Simulation and optimization in transportation: an overview

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Outline

Transport policies

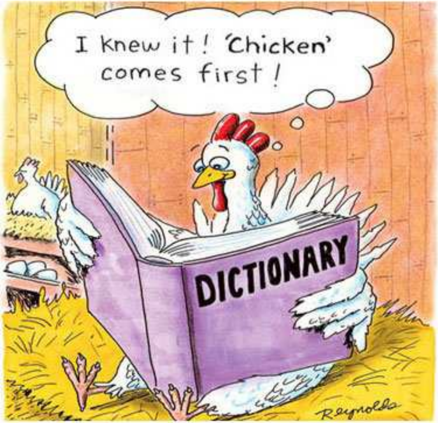
Complexity

- Transport systems are complex
- Many elements interact
- Presence of uncertainty

Transport policies

Causal effects

- Very important to identify the causal effects
- Failure to do so may generate wrong conclusions



Example: improving safety

Accidents in Kid City

- The mayor of Kid City has commissioned a consulting company
- Objective: assess the effectiveness of safety campaigns
- Before and after analysis

Example: improving safety

Accidents in Kid City

Example: improving safety

Accidents in Kid City: 

Example: improving safety

Accidents in Kid City

Example: improving safety

Accidents in Kid City

Example: improving safety

Accidents in Kid City: 

Example: improving safety

Conclusions

- The "Drive safely" signs have a significant impact on safety
- The number of accidents has been reduced by 57%, from 21 down to 9.

Example: improving safety

Two major flaws

- Causal effects are not modeled
- Simulation performed with only one draw

Capturing the complexity

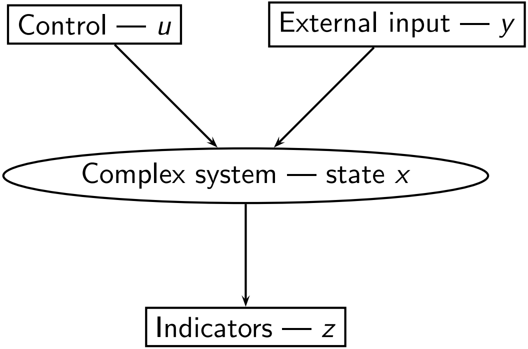
Simulation

the act of imitating the behavior of some situation or some process by means of something suitably analogous

Simulation: what it is not in engineering

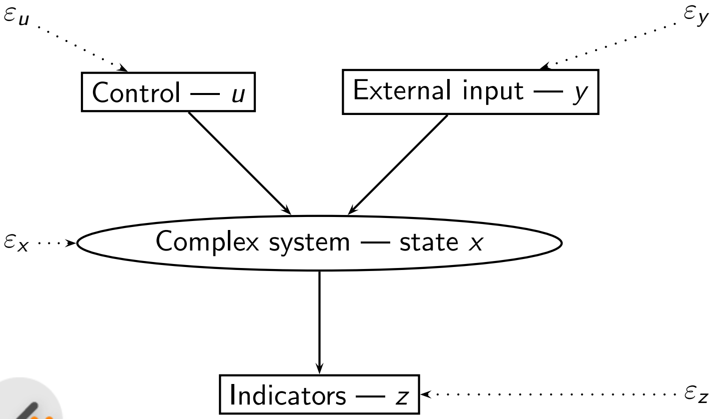
Simulation

$$z = h(x, y, u)$$



Simulation

$$Z = h(X, Y, U) + \varepsilon_z$$



Simulation

Propagation of uncertainty

$$Z = h(X, Y, U) + \varepsilon_z$$

- Given the distribution of X, Y, U and ε_z
- what is the distribution of Z?

Derivation of indicators

- Mean
- Variance
- Modes
- Quantiles

Simulation

Sampling

- Draw realizations of X, Y, U, ε_z
- Call them $x^r, y^r, u^u, \varepsilon_z^r$
- For each r, compute

