimate of the forecasting error ς.
- The forecasting error is the standard deviation of the residuals based on a historical fit.
- Its distribution can be approximated as a normal, and is used to calculate probabilities of container overflows and route failures.
- The probabilities are dynamic and conditional, and depend on:
  - the evolution of container states over the planning horizon,
  - and the vehicle visits on each day.

Objective Function



Objective Function

Routing cost + Expected overflow and emergency collection cost + Expected route failure cost

- Lower routing cost is counterbalanced by more overflows and route failures, and vice versa.
- Our goal is to minimize the expected monetary value of all components.

Objective Function: Main Concepts

- Two container states:
  - $\sigma_{it} = 0$: not full,
  - $\sigma_{it} = 1$: full and overflowing.

Objective Function: Main Concepts

- Two container states:
  - $\sigma_{it} = 0$: not full,
  - $\sigma_{it} = 1$: full and overflowing.

- Two types of container collection:
  - regular collection of container i on day t: $\exists k \in \mathcal{K} : y_{ikt} = 1$,
  - emergency collection of container i on day t: $\sigma_{it} = 1$ and $y_{ikt} = 0, \forall k \in \mathcal{K}$.

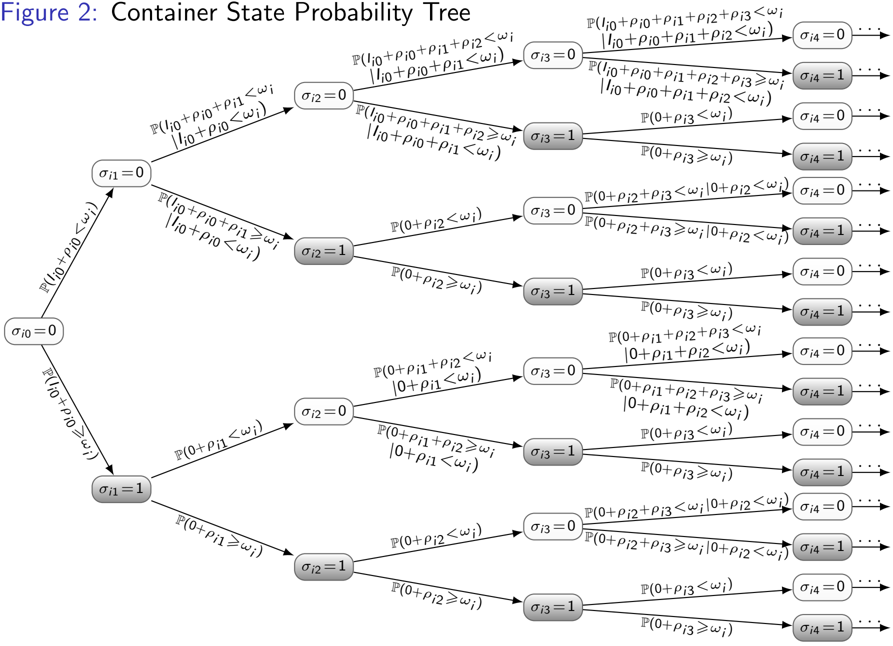
Objective Function: Main Concepts

- Two container states:
  - $\sigma_{it} = 0$: not full,
  - $\sigma_{it} = 1$: full and overflowing.

- Two types of container collection:
  - regular collection of container i on day t: $\exists k \in \mathcal{K} : y_{ikt} = 1$,
  - emergency collection of container i on day t: $\sigma_{it} = 1$ and $y_{ikt} = 0, \forall k \in \mathcal{K}$.

- Related costs:
  - overflow cost χ: paid in state $\sigma_{it} = 1$,
  - emergency collection cost ζ: paid in state $\sigma_{it} = 1$ when $y_{ikt} = 0, \forall k \in \mathcal{K}$.

Figure 2: Container State Probability Tree



Objective Function: Formulation

- Routing Cost (RC):

$$\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\left(\varphi_k z_{kt} + \beta_k \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} \pi_{ij} x_{ijkt} + \theta_k \left(S_{dkt} - S_{okt}\right)\right) \quad (1)$$

Objective Function: Formulation

- Routing Cost (RC):

$$\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\left(\varphi_k z_{kt} + \beta_k \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} \pi_{ij} x_{ijkt} + \theta_k \left(S_{dkt} - S_{okt}\right)\right) \tag{1}$$

- Expected Overflow and Emergency Collection Cost (EOECC):

$$\sum_{t\in\mathcal{T}\cup\mathcal{T}^+}\sum_{i\in\mathcal{P}}\left(\mathbb{P}\left(\sigma_{it} = 1 \mid \max\left(0, g < t\colon \exists k \in \mathcal{K}\colon y_{ikg} = 1\right)\right)\left(\chi + \zeta - \zeta \sum_{k\in\mathcal{K}} y_{ikt}\right)\right) \tag{2}$$

Objective Function: Formulation

- Expected Route Failure Cost (ERFC):

$$\sum_{t \in \mathcal{T} \setminus 0} \sum_{k \in \mathcal{K}} \sum_{\mathcal{S} \in \mathscr{S}_{kt}} \left(\psi C_{\mathcal{S}} \, \mathbb{P} \left(\sum_{s \in \mathcal{S}} (I_{st} > \Omega_k \mid \max(0, g < t \colon y_{skg} = 1)) \right) \right), \quad (3)$$

where

- $\mathscr{S}_{kt}$ is the set of depot-to-dump or dump-to-dump trips for vehicle k on day t,
- $\mathcal{S}$ is the set of containers in a particular trip,
- $C_{\mathcal{S}}$ is the average routing cost of going from $\mathcal{S}$ to the nearest dump and back to $\mathcal{S}$,
- ψ is the Route Failure Cost Multiplier (RFCM), controlling the degree of conservatism wrt this component.

