Vehicle routing and demand forecasting in recyclable waste collection

Iliya Markov

Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
École Polytechnique Fédérale de Lausanne

Joint work with Yousef Maknoon, Matthieu de Lapparent,
Jean-François Cordeau, Sacha Varone, Michel Bierlaire

CIRRELT
November 26, 2015

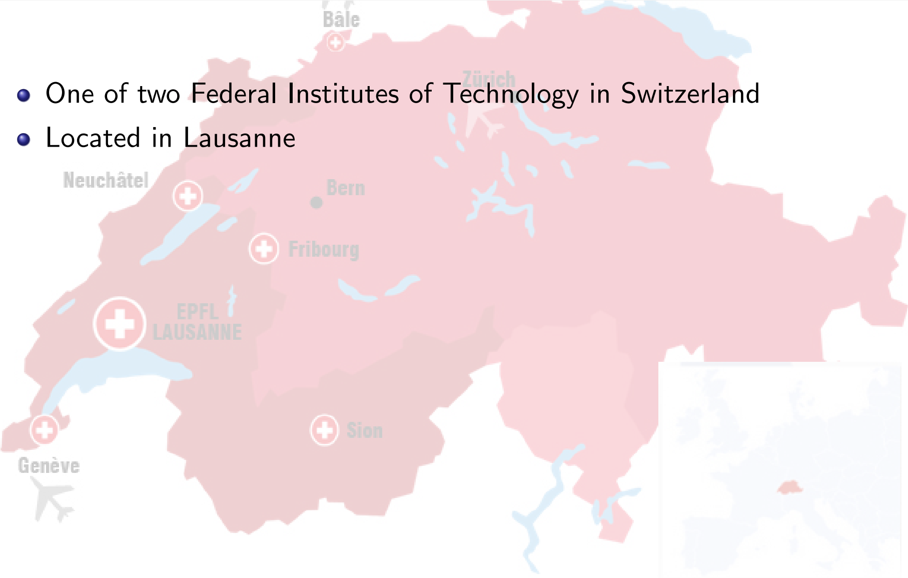

Source: EPFL Facebook page

Source: EPFL Facebook page

Source: EPFL website

EPFL facts and figures

- One of two Federal Institutes of Technology in Switzerland
- Located in Lausanne



EPFL facts and figures

- One of two Federal Institutes of Technology in Switzerland
- Located in Lausanne
- Comprises 5 schools, 2 colleges, 26 institutes, and 350 laboratories

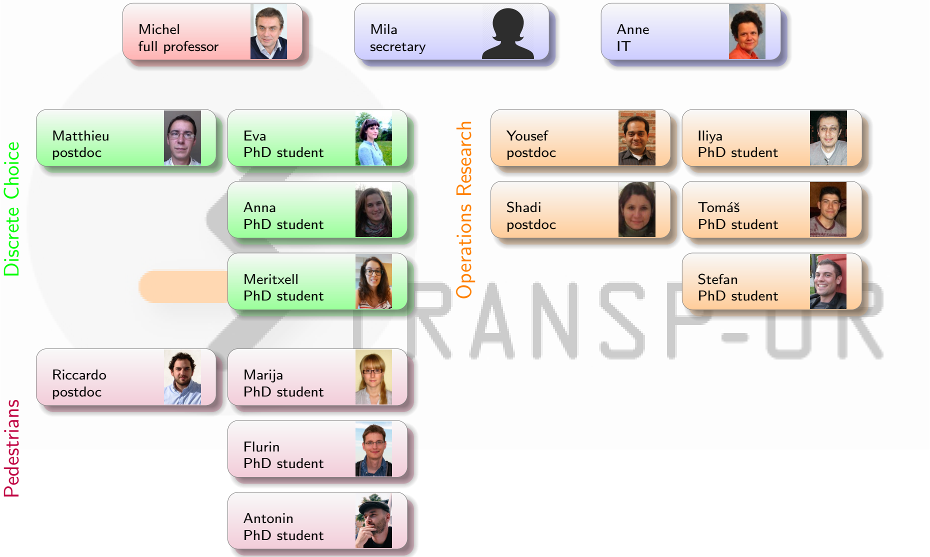
EPFL facts and figures

- One of two Federal Institutes of Technology in Switzerland
- Located in Lausanne
- Comprises 5 schools, 2 colleges, 26 institutes, and 350 laboratories
- 10,124 students of over 115 nationalities
  - 5,205 bachelor students (40% international)
  - 2,646 master students (50% international)
  - 2,077 PhD students (80% international)

EPFL facts and figures

- One of two Federal Institutes of Technology in Switzerland
- Located in Lausanne
- Comprises 5 schools, 2 colleges, 26 institutes, and 350 laboratories
- 10,124 students of over 115 nationalities
  - 5,205 bachelor students (40% international)
  - 2,646 master students (50% international)
  - 2,077 PhD students (80% international)
- 5,630 staff
  - 338 faculty
  - 1,228 postdocs

TRANSP-OR


Michel
full professor
Mila
secretary
Anne
IT
Discrete Choice
Matthieu
postdoc
Eva
PhD student
Anna
PhD student
Meritxell
PhD student
Operations Research
Yousef
postdoc
Iliya
PhD student
Shadi
postdoc
Tomáš
PhD student
Stefan
PhD student
TRANSP-OR
Pedestrians
Riccardo
postdoc
Marija
PhD student
Flurin
PhD student
Antonin
PhD student

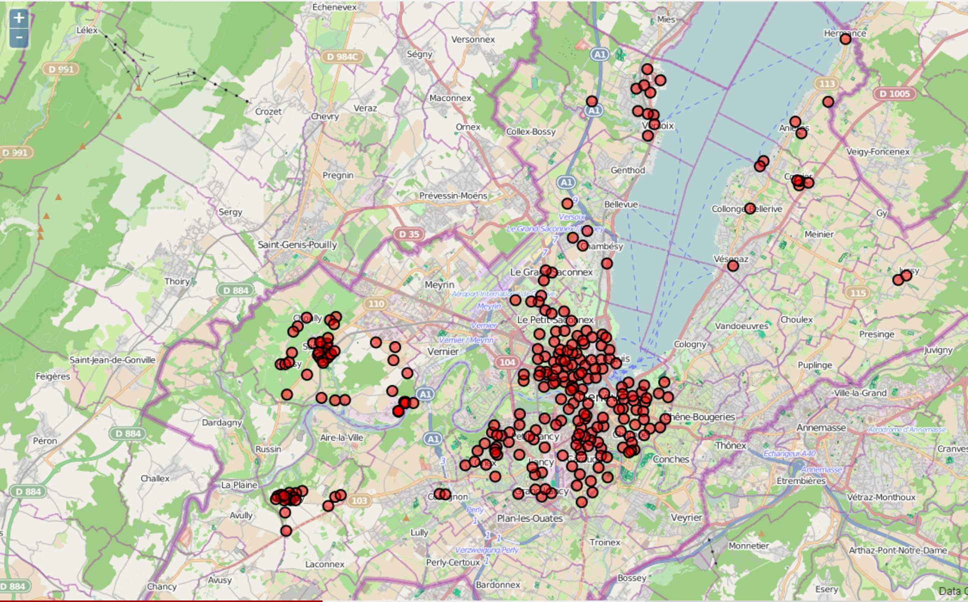
Contents

Contents

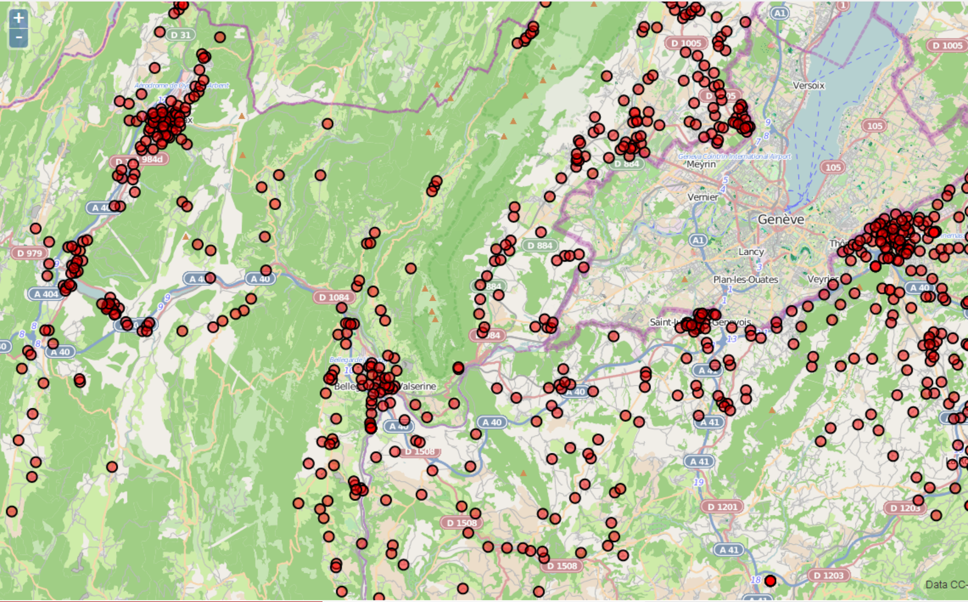
Ecological waste management (CTI project)

Ecopoint in Rue de Neuchâtel, Geneva; Source: self

Ecological waste management (CTI project)

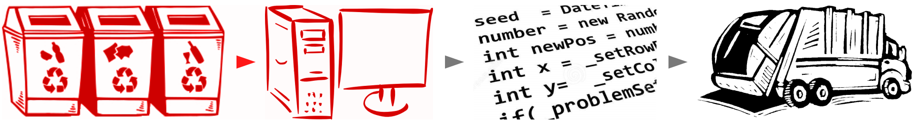


Ecological waste management (CTI project)



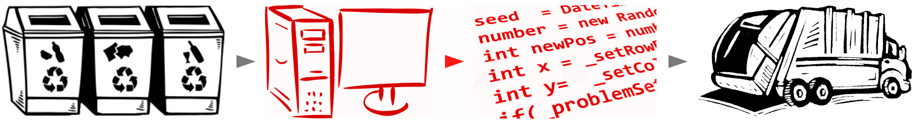
Introduction

- Sensorized containers for recyclables periodically send waste level data to a central database.



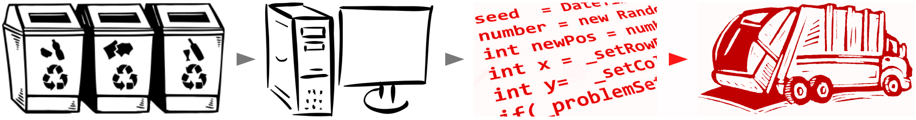
Introduction

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and vehicle routing, with tours *often planned several days in advance*.



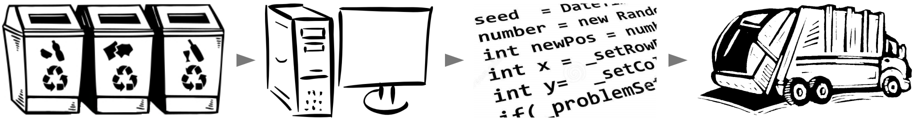
Introduction

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and vehicle routing, with tours *often planned several days in advance*.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.