$$z^r = h(x^r, y^r, u^r) + \varepsilon_z^r$$

- z^r are draws from the random variable Z

Simulation

Empirical distribution function

$$F_e(x) = \frac{1}{R}\,\#\{z^r \leq x\},$$

For any $x \in \mathbb{R}$,

$$\mathrm{E}[F_e(x)] = F(x)$$

and

$$\mathrm{var}(F_e(x)) = \frac{1}{R}F(x)(1 - F(x)).$$

Statistics

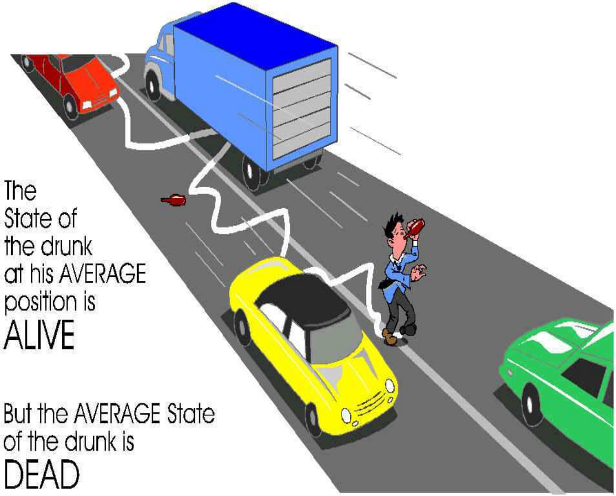
"Numbers don't lie. That's where we come in."

Indicators

- Mean: $\mathrm{E}[Z] \approx \bar{Z}_R = \frac{1}{R}\sum_{r=1}^{R} z^r$
- Variance: $\mathrm{Var}(Z) \approx \frac{1}{R}\sum_{r=1}^{R}(z^r - \bar{Z}_R)^2$.
- Modes: based on the histogram
- Quantiles: sort and select

Important: there is more than the mean

The mean



Savage et al. (2012)

The mean

The flaw of averages

Savage et al. (2012)

$$E[Z] = E[h(X, Y, U) + \varepsilon_z] \neq h(E[X], E[Y], E[U]) + E[\varepsilon_z]$$

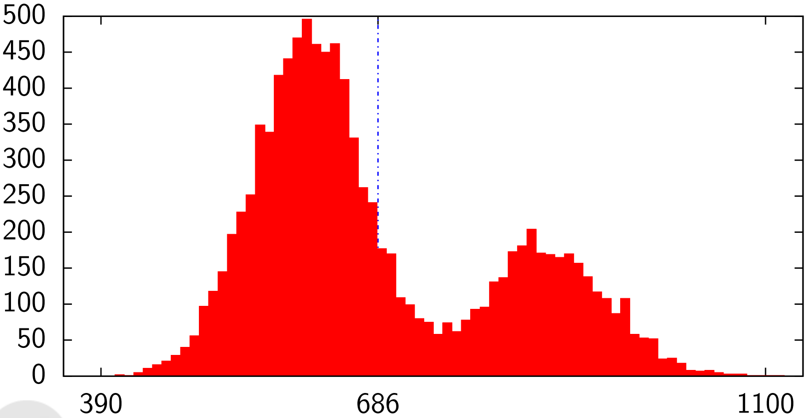
... except if h is linear.

There is more than the mean

Example

- Intersection with capacity 2000 veh/hour
- Traffic light: 30 sec green / 30 sec red
- Constant arrival rate: 2000 veh/hour during 30 minutes
- With 30% probability, capacity at 80%.
- Indicator: Average time spent by travelers

There is more than the mean



Pitfalls of simulation

Few number of runs

- Run time is prohibitive
- Tempting to generate partial results rather than no result

Focus on the mean

- The mean is useful, but not sufficient.
- For complex distributions, it may be misleading.
- Intuition from normal distribution (mode = mean, symmetry) do not hold in general.
- Important to investigate the whole distribution.
- Simulation allows to do it easily.

Outline

Optimization

Assumptions

- U is deterministic.
- $S^R(Z)$ is the statistic of Z under interest (mean, quantile, etc.)
- R is the number of draws generated to obtain the statistics
- Distributions of X, Y and ε_z are known.

Optimization problem

$$\min_u f(u) = S^R(Z) = S^R(h(X, Y, u) + \varepsilon_z)$$

subject to

$$g(u) = 0.$$

Optimization problem

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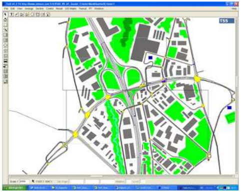
subject to

$$g(u) = 0.$$

Difficulties

- R must be large, so calculating f is computationally intensive
- The derivatives of f are unavailable or very difficult to obtain

Traffic simulation

Parameters calibration

- X: state of traffic
- Y: observed link flows
- u: parameters of the simulator
- h: traffic simulator
- Z: total squared difference between modeled and observed flows
- $S^R(Z)$: mean squared error

Traffic simulation

Traffic light optimization

- X: state of traffic
- Y: OD matrices
- u: traffic light configuration
- h: traffic simulator
- Z: total travel time
- $S^R(Z)$: mean of total travel time Osorio and Bierlaire (2013)
- $S^R(Z)$: std. dev. of total travel time Chen et al. (2013)

Outline

Scenario based optimization

"Of course, this is a worst case scenario."