Objective Function: Formulation

- Expected Route Failure Cost (ERFC):

$$\sum_{t \in \mathcal{T} \setminus 0} \sum_{k \in \mathcal{K}} \sum_{\mathcal{S} \in \mathscr{S}_{kt}} \left(\psi C_{\mathcal{S}} \, \mathbb{P} \left(\sum_{s \in \mathcal{S}} (I_{st} > \Omega_k \mid \max(0, g < t \colon y_{skg} = 1)) \right) \right), \quad (3)$$

where

- $\mathscr{S}_{kt}$ is the set of depot-to-dump or dump-to-dump trips for vehicle k on day t,
- $\mathcal{S}$ is the set of containers in a particular trip,
- $C_{\mathcal{S}}$ is the average routing cost of going from $\mathcal{S}$ to the nearest dump and back to $\mathcal{S}$,
- ψ is the Route Failure Cost Multiplier (RFCM), controlling the degree of conservatism wrt this component.

- The objective function becomes

$$\min z = \text{RC} + \text{EOECC} + \text{ERFC} \quad (4)$$

and is non-linear, thus resulting in an MINLP.

Constraints: Basic routing

$$\sum_{j\in\mathcal{N}} x_{ojkt} = \alpha_{kt} z_{kt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{5}$$

$$\sum_{i\in\mathcal{D}} x_{idkt} = \alpha_{kt} z_{kt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{6}$$

$$y_{ikt} = \sum_{j\in\mathcal{N}} x_{ijkt} = \sum_{j\in\mathcal{N}} x_{jikt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{7}$$

$$\sum_{k\in\mathcal{K}} y_{ikt} \leqslant 1, \qquad \forall t \in \mathcal{T}, i \in \mathcal{P} \tag{8}$$

$$y_{ikt} \leqslant \alpha_{ik}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{9}$$

$$\sum_{i\in\mathcal{N}} x_{ijkt} = \sum_{i\in\mathcal{N}} x_{jikt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{D} \cup \mathcal{P} \tag{10}$$

$$I_{it} = I_{i(t-1)} - \sum_{k\in\mathcal{K}} q_{ik(t-1)} + \mathbb{E}(\rho_{i(t-1)}), \qquad \forall t \in \mathcal{T}^+, i \in \mathcal{P} \tag{11}$$

$$I_{it} \leqslant \omega_i, \qquad \forall t \in \mathcal{T}^+, i \in \mathcal{P} \tag{12}$$

$$I_{i0} - \omega_i \leqslant \omega_i \sum_{k\in\mathcal{K}} y_{ik0}, \qquad \forall i \in \mathcal{P} \tag{13}$$

$$q_{ikt} \leqslant M y_{ikt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{14}$$

$$q_{ikt} \leqslant I_{it}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{15}$$

$$q_{ikt} \geqslant I_{it} - M(1 - y_{ikt}), \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{16}$$

Constraints: Inventory balance

$$\sum_{j \in \mathcal{N}} x_{ojkt} = \alpha_{kt} z_{kt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{5}$$

$$\sum_{i \in \mathcal{D}} x_{idkt} = \alpha_{kt} z_{kt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{6}$$

$$y_{ikt} = \sum_{j \in \mathcal{N}} x_{ijkt} = \sum_{j \in \mathcal{N}} x_{jikt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{7}$$

$$\sum_{k \in \mathcal{K}} y_{ikt} \leqslant 1, \quad \forall t \in \mathcal{T}, i \in \mathcal{P} \tag{8}$$

$$y_{ikt} \leqslant \alpha_{ik}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{9}$$

$$\sum_{i \in \mathcal{N}} x_{ijkt} = \sum_{i \in \mathcal{N}} x_{jikt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{D} \cup \mathcal{P} \tag{10}$$

$$I_{it} = I_{i(t-1)} - \sum_{k \in \mathcal{K}} q_{ik(t-1)} + \mathbb{E}(\rho_{i(t-1)}), \quad \forall t \in \mathcal{T}^+, i \in \mathcal{P} \tag{11}$$

$$I_{it} \leqslant \omega_i, \quad \forall t \in \mathcal{T}^+, i \in \mathcal{P} \tag{12}$$

$$I_{i0} - \omega_i \leqslant \omega_i \sum_{k \in \mathcal{K}} y_{ik0}, \quad \forall i \in \mathcal{P} \tag{13}$$

$$q_{ikt} \leqslant M y_{ikt}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{14}$$

$$q_{ikt} \leqslant I_{it}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{15}$$

$$q_{ikt} \geqslant I_{it} - M(1 - y_{ikt}), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{16}$$

Constraints: Capacity related

$$q_{ikt} \leqslant Q_{ikt} \leqslant \Omega_k, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{17}$$

$$Q_{ikt} = 0, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \mathcal{P} \tag{18}$$

$$Q_{ikt} + q_{jkt} \leqslant Q_{jkt} + \Omega_k \left(1 - x_{ijkt}\right), \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{P} \tag{19}$$

$$S_{ikt} + \delta_i + \tau_{ijk} \leqslant S_{jkt} + \left(\mu_i + \delta_i + \tau_{ijk}\right)\left(1 - x_{ijkt}\right), \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{N} \setminus \{o\} \tag{20}$$

$$\lambda_i \sum_{j \in \mathcal{N}} x_{ijkt} \leqslant S_{ikt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\} \tag{21}$$

$$S_{jkt} \leqslant \mu_j \sum_{i \in \mathcal{N}} x_{ijkt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{N} \setminus \{o\} \tag{22}$$

$$0 \leqslant S_{dkt} - S_{okt} \leqslant \mathrm{H} \qquad \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{23}$$

$$x_{ijkt}, y_{ikt}, z_{kt} \in \{0, 1\}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i, j \in \mathcal{N} \tag{24}$$

$$q_{ikt}, Q_{ikt}, I_{it}, S_{ikt} \geqslant 0, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \tag{25}$$