Introduction

- Sensorized containers for recyclables periodically send waste level data to a central database.
- Level data is used for container selection and vehicle routing, with tours *often planned several days in advance*.
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.
- Efficient waste collection thus depends on the ability to
  - make **good forecasts** of the container levels at the time of collection,
  - and **optimally route** the vehicles to serve the selected containers.



Contents

Problem description

- Multiple depots, containers, and dumps (recycling plants) with TW
- Maximum tour duration, interrupted by a break
- Site dependencies (accessibility restrictions)
- Tours are sequences of collections and disposals at the available dumps, with a mandatory disposal before the end.

Problem description

- Multiple depots, containers, and dumps (recycling plants) with TW
- Maximum tour duration, interrupted by a break
- Site dependencies (accessibility restrictions)
- Tours are sequences of collections and disposals at the available dumps, with a mandatory disposal before the end.

- *Tours need not finish at the depot they started from.*
  - flexible assignment of destination depots
  - practiced in sparsely populated rural areas

Problem description

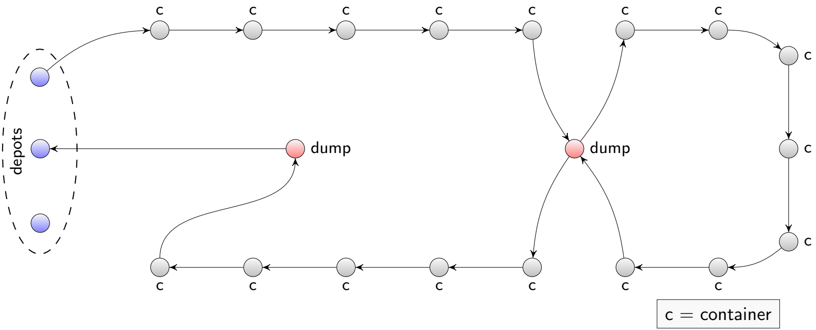
- Multiple depots, containers, and dumps (recycling plants) with TW
- Maximum tour duration, interrupted by a break
- Site dependencies (accessibility restrictions)
- Tours are sequences of collections and disposals at the available dumps, with a mandatory disposal before the end.

- *Tours need not finish at the depot they started from*.
  - flexible assignment of destination depots
  - practiced in sparsely populated rural areas

- *There is a heterogeneous fixed fleet*.
  - different volume and weight capacities, speeds, costs, etc...

Problem description

Figure 1: Tour illustration



State of the art

- VRP with intermediate facilities (VRP-IF)
  - Bard et al. (1998), Kim et al. (2006), Crevier et al. (2007)
- Electric and alternative fuel VRP
  - Conrad and Figliozzi (2011), Erdoğan and Miller-Hooks (2012), Schneider et al. (2014), Schneider et al. (2015)
- Heterogeneous fixed fleet VRP
  - Taillard (1996), Baldacci and Mingozzi (2009), Subramanian et al. (2012), Penna et al. (2013)
  - Hiermann et al. (2014) and Goeke and Schneider (2014) use some form of heterogeneity in the electric VRP
- Flexible assignment of depots
  - Kek et al. (2008)

Contributions

- Integration of dynamic destination depot assignment into the VRP-IF
  - consideration of relocation costs
- Integration of heterogeneous fixed fleet into the VRP-IF
  - challenges posed by intermediate facility visits
- Benchmarking to several classes of simpler problems from the literature and state of practice
  - E-VRPTW (modified from Schneider et al., 2014)
  - MDVRPI (Crevier et al., 2007)
  - optimal solutions, state of practice, etc...

Formulation

Sets

$\mathcal{O}'$	= set of origins	$\mathcal{O}''$	= set of destinations
$\mathcal{D}$	= set of dumps	$\mathcal{P}$	= set of containers
$\mathcal{N}$	$= \mathcal{O}' \cup \mathcal{O}'' \cup \mathcal{D} \cup \mathcal{P}$	$\mathcal{K}$	= set of vehicles

Parameters

π_{ij}	= length of arc (i, j)
τ_{ijk}	= travel time of vehicle k on arc (i, j)
$[\lambda_i, \mu_i]$	= time window lower and upper bound at point i
ε_i	= service duration at point i
ρ_i	= pickup quantity at point i
Ω_k	= capacity of vehicle k
ϕ_k	= fixed cost of vehicle k
β_k	= unit-distance running cost of vehicle k
θ_k	= unit-time running cost of vehicle k
α_{ik}	= 1 if point i is accessible for vehicle k, 0 otherwise
H	= maximum tour duration
η	= maximum continuous work limit after which a break is due
δ	= break duration
Ψ	= weight of relocation cost term

Formulation

Decision variables: binary

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ traverses arc } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ijk} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are, respectively, the origin and destination of vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

$$b_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ takes a break on arc } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

$$y_{k} = \begin{cases} 1 & \text{if vehicle } k \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Decision variables: continuous

S_{ik} = start-of-service time of vehicle k at point i

Q_{ik} = cumulative quantity on vehicle k at point i

Formulation

$$\min \quad r = \sum_{k \in \mathcal{K}} \left(\phi_k y_k + \beta_k \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij} x_{ijk} + \theta_k \left(\sum_{j \in \mathcal{O}''_k} S_{jk} - \sum_{i \in \mathcal{O}'_k} S_{ik} \right) \right) + \Psi \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{O}'_k} \sum_{j \in \mathcal{O}''_k} \left(\beta_k \pi_{ji} + \theta_k \tau_{jik} \right) z_{ijk} \quad (1)$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{D} \cup \mathcal{P}} x_{ijk} = 1, \qquad \forall i \in \mathcal{P} \quad (2)$$

$$\sum_{i \in \mathcal{O}'_k} \sum_{j \in \mathcal{N}} x_{ijk} = y_k, \qquad \forall k \in \mathcal{K} \quad (3)$$

$$\sum_{i \in \mathcal{D}} \sum_{j \in \mathcal{O}''_k} x_{ijk} = y_k, \qquad \forall k \in \mathcal{K} \quad (4)$$

$$\sum_{i \in \mathcal{N}} x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, j \in \mathcal{O}' \cup (\mathcal{O}'' \setminus \mathcal{O}''_k) \quad (5)$$

$$\sum_{j \in \mathcal{N}} x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'' \cup (\mathcal{O}' \setminus \mathcal{O}'_k) \quad (6)$$

Formulation

$$\min \quad r = \sum_{k \in \mathcal{K}} \left(\phi_k y_k + \beta_k \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \pi_{ij} x_{ijk} + \theta_k \left(\sum_{j \in \mathcal{O}''_k} S_{jk} - \sum_{i \in \mathcal{O}'_k} S_{ik} \right) \right) + \Psi \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{O}'_k} \sum_{j \in \mathcal{O}''_k} \left(\beta_k \pi_{ji} + \theta_k \tau_{jik} \right) z_{ijk} \tag{1}$$

$$\text{s.t.} \quad \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{D} \cup \mathcal{P}} x_{ijk} = 1, \qquad \forall i \in \mathcal{P} \tag{2}$$

$$\sum_{i \in \mathcal{O}'_k} \sum_{j \in \mathcal{N}} x_{ijk} = y_k, \qquad \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{i \in \mathcal{D}} \sum_{j \in \mathcal{O}''_k} x_{ijk} = y_k, \qquad \forall k \in \mathcal{K} \tag{4}$$

$$\sum_{i \in \mathcal{N}} x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, j \in \mathcal{O}' \cup (\mathcal{O}'' \setminus \mathcal{O}''_k) \tag{5}$$

$$\sum_{j \in \mathcal{N}} x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'' \cup (\mathcal{O}' \setminus \mathcal{O}'_k) \tag{6}$$

Formulation

$$\min \quad r = \sum_{k\in\mathcal{K}} \left(\phi_k y_k + \beta_k \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}} \pi_{ij} x_{ijk} + \theta_k \left(\sum_{j\in\mathcal{O}''_k} S_{jk} - \sum_{i\in\mathcal{O}'_k} S_{ik} \right) \right)$$
$$+ \Psi \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{O}'_k} \sum_{j\in\mathcal{O}''_k} \left(\beta_k \pi_{ji} + \theta_k \tau_{jik}\right) z_{ijk} \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{D}\cup\mathcal{P}} x_{ijk} = 1, \qquad \forall i \in \mathcal{P} \tag{2}$$

$$\sum_{i\in\mathcal{O}'_k} \sum_{j\in\mathcal{N}} x_{ijk} = y_k, \qquad \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{i\in\mathcal{D}} \sum_{j\in\mathcal{O}''_k} x_{ijk} = y_k, \qquad \forall k \in \mathcal{K} \tag{4}$$

$$\sum_{i\in\mathcal{N}} x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, j \in \mathcal{O}' \cup (\mathcal{O}'' \setminus \mathcal{O}''_k) \tag{5}$$

$$\sum_{j\in\mathcal{N}} x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'' \cup (\mathcal{O}' \setminus \mathcal{O}'_k) \tag{6}$$

Formulation

$$\min \quad r = \sum_{k\in\mathcal{K}} \left(\phi_k y_k + \beta_k \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}} \pi_{ij} x_{ijk} + \theta_k \left(\sum_{j\in\mathcal{O}''_k} S_{jk} - \sum_{i\in\mathcal{O}'_k} S_{ik} \right)\right)$$
$$+ \Psi \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{O}'_k} \sum_{j\in\mathcal{O}''_k} \left(\beta_k \pi_{ji} + \theta_k \tau_{jik}\right) z_{ijk} \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{D}\cup\mathcal{P}} x_{ijk} = 1, \qquad \forall i \in \mathcal{P} \tag{2}$$

$$\sum_{i\in\mathcal{O}'_k} \sum_{j\in\mathcal{N}} x_{ijk} = y_k, \qquad \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{i\in\mathcal{D}} \sum_{j\in\mathcal{O}''_k} x_{ijk} = y_k, \qquad \forall k \in \mathcal{K} \tag{4}$$

$$\sum_{i\in\mathcal{N}} x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, j \in \mathcal{O}' \cup (\mathcal{O}'' \setminus \mathcal{O}''_k) \tag{5}$$

$$\sum_{j\in\mathcal{N}} x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'' \cup (\mathcal{O}' \setminus \mathcal{O}'_k) \tag{6}$$