Method

- Identify a list of scenarios $u_1, \ldots, u_N$
- Compute $f(u_i)$ for each i

Comments

- Solution is feasible and realistic
- Limited computational effort
- No systematic investigation
- Relies only on the creativity of the analyst

Nonlinear programming

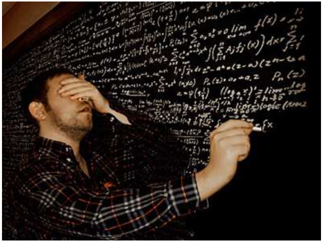
General approach

- $f(u) = S^R(h(X, Y, u) + \varepsilon_z)$ is a nonlinear function of u
- In general, it is continuous and differentiable
- As h is a computer program, the derivatives are not available

Methods

- Automatic differentiation Griewank (2000)
- Derivative-free optimization Conn et al. (2009)
- Direct search Lewis et al. (2000)

Automatic differentiation

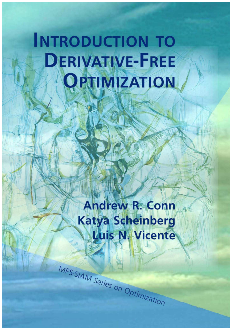


Method

Griewank (2000), Naumann (2012)

- A software is a sequence of a finite set of elementary operations
- Each of them is easy to differentiate
- Use chain rule to propagate

Derivative-free optimization



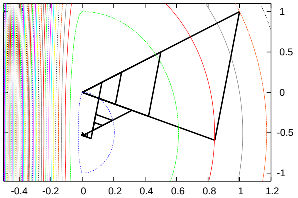
Method

- Build a model of the function using interpolation
  - Lagrange polynomials
  - Splines
  - Kriging
- Use a trust region framework to guarantee global convergence

Comments

- Convergence theory
- Numerical issues with interpolation
- Need for a large number of interpolation points

Direct search



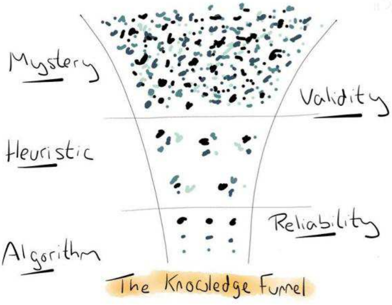
Method

- Generate a sequence of simplices
- using geometrical transformations maintaining the simplex structure

Comments

- Some do not always converge (Nelder-Nead)
- Convergence may be slow

Heuristics



Neighborhood

- Simple modifications of u
- Feasible or infeasible

Local search

- Select a better neighbor
- Stop at a local optimum

Meta heuristics

- Escape from local optima
- Simulated annealing
- Variable neighborhood search
- and many others...

