

# Stochastic optimization with adaptive batch size

## Discrete choice models as a case study

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# Outline

1. Motivation
2. In the literature
3. WMA-ABS
4. Results
5. Conclusion

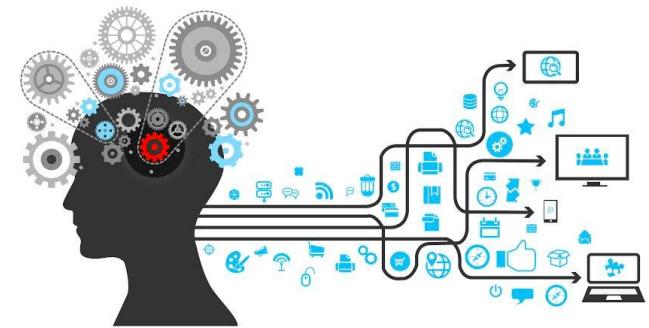
# Motivation

# DCMs in a nutshell

- Many success stories until now...



# Solution: Machine Learning



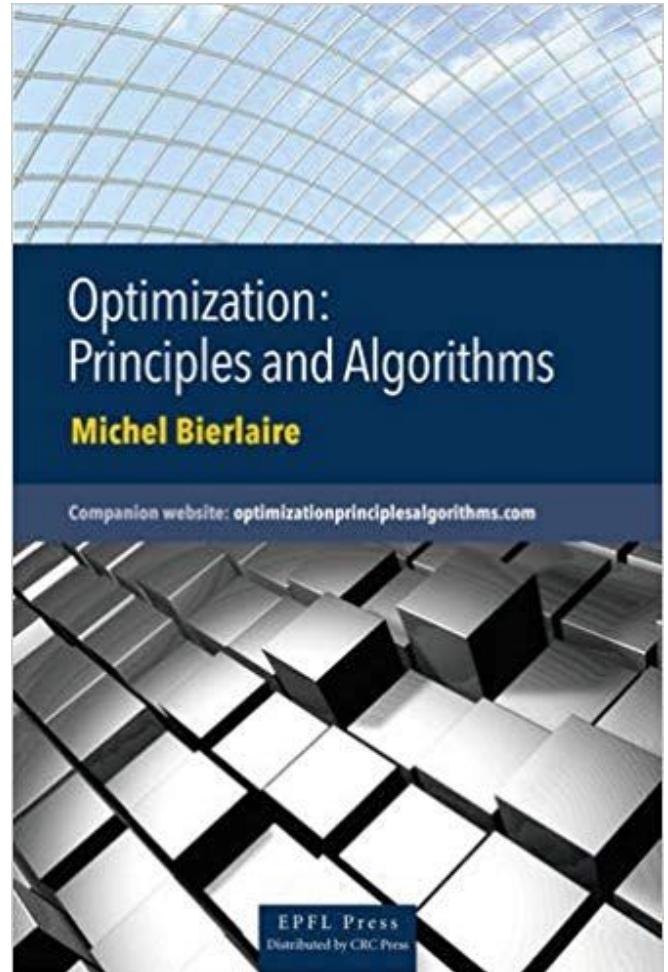
- ML is THE field dealing with a lot of data!

# Third direction

- Mix of Choice Modeling and Machine Learning:

# In the literature

# Optimization of DCM and ML



# Stochastic Iterative Optimization Algorithms

```
optimize():
```

```
    while stop_crit == False:
```

Select a batch of data

Compute  $p_k$

1. Order:

$$\nabla^2 f(x_k) \mathbf{B}_{P_k}^{-1} - \nabla f(x_k)$$

Compute alpha

$x_{k+1} = x_k + \text{alpha} * p_k$

Check stop\_crit

```
return x_k+1
```

# Some state-of-the-art algorithms

- 1st order:  
Adagrad (**Duchi et al., 2011**), AMSGrad (**Reddi et al., 2019**)
- 1.5th order:  
adaQN (**Keskar & Berahas, 2016**), SGD-QN (**Bordes et al., 2009**)
- 2nd order:  
Stochastic Hessian (**Byrd et al., 2011**)

# **WMA-ABS**

# Window Moving Average - Adaptive Batch Size

# WMA-ABS: the algorithm

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## Algorithm 2 Window Moving Average - Adaptive Batch Size (WMA-ABS)

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**Input:** Current iteration index ( $M$ ), function value at iteration  $M$  ( $f_M$ ), batch size ( $n$ ), and full size ( $N$ )