Formulation

s.t.
$$\sum_{i\in\mathcal{N}:\, i\neq j} x_{ijk} = \sum_{i\in\mathcal{N}:\, i\neq j} x_{jik}, \quad \forall k \in \mathcal{K}, j \in \mathcal{P}\cup\mathcal{D} \tag{7}$$

$$\sum_{m\in\mathcal{N}} x_{imk} + \sum_{m\in\mathcal{D}} x_{mjk} - 1 \leqslant z_{ijk}, \quad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k, j \in \mathcal{O}''_k \tag{8}$$

$$\sum_{j\in\mathcal{N}} x_{ijk} \leqslant \alpha_{ik}, \quad \forall k \in \mathcal{K}, i \in \mathcal{P}\cup\mathcal{D} \tag{9}$$

$$\rho_i \leqslant Q_{ik} \leqslant \Omega_k, \quad \forall k \in \mathcal{K}, i \in \mathcal{P} \tag{10}$$

$$Q_{ik} = 0, \quad \forall k \in \mathcal{K}, i \in \mathcal{N}\setminus\mathcal{P} \tag{11}$$

$$Q_{ik} + \rho_j \leqslant Q_{jk} + \Omega_k\left(1 - x_{ijk}\right), \quad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup\mathcal{P}\cup\mathcal{D}, j \in \mathcal{P} \tag{12}$$

$$S_{ik} + \varepsilon_i + \delta b_{ijk} + \tau_{ijk} \leqslant S_{jk} + M\left(1 - x_{ijk}\right), \quad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup\mathcal{P}\cup\mathcal{D}, j \in \mathcal{P}\cup\mathcal{D}\cup\mathcal{O}''_k \tag{13}$$

$$\lambda_i \sum_{j\in\mathcal{N}} x_{ijk} \leqslant S_{ik}, \quad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup\mathcal{P}\cup\mathcal{D} \tag{14}$$

$$S_{jk} \leqslant \mu_j \sum_{i\in\mathcal{N}} x_{ijk}, \quad \forall k \in \mathcal{K}, j \in \mathcal{P}\cup\mathcal{D}\cup\mathcal{O}''_k \tag{15}$$

$$0 \leqslant \sum_{j\in\mathcal{O}''_k} S_{jk} - \sum_{i\in\mathcal{O}'_k} S_{ik} \leqslant \mathrm{H}, \quad \forall k \in \mathcal{K} \tag{16}$$

Formulation

$$\text{s.t.} \quad \sum_{i \in \mathcal{N}:\, i \neq j} x_{ijk} = \sum_{i \in \mathcal{N}:\, i \neq j} x_{jik}, \qquad \forall k \in \mathcal{K}, j \in \mathcal{P} \cup \mathcal{D} \tag{7}$$

$$\sum_{m \in \mathcal{N}} x_{imk} + \sum_{m \in \mathcal{D}} x_{mjk} - 1 \leqslant z_{ijk}, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k, j \in \mathcal{O}''_k \tag{8}$$

$$\sum_{j \in \mathcal{N}} x_{ijk} \leqslant \alpha_{ik}, \qquad \forall k \in \mathcal{K}, i \in \mathcal{P} \cup \mathcal{D} \tag{9}$$

$$\rho_i \leqslant Q_{ik} \leqslant \Omega_k, \qquad \forall k \in \mathcal{K}, i \in \mathcal{P} \tag{10}$$

$$Q_{ik} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{N} \setminus \mathcal{P} \tag{11}$$

$$Q_{ik} + \rho_j \leqslant Q_{jk} + \Omega_k \left(1 - x_{ijk}\right), \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \tag{12}$$

$$S_{ik} + \varepsilon_i + \delta b_{ijk} + \tau_{ijk} \leqslant S_{jk} + M\left(1 - x_{ijk}\right), \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k \tag{13}$$

$$\lambda_i \sum_{j \in \mathcal{N}} x_{ijk} \leqslant S_{ik}, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D} \tag{14}$$

$$S_{jk} \leqslant \mu_j \sum_{i \in \mathcal{N}} x_{ijk}, \qquad \forall k \in \mathcal{K}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k \tag{15}$$

$$0 \leqslant \sum_{j \in \mathcal{O}''_k} S_{jk} - \sum_{i \in \mathcal{O}'_k} S_{ik} \leqslant \mathrm{H}, \qquad \forall k \in \mathcal{K} \tag{16}$$

Formulation

s.t.

$$\sum_{i\in\mathcal{N}:\, i\neq j} x_{ijk} = \sum_{i\in\mathcal{N}:\, i\neq j} x_{jik}, \qquad \forall k\in\mathcal{K}, j\in\mathcal{P}\cup\mathcal{D} \tag{7}$$

$$\sum_{m\in\mathcal{N}} x_{imk} + \sum_{m\in\mathcal{D}} x_{mjk} - 1 \leqslant z_{ijk}, \qquad \forall k\in\mathcal{K}, i\in\mathcal{O}'_k, j\in\mathcal{O}''_k \tag{8}$$

$$\sum_{j\in\mathcal{N}} x_{ijk} \leqslant \alpha_{ik}, \qquad \forall k\in\mathcal{K}, i\in\mathcal{P}\cup\mathcal{D} \tag{9}$$

$$\rho_i \leqslant Q_{ik} \leqslant \Omega_k, \qquad \forall k\in\mathcal{K}, i\in\mathcal{P} \tag{10}$$

$$Q_{ik} = 0, \qquad \forall k\in\mathcal{K}, i\in\mathcal{N}\setminus\mathcal{P} \tag{11}$$

$$Q_{ik} + \rho_j \leqslant Q_{jk} + \Omega_k\left(1 - x_{ijk}\right), \qquad \forall k\in\mathcal{K}, i\in\mathcal{O}'_k\cup\mathcal{P}\cup\mathcal{D}, j\in\mathcal{P} \tag{12}$$

$$S_{ik} + \varepsilon_i + \delta b_{ijk} + \tau_{ijk} \leqslant S_{jk} + M\left(1 - x_{ijk}\right), \forall k\in\mathcal{K}, i\in\mathcal{O}'_k\cup\mathcal{P}\cup\mathcal{D}, j\in\mathcal{P}\cup\mathcal{D}\cup\mathcal{O}''_k \tag{13}$$

$$\lambda_i \sum_{j\in\mathcal{N}} x_{ijk} \leqslant S_{ik}, \qquad \forall k\in\mathcal{K}, i\in\mathcal{O}'_k\cup\mathcal{P}\cup\mathcal{D} \tag{14}$$

$$S_{jk} \leqslant \mu_j \sum_{i\in\mathcal{N}} x_{ijk}, \qquad \forall k\in\mathcal{K}, j\in\mathcal{P}\cup\mathcal{D}\cup\mathcal{O}''_k \tag{15}$$

$$0 \leqslant \sum_{j\in\mathcal{O}''_k} S_{jk} - \sum_{i\in\mathcal{O}'_k} S_{ik} \leqslant \mathrm{H}, \qquad \forall k\in\mathcal{K} \tag{16}$$

Formulation

s.t.

$$\sum_{i\in\mathcal{N}:\ i\neq j} x_{ijk} = \sum_{i\in\mathcal{N}:\ i\neq j} x_{jik}, \qquad \forall k\in\mathcal{K}, j\in\mathcal{P}\cup\mathcal{D} \tag{7}$$

$$\sum_{m\in\mathcal{N}} x_{imk} + \sum_{m\in\mathcal{D}} x_{mjk} - 1 \leqslant z_{ijk}, \qquad \forall k\in\mathcal{K}, i\in\mathcal{O}'_k, j\in\mathcal{O}''_k \tag{8}$$

$$\sum_{j\in\mathcal{N}} x_{ijk} \leqslant \alpha_{ik}, \qquad \forall k\in\mathcal{K}, i\in\mathcal{P}\cup\mathcal{D} \tag{9}$$

$$\rho_i \leqslant Q_{ik} \leqslant \Omega_k, \qquad \forall k\in\mathcal{K}, i\in\mathcal{P} \tag{10}$$

$$Q_{ik} = 0, \qquad \forall k\in\mathcal{K}, i\in\mathcal{N}\setminus\mathcal{P} \tag{11}$$

$$Q_{ik} + \rho_j \leqslant Q_{jk} + \Omega_k\left(1 - x_{ijk}\right), \qquad \forall k\in\mathcal{K}, i\in\mathcal{O}'_k\cup\mathcal{P}\cup\mathcal{D}, j\in\mathcal{P} \tag{12}$$

$$S_{ik} + \varepsilon_i + \delta b_{ijk} + \tau_{ijk} \leqslant S_{jk} + M\left(1 - x_{ijk}\right), \forall k\in\mathcal{K}, i\in\mathcal{O}'_k\cup\mathcal{P}\cup\mathcal{D}, j\in\mathcal{P}\cup\mathcal{D}\cup\mathcal{O}''_k \tag{13}$$

$$\lambda_i \sum_{j\in\mathcal{N}} x_{ijk} \leqslant S_{ik}, \qquad \forall k\in\mathcal{K}, i\in\mathcal{O}'_k\cup\mathcal{P}\cup\mathcal{D} \tag{14}$$

$$S_{jk} \leqslant \mu_j \sum_{i\in\mathcal{N}} x_{ijk}, \qquad \forall k\in\mathcal{K}, j\in\mathcal{P}\cup\mathcal{D}\cup\mathcal{O}''_k \tag{15}$$

$$0 \leqslant \sum_{j\in\mathcal{O}''_k} S_{jk} - \sum_{i\in\mathcal{O}'_k} S_{ik} \leqslant \mathrm{H}, \qquad \forall k\in\mathcal{K} \tag{16}$$

Formulation

$$\text{s.t.} \quad \sum_{i \in \mathcal{N}:\, i \neq j} x_{ijk} = \sum_{i \in \mathcal{N}:\, i \neq j} x_{jik}, \qquad \forall k \in \mathcal{K}, j \in \mathcal{P} \cup \mathcal{D} \tag{7}$$

$$\sum_{m \in \mathcal{N}} x_{imk} + \sum_{m \in \mathcal{D}} x_{mjk} - 1 \leqslant z_{ijk}, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k, j \in \mathcal{O}''_k \tag{8}$$

$$\sum_{j \in \mathcal{N}} x_{ijk} \leqslant \alpha_{ik}, \qquad \forall k \in \mathcal{K}, i \in \mathcal{P} \cup \mathcal{D} \tag{9}$$

$$\rho_i \leqslant Q_{ik} \leqslant \Omega_k, \qquad \forall k \in \mathcal{K}, i \in \mathcal{P} \tag{10}$$

$$Q_{ik} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{N} \setminus \mathcal{P} \tag{11}$$

$$Q_{ik} + \rho_j \leqslant Q_{jk} + \Omega_k \left(1 - x_{ijk}\right), \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \tag{12}$$

$$S_{ik} + \varepsilon_i + \delta b_{ijk} + \tau_{ijk} \leqslant S_{jk} + M \left(1 - x_{ijk}\right), \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k \tag{13}$$

$$\lambda_i \sum_{j \in \mathcal{N}} x_{ijk} \leqslant S_{ik}, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D} \tag{14}$$

$$S_{jk} \leqslant \mu_j \sum_{i \in \mathcal{N}} x_{ijk}, \qquad \forall k \in \mathcal{K}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k \tag{15}$$

$$0 \leqslant \sum_{j \in \mathcal{O}''_k} S_{jk} - \sum_{i \in \mathcal{O}'_k} S_{ik} \leqslant \mathrm{H}, \qquad \forall k \in \mathcal{K} \tag{16}$$

Formulation

$$\text{s.t.} \left(S_{ik} - \sum_{m \in \mathcal{O}'_k} S_{mk} \right) + \varepsilon_i - \eta \leqslant M \left(1 - b_{ijk}\right), \quad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k \quad (17)$$

$$\eta - \left(S_{jk} - \sum_{m \in \mathcal{O}'_k} S_{mk} \right) \leqslant M \left(1 - b_{ijk}\right), \quad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k \quad (18)$$

$$b_{ijk} \leqslant x_{ijk}, \quad \forall k \in \mathcal{K}, i, j \in \mathcal{N} \quad (19)$$

$$\left(\sum_{j \in \mathcal{O}''_k} S_{jk} - \sum_{i \in \mathcal{O}'_k} S_{ik} \right) - \eta \leqslant (\mathrm{H} - \eta) \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} b_{ijk}, \quad \forall k \in \mathcal{K} \quad (20)$$

$$x_{ijk}, z_{ijk}, b_{ijk}, y_k \in \{0, 1\}, \quad \forall k \in \mathcal{K}, i, j \in \mathcal{N} \quad (21)$$

$$S_{ik} \, Q_{ik} \geqslant 0, \quad \forall k \in \mathcal{K}, i \in \mathcal{N} \quad (22)$$