Outline

Example of simulation

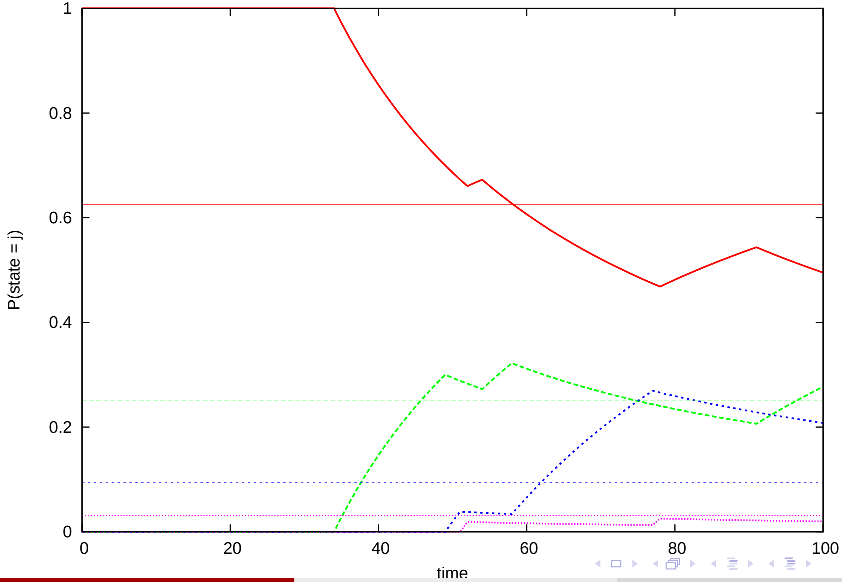
Machine with 4 states wrt wear

- perfect condition,
- partially damaged,
- seriously damaged,
- completely useless.

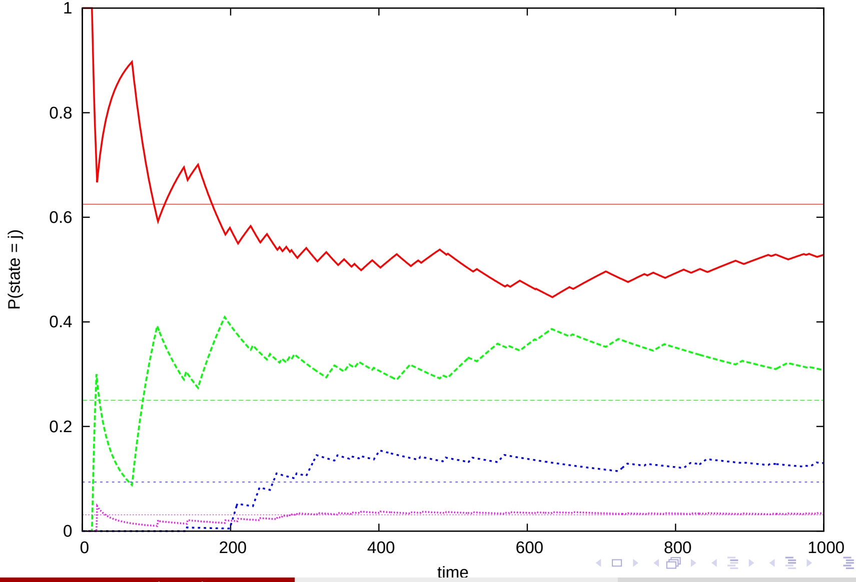
Transition

$$\begin{pmatrix} 0.95 & 0.04 & 0.01 & 0.0 \\ 0.0 & 0.90 & 0.05 & 0.05 \\ 0.0 & 0.0 & 0.80 & 0.20 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}$$