Constraints: Time related

$$q_{ikt} \leqslant Q_{ikt} \leqslant \Omega_k, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{17}$$

$$Q_{ikt} = 0, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \mathcal{P} \tag{18}$$

$$Q_{ikt} + q_{jkt} \leqslant Q_{jkt} + \Omega_k \left(1 - x_{ijkt}\right), \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{P} \tag{19}$$

$$S_{ikt} + \delta_i + \tau_{ijk} \leqslant S_{jkt} + \left(\mu_i + \delta_i + \tau_{ijk}\right) \left(1 - x_{ijkt}\right), \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{N} \setminus \{o\} \tag{20}$$

$$\lambda_i \sum_{j \in \mathcal{N}} x_{ijkt} \leqslant S_{ikt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\} \tag{21}$$

$$S_{jkt} \leqslant \mu_j \sum_{i \in \mathcal{N}} x_{ijkt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{N} \setminus \{o\} \tag{22}$$

$$0 \leqslant S_{dkt} - S_{okt} \leqslant \mathrm{H} \qquad \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{23}$$

$$x_{ijkt}, y_{ikt}, z_{kt} \in \{0, 1\}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i, j \in \mathcal{N} \tag{24}$$

$$q_{ikt}, Q_{ikt}, I_{it}, S_{ikt} \geqslant 0, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \tag{25}$$

Constraints: Domain

$$q_{ikt} \leqslant Q_{ikt} \leqslant \Omega_k, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{P} \tag{17}$$

$$Q_{ikt} = 0, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \mathcal{P} \tag{18}$$

$$Q_{ikt} + q_{jkt} \leqslant Q_{jkt} + \Omega_k \left(1 - x_{ijkt}\right), \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{P} \tag{19}$$

$$S_{ikt} + \delta_i + \tau_{ijk} \leqslant S_{jkt} + \left(\mu_i + \delta_i + \tau_{ijk}\right)\left(1 - x_{ijkt}\right), \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\}, j \in \mathcal{N} \setminus \{o\} \tag{20}$$

$$\lambda_i \sum_{j \in \mathcal{N}} x_{ijkt} \leqslant S_{ikt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \setminus \{d\} \tag{21}$$

$$S_{jkt} \leqslant \mu_j \sum_{i \in \mathcal{N}} x_{ijkt}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{N} \setminus \{o\} \tag{22}$$

$$0 \leqslant S_{dkt} - S_{okt} \leqslant \mathrm{H} \qquad \forall t \in \mathcal{T}, k \in \mathcal{K} \tag{23}$$

$$x_{ijkt}, y_{ikt}, z_{kt} \in \{0, 1\}, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i, j \in \mathcal{N} \tag{24}$$

$$q_{ikt}, Q_{ikt}, I_{it}, S_{ikt} \geqslant 0, \qquad \forall t \in \mathcal{T}, k \in \mathcal{K}, i \in \mathcal{N} \tag{25}$$

Solution Methodology

- The resulting problem is $\mathcal{NP}$-hard and has a non-linear objective function.
- To solve it, we develop an Adaptive Large Neighborhood Search (ALNS) algorithm with solution acceptance by Simulated Annealing (SA).
- The ALNS accepts infeasible intermediate solutions with dynamic penalty management for various types of infeasibilities.

Outline

Benchmarking: Archetti et al. (2007) Instances

- First classical IRP testbed.
- 160 instances in total.
- 5 to 50 customers.
- 3 or 6 periods in the planning horizon.
- Single vehicle.
- Low and high inventory holding costs.

Benchmarking: Archetti et al. (2007) Instances

- First classical IRP testbed.
- 160 instances in total.
- 5 to 50 customers.
- 3 or 6 periods in the planning horizon.
- Single vehicle.
- Low and high inventory holding costs.

- Optimal solutions (branch-and-cut) by Archetti et al. (2007).
- Heuristic solutions by Archetti et al. (2012), Coelho et al. (2012a), Coelho et al. (2012b), etc...
- We solve each instance 10 times and report best and average results.

Benchmarking: Archetti et al. (2007) Instances

Table 1: Results on Instances with High Inventory Holding Cost

		ALNS Fast Version			ALNS Slow Version		
$\|\mathcal{T}\|$	n	Runtime(s.)	Best Gap(%)	Avg Gap(%)	Runtime(s.)	Best Gap(%)	Avg Gap(%)
3	5	8	0.00	0.00	32	0.00	0.00
3	10	14	0.00	0.00	59	0.00	0.00
3	15	22	0.00	0.00	93	0.00	0.00
3	20	36	0.00	0.01	149	0.00	0.00
3	25	53	0.00	0.06	221	0.00	0.01
3	30	77	0.00	0.27	318	0.00	0.06
3	35	108	0.01	0.15	440	0.00	0.04
3	40	149	0.12	0.48	602	0.01	0.23
3	45	199	0.17	0.47	808	0.10	0.25
3	50	276	0.15	0.52	1074	0.07	0.25
6	5	14	0.00	0.00	55	0.00	0.00
6	10	28	0.00	0.01	113	0.00	0.00
6	15	53	0.00	0.07	198	0.00	0.01
6	20	81	0.04	0.14	331	0.01	0.08
6	25	128	0.19	0.64	513	0.10	0.38
6	30	189	0.08	0.70	772	0.07	0.38
Average		90	0.05	0.22	361	0.02	0.11