Formulation

$$\text{s.t.} \left(S_{ik} - \sum_{m \in \mathcal{O}'_k} S_{mk} \right) + \varepsilon_i - \eta \leqslant M\left(1 - b_{ijk}\right), \ \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k \quad (17)$$

$$\eta - \left(S_{jk} - \sum_{m \in \mathcal{O}'_k} S_{mk} \right) \leqslant M\left(1 - b_{ijk}\right), \quad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k \quad (18)$$

$$b_{ijk} \leqslant x_{ijk}, \quad \forall k \in \mathcal{K}, i, j \in \mathcal{N} \quad (19)$$

$$\left(\sum_{j \in \mathcal{O}''_k} S_{jk} - \sum_{i \in \mathcal{O}'_k} S_{ik} \right) - \eta \leqslant (\mathrm{H} - \eta) \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} b_{ijk}, \ \forall k \in \mathcal{K} \quad (20)$$

$$x_{ijk}, z_{ijk}, b_{ijk}, y_k \in \{0, 1\}, \quad \forall k \in \mathcal{K}, i, j \in \mathcal{N} \quad (21)$$

$$S_{ik} Q_{ik} \geqslant 0, \quad \forall k \in \mathcal{K}, i \in \mathcal{N} \quad (22)$$

Solution methodology: Exact approach

- We apply variable fixing and several additional inequalities
- Impossible traversals

$$x_{iik} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{N} \tag{23}$$
$$x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k, j \in \mathcal{D} \cup \mathcal{O}''_k \tag{24}$$
$$x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{P}, j \in \mathcal{O}''_k \tag{25}$$
$$x_{ijk} = 0, \qquad \forall k \in \mathcal{K}, i \in \mathcal{D}, j \in \mathcal{D} \colon i \neq j \tag{26}$$

- Time-window infeasible traversals

$$x_{ijk} = 0, \qquad \forall k \in K, i \in \mathcal{O}'_k \cup \mathcal{P} \cup \mathcal{D}, j \in \mathcal{P} \cup \mathcal{D} \cup \mathcal{O}''_k : \lambda_i + \varepsilon_i + \tau_{ijk} > \mu_j \tag{27}$$

- Bounds on time

$$\sum_{j \in \mathcal{O}''_k} S_{jk} - \sum_{i \in \mathcal{O}'_k} S_{ik} \geqslant \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} x_{ijk}(\varepsilon_i + \tau_{ijk}), \qquad \forall k \in \mathcal{K} \tag{28}$$
$$S_{ik} \leqslant \max_{m \in \mathcal{P}} (\mu_m - \tau_{imk})\, y_k, \qquad \forall k \in \mathcal{K}, i \in \mathcal{O}'_k \tag{29}$$
$$S_{jk} \geqslant \min_{m \in \mathcal{D}} (\lambda_m + \varepsilon_m + \tau_{mjk}) \sum_{m \in \mathcal{D}} x_{mjk}, \qquad \forall k \in \mathcal{K}, j \in \mathcal{O}''_k \tag{30}$$

Solution methodology: Exact approach

- Symmetry breaking for subsets $\mathcal{K}'$ of identical vehicles

$$\sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{P} \cup \mathcal{D}} \rho_i^v x_{ijk'_g} \geqslant \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{P} \cup \mathcal{D}} \rho_i^v x_{ijk'_{g+1}}, \qquad \forall g \in 1, \dots, (|\mathcal{K}'| - 1) \qquad (31)$$

- Symmetry breaking for replications of the same dump $\mathcal{D}'$

$$\sum_{i \in \mathcal{P}} i x_{ij'_g k} \leqslant \sum_{i \in \mathcal{P}} i x_{ij'_{g+1} k}, \qquad \forall k \in \mathcal{K}, g \in 1, \dots, (|\mathcal{D}'| - 1) \qquad (32)$$

- Bounds on dump visits

$$\sum_{i \in \mathcal{P}} x_{ijk} \leqslant 1, \qquad \forall k \in \mathcal{K}, j \in \mathcal{D} \qquad (33)$$

$$\sum_{i \in \mathcal{D}} \sum_{j \in \mathcal{P}} x_{ijk} \leqslant \min\left(|\mathcal{D}| - 1, |\mathcal{P}| - 1\right), \qquad \forall k \in \mathcal{K} \qquad (34)$$

Solution methodology: Heuristic approach

- To solve instances of realistic size, we developed a heuristic algorithm.
- It constructs a feasible initial solution using an insertion procedure.
- It improves the initial solution through a multiple neighborhood search procedure admitting intermediate infeasibility with a dynamically evolving penalty.
- Periodically, we restart from the best feasible solution because feasibility may be hard to restore.
- Periodically, we also reassign dump visits and evaluate vehicle reassignments because the fleet is heterogeneous and fixed.

Results

- We test the heuristic against the mathematical model on modifications of the Schneider et al. (2014) E-VRPTW instances.
  - recharging facilities are regarded as dumps
  - additional features relevant to our problem are added
- Additionally, we test the heuristic on
  - the Crevier et al. (2007) instances for the purpose of evaluating the benefit of flexible depot assignment,
  - and on state-of-practice data
- For each instance, the heuristic is run 10 times.

Results: Modified Schneider et al. (2014) instances

- 36 instances derived from the Solomon (1987) VRPTW instances
- 3 groups of 12 instances with 5, 10, and 15 customers
- Number of recharging stations: 2 to 8

Results: Modified Schneider et al. (2014) instances

- 36 instances derived from the Solomon (1987) VRPTW instances
- 3 groups of 12 instances with 5, 10, and 15 customers
- Number of recharging stations: 2 to 8
- Modifications
  - regard recharging stations as dumps
  - use 2 vehicle classes with different capacities, costs, and site dependencies
  - apply a maximum tour duration, a maximum working time limit, and a break duration

Results: Modified Schneider et al. (2014) instances

- 36 instances derived from the Solomon (1987) VRPTW instances
- 3 groups of 12 instances with 5, 10, and 15 customers
- Number of recharging stations: 2 to 8

- Modifications
  - regard recharging stations as dumps
  - use 2 vehicle classes with different capacities, costs, and site dependencies
  - apply a maximum tour duration, a maximum working time limit, and a break duration

- We compare the heuristic against the mathematical model.
- For each instance, the heuristic is run 10 times.

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 5 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c101C5	4	489.70	489.70	0.05	489.70	0.00	0.39	0.00	489.70	35.71	7,200.01	0.00
c103C5	2	281.33	281.33	0.04	268.09	0.00	0.17	-4.94	268.09	0.00	4,910.40	-4.94
c206C5	2	374.67	374.67	0.06	360.09	0.00	0.24	-4.05	360.09	0.00	90.27	-4.05
c208C5	2	343.20	343.20	0.06	343.20	0.00	0.49	0.00	343.20	38.57	7,200.04	0.00
r104C5	1	182.81	182.81	0.02	182.81	0.00	0.04	0.00	182.81	0.00	1.90	0.00
r105C5	2	251.15	251.15	0.05	251.15	0.00	0.08	0.00	251.15	0.00	0.22	0.00
r202C5	1	176.52	176.52	0.02	176.52	0.00	0.05	0.00	176.52	0.00	2.67	0.00
r203C5	1	228.05	228.05	0.03	228.05	0.00	0.12	0.00	228.05	0.00	1,504.85	0.00
rc105C5	2	327.19	327.19	0.05	327.19	0.00	0.15	0.00	327.19	25.27	7,200.04	0.00
rc108C5	2	345.87	345.87	0.04	345.87	0.00	0.15	0.00	345.87	0.00	2,069.22	0.00
rc204C5	1	223.17	223.17	0.09	223.17	0.00	0.17	0.00	223.17	0.00	1,327.76	0.00
rc208C5	1	212.67	212.67	0.02	212.67	0.00	0.25	0.00	212.67	0.00	1,156.35	0.00

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 5 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)
c101C5	4	489.70	489.70	0.05	489.70	0.00	0.39	0.00	489.70	35.71	7,200.01	0.00
c103C5	2	281.33	281.33	0.04	268.09	0.00	0.17	-4.94	268.09	0.00	4,910.40	-4.94
c206C5	2	374.67	374.67	0.06	360.09	0.00	0.24	-4.05	360.09	0.00	90.27	-4.05
c208C5	2	343.20	343.20	0.06	343.20	0.00	0.49	0.00	343.20	38.57	7,200.04	0.00
r104C5	1	182.81	182.81	0.02	182.81	0.00	0.04	0.00	182.81	0.00	1.90	0.00
r105C5	2	251.15	251.15	0.05	251.15	0.00	0.08	0.00	251.15	0.00	0.22	0.00
r202C5	1	176.52	176.52	0.02	176.52	0.00	0.05	0.00	176.52	0.00	2.67	0.00
r203C5	1	228.05	228.05	0.03	228.05	0.00	0.12	0.00	228.05	0.00	1,504.85	0.00
rc105C5	2	327.19	327.19	0.05	327.19	0.00	0.15	0.00	327.19	25.27	7,200.04	0.00
rc108C5	2	345.87	345.87	0.04	345.87	0.00	0.15	0.00	345.87	0.00	2,069.22	0.00
rc204C5	1	223.17	223.17	0.09	223.17	0.00	0.17	0.00	223.17	0.00	1,327.76	0.00
rc208C5	1	212.67	212.67	0.02	212.67	0.00	0.25	0.00	212.67	0.00	1,156.35	0.00

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 5 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c101C5	4	489.70	489.70	0.05	489.70	0.00	0.39	0.00	489.70	35.71	7,200.01	0.00
c103C5	2	281.33	281.33	0.04	268.09	0.00	0.17	-4.94	268.09	0.00	4,910.40	-4.94
c206C5	2	374.67	374.67	0.06	360.09	0.00	0.24	-4.05	360.09	0.00	90.27	-4.05
c208C5	2	343.20	343.20	0.06	343.20	0.00	0.49	0.00	343.20	38.57	7,200.04	0.00
r104C5	1	182.81	182.81	0.02	182.81	0.00	0.04	0.00	182.81	0.00	1.90	0.00
r105C5	2	251.15	251.15	0.05	251.15	0.00	0.08	0.00	251.15	0.00	0.22	0.00
r202C5	1	176.52	176.52	0.02	176.52	0.00	0.05	0.00	176.52	0.00	2.67	0.00
r203C5	1	228.05	228.05	0.03	228.05	0.00	0.12	0.00	228.05	0.00	1,504.85	0.00
rc105C5	2	327.19	327.19	0.05	327.19	0.00	0.15	0.00	327.19	25.27	7,200.04	0.00
rc108C5	2	345.87	345.87	0.04	345.87	0.00	0.15	0.00	345.87	0.00	2,069.22	0.00
rc204C5	1	223.17	223.17	0.09	223.17	0.00	0.17	0.00	223.17	0.00	1,327.76	0.00
rc208C5	1	212.67	212.67	0.02	212.67	0.00	0.25	0.00	212.67	0.00	1,156.35	0.00