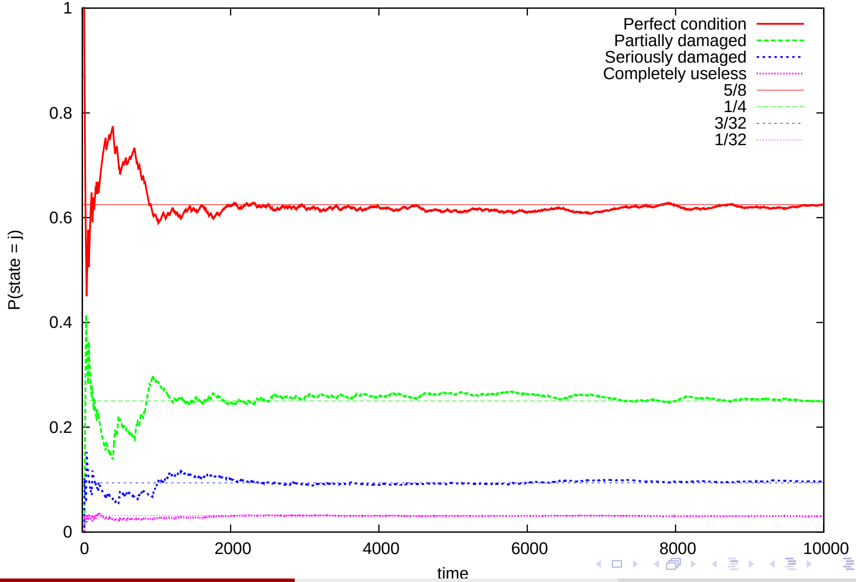
Noise reduction: $R = 100$



Noise reduction: $R = 1000$



Noise reduction: $R = 10000$



Noise reduction methods

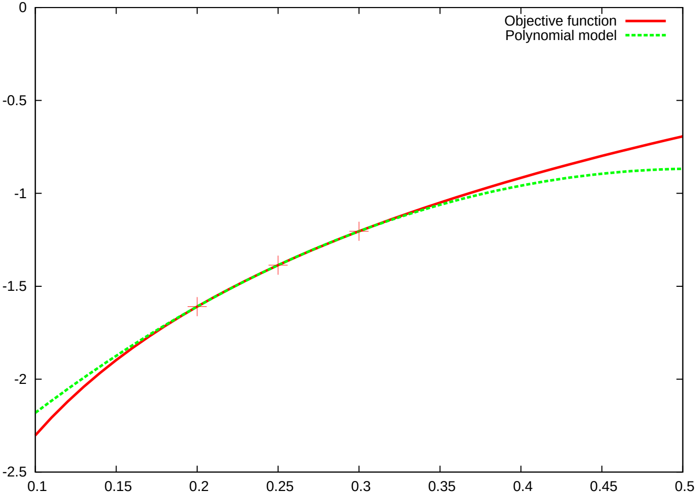
Adaptive Monte-Carlo

Bastin et al. (2006)

- R varies across iterations
- Small R in early iterations
- R increases as the algorithm converges

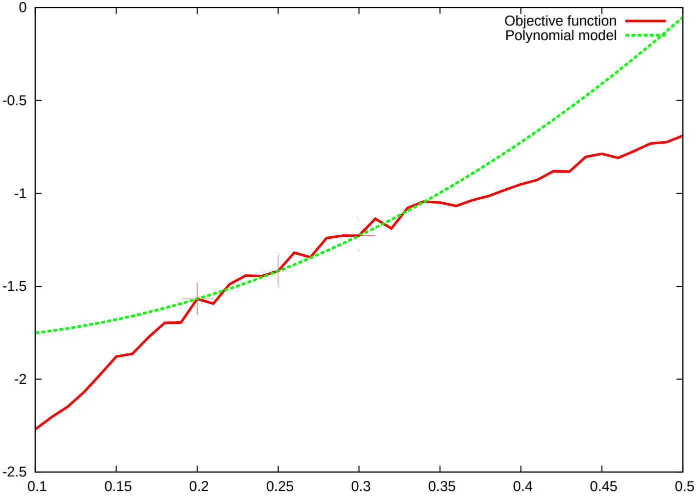
Noise reduction methods

Interpolation: true function



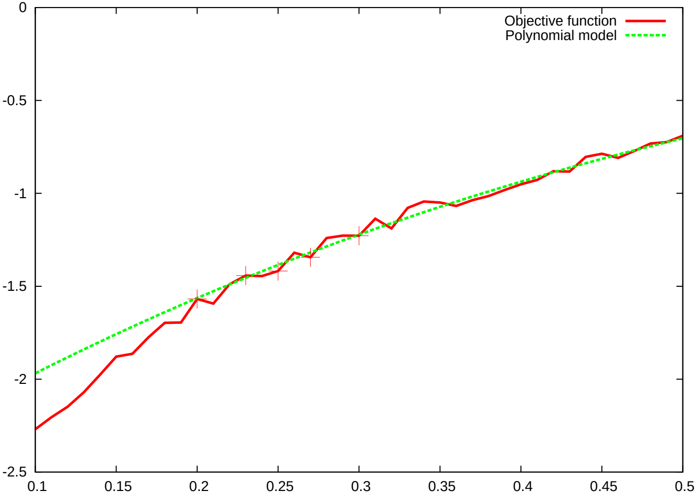
Noise reduction methods

Interpolation: simulated function



Noise reduction methods

Least-square fitting: simulated function



Noise reduction methods

Least square fitting

Bierlaire et al. (2007), Bierlaire and Crittin (2006)

- Interpolation model + adaptive Monte-Carlo
- Each iterate considered as a sample
- Regression is used instead of interpolation

Comments

- Originally for systems of nonlinear equations
- An update formula à la Broyden can be derived
- Appropriate for large-scale applications (2 millions variables)

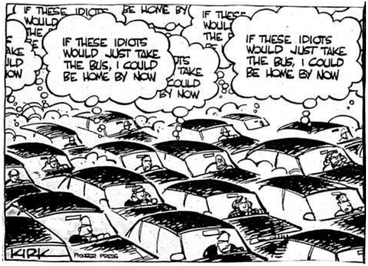
Outline

Open box algorithms

What are we simulating?