Benchmarking: Archetti et al. (2007) Instances

Table 2: Results on Instances with Low Inventory Holding Cost

		ALNS Fast Version			ALNS Slow Version		
$\|\mathcal{T}\|$	n	Runtime(s.)	Best Gap(%)	Avg Gap(%)	Runtime(s.)	Best Gap(%)	Avg Gap(%)
3	5	7	0.00	0.00	30	0.00	0.00
3	10	14	0.00	0.00	55	0.00	0.00
3	15	22	0.00	0.00	89	0.00	0.00
3	20	34	0.00	0.04	141	0.00	0.01
3	25	52	0.00	0.17	210	0.00	0.04
3	30	71	0.02	0.56	295	0.00	0.14
3	35	101	0.01	0.53	423	0.00	0.18
3	40	140	0.37	1.20	567	0.12	0.48
3	45	191	0.59	1.71	751	0.26	1.03
3	50	247	0.30	1.52	1009	0.25	1.00
6	5	13	0.00	0.00	54	0.00	0.00
6	10	28	0.00	0.02	109	0.00	0.01
6	15	49	0.00	0.03	188	0.00	0.00
6	20	77	0.08	0.26	315	0.05	0.15
6	25	121	0.25	1.29	493	0.24	0.65
6	30	182	0.67	1.90	726	0.07	1.06
Average		84	0.14	0.58	341	0.06	0.30

Case Study: Instances

- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Maximum tour duration of 4 hours.
- Time windows from 8h00 to 12h00.
- Planning horizon of 7 days.
- Up to 2 heterogeneous vehicles.
- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.

Case Study: Instances

- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Maximum tour duration of 4 hours.
- Time windows from 8h00 to 12h00.
- Planning horizon of 7 days.
- Up to 2 heterogeneous vehicles.
- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.

- 10 runs for each instance.
- Simulation of the forecasting error realizations for each solution.
- Evaluation of the relevance of the probability information captured by the objective function.

Case Study: Probabilistic Policies

- We consider two types of objective function:
  - Complete: minimizes the full probabilistic objective defined by expression (4).
  - Routing-only: minimizes the routing cost only, as defined by expression (1), disregarding all probability information.

Case Study: Probabilistic Policies

- We consider two types of objective function:
  - Complete: minimizes the full probabilistic objective defined by expression (4).
  - Routing-only: minimizes the routing cost only, as defined by expression (1), disregarding all probability information.

- Probability-related costs:
  - overflow cost χ: 100 CHF (fixed by municipality),
  - emergency collection cost ζ: 100 CHF, 50 CHF, 25 CHF (does not apply to routing-only $=$ 0 CHF),
  - Route Failure Cost Multiplier (RFCM) ψ: 1.00, 0.50, 0.25 (does not apply to routing-only $=$ 0 CHF).

Case Study: Probabilistic Policies

Table 3: Basic Results for Cost Analysis

Objective	ECC	RFCM	Runtime(s.)	Avg Num Tours	Avg Num Containers	Avg Num Dump Visits	Best Cost (CHF)	Avg Cost (CHF)	Gap Avg-Best(%)
Complete	100.00	1.00	781.71	1.96	43.44	2.31	664.76	679.54	2.22
Complete	100.00	0.50	862.13	1.96	43.43	2.30	664.82	678.84	2.11
Complete	100.00	0.25	806.52	1.95	43.52	2.28	664.34	677.81	2.03
Complete	100.00	0.00	715.82	1.95	43.80	2.28	664.00	677.11	1.97
Complete	50.00	1.00	915.61	1.92	41.08	2.20	650.86	662.18	1.74
Complete	50.00	0.50	812.67	1.91	41.22	2.21	650.55	662.28	1.80
Complete	50.00	0.25	809.76	1.91	41.19	2.19	650.72	661.88	1.71
Complete	50.00	0.00	790.21	1.91	41.07	2.19	651.09	661.93	1.66
Complete	25.00	1.00	814.44	1.90	39.56	2.13	641.43	651.24	1.53
Complete	25.00	0.50	789.00	1.90	39.56	2.14	641.79	652.04	1.60
Complete	25.00	0.25	789.40	1.90	39.57	2.15	641.42	651.85	1.63
Complete	25.00	0.00	783.33	1.89	39.59	2.13	642.71	651.71	1.40
Routing-only	0.00	0.00	725.46	1.83	16.77	1.87	422.64	425.08	0.58