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 5 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c101C5	4	489.70	489.70	0.05	489.70	0.00	0.39	0.00	489.70	35.71	7,200.01	0.00
c103C5	2	281.33	281.33	0.04	268.09	0.00	0.17	-4.94	268.09	0.00	4,910.40	-4.94
c206C5	2	374.67	374.67	0.06	360.09	0.00	0.24	-4.05	360.09	0.00	90.27	-4.05
c208C5	2	343.20	343.20	0.06	343.20	0.00	0.49	0.00	343.20	38.57	7,200.04	0.00
r104C5	1	182.81	182.81	0.02	182.81	0.00	0.04	0.00	182.81	0.00	1.90	0.00
r105C5	2	251.15	251.15	0.05	251.15	0.00	0.08	0.00	251.15	0.00	0.22	0.00
r202C5	1	176.52	176.52	0.02	176.52	0.00	0.05	0.00	176.52	0.00	2.67	0.00
r203C5	1	228.05	228.05	0.03	228.05	0.00	0.12	0.00	228.05	0.00	1,504.85	0.00
rc105C5	2	327.19	327.19	0.05	327.19	0.00	0.15	0.00	327.19	25.27	7,200.04	0.00
rc108C5	2	345.87	345.87	0.04	345.87	0.00	0.15	0.00	345.87	0.00	2,069.22	0.00
rc204C5	1	223.17	223.17	0.09	223.17	0.00	0.17	0.00	223.17	0.00	1,327.76	0.00
rc208C5	1	212.67	212.67	0.02	212.67	0.00	0.25	0.00	212.67	0.00	1,156.35	0.00

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 10 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c101C10	6	846.10	846.10	0.61	837.13	0.00	5,489.48	-1.07	846.10	77.21	7,200.48	0.00
c104C10	3	456.86	456.86	0.43	456.86	0.00	37.28	0.00	456.86	53.05	7,200.08	0.00
c202C10	4	549.74	549.74	0.42	549.74	18.44	7,200.09	0.00	549.74	67.71	7,200.31	0.00
c205C10	4	568.92	568.92	0.58	568.58	0.00	2,788.37	-0.06	568.92	64.77	7,200.11	0.00
r102C10	3	391.14	391.14	0.40	391.14	0.00	158.70	0.00	391.14	47.69	7,200.16	0.00
r103C10	2	288.67	288.67	0.50	288.67	0.00	18.39	0.00	288.67	43.72	7,200.04	0.00
r201C10	2	310.16	310.16	0.45	310.16	0.00	45.22	0.00	310.16	43.64	7,200.46	0.00
r203C10	2	329.78	329.78	1.13	329.78	0.00	5,757.28	0.00	329.78	47.26	7,200.08	0.00
rc102C10	3	534.75	534.75	0.40	534.75	0.00	6.25	0.00	534.75	38.77	7,200.09	0.00
rc108C10	2	429.79	429.79	0.42	429.79	0.00	6.94	0.00	429.79	25.30	7,200.09	0.00
rc201C10	2	502.45	502.45	0.40	499.88	0.00	147.43	-0.51	502.45	58.48	7,200.09	0.00
rc205C10	2	428.80	428.80	0.45	421.36	0.00	26.00	-1.77	428.80	40.52	7,201.11	0.00

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 10 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)
c101C10	6	846.10	846.10	0.61	837.13	0.00	5,489.48	-1.07	846.10	77.21	7,200.48	0.00
c104C10	3	456.86	456.86	0.43	456.86	0.00	37.28	0.00	456.86	53.05	7,200.08	0.00
c202C10	4	549.74	549.74	0.42	549.74	18.44	7,200.09	0.00	549.74	67.71	7,200.31	0.00
c205C10	4	568.92	568.92	0.58	568.58	0.00	2,788.37	-0.06	568.92	64.77	7,200.11	0.00
r102C10	3	391.14	391.14	0.40	391.14	0.00	158.70	0.00	391.14	47.69	7,200.16	0.00
r103C10	2	288.67	288.67	0.50	288.67	0.00	18.39	0.00	288.67	43.72	7,200.04	0.00
r201C10	2	310.16	310.16	0.45	310.16	0.00	45.22	0.00	310.16	43.64	7,200.46	0.00
r203C10	2	329.78	329.78	1.13	329.78	0.00	5,757.28	0.00	329.78	47.26	7,200.08	0.00
rc102C10	3	534.75	534.75	0.40	534.75	0.00	6.25	0.00	534.75	38.77	7,200.09	0.00
rc108C10	2	429.79	429.79	0.42	429.79	0.00	6.94	0.00	429.79	25.30	7,200.09	0.00
rc201C10	2	502.45	502.45	0.40	499.88	0.00	147.43	-0.51	502.45	58.48	7,200.09	0.00
rc205C10	2	428.80	428.80	0.45	421.36	0.00	26.00	-1.77	428.80	40.52	7,201.11	0.00

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 10 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c101C10	6	846.10	846.10	0.61	837.13	0.00	5,489.48	-1.07	846.10	77.21	7,200.48	0.00
c104C10	3	456.86	456.86	0.43	456.86	0.00	37.28	0.00	456.86	53.05	7,200.08	0.00
c202C10	4	549.74	549.74	0.42	549.74	18.44	7,200.09	0.00	549.74	67.71	7,200.31	0.00
c205C10	4	568.92	568.92	0.58	568.58	0.00	2,788.37	-0.06	568.92	64.77	7,200.11	0.00
r102C10	3	391.14	391.14	0.40	391.14	0.00	158.70	0.00	391.14	47.69	7,200.16	0.00
r103C10	2	288.67	288.67	0.50	288.67	0.00	18.39	0.00	288.67	43.72	7,200.04	0.00
r201C10	2	310.16	310.16	0.45	310.16	0.00	45.22	0.00	310.16	43.64	7,200.46	0.00
r203C10	2	329.78	329.78	1.13	329.78	0.00	5,757.28	0.00	329.78	47.26	7,200.08	0.00
rc102C10	3	534.75	534.75	0.40	534.75	0.00	6.25	0.00	534.75	38.77	7,200.09	0.00
rc108C10	2	429.79	429.79	0.42	429.79	0.00	6.94	0.00	429.79	25.30	7,200.09	0.00
rc201C10	2	502.45	502.45	0.40	499.88	0.00	147.43	-0.51	502.45	58.48	7,200.09	0.00
rc205C10	2	428.80	428.80	0.45	421.36	0.00	26.00	-1.77	428.80	40.52	7,201.11	0.00

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 10 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c101C10	6	846.10	846.10	0.61	837.13	0.00	5,489.48	-1.07	846.10	77.21	7,200.48	0.00
c104C10	3	456.86	456.86	0.43	456.86	0.00	37.28	0.00	456.86	53.05	7,200.08	0.00
c202C10	4	549.74	549.74	0.42	549.74	18.44	7,200.09	0.00	549.74	67.71	7,200.31	0.00
c205C10	4	568.92	568.92	0.58	568.58	0.00	2,788.37	-0.06	568.92	64.77	7,200.11	0.00
r102C10	3	391.14	391.14	0.40	391.14	0.00	158.70	0.00	391.14	47.69	7,200.16	0.00
r103C10	2	288.67	288.67	0.50	288.67	0.00	18.39	0.00	288.67	43.72	7,200.04	0.00
r201C10	2	310.16	310.16	0.45	310.16	0.00	45.22	0.00	310.16	43.64	7,200.46	0.00
r203C10	2	329.78	329.78	1.13	329.78	0.00	5,757.28	0.00	329.78	47.26	7,200.08	0.00
rc102C10	3	534.75	534.75	0.40	534.75	0.00	6.25	0.00	534.75	38.77	7,200.09	0.00
rc108C10	2	429.79	429.79	0.42	429.79	0.00	6.94	0.00	429.79	25.30	7,200.09	0.00
rc201C10	2	502.45	502.45	0.40	499.88	0.00	147.43	-0.51	502.45	58.48	7,200.09	0.00
rc205C10	2	428.80	428.80	0.45	421.36	0.00	26.00	-1.77	428.80	40.52	7,201.11	0.00

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 15 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c103C15	5	823.82	823.82	0.92	823.82	34.38	7,200.18	0.00	823.82	73.45	7,200.84	0.00
c106C15	5	653.46	653.46	0.69	653.46	17.67	7,200.19	0.00	653.46	63.86	7,200.07	0.00
c202C15	6	932.30	932.30	0.77	932.30	36.39	7,200.23	0.00	932.30	68.58	7,200.51	0.00
c208C15	5	725.23	725.23	1.55	725.23	25.75	7,200.17	0.00	725.23	68.69	7,200.38	0.00
r102C15	5	678.40	678.40	0.83	678.40	27.89	7,200.17	0.00	678.40	64.94	7,200.22	0.00
r105C15	3	462.52	462.52	0.70	462.52	0.00	56.82	0.00	462.52	53.50	7,200.10	0.00
r202C15	3	528.59	535.08	1.41	528.59	30.25	7,200.11	0.00	528.59	54.05	7,200.95	0.00
r209C15	2	369.29	371.60	1.26	369.29	7.10	7,200.11	0.00	369.29	37.62	7,201.49	0.00
rc103C15	3	556.87	556.87	0.83	556.87	16.41	7,200.10	0.00	556.87	58.12	7,200.06	0.00
rc108C15	3	510.41	511.03	1.19	510.41	3.47	7,200.07	0.00	510.41	49.31	7,200.14	0.00
rc202C15	3	601.71	601.71	1.30	598.83	27.55	7,200.18	-0.48	601.71	58.77	7,200.24	0.00
rc204C15	2	421.54	422.22	5.67	421.54	25.44	7,201.01	0.00	421.54	49.29	7,201.67	0.00
Average		453.82	454.10	0.66	452.43	7.52	2,803.97	-0.36	453.05	39.11	5,707.60	-0.25

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 15 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)
c103C15	5	823.82	823.82	0.92	823.82	34.38	7,200.18	0.00	823.82	73.45	7,200.84	0.00
c106C15	5	653.46	653.46	0.69	653.46	17.67	7,200.19	0.00	653.46	63.86	7,200.07	0.00
c202C15	6	932.30	932.30	0.77	932.30	36.39	7,200.23	0.00	932.30	68.58	7,200.51	0.00
c208C15	5	725.23	725.23	1.55	725.23	25.75	7,200.17	0.00	725.23	68.69	7,200.38	0.00
r102C15	5	678.40	678.40	0.83	678.40	27.89	7,200.17	0.00	678.40	64.94	7,200.22	0.00
r105C15	3	462.52	462.52	0.70	462.52	0.00	56.82	0.00	462.52	53.50	7,200.10	0.00
r202C15	3	528.59	535.08	1.41	528.59	30.25	7,200.11	0.00	528.59	54.05	7,200.95	0.00
r209C15	2	369.29	371.60	1.26	369.29	7.10	7,200.11	0.00	369.29	37.62	7,201.49	0.00
rc103C15	3	556.87	556.87	0.83	556.87	16.41	7,200.10	0.00	556.87	58.12	7,200.06	0.00
rc108C15	3	510.41	511.03	1.19	510.41	3.47	7,200.07	0.00	510.41	49.31	7,200.14	0.00
rc202C15	3	601.71	601.71	1.30	598.83	27.55	7,200.18	-0.48	601.71	58.77	7,200.24	0.00
rc204C15	2	421.54	422.22	5.67	421.54	25.44	7,201.01	0.00	421.54	49.29	7,201.67	0.00
Average		453.82	454.10	0.66	452.43	7.52	2,803.97	-0.36	453.05	39.11	5,707.60	-0.25