- $h(\cdot)$ is a detailed description of our system
- We need simulation because it is complicated
- We open the box, an build a simpler representation of the system

Deterministic model



Congestion

Osorio and Bierlaire (2009)

- Queuing theory
- Closed form analytical equations
- Simplifying assumptions (e.g. stationarity)

Metamodel

Osorio and Bierlaire (2013)

$$m(u, x; \alpha, \beta, q) = \alpha T(u, x, q) + \phi(u, \beta)$$

- $T(\cdot)$ analytical model
- $\phi(\cdot)$ interpolation model
- u control (traffic lights)
- x state variables

Metamodel

Osorio and Bierlaire (2013)

$$m(u, x; \alpha, \beta, q) = \alpha T(u, x, q) + \phi(u, \beta)$$

- $T(\cdot)$ analytical model
- $\phi(\cdot)$ interpolation model
- u control (traffic lights)
- x state variables

- engineering
- mathematics

Metamodel approach

Ongoing research

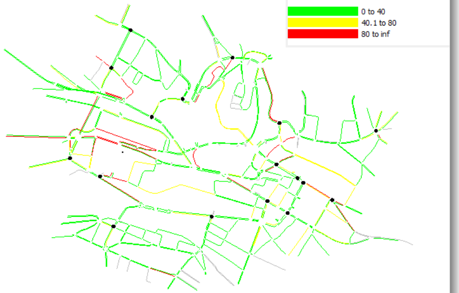
- Large scale problems Osorio and Chong (ta)
- Fuel consumption Osorio and Nanduri (ta)
- Emissions Osorio and Nanduri (2013)

Large scale problems

Simulated travel time (with 50 draws) Osorio and Chong (ta)

Initial signal plan

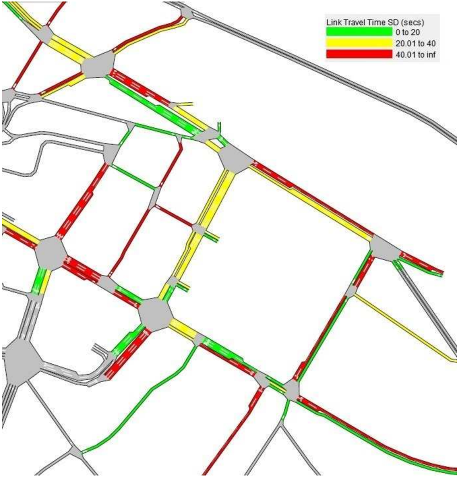
Optimized signal plan



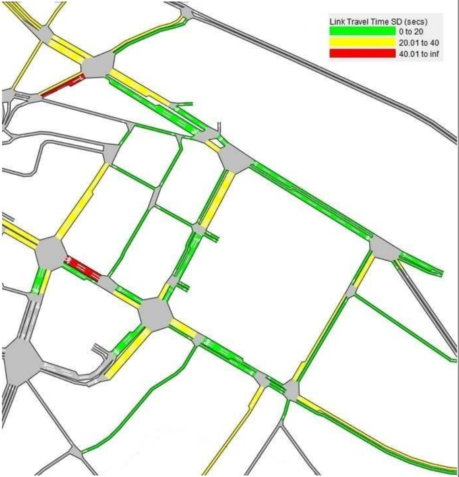
Reliability

Simulated standard deviation (with 50 draws)Chen et al. (2013)

Initial signal plan



Optimized signal plan



Outline

Summary

Black box algorithms

- Scenarios
- Automatic differentiation
- Derivative-free
- Direct search
- Heuristics
- Noise reduction

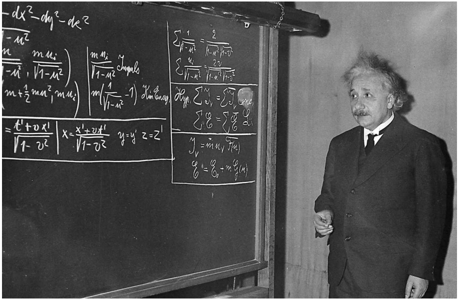
Simulation

- Number of draws
- Beyond the mean

Open box algorithms

- Deterministic engineering model
- Metamodel

Conclusion



Everything should be made as simple as possible, but no simpler

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Albert Einstein

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