Case Study: Probabilistic Policies

Table 4: Key Performance Indicators for Cost Analysis

Objective	ECC	RFCM	Avg Routing Cost (CHF)	Avg Overflow Cost (CHF)	Avg Rte Failure Cost (CHF)	Avg Collected Volume (L)	Liters Per Unit Cost	Liters Per Unit Routing Cost
Complete	100.00	1.00	579.78	99.73	0.03	47,234.59	69.51	81.47
Complete	100.00	0.50	579.46	99.33	0.05	47,225.62	69.57	81.50
Complete	100.00	0.25	577.84	99.93	0.04	47,455.19	70.01	82.13
Complete	100.00	0.00	578.83	98.28	0.00	47,662.90	70.39	82.34
Complete	50.00	1.00	559.44	102.72	0.02	45,646.48	68.93	81.59
Complete	50.00	0.50	558.37	103.82	0.09	45,852.89	69.24	82.12
Complete	50.00	0.25	558.47	103.35	0.07	45,949.94	69.42	82.28
Complete	50.00	0.00	557.16	104.77	0.00	45,788.15	69.17	82.18
Complete	25.00	1.00	547.74	103.46	0.04	44,682.00	68.61	81.57
Complete	25.00	0.50	548.10	103.83	0.11	44,653.66	68.48	81.47
Complete	25.00	0.25	547.75	104.05	0.06	44,678.38	68.54	81.57
Complete	25.00	0.00	546.34	105.37	0.00	44,773.34	68.70	81.95
Routing-only	0.00	0.00	425.08	0.00	0.00	25,286.94	59.49	59.49

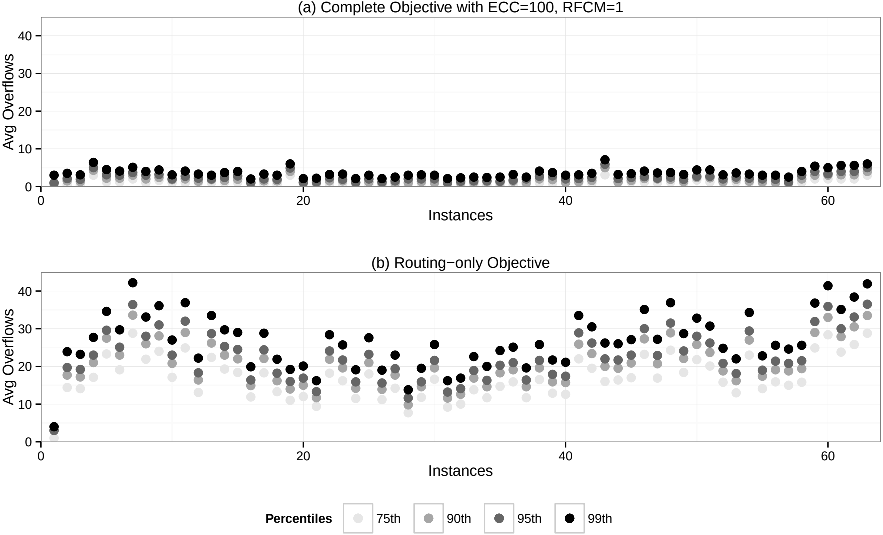
Case Study: Probabilistic Policies

Table 5: Container Overflows and Route Failures

			Avg Num Overflows				Avg Num Route Failures			
Objective	ECC	RFCM	75th Perc.	90th Perc.	95th Perc.	99th Perc.	75th Perc.	90th Perc.	95th Perc.	99th Perc.
Complete	100.00	1.00	0.98	1.78	2.40	3.58	0.03	0.03	0.04	0.05
Complete	100.00	0.50	0.99	1.78	2.39	3.55	0.04	0.05	0.05	0.07
Complete	100.00	0.25	0.97	1.80	2.38	3.56	0.04	0.05	0.06	0.10
Complete	100.00	0.00	0.94	1.77	2.33	3.54	0.08	0.10	0.12	0.16
Complete	50.00	1.00	1.26	2.19	2.82	4.14	0.05	0.05	0.05	0.05
Complete	50.00	0.50	1.28	2.19	2.84	4.16	0.06	0.07	0.08	0.09
Complete	50.00	0.25	1.28	2.18	2.83	4.15	0.04	0.06	0.07	0.10
Complete	50.00	0.00	1.31	2.23	2.85	4.18	0.07	0.09	0.10	0.12
Complete	25.00	1.00	1.48	2.46	3.14	4.58	0.05	0.05	0.05	0.07
Complete	25.00	0.50	1.48	2.46	3.14	4.58	0.05	0.07	0.07	0.10
Complete	25.00	0.25	1.51	2.50	3.18	4.61	0.04	0.07	0.07	0.09
Complete	25.00	0.00	1.54	2.51	3.19	4.64	0.08	0.10	0.10	0.12
Routing-only	0.00	0.00	16.97	20.45	22.56	26.70	0.01	0.03	0.04	0.05

Case Study: Probabilistic Policies

Figure 3: Average Number of Overflows for All Instances



Case Study: Probabilistic Policies

Table 6: Average Number of Collections by Day

Type	EC	RFCM	day $t = 0$	day $t = 1$	day $t = 2$	day $t = 3$	day $t = 4$	day $t = 5$	day $t = 6$	day $t = 7$
Complete	100.00	1.00	60	4	15	53	49	—	—	—
Complete	100.00	0.50	60	6	17	54	56	—	—	—
Complete	100.00	0.25	60	5	16	56	52	—	—	—
Complete	100.00	0.00	60	4	14	53	53	—	—	—
Complete	50.00	1.00	59	6	25	56	44	—	—	—
Complete	50.00	0.50	59	7	18	57	44	—	—	—
Complete	50.00	0.25	59	6	20	54	37	—	—	—
Complete	50.00	0.00	59	6	23	55	43	—	—	—
Complete	25.00	1.00	57	8	27	54	31	—	—	—
Complete	25.00	0.50	57	8	24	56	26	—	—	—
Complete	25.00	0.25	57	8	24	55	29	—	—	—
Complete	25.00	0.00	57	9	28	54	34	—	—	—
Routing-only	0.00	0.00	53	60	45	7	3	—	—	—

Case Study: Alternative Policies

- An alternative practical policy is the use of artificially low capacities in the solution process:
  - Container Effective Capacity (CEC): the fraction of the usable container capacity,
  - Truck Effective Capacity (TEC): the fraction of the usable truck capacity,
  - tests for values of 1.00, 0.90 and 0.75.