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 15 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)
c103C15	5	823.82	823.82	0.92	823.82	34.38	7,200.18	0.00	823.82	73.45	7,200.84	0.00
c106C15	5	653.46	653.46	0.69	653.46	17.67	7,200.19	0.00	653.46	63.86	7,200.07	0.00
c202C15	6	932.30	932.30	0.77	932.30	36.39	7,200.23	0.00	932.30	68.58	7,200.51	0.00
c208C15	5	725.23	725.23	1.55	725.23	25.75	7,200.17	0.00	725.23	68.69	7,200.38	0.00
r102C15	5	678.40	678.40	0.83	678.40	27.89	7,200.17	0.00	678.40	64.94	7,200.22	0.00
r105C15	3	462.52	462.52	0.70	462.52	0.00	56.82	0.00	462.52	53.50	7,200.10	0.00
r202C15	3	528.59	535.08	1.41	528.59	30.25	7,200.11	0.00	528.59	54.05	7,200.95	0.00
r209C15	2	369.29	371.60	1.26	369.29	7.10	7,200.11	0.00	369.29	37.62	7,201.49	0.00
rc103C15	3	556.87	556.87	0.83	556.87	16.41	7,200.10	0.00	556.87	58.12	7,200.06	0.00
rc108C15	3	510.41	511.03	1.19	510.41	3.47	7,200.07	0.00	510.41	49.31	7,200.14	0.00
rc202C15	3	601.71	601.71	1.30	598.83	27.55	7,200.18	-0.48	601.71	58.77	7,200.24	0.00
rc204C15	2	421.54	422.22	5.67	421.54	25.44	7,201.01	0.00	421.54	49.29	7,201.67	0.00
Average		453.82	454.10	0.66	452.43	7.52	2,803.97	-0.36	453.05	39.11	5,707.60	-0.25

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances with 15 customers

		Heuristic			Solver on model with adtl inequalities				Solver on model without adtl inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c103C15	5	823.82	823.82	0.92	823.82	34.38	7,200.18	0.00	823.82	73.45	7,200.84	0.00
c106C15	5	653.46	653.46	0.69	653.46	17.67	7,200.19	0.00	653.46	63.86	7,200.07	0.00
c202C15	6	932.30	932.30	0.77	932.30	36.39	7,200.23	0.00	932.30	68.58	7,200.51	0.00
c208C15	5	725.23	725.23	1.55	725.23	25.75	7,200.17	0.00	725.23	68.69	7,200.38	0.00
r102C15	5	678.40	678.40	0.83	678.40	27.89	7,200.17	0.00	678.40	64.94	7,200.22	0.00
r105C15	3	462.52	462.52	0.70	462.52	0.00	56.82	0.00	462.52	53.50	7,200.10	0.00
r202C15	3	528.59	535.08	1.41	528.59	30.25	7,200.11	0.00	528.59	54.05	7,200.95	0.00
r209C15	2	369.29	371.60	1.26	369.29	7.10	7,200.11	0.00	369.29	37.62	7,201.49	0.00
rc103C15	3	556.87	556.87	0.83	556.87	16.41	7,200.10	0.00	556.87	58.12	7,200.06	0.00
rc108C15	3	510.41	511.03	1.19	510.41	3.47	7,200.07	0.00	510.41	49.31	7,200.14	0.00
rc202C15	3	601.71	601.71	1.30	598.83	27.55	7,200.18	-0.48	601.71	58.77	7,200.24	0.00
rc204C15	2	421.54	422.22	5.67	421.54	25.44	7,201.01	0.00	421.54	49.29	7,201.67	0.00
Average		453.82	454.10	0.66	452.43	7.52	2,803.97	-0.36	453.05	39.11	5,707.60	-0.25

Results: Crevier et al. (2007) instances

- 22 instances, with a limited homogeneous fleet stationed at one depot
- All depots can act as intermediate facilities.
- BKS by Hemmelmayr et al. (2013)
- We apply the MNS heuristic to evaluate the benefits from flexible destination depot assignments.

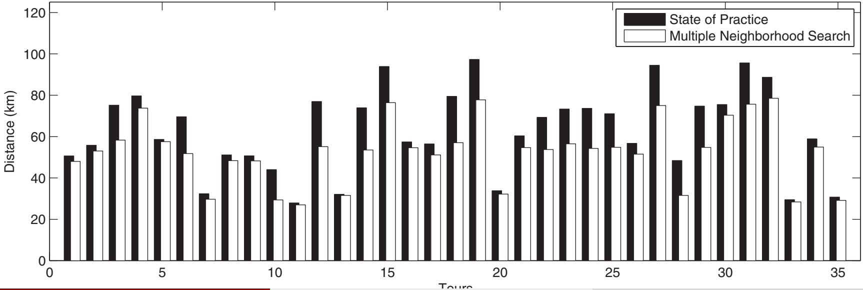
- Keeping the home depot and optimizing the destination depot obtains
  - *0.37%* average savings over 10 runs
  - *1.77%* savings in the best case

- Optimizing the home depot and the destination depot obtains
  - *1.37%* average savings over 10 runs
  - *2.54%* savings in the best case

Results: Comparison to the state of practice

- 35 tours planned by specialized software for the canton of Geneva
- 7 to 38 containers per tour, up to 4 dump visits per tour
- MNS heuristic improves tours by *1.73%* to *34.91%*, on avg *14.75%*
- Extrapolating annually, cost reductions of at least *300,000* USD

Figure 2: Comparison to the state of practice (average of 10 runs per tour)



Contents

State of the art

- Much of it is focused on city and regional level.
- And a fairly small amount on the container (micro) level, e.g.
  - inventory levels in pharmacies (Nolz et al., 2011, 2014)
  - recyclable materials from old cars (Krikke et al., 2008)
  - charity donation banks (McLeod et al., 2013)
  - waste container levels (Johansson, 2006; Faccio et al., 2011; Mes, 2012; Mes et al., 2014)
- Contributions
  - operational level forecasting rather than critical levels
  - estimated and validated on real data, compared to most of the literature which uses simulated data

Methodology

- Let n_{itg} denote the number of deposits in container i on day t of size q_g. The data generating process of the daily demands is

$$\rho_{it} = \sum_{g=1}^{m} q_g n_{itg} \tag{35}$$

- Let $n_{itg} \overset{\text{iid}}{\sim} \text{Pois}\left(\lambda_{itg}\right)$ and have a probability π_{itg}. Then we obtain

$$\mathbb{E}\left(\rho_{it}\right) = \sum_{g=1}^{m} q_g \lambda_{itg} \pi_{itg} \tag{36}$$

- We minimize the sum of squared errors between observed ρ_{it}° and expected $\mathbb{E}\left(\rho_{it}\right)$ for p containers and h days

$$\min_{\boldsymbol{\lambda},\boldsymbol{\pi}} \sum_{i=1}^{p} \sum_{t=1}^{h} \left(\rho_{it}^{\circ} - \sum_{g=1}^{m} q_g \lambda_{itg} \pi_{itg}\right)^2 \tag{37}$$

assuming strict exogeneity and errors modeled as white noise.

Methodology

- Given vectors of covariates $\mathbf{c}_{it}$ and vectors of parameters $\boldsymbol{\beta}_g^{\lambda}$ and $\boldsymbol{\beta}_g^{\pi}$, we define Poisson rates and logit-type probabilities

$$\lambda_{itg} = \exp\left(\mathbf{c}_{it}^{\mathsf{T}} \boldsymbol{\beta}_g^{\lambda}\right) \tag{38}$$

$$\pi_{itg} = \frac{\exp\left(\mathbf{c}_{it}^{\mathsf{T}} \boldsymbol{\beta}_g^{\pi}\right)}{\sum_{j=1}^{m} \exp\left(\mathbf{c}_{it}^{\mathsf{T}} \boldsymbol{\beta}_j^{\pi}\right)} \tag{39}$$

- Then, in compact form, the minimization problem writes as

$$\min_{\mathbf{B}} \sum_{i=1}^{p} \sum_{t=1}^{h} \left(\rho_{it}^{o} - \sum_{g=1}^{m} \frac{\exp\left(\mathbf{c}_{it}^{\mathsf{T}} \boldsymbol{\beta}_g^{\lambda} + \mathbf{c}_{it}^{\mathsf{T}} \boldsymbol{\beta}_g^{\pi} + \ln\left(q_g\right)\right)}{\sum_{j=1}^{m} \exp\left(\mathbf{c}_{it}^{\mathsf{T}} \boldsymbol{\beta}_j^{\pi}\right)} \right)^2 \tag{40}$$

- $\mathbf{B} = (\boldsymbol{\beta}_g^{\lambda}, \boldsymbol{\beta}_g^{\pi} : \forall g)$, and $\boldsymbol{\beta}_{g^\star}^{\pi} = \mathbf{0}$ for one arbitrarily chosen $g^\star$.
- We will refer to this minimization problem as the *mixture model*.

Methodology

- In case of only one deposit quantity, it degenerates to a pseudo-count data process

$$\min_{\mathbf{B}} \sum_{i=1}^{p} \sum_{t=1}^{h} \left(\rho_{it}^{\circ} - \exp\left(\mathbf{c}_{it}^{\mathsf{T}} \boldsymbol{\beta} + \ln(q) \right) \right)^2 \tag{41}$$

- We will refer to this minimization problem as the *simple model*.

Methodology

- Using new sets of covariates $\dot{\mathbf{c}}_{it}$, we can generate a forecast as

$$\mathbb{E}(\rho_{it}) = \sum_{g=1}^{m} \frac{\exp\left(\dot{\mathbf{c}}_{it}^{\mathsf{T}}\boldsymbol{\beta}_g^{\lambda} + \dot{\mathbf{c}}_{it}^{\mathsf{T}}\boldsymbol{\beta}_g^{\pi} + \ln\left(q_g\right)\right)}{\sum_{j=1}^{m}\exp\left(\dot{\mathbf{c}}_{it}^{\mathsf{T}}\boldsymbol{\beta}_j^{\pi}\right)} \tag{42}$$

- Given the operational nature of the problem, the covariates should be quick and easy to obtain.
- Examples include days of the week, months, weather data, holidays, etc...

Data

- 36 containers for PET in the canton of Geneva with capacity of 3,040 or 3,100 liters
- Balanced panel covering March to June, 2014 (122 days), which brings the total number of observations to 4,392
- The final sample excludes unreliable level data (removed after visual inspection).
- Missing data is linearly interpolated for the values of ρ^{o}_{it}.

