Reactive Methods to Solve the Berth Allocation Problem with Stochastic Arrival and Handling Times

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Motivation

- High level of uncertainty in port operations due to weather conditions, mechanical problems etc.
  
  - Disrupt the normal functioning of the port
  
  - Require quick real time action.

- Very few studies address the problem of real time recovery in port operations, while the problem has not been studied at all in context of bulk ports.

- Our research problem derives from the realistic requirements at the SAQR port, Ras Al Khaimah, UAE
Research Objectives

• Develop real time algorithms for disruption recovery in berth allocation problem (BAP)

• For a given baseline berthing schedule, minimize the total realized costs of the updated schedule as actual arrival and handling time data is revealed in real time.
Literature Review

- Very scarce studies on real time and robust algorithms in container terminals. To the best of our knowledge, no literature on bulk ports.

- OR literature related to BAP under uncertainty in container terminals

  - **Pro-active Robustness**
    - Stochastic programming approach used by Zhen et al. (2011), Han et al. (2010)
    - Define surrogate problems to define the stochastic nature of the problem: Moorthy and Teo (2006), Zhen and Chang (2012), Xu et al. (2012) and Hendriks et al. (2010)

  - **Reactive approach or disruption management**
    - Zeng et al. (2012) and Du et al. (2010) propose reactive strategies to minimize the impact of disruptions.
Baseline Schedule

- Any feasible berthing assignment and schedule of vessels along the quay respecting the spatial and temporal constraints on the individual vessels

- Best case: Optimal solution of the deterministic berth allocation problem (without accounting for any uncertainty in information)
Deterministic BAP: Problem Definition

- **Find**
  - Optimal assignment and schedule of vessels along the quay (without accounting for any uncertainty)

- **Given**
  - Expected arrival times of vessels
  - Estimated handling times of vessels dependent on cargo type on the vessel (the relative location of the vessel along the quay with respect to the cargo location on the yard) and the number of cranes operating on the vessel

- **Objective**
  - Minimize total service times (waiting time + handling time) of vessels berthing at the port

- **Results**
  - Near optimal solution obtained using set partitioning method or heuristic based on squeaky wheel optimization for instances containing up to 40 vessels
Real Time Recovery in Berth Allocation Problem
Problem Definition: Real time recovery in BAP

- **Objective:** For a given baseline berthing schedule, minimize the total realized costs of the actual berthing schedule as actual data is revealed in real time

\[
\begin{align*}
\min \quad & Z_t = Z_{1t} + Z_{2t} + Z_{3t} \\
Z_{1t} = & \sum_{i \in N_i^u} (m_i' - a_i' + h_i'(k')) \\  
Z_{2t} = & \sum_{i \in N_i^u} (c_1 |b_i'(k') - b_i(k)| + c_2 \mu_i |e_i' - e_i|) \\
Z_{3t} = & \sum_{i \in N_i^o} (c_3 w_i')
\end{align*}
\]

- **Service cost of unassigned vessels**
- **Cost of re-allocation of unassigned vessels**
- **Berthing delays to vessels arriving on-time**
Problem Definition: Real time recovery in BAP

- Key arrival disruption pattern in real time
  - For each vessel \( i \in N \), we are given an expected arrival time \( A_i \) which is known in advance.
  - The expected arrival time of a given vessel may be updated \( |F| \) times during the planning horizon of length \( |H| \) at time instants \( t_{i1}, t_{i2}, \ldots, t_{iF} \) such that
    \[
    0 \leq t_{i1} < t_{i2} < t_{i3} < \ldots < t_{i(F-1)} < t_{iF} < a_i
    \]
    where \( a_i \) is the actual arrival time of the vessel, and the corresponding arrival time update at time instant \( t_{iF} \) is \( A_{iF} \) for all \( i \in N \).

- Actual handling time of a vessel is revealed at the time instant when the handling of the vessel is actually finished.
Modeling the Uncertainty

- Uncertainty in arrival times
  
  - Arrival times are modeled using a uniform distribution. Actual arrival time $a_i$ of vessel $i$ lies in the range $[A_i-V, A_i+V]$, where $A_i$ is the expected arrival time of vessel $i$ at the start of the planning horizon.
  