Case Study: Alternative Policies

- An alternative practical policy is the use of artificially low capacities in the solution process:
  - Container Effective Capacity (CEC): the fraction of the usable container capacity,
  - Truck Effective Capacity (TEC): the fraction of the usable truck capacity,
  - tests for values of 1.00, 0.90 and 0.75.
- The simulation experiments are wrt the original capacities.
- The objective is always routing-only.

Case Study: Alternative Policies

Table 7: Basic Results for Cost Analysis

Objective	CEC	TEC	Runtime(s.)	Avg Num Tours	Avg Num Containers	Avg Num Dump Visits	Best Cost (CHF)	Avg Cost (CHF)	Gap Avg-Best(%)
Routing-only	1.00	1.00	812.43	1.83	16.77	1.87	422.72	425.48	0.65
Routing-only	1.00	0.90	845.99	1.84	16.72	1.88	422.73	426.94	0.99
Routing-only	1.00	0.75	865.26	1.83	16.81	1.93	424.29	428.02	0.88
Routing-only	0.90	1.00	882.96	2.00	22.69	2.04	486.88	488.76	0.39
Routing-only	0.90	0.90	853.53	2.00	22.69	2.06	487.38	489.20	0.37
Routing-only	0.90	0.75	860.20	2.00	22.71	2.17	489.55	491.91	0.48
Routing-only	0.75	1.00	1003.83	2.10	33.80	2.57	547.48	564.83	3.17
Routing-only	0.75	0.90	1010.03	2.11	33.87	2.73	553.27	570.32	3.08
Routing-only	0.75	0.75	1010.74	2.11	33.89	2.97	558.16	575.98	3.19

Case Study: Alternative Policies

Table 8: Key Performance Indicators for Cost Analysis

Objective	CEC	TEC	Avg Routing Cost (CHF)	Avg Overflow Cost (CHF)	Avg Rte Failure Cost (CHF)	Avg Collected Volume (L)	Liters Per Unit Cost	Liters Per Unit Routing Cost
Routing-only	1.00	1.00	425.48	0.00	0.00	25,311.81	59.49	59.49
Routing-only	1.00	0.90	426.94	0.00	0.00	25,233.43	59.10	59.10
Routing-only	1.00	0.75	428.02	0.00	0.00	25,371.43	59.28	59.28
Routing-only	0.90	1.00	488.76	0.00	0.00	31,532.12	64.51	64.51
Routing-only	0.90	0.90	489.20	0.00	0.00	31,611.40	64.62	64.62
Routing-only	0.90	0.75	491.91	0.00	0.00	31,732.72	64.51	64.51
Routing-only	0.75	1.00	564.83	0.00	0.00	44,134.12	78.14	78.14
Routing-only	0.75	0.90	570.32	0.00	0.00	44,084.86	77.30	77.30
Routing-only	0.75	0.75	575.98	0.00	0.00	44,079.24	76.53	76.53

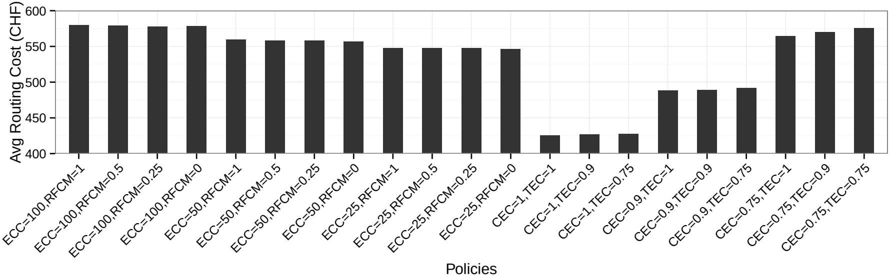
Case Study: Alternative Policies

Table 9: Container Overflows and Route Failures

			Avg Num Overflows				Avg Num Route Failures			
Objective	CEC	TEC	75th Perc.	90th Perc.	95th Perc.	99th Perc.	75th Perc.	90th Perc.	95th Perc.	99th Perc.
Routing-only	1.00	1.00	16.97	20.45	22.58	26.72	0.01	0.03	0.03	0.04
Routing-only	1.00	0.90	17.02	20.51	22.65	26.80	0.00	0.00	0.00	0.00
Routing-only	1.00	0.75	16.91	20.40	22.54	26.65	0.00	0.00	0.00	0.00
Routing-only	0.90	1.00	10.32	13.14	14.85	18.29	0.02	0.02	0.02	0.02
Routing-only	0.90	0.90	10.30	13.09	14.81	18.24	0.00	0.00	0.00	0.00
Routing-only	0.90	0.75	10.32	13.09	14.85	18.28	0.00	0.00	0.00	0.00
Routing-only	0.75	1.00	4.24	6.08	7.27	9.68	0.03	0.03	0.03	0.03
Routing-only	0.75	0.90	4.24	6.06	7.26	9.68	0.00	0.00	0.00	0.00
Routing-only	0.75	0.75	4.22	6.04	7.26	9.67	0.00	0.00	0.00	0.00