Residual plots

Figure 3: Residual plot of the mixture model

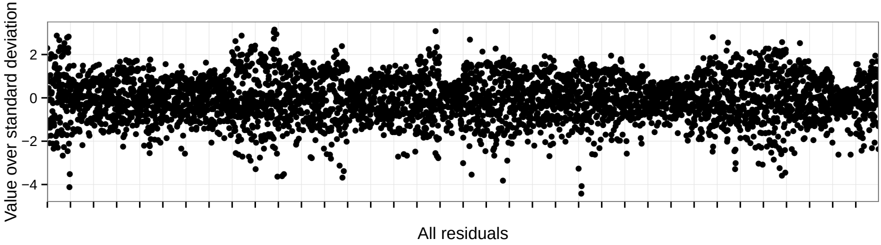
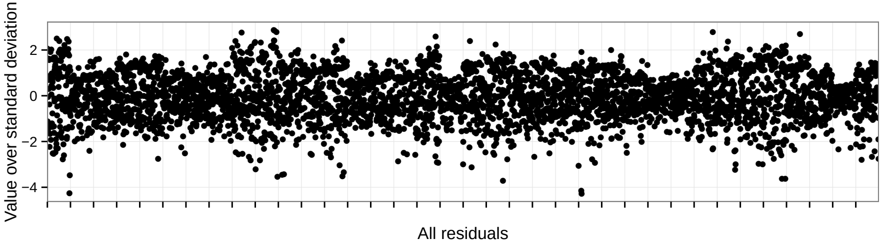


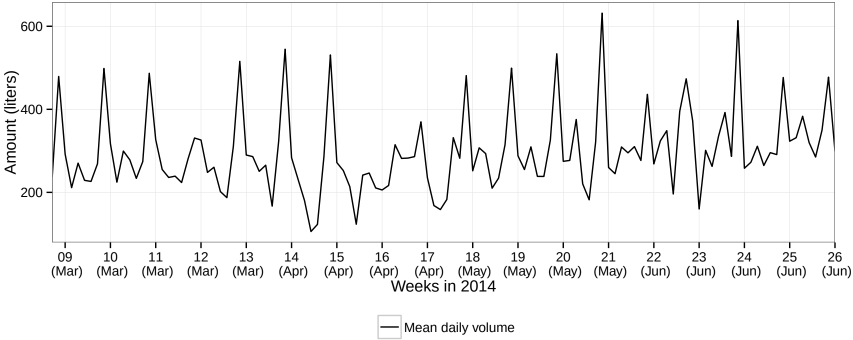
Figure 4: Residual plot of the simple model



Seasonality pattern

- Waste generation exhibits strong weekly seasonality.
- Peaks are observed during the weekends.
- There also appear to be longer-term effects for months.

Figure 5: Mean daily volume deposited in the containers



Covariates

Table 2: Table of covariates

Variable	Type
Container fixed effect	dummy
Day of the week	dummy
Month	dummy
Minimum temperature in Celsius	continuous
Precipitation in mm	continuous
Pressure in hPa	continuous
Wind speed in kmph	continuous

Evaluating the fits

- Coefficient of determination

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}} \quad (43)$$

with higher values for a better model.

- Akaike information criterion (AIC):

$$\text{AIC} = \left(\frac{SS_{\text{res}}}{\#\text{obs}}\right) \exp\left(\frac{2 * \#\text{param}}{\#\text{obs}}\right) \quad (44)$$

with lower values for a better model. The exponential penalizes model complexity.

- SS_{res} is the residual sum of squares.
- SS_{tot} is the total sum of squares.

Estimation on full sample

- Mixture model: R^2 of *0.341* (AIC *52,900*) with 5L and 15L
- Simple model: R^2 of *0.300* (AIC *53,700*) with 10L

Table 3: Estimated coefficients of mixture model

	$\hat{\beta}_1^{\lambda}$ (5L)***	$\hat{\beta}_2^{\lambda}$ (15L)***	$\hat{\beta}_2^{\pi}$ (15L)***
Minimum temperature in Celsius	1,461.356	0.022	-0.037
Precipitation in mm	-0.821	-0.009	0.018
Pressure in hPa	-13.724	-0.001	0.010
Wind speed in kmph	7.580	-0.004	0.020
Monday	402.235	2.166	-9.693
Tuesday	1,908.233	2.293	-9.977
Wednesday	-844.662	1.432	0.202
Thursday	1,937.385	1.198	1.453
Friday	1,876.162	1.239	4.419
Saturday	-6,981.339	1.358	4.723
Sunday	1,831.715	1.905	2.832
March	-27.136	2.955	-1.453
April	1,071.406	2.746	-1.532
May	1,689.979	2.988	-1.603
June	-2,604.520	2.901	-1.452

Validation

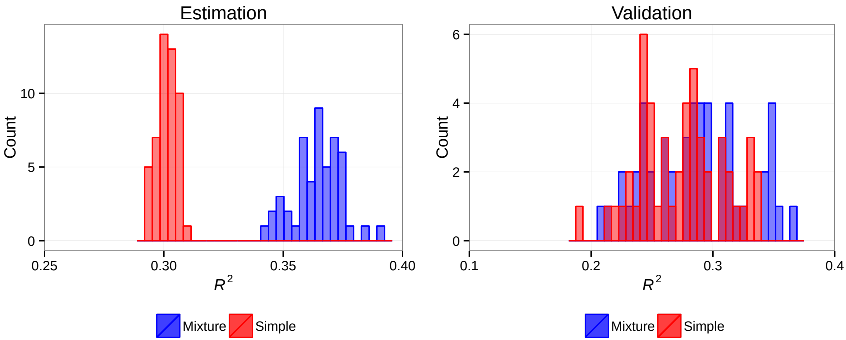
- 50 experiments
- The mixture and the simple model are estimated on a random sample of 90% of the panel, and validated on the remaining 10%.
- In both cases the values are significantly different at 90% confidence level.

Table 4: Mean R^2 for estimation and validation sets

	Mixture model mean R^2	Simple model mean R^2
Estimation	0.364 (AIC 51,400)	0.302 (AIC 53,600)
Validation	0.286	0.274

Validation

Figure 6: Histograms for estimation and validation samples



Work so far

- Markov, I., Varone, S., and Bierlaire, M. (under review). Integrating a heterogeneous fixed fleet and a flexible assignment of destination depots in the waste collection VRP with intermediate facilities.
- Markov, I., Lapparent, M. (de), Bierlaire, M., and Varone, S. (2015). Modeling a waste disposal process via a discrete mixture of count data models. *Proceedings of the 15th Swiss Transport Research Conference (STRC)*, April 15-17, 2015, Ascona, Switzerland.

Contents

Inventory routing: Setup

- Integration of forecasting, container selection and routing over a planning horizon adds up to solving an IRP.

Inventory routing: Setup

- Integration of forecasting, container selection and routing over a planning horizon adds up to solving an IRP.
- We do not allow overflows in expected terms.

Inventory routing: Setup

- Integration of forecasting, container selection and routing over a planning horizon adds up to solving an IRP.
- We do not allow overflows in expected terms.
- But container demands are stochastic and we *cannot* ignore the probability of overflow.

lower routing cost
higher overflow probability

↔

higher routing cost
lower overflow probability

Inventory routing: Setup

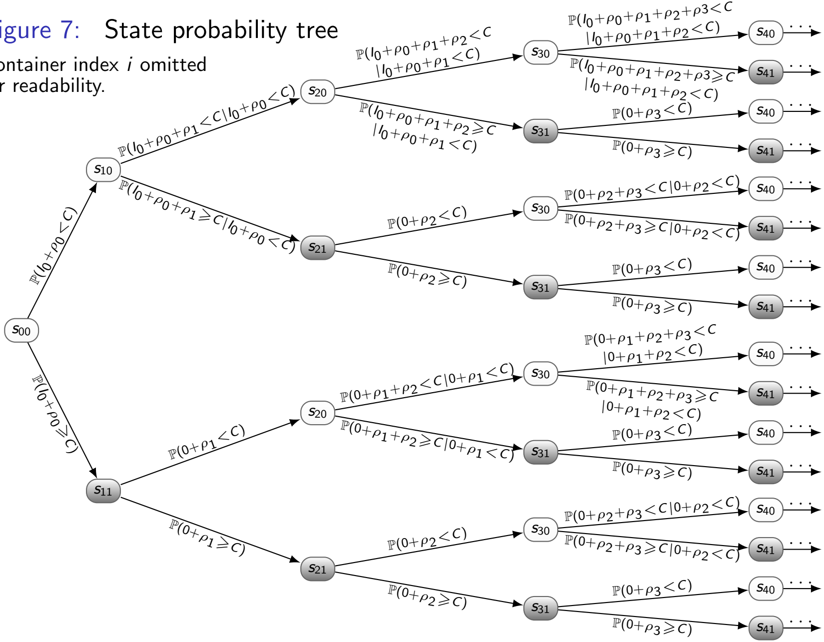
- Integration of forecasting, container selection and routing over a planning horizon adds up to solving an IRP.
- We do not allow overflows in expected terms.
- But container demands are stochastic and we *cannot* ignore the probability of overflow.

lower routing cost
higher overflow probability ↔ higher routing cost
lower overflow probability

- To achieve this balance, we need to penalize a state of overflow weighted by its probability.
- Notation
  - I_{it} inventory of container i on day t
  - s_{it0} container i on day t is in a state of *no overflow*
  - s_{it1} container i on day t is in a state of *overflow*

Figure 7: State probability tree

Container index i omitted for readability.



Calculating the probabilities

- The forecasting errors are assumed to be iid normal, therefore

$$\rho_{it}^{\circ} = \mathbb{E}(\rho_{it}) + \varepsilon_{it}, \quad \varepsilon_{it} \overset{\text{iid}}{\sim} \text{Normal}(0, \sigma^2) \tag{45}$$

- An unbiased and consistent estimate of the variance, with p containers and h days of historical data, is given by

$$\sigma^2 = \frac{\sum_{i=1}^{p} \sum_{t=1}^{h} (\rho_{it}^{\circ} - \mathbb{E}(\rho_{it}))^2}{ph - \#\text{params}} \tag{46}$$

Calculating the probabilities

- The forecasting errors are assumed to be iid normal, therefore

$$\rho^{\circ}_{it} = \mathbb{E}(\rho_{it}) + \varepsilon_{it}, \quad \varepsilon_{it} \overset{\text{iid}}{\sim} \text{Normal}(0, \sigma^2) \tag{45}$$

- An unbiased and consistent estimate of the variance, with p containers and h days of historical data, is given by

$$\sigma^2 = \frac{\sum_{i=1}^{p} \sum_{t=1}^{h} (\rho^{\circ}_{it} - \mathbb{E}(\rho_{it}))^2}{ph - \#\text{params}} \tag{46}$$

- An unconditional probability can be calculated simply as

$$\begin{aligned} \mathbb{P}(I_{i0} + \rho_{i0} \geqslant C_i) &= \mathbb{P}\left(\varepsilon_{it} \geqslant C_i - I_{i0} - \mathbb{E}(\rho_{i0})\right) \\ &= 1 - \Phi\left(\frac{C_i - I_{i0} - \mathbb{E}(\rho_{i0})}{\sigma}\right) \end{aligned} \tag{47}$$

Calculating the probabilities

- To calculate a conditional probability, we need to evaluate

$$\mathbb{P}\left(I_{i0} + \sum_{t=0}^{h} \rho_{it} \geqslant C_i \,\middle|\, I_{i0} + \sum_{t=0}^{h-1} \rho_{it} < C_i\right) = \\ = \mathbb{P}\left(\sum_{t=0}^{h} \varepsilon_{it} \geqslant C_i - I_{i0} - \sum_{t=0}^{h} \mathbb{E}(\rho_{it}) \,\middle|\, \sum_{t=0}^{h-1} \varepsilon_{it} < C_i - I_{i0} - \sum_{t=0}^{h-1} \mathbb{E}(\rho_{it})\right) \quad (48)$$