  - At any given time instant $t$ in the planning horizon, the following 3 cases arise
    
    - Case I: vessel $i$ has arrived and the actual arrival time $a_i$ is known
    - Case II: the vessel hasn’t arrived yet but the expected arrival time $A_i$ is known
    - Case III: neither the actual nor the expected arrival time is known at time instant $t$, then the arrival time estimate $a_i^t$ at time instant $t$ is such that $a_i^t \in [t, A_i + V]$, and is determined from the following equation

\[
\Pr(a_i \leq a_i^t) = \rho_a
\]

Since the arrival time of vessel $i$ is assumed to be uniformly distributed,

\[
a_i^t = t + \rho_a (A_i + V - t)
\]
Modeling the Uncertainty

- Uncertainty in handling times
  
  Handling times are modeled using a truncated exponential distribution. Handling time $h_i(k)$ of vessel $i$ berthed at starting section $k$ lies in the range $[H_i(k), \gamma H_i(k)]$, where $H_i(k)$ is the estimated (deterministic) handling time of vessel $i$ berthed at starting section $k$.

- At any given time instant $t$ in the planning horizon, the following 3 cases arise
  
  - Case I: the handling of vessel $i$ berthed at starting section $k'$ is finished, then the actual handling time $h_i(k')$ is known.

  - Case II: the vessel is being handled at time instant $t$, thus the actual berthing position $k'$ of the vessel is known, but the actual handling time is unknown. The handling time estimate at time instant $t$ is given by
    
    \[ \Pr \text{ob} (h_i(k') \leq h_i^t(k')) = \rho_h \]

  - Case III: the vessel is not assigned yet, in which case the handling time of the vessel at time instant $t$ for any berthing position $k$ is given by
    
    \[ \Pr \text{ob} (h_i(k) \leq h_i^t(k)) = \rho_h \]

  Since the handling times follow a truncated exponentially distribution,

  \[
  h_i^t(k) = -\frac{1}{\lambda} \ln( e^{-\lambda h_i^L(k)} - \rho_h (e^{-\lambda h_i^L(k)} - e^{-\lambda h_i^U(k)}))
  \]
Solution Algorithms

- **Optimization Based Recovery Algorithm**
  - Re-optimize the berthing schedule of all unassigned vessels using set-partitioning approach every time there is a disruption
    - arrival time of any vessel is updated and it deviates from its previous expected value.
    - handling of any vessel is finished and it deviates from the estimated value
  - the future vessel arrival and handling times provided as input parameters are modeled as discussed earlier
  - the berthing assignment of all vessels that have already been assigned to the quay is considered frozen and unchangeable

- **Smart Greedy Recovery Algorithm**
  - Assign an incoming vessel to the quay as it arrives as soon as berthing space is available, to the section(s) at which the total realized cost of all the unassigned vessels at that instant is minimized by modeling the uncertainty in future vessel arrival and handling times of other vessels
    - Vessel is assigned at or after the estimated berthing time of the vessel (as per the baseline schedule)
  - In the determination of the total realized cost to assign a given vessel at a given set of section(s), all other unassigned vessels are assigned to the estimated berthing sections as per the baseline schedule
Benchmark Solutions

- **Greedy Recovery Algorithm**
  - Assign the vessels as they arrive as soon as berthing space is available. Any given vessel is assigned at those set of sections where the realized cost of assigning it is minimized
  - No need to model uncertainty in future arrival and handling times
  - Closely represents the ongoing practice at the port

- **Apriori Optimization Approach**
  - Assume that all arrival and handling delay information is available at the start of the planning horizon
  - Problem of real time recovery reduces to solving the deterministic berth allocation problem with the objective function to minimize total realized cost of the schedule
  - Provides a lower bound to the minimization problem of real time recovery being solved
## Arrival Disruption Scenario