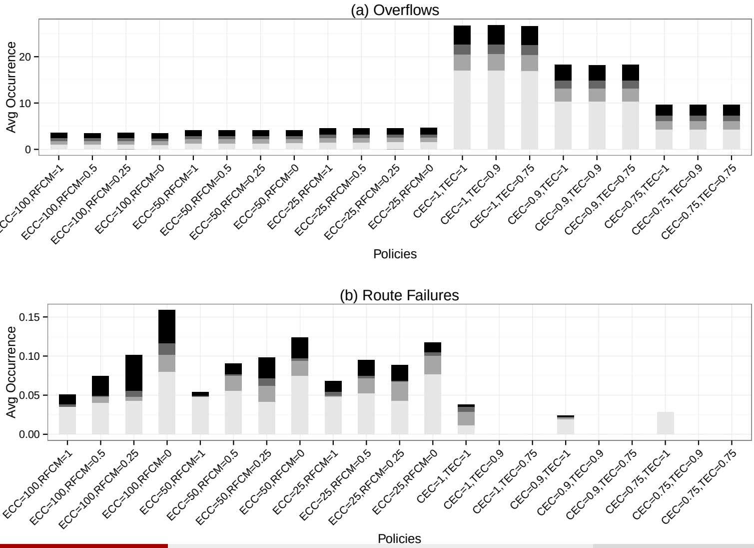
Case Study: Policy Comparison

Figure 4: Comparison of Routing Cost for Probabilistic and Alternative Policies



Case Study: Policy Comparison

Figure 5: Comparison of Container Overflows and Route Failures



Case Study: Rolling Horizon Approach

- In practice, our SIRP will be solved on a rolling horizon basis:
  - container information is dynamically revealed each day,
  - the problem is solved for a planning horizon $\mathcal{T}$,
  - the tours planned for day $t = 0$ are executed,
  - the horizon is rolled over by a day and the procedure is repeated.
- The problem described above is referred to as a Dynamic and Stochastic Inventory Routing Problem (DSIRP).

Case Study: Rolling Horizon Approach

- In practice, our SIRP will be solved on a rolling horizon basis:
  - container information is dynamically revealed each day,
  - the problem is solved for a planning horizon $\mathcal{T}$,
  - the tours planned for day $t = 0$ are executed,
  - the horizon is rolled over by a day and the procedure is repeated.
- The problem described above is referred to as a Dynamic and Stochastic Inventory Routing Problem (DSIRP).
- We hypothesize that the solution cost of a DSIRP is bounded:
  - below by the solution of a static IRP with true demands,
  - above by the solution of a static SIRP with forecast demands.

Case Study: Rolling Horizon Approach

- In practice, our SIRP will be solved on a rolling horizon basis:
  - container information is dynamically revealed each day,
  - the problem is solved for a planning horizon $\mathcal{T}$,
  - the tours planned for day $t = 0$ are executed,
  - the horizon is rolled over by a day and the procedure is repeated.
- The problem described above is referred to as a Dynamic and Stochastic Inventory Routing Problem (DSIRP).
- We hypothesize that the solution cost of a DSIRP is bounded:
  - below by the solution of a static IRP with true demands,
  - above by the solution of a static SIRP with forecast demands.
- Tests on 41 instances, each covering two weeks of white glass collections in the canton of Geneva, Switzerland in 2014, 2015, or 2016.

Case Study: Rolling Horizon Approach

Table 10: Analysis of Rolling Horizon DSIRP Bounds

Instance	Static IRP with True Demand	Rolling DSIRP with Forecast Demand	Static SIRP with Forecast Demand
Inst_1	276.44	582.89	665.19
Inst_2	448.67	784.55	854.49
Inst_3	307.95	653.60	819.79
Inst_4	266.15	574.23	700.36
Inst_5	454.61	682.24	824.57
Inst_6	485.30	677.92	764.86
Inst_7	268.65	569.11	649.57
Inst_8	429.56	585.42	681.23
Inst_9	442.34	599.24	659.30
Inst_10	448.70	564.04	650.88
Inst_11	467.88	549.61	670.36
Inst_12	**449.20**	**674.53**	**626.18**
Inst_13	254.66	556.94	629.93
Inst_14	276.60	585.77	683.65
Inst_15	431.08	548.56	790.39
Inst_16	529.60	635.37	701.64
Inst_17	423.07	578.84	662.76
Inst_18	458.18	595.36	680.75
Inst_19	448.66	524.63	611.56
Inst_20	418.12	520.30	653.18
Inst_21	**276.32**	**791.63**	**626.29**
Inst_22	429.20	531.04	607.63
Inst_23	241.44	551.58	690.62
Inst_24	**547.92**	**758.84**	**748.71**
Inst_25	446.31	618.80	696.75
Inst_26	442.38	589.53	695.11
Inst_27	441.36	589.07	707.30
Inst_28	468.46	616.53	738.58
Inst_29	436.25	575.25	701.73
Inst_30	414.41	677.65	690.37
Inst_31	442.87	544.75	668.51
Inst_32	255.32	612.44	694.35
Inst_33	460.04	677.54	808.74
Inst_34	505.55	682.90	711.62
Inst_35	**490.37**	**989.21**	**785.51**
Inst_36	454.60	646.95	805.95
Inst_37	465.31	607.52	746.64
Inst_38	520.38	721.23	815.21
Inst_39	243.94	613.96	705.10
Inst_40	450.94	624.76	759.97
Inst_41	403.01	575.80	688.24

Note: The four instances for which the hypothesized bounds do not hold are shown in bold.

Outline

Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- An ALNS that produces excellent results on IRP benchmarks.
- Computational experiments on real-data instances demonstrate:
  - the relevance of the probabilistic information captured in the objective,
  - the superiority of the probabilistic approach in comparison to alternative policies.
- Empirical bounds on the solution cost of a rolling horizon approach.

Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.
- An ALNS that produces excellent results on IRP benchmarks.
- Computational experiments on real-data instances demonstrate:
  - the relevance of the probabilistic information captured in the objective,
  - the superiority of the probabilistic approach in comparison to alternative policies.
- Empirical bounds on the solution cost of a rolling horizon approach.
- Future research directions:
  - decomposition methods,
  - chance constraints,
  - value of stochastic information.

Thank you.

Questions?

Abdollahi, M., Arvan, M., Omidvar, A., and Ameri, F. (2014). A simulation optimization approach to apply value at risk analysis on the inventory routing problem with backlogged demand. *International Journal of Industrial Engineering Computations*, 5(4):603–620.

Adulyasak, Y., Cordeau, J.-F., and Jans, R. (2015). Benders decomposition for production routing under demand uncertainty. *Operations Research*, 63(4):851–867.

Archetti, C., Bertazzi, L., Hertz, A., and Speranza, M. G. (2012). A hybrid heuristic for an inventory routing problem. *INFORMS Journal on Computing*, 24(1):101–116.

Archetti, C., Bertazzi, L., Laporte, G., and Speranza, M. G. (2007). A branch-and-cut algorithm for a vendor-managed inventory-routing problem. *Transportation Science*, 41(3):382–391.

Baldacci, R. and Mingozzi, A. (2009). A unified exact method for solving different classes of vehicle routing problems. *Mathematical Programming*, 120(2):347–380.

Bard, J. F., Huang, L., Dror, M., and Jaillet, P. (1998a). A branch and cut algorithm for the VRP with satellite facilities. *IIE Transactions*, 30(9):821–834.

Bard, J. F., Huang, L., Jaillet, P., and Dror, M. (1998b). A decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science*, 32(2):189–203.

Bertazzi, L., Bosco, A., Guerriero, F., and Laganà, D. (2013). A stochastic inventory routing problem with stock-out. *Transportation Research Part C: Emerging Technologies*, 27:89–107.

Bertazzi, L., Bosco, A., and Laganà, D. (2015). Managing stochastic demand in an inventory routing problem with transportation procurement. *Omega*, 56:112–121.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2012a). Consistency in multi-vehicle inventory-routing. *Transportation Research Part C: Emerging Technologies*, 24:270–287.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2012b). The inventory-routing problem with transshipment. *Computers & Operations Research*, 39(11):2537–2548.

Conrad, R. G. and Figliozzi, M. A. (2011). The recharging vehicle routing problem. In Doolen, T. and Aken, E. V., editors, *Proceedings of the 2011 Industrial Engineering Research Conference*, Reno, NV, USA.

Crevier, B., Cordeau, J.-F., and Laporte, G. (2007). The multi-depot vehicle routing problem with inter-depot routes. *European Journal of Operational Research*, 176(2):756–773.

Erdoğan, S. and Miller-Hooks, E. (2012). A green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, 48(1):100–114.

Goeke, D. and Schneider, M. (2015). Routing a mixed fleet of electric and conventional vehicles. *European Journal of Operational Research*, 245(1):81–99.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Savelsbergh, M. W. (2010). Vendor managed inventory for environments with stochastic product usage. *European Journal of Operational Research*, 202(3):686–695.

Hiermann, G., Puchinger, J., and Hartl, R. F. (2014). The electric fleet size and mix vehicle routing problem with time windows and recharging stations. Working paper, Austrian Institute of Technology and University of Vienna, Austria.

Jaillet, P., Bard, J. F., Huang, L., and Dror, M. (2002). Delivery cost approximations for inventory routing problems in a rolling horizon framework. *Transportation Science*, 36(3):292–300.

Kim, B. I., Kim, S., and Sahoo, S. (2006). Waste collection vehicle routing problem with time windows. *Computers & Operations Research*, 33(12):3624–3642.

Nolz, P. C., Absi, N., and Feillet, D. (2014). A stochastic inventory routing problem for infectious medical waste collection. *Networks*, 63(1):82–95.

Penna, P. H. V., Subramanian, A., and Ochi, L. S. (2013). An iterated local search heuristic for the heterogeneous fleet vehicle routing problem. *Journal of Heuristics*, 19(2):201–232.

Schneider, M., Stenger, A., and Goeke, D. (2014). The electric vehicle-routing problem with time windows and recharging stations. *Transportation Science*, 48(4):500–520.

Schneider, M., Stenger, A., and Hof, J. (2015). An adaptive VNS algorithm for vehicle routing problems with intermediate stops. *OR Spectrum*, 37(2):353–387.

Solyalı, O., Cordeau, J.-F., and Laporte, G. (2012). Robust inventory routing under demand uncertainty. *Transportation Science*, 46(3):327–340.

Soysal, M., Bloemhof-Ruwaard, J. M., Haijema, R., and van der Vorst, J. G. (2015). Modeling an inventory routing problem for perishable products with environmental considerations and demand uncertainty. *International Journal of Production Economics*, 164:118–133.

Subramanian, A., Penna, P. H. V., Uchoa, E., and Ochi, L. S. (2012). A hybrid algorithm for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research*, 221(2):285–295.

Taillard, É. D. (1999). A heuristic column generation method for the heterogeneous fleet VRP. *RAIRO - Operations Research*, 33(1):1–14.

Trudeau, P. and Dror, M. (1992). Stochastic inventory routing: Route design with stockouts and route failures. *Transportation Science*, 26(3):171–184.

Yu, Y., Chu, C., Chen, H., and Chu, F. (2012). Large scale stochastic inventory routing problems with split delivery and service level constraints. *Annals of Operations Research*, 197(1):135–158.