- Substitute $a = C_i - I_{i0} - \sum_{t=0}^{h-1} \mathbb{E}(\rho_{it})$ and $X = \sum_{t=0}^{h-1} \varepsilon_{it}$, where $X \sim \mathcal{N}(0, h\sigma^2)$ and X is independent of ε_{ih}. After a standardization, expression (48) rewrites as

$$\mathbb{P}\left(X + \varepsilon_{ih} \geqslant a - \mathbb{E}(\rho_{ih}) \mid X < a\right) = \frac{\mathbb{P}(\varepsilon_{ih} \geqslant a - \mathbb{E}(\rho_{ih}) - X,\ X < a)}{\mathbb{P}(X < a)} = \\ = \frac{1}{2\pi\Phi(a/\sigma\sqrt{h})} \int_{-\infty}^{a/\sigma\sqrt{h}} \int_{\frac{a - \mathbb{E}(\rho_{ih}) - x\sigma\sqrt{h}}{\sigma}}^{\infty} \exp(-x^2/2)\exp(-y^2/2)dxdy = \\ = \frac{1}{2\sqrt{2\pi}\Phi(a/\sigma\sqrt{h})} \int_{-\infty}^{a/\sigma\sqrt{h}} \exp(-x^2/2)\,\mathrm{erfc}\left(\frac{a - \mathbb{E}(\rho_{ih}) - x\sigma\sqrt{h}}{\sigma\sqrt{2}}\right) dx \quad (49)$$

Formulation outline

- All unconditional and conditional probabilities in the tree can be precomputed.

Formulation outline

- All unconditional and conditional probabilities in the tree can be precomputed.
- We are now in a position to formulate a new objective function that captures the probabilistic information.

Expected overflow and emergency visit cost + Routing cost + Expected route failure cost

Formulation outline

- All unconditional and conditional probabilities in the tree can be precomputed.
- We are now in a position to formulate a new objective function that captures the probabilistic information.

Expected overflow and emergency visit cost + Routing cost + Expected route failure cost

- Constraints use expected values only.
- We add a day-index and inventory constraints to the VRP model.

Formulation outline

- Expected overflow and emergency visit cost

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{P}} \left(\mathbb{P}\left(s_{it1} \mid \max\left(0, g < t \colon \exists k \in \mathcal{K} \colon y_{ikg} = 1\right)\right) \left(B + \chi - B \sum_{k \in \mathcal{K}} y_{ikt} \right) \right) \quad (50)$$

where

- $\mathcal{T}$ set of days in the planning horizon
- $\mathcal{P}$ set of containers
- $\mathcal{K}$ set of vehicles
- χ overflow cost
- B emergency visit cost
- y_{ikt} $= 1$ if vehicle k visits container i on day t, 0 otherwise

Formulation outline

- Expected overflow and emergency visit cost

$$\sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{P}} \left(\mathbb{P}\left(s_{it1} \mid \max\left(0, g < t \colon \exists k \in \mathcal{K} \colon y_{ikg} = 1\right)\right) \left(B + \chi - B \sum_{k \in \mathcal{K}} y_{ikt} \right) \right) \quad (50)$$

where

- $\mathcal{T}$ set of days in the planning horizon
- $\mathcal{P}$ set of containers
- $\mathcal{K}$ set of vehicles
- χ overflow cost
- B emergency visit cost
- y_{ikt} $= 1$ if vehicle k visits container i on day t, 0 otherwise

- Routing cost remains the same.

Formulation outline

- Expected route failure cost

$$\sum_{t\in\mathcal{T}\backslash 0}\sum_{k\in\mathcal{K}}\sum_{\mathcal{S}\in\mathscr{S}_{kt}}\left(C_S\,\mathbb{P}\left(\sum_{s\in\mathcal{S}} I_{st} > \Omega_k \;\middle|\; \max(0, g < t\colon y_{skg} = 1)\right)\right) \tag{51}$$

where

- $\mathcal{T}$ set of days in the planning horizon
- $\mathcal{K}$ set of vehicles
- Ω_k capacity of vehicle k
- $\mathscr{S}_{kt}$ set of depot-to-dump or dump-to-dump trips for vehicle k on day t
- $\mathcal{S}$ set of containers in a particular trip
- C_S average cost of a BF visit to nearest dump
- y_{ikt} $= 1$ if vehicle k visits container i on day t, 0 otherwise

Classification: Coelho et al. (2014b) scheme

- Structural classification

Table 5: Structural classification

Criterion	Classification
Time horizon	Finite (Rolling)
Structure	Many-to-one
Routing	Multiple
Inventory policy	Order-up-to
Inventory decisions	Back-order with penalty and limit
Fleet composition	Heterogeneous
Fleet size	Multiple (Fixed)

- Information-based classification
  - stochastic
  - dynamic with information revealed each day

Contributions

- With respect to related research
  - Trudeau and Dror (1992): we do not impose the assumption of a single container visit and single overflow in the planning horizon
  - Coelho et al. (2014a): we include forecasting uncertainty
  - Campbell and Savelsbergh (2004): we use a similar decomposition approach, including uncertainty in both the short and long-term
- Our IRP includes a lot of rich features.
- It integrates real-time forecasting.
  - much of the literature focuses on known distributions
  - in our case the rates are non-stationary and there is no unique optimal service frequency

Contents

Conclusion

- At the moment, the forecasting model can produce future levels, for which the routing problem is solved.
- We have almost finalized the development of an ALNS for the IRP.

Conclusion

- At the moment, the forecasting model can produce future levels, for which the routing problem is solved.
- We have almost finalized the development of an ALNS for the IRP.
- Future research will focus on:
  - analyzing alternative formulations of the forecasting model, e.g. more deposit sizes or a continuous distribution
  - analyzing alternative IRP formulation, e.g. chance constraints, robust optimization, two-stage stochastic model
  - testing the ALNS on benchmark and real-life data
  - hybridizing the ALNS with some exact operators (especially for dump insertion)
- Once integrated at the partnering company, the available data will allow for additional extensive testing and results.

Thank you.

Questions?

Baldacci, R. and Mingozzi, A. (2009). A unified exact method for solving different classes of vehicle routing problems. *Mathematical Programming*, 120(2):347–380.

Bard, J. F., Huang, L., Dror, M., and Jaillet, P. (1998). A branch and cut algorithm for the VRP with satellite facilities. *IIE Transactions*, 30(9):821–834.

Campbell, A. M. and Savelsbergh, M. W. P. (2004). A decomposition approach for the inventory-routing problem. *Transportation Science*, 38(4):488–502.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2014a). Heuristics for dynamic and stochastic inventory-routing. *Computers & Operations Research*, 52(A):55–67.

Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2014b). Thirty years of inventory routing. *Transportation Science*, 48(1):1–19.

Conrad, R. G. and Figliozzi, M. A. (2011). The recharging vehicle routing problem. In Doolen, T. and Aken, E. V., editors, *Proceedings of the 2011 Industrial Engineering Research Conference*, Reno, NV, USA.

Crevier, B., Cordeau, J.-F., and Laporte, G. (2007). The multi-depot vehicle routing problem with inter-depot routes. *European Journal of Operational Research*, 176(2):756–773.

Erdoğan, S. and Miller-Hooks, E. (2012). A green vehicle routing problem. *Transportation Research Part E: Logistics and Transportation Review*, 48(1):100–114.

Faccio, M., Persona, A., and Zanin, G. (2011). Waste collection multi objective model with real time traceability data. *Waste Management*, 31(12):2391 – 2405.

Goeke, D. and Schneider, M. (2014). Routing a mixed fleet of electric and conventional vehicles. Technical Report LPIS-02/2014, TU Darmstadt, Germany.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., and Rath, S. (2013). A heuristic solution method for node routing based solid waste collection problems. *Journal of Heuristics*, 19(2):129–156.

Hiermann, G., Puchinger, J., and Hartl, R. F. (2014). The electric fleet size and mix vehicle routing problem with time windows and recharging stations. Working paper, Austrian Institute of Technology and University of Vienna, Austria.

Johansson, O. M. (2006). The effect of dynamic scheduling and routing in a solid waste management system. *Waste Management*, 26(8):875–885.

Kek, A. G., Cheu, R. L., and Meng, Q. (2008). Distance-constrained capacitated vehicle routing problems with flexible assignment of start and end depots. *Mathematical and Computer Modelling*, 47:140–152.

Kim, B. I., Kim, S., and Sahoo, S. (2006). Waste collection vehicle routing problem with time windows. *Computers & Operations Research*, 33(12):3624–3642.

Krikke, H., le Blanc, I., van Krieken, M., and Fleuren, H. (2008). Low-frequency collection of materials disassembled from end-of-life vehicles: On the value of on-line monitoring in optimizing route planning. *International Journal of Production Economics*, 111(2):209 – 228. Special Section on Sustainable Supply Chain.

McLeod, F., Erdoğan, G., Cherrett, T., Bektaş, T., Davies, N., Speed, C., Dickinson, J., and Norgate, S. (2013). Dynamic collection scheduling using remote asset monitoring. *Transportation Research Record*, 2378:65–72.

Mes, M. (2012). Using simulation to assess the opportunities of dynamic waste collection. In Bangsow, S., editor, *Use Cases of Discrete Event Simulation*, pages 277–307. Springer Berlin Heidelberg.

Mes, M., Schutten, M., and Rivera, A. P. (2014). Inventory routing for dynamic waste collection. *Waste Management*, 34(9):1564–1576.

Nolz, P. C., Absi, N., and Feillet, D. (2011). Optimization of infectious medical waste collection using RFID. In Böse, J. W., Hu, H., Jahn, C., Shi, X., Stahlbock, R., and Voß, S., editors, *Computational Logistics*, volume 6971 of *Lecture Notes in Computer Science*, pages 86–100. Springer Berlin Heidelberg.

Nolz, P. C., Absi, N., and Feillet, D. (2014). A stochastic inventory routing problem for infectious medical waste collection. *Networks*, 63(1):82–95.

Penna, P. H. V., Subramanian, A., and Ochi, L. S. (2013). An iterated local search heuristic for the heterogeneous fleet vehicle routing problem. *Journal of Heuristics*, 19(2):201–232.

Schneider, M., Stenger, A., and Goeke, D. (2014). The electric vehicle-routing problem with time windows and recharging stations. *Transportation Science*, 48(4):500–520.

Schneider, M., Stenger, A., and Hof, J. (2015). An adaptive VNS algorithm for vehicle routing problems with intermediate stops. *OR Spectrum*, 37(2):353–387.

Solomon, M. M. (1987). Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research*, 35(2):254–265.

Subramanian, A., Penna, P. H. V., Uchoa, E., and Ochi, L. S. (2012). A hybrid algorithm for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research*, 221(2):285–295.

Taillard, É. D. (1996). A heuristic column generation method for the heterogeneous fleet VRP. *Publication CRT-96-13*.

Trudeau, P. and Dror, M. (1992). Stochastic inventory routing: Route design with stockouts and route failures. *Transportation Science*, 26(3):171–184.