**Output:** New batch size  $n'$

```
1: while stop_crit == False:
2:   1. Compute the WMA using previous function values → Window (10)
3:   2. Compute the improvement using previous averages
4:   3. Check if under threshold → Threshold (1%)
5:   4. If yes, update counter. Otherwise, reset counter.
6:   5. Check if counter finished → Count (1 or 2)
7:   6. If yes, update the batch size → Factor (2)
8:   USE WMA-ABS
9:   else:
10:    c = c + 1
11:    if c == C then
12:      if n' >= N then
13:        if n' >= N then
14:          if n' >= N then
15:            if n' >= N then
16:              if n' >= N then
17:                n' = n
18: return n'
19: return x_k+1
```

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# Results

# Case Study

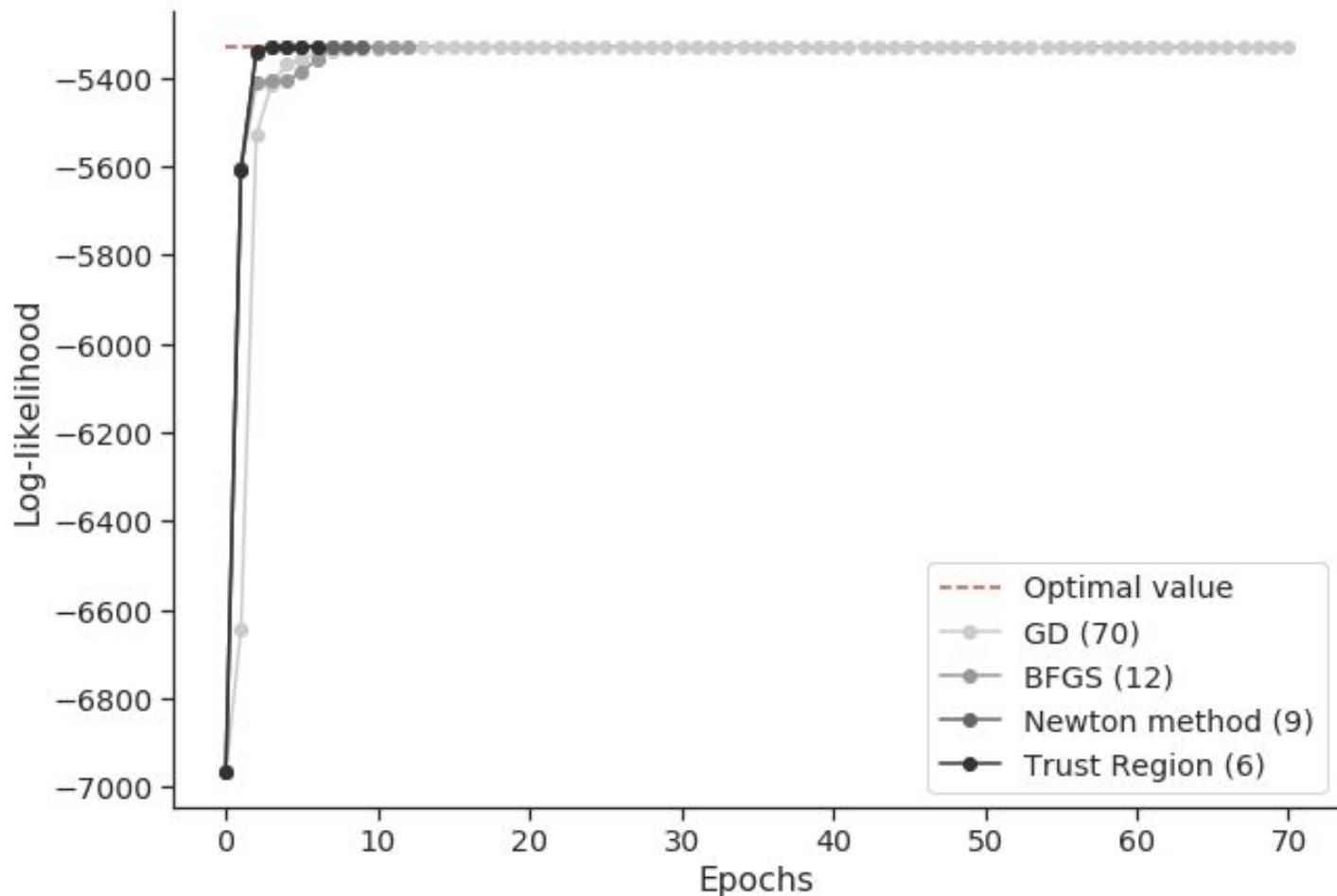
- 5 different models

Name	Type	#Parameters	Data set	#Observations
MNL-SM	MNL	4	Swissmetro	6'768
Nested-SM	Nested	5	Swissmetro	6'768
LogReg-BS	Logistic Regression	12	Bike Sharing	16'637
small MNL-CLT	MNL	100	London Passenger Mode Choice	27'478
MNL-CLT	MNL	100	London Passenger Mode Choice	54'766

- 4 different IOAs:

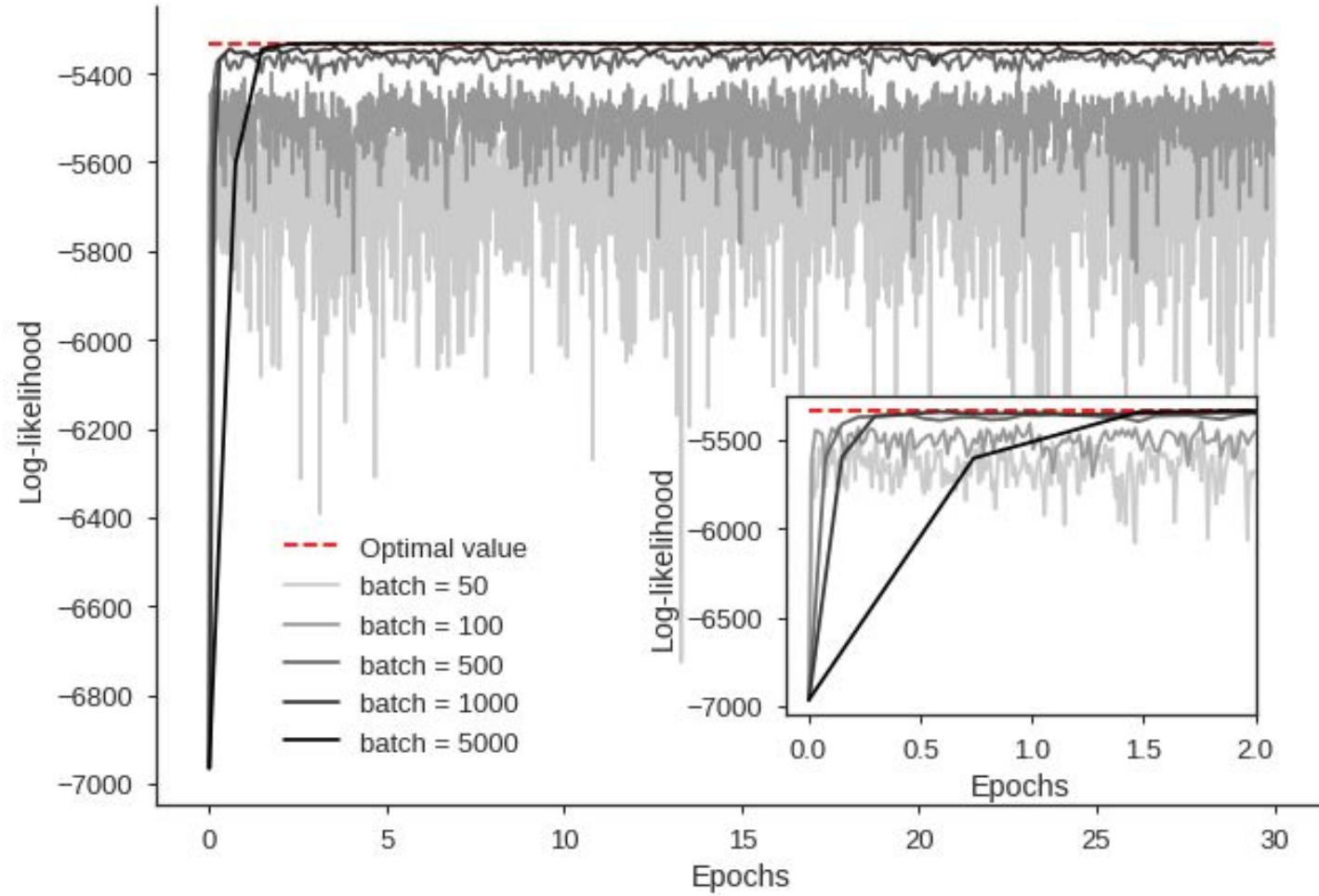
**GD, BFGS, Newton Method, Trust Region**