<table>
<thead>
<tr>
<th>Vessel</th>
<th>EAT</th>
<th>Vessel 0:</th>
<th>ATA:26</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18</td>
<td>23(21)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>9(2)</td>
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</tr>
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<td>2</td>
<td>19</td>
<td>24(3)</td>
<td>31(7)</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>22(8)</td>
<td>ATA:10</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>16(1)</td>
<td>16(2)</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>19(8)</td>
<td>12(10)</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3(1)</td>
<td>ATA:21</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>3(2)</td>
<td>ATA:20</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
<td>29(1)</td>
<td>20(2)</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>3(2)</td>
<td>ATA:20</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>15(6)</td>
<td>8(7)</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>23(6)</td>
<td>18(7)</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td>29(1)</td>
<td>ATA:10</td>
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<td>13</td>
<td>2</td>
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<td>14</td>
<td>19</td>
<td>17(2)</td>
<td>27(4)</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>19(2)</td>
<td>12(4)</td>
</tr>
<tr>
<td>16</td>
<td>14</td>
<td>15(6)</td>
<td>10(8)</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>Vessel 17:</td>
<td>ATA:-12</td>
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<tr>
<td>18</td>
<td>19</td>
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</tr>
<tr>
<td>19</td>
<td>0</td>
<td>Vessel 19:</td>
<td>ATA:-15</td>
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<tr>
<td>20</td>
<td>14</td>
<td>Vessel 20:</td>
<td>ATA:-1</td>
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<tr>
<td>21</td>
<td>12</td>
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</tr>
<tr>
<td>24</td>
<td>10</td>
<td>Vessel 24:</td>
<td>ATA:-1</td>
</tr>
</tbody>
</table>

**Vessel 0:**
23(21) ATA:26

**Vessel 1:**
9(2) 14(4) 17(5) ATA:8

**Vessel 2:**
24(3) 31(7) 15(9) 21(12) 24(13) 16(14) 30(15) 32(16) 21(17) 20(18) 20(19) 21(20) ATA:21

**Vessel 3:**
22(8) ATA:10

**Vessel 4:**
16(1) 16(2) ATA:6

**Vessel 5:**
19(8) 12(10) 15(13) 24(14) 24(15) 18(16) 20(17) 24(18) 22(19) 22(20) ATA:21

**Vessel 6:**
15(8) ATA:16

**Vessel 7:**
3(1) 10(6) 13(7) 19(10) 32(11) 23(12) 22(13) 19(14) 26(15) 32(16) 31(17) 31(18) 29(19) 21(20) ATA:21

**Vessel 8:**
29(1) 20(2) 19(4) 9(5) ATA:7

**Vessel 9:**
3(2) ATA:20

**Vessel 10:**
10(1) 15(6) 8(7) 14(8) 13(9) 16(10) ATA:11

**Vessel 11:**
23(6) 18(7) 15(9) 12(10) 16(11) 20(12) ATA:13

**Vessel 12:**
29(1) ATA:10

**Vessel 13:**
5(0) 8(6) ATA:9

**Vessel 14:**
17(2) 27(4) 13(9) 26(15) 22(16) 27(17) 27(18) 33(19) 25(20) 23(21) 34(22) ATA:23

**Vessel 15:**
19(2) 12(4) 7(5) 7(6) 29(7) 29(9) 16(10) 20(11) 20(12) 24(13) 28(14) ATA:15

**Vessel 16:**
15(6) 10(8) 11(9) 28(10) 27(11) 29(12) 16(13) 15(14) ATA:15

**Vessel 17:**
ATA:-12

**Vessel 18:**
29(8) 13(9) 25(10) 30(12) 34(13) 18(14) 25(15) 20(16) 29(17) 34(18) 34(19) ATA:20

**Vessel 19:**
ATA:-15

**Vessel 20:**
ATA:-1

**Vessel 21:**
7(6) 20(9) 25(14) 24(19) 22(20) 27(21) 23(22) 26(23) ATA:24

**Vessel 22:**
12(0) ATA:5

**Vessel 23:**
21(5) 14(6) 13(7) 10(8) 10(9) 24(13) 19(14) 17(15) 27(16) ATA:17

**Vessel 24:**
ATA:-1
Computational Results

- $|N|=10$ vessels, $|M|=10$ sections, $c_1 = c_3 = 1.0$, $c_2 = 0.002$, $U=4$ hours, $V=5$, $\gamma = 1.1$

- Mean Gap with respect to the apriori optimization solution

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Mean Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greedy Approach</td>
<td>7.65%</td>
</tr>
<tr>
<td>Optimization based Approach</td>
<td>3.16%</td>
</tr>
<tr>
<td>Smart Greedy Approach</td>
<td>7.81%</td>
</tr>
</tbody>
</table>
Computational Results

- $|N| = 25$ vessels, $|M| = 10$ sections, $c_1 = c_3 = 1.0$, $c_2 = 0.002$, $U = 4$ hours, $V = 5$, $\gamma = 1.1$

- Mean Gap with respect to the apriori optimization solution

<table>
<thead>
<tr>
<th>Approach</th>
<th>Gap Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greedy Approach</td>
<td>54.41%</td>
</tr>
<tr>
<td>Optimization based Approach</td>
<td>27.40%</td>
</tr>
<tr>
<td>Smart Greedy Approach</td>
<td>37.11%</td>
</tr>
</tbody>
</table>
Computational Results

- $|N|=10$ vessels, $|M|=10$ sections, $c_1 = c_3 = 1.0$, $c_2 = 0.002$, $U=4$ hours, $V=10$, $\gamma = 1.1$

- Mean Gap with respect to the apriori optimization solution

<table>
<thead>
<tr>
<th>Approach</th>
<th>Greedy Approach</th>
<th>Optimization based Approach</th>
<th>Smart Greedy Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.37 %</td>
<td>3.06 %</td>
<td>8.56 %</td>
</tr>
</tbody>
</table>
Computational Results

- $|N|=25$ vessels, $|M|=10$ sections, $c_1 = c_3 = 1.0$, $c_2 = 0.002$, $U=4$ hours, $V=10$, $\gamma=1.1$

- Mean Gap with respect to the apriori optimization solution

<table>
<thead>
<tr>
<th>Approach</th>
<th>Greedy Approach</th>
<th>Optimization based Approach</th>
<th>Smart Greedy Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage</td>
<td>45.40 %</td>
<td>28.87 %</td>
<td>34.71 %</td>
</tr>
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</table>
Computational Results

- $|N| = 25$ vessels, $|M| = 10$ sections, $c_1 = c_3 = 1.0$, $c_2 = 0.002$, $U = 4$ hours, $V = 24$, $\gamma = 1.2$

Mean Gap with respect to the apriori optimization solution

<table>
<thead>
<tr>
<th>Approach</th>
<th>Objective Function Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greedy Approach</td>
<td>1000 - 1500</td>
</tr>
<tr>
<td>Optimization based Approach</td>
<td>1200 - 1700</td>
</tr>
<tr>
<td>Smart Greedy Approach</td>
<td>1600 - 2100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Approach</th>
<th>Mean Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greedy Approach</td>
<td>40.58 %</td>
</tr>
<tr>
<td>Optimization based Approach</td>
<td>43.26 %</td>
</tr>
<tr>
<td>Smart Greedy Approach</td>
<td>44.65 %</td>
</tr>
</tbody>
</table>
Conclusions and Future Work

- Modeling the uncertainty in future vessel arrival and handling times can significantly reduce the total realized costs of the schedule, in comparison to the ongoing practice of re-assigning vessels at the port.

- The optimization based recovery algorithm outperforms the heuristic based smart greedy recovery algorithm, but is computationally expensive.

- Limitation: Modeling of uncertainty fails to produce good results for larger instance size or when the stochasticity in arrival times and/or handling times is too high.

- As part of future work, plan to develop a robust formulation of the berth allocation problem with a certain degree of anticipation of variability in information.
Thank you!
Problem Definition: Real time recovery in BAP

- **Penalty Cost on late arriving vessels:** Impose a penalty fees on vessels arriving beyond the right end of the arrival window, $A_i + U_i$

![Diagram](image_url)

- $c_3g_i$
- $A_i - U_i$ to $A_i$ to $A_i + U_i$ to $a_i$
- Slope $= c_3$
- Arrival Time Window $= 2U_i$