# Iterative Optimization Algorithms



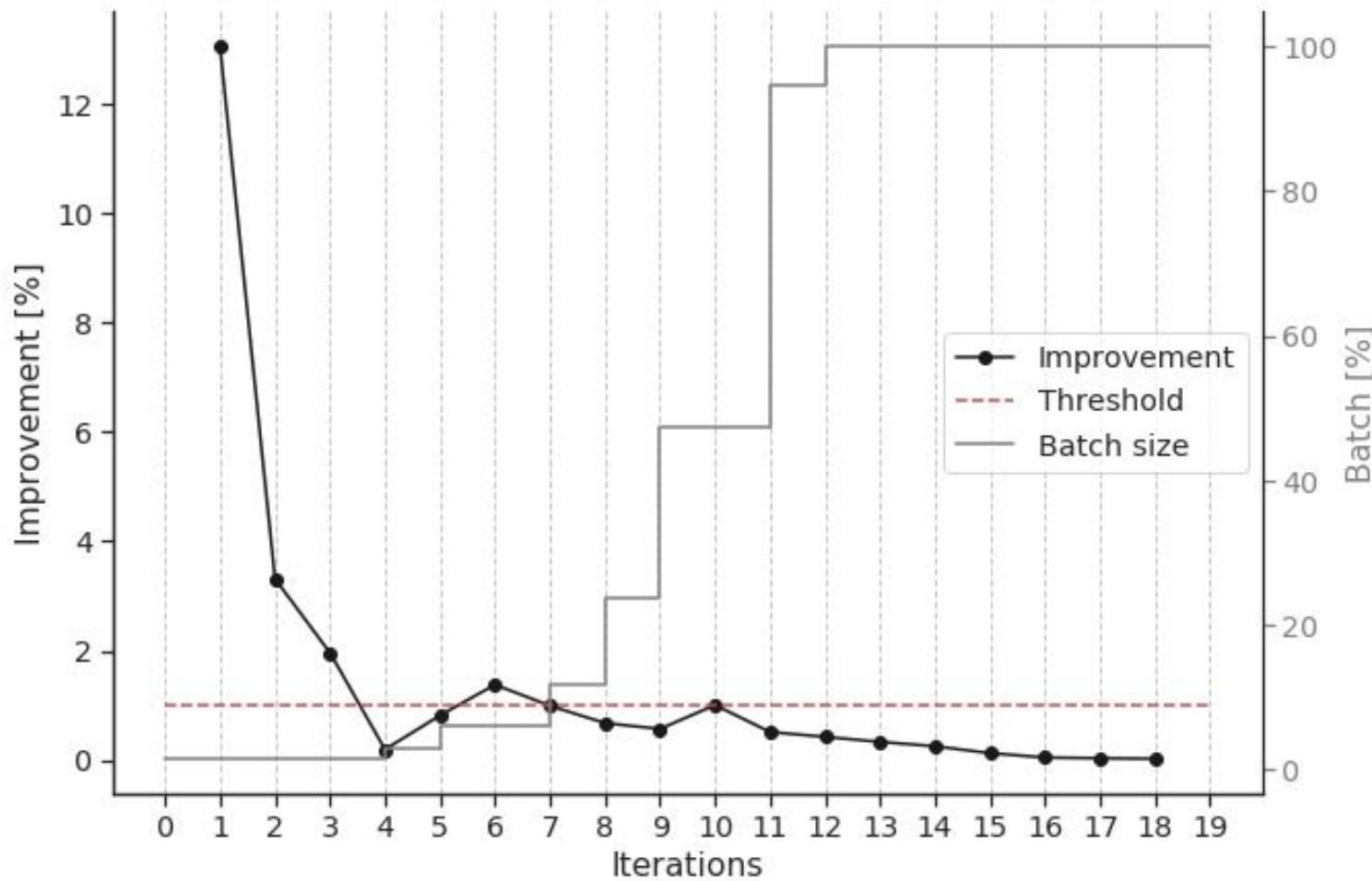
Optimization of model MNL-SM. (# of epochs until convergence)

# Stochastic Newton Method



Optimization of model MNL-SM. Lines = avg value over 10 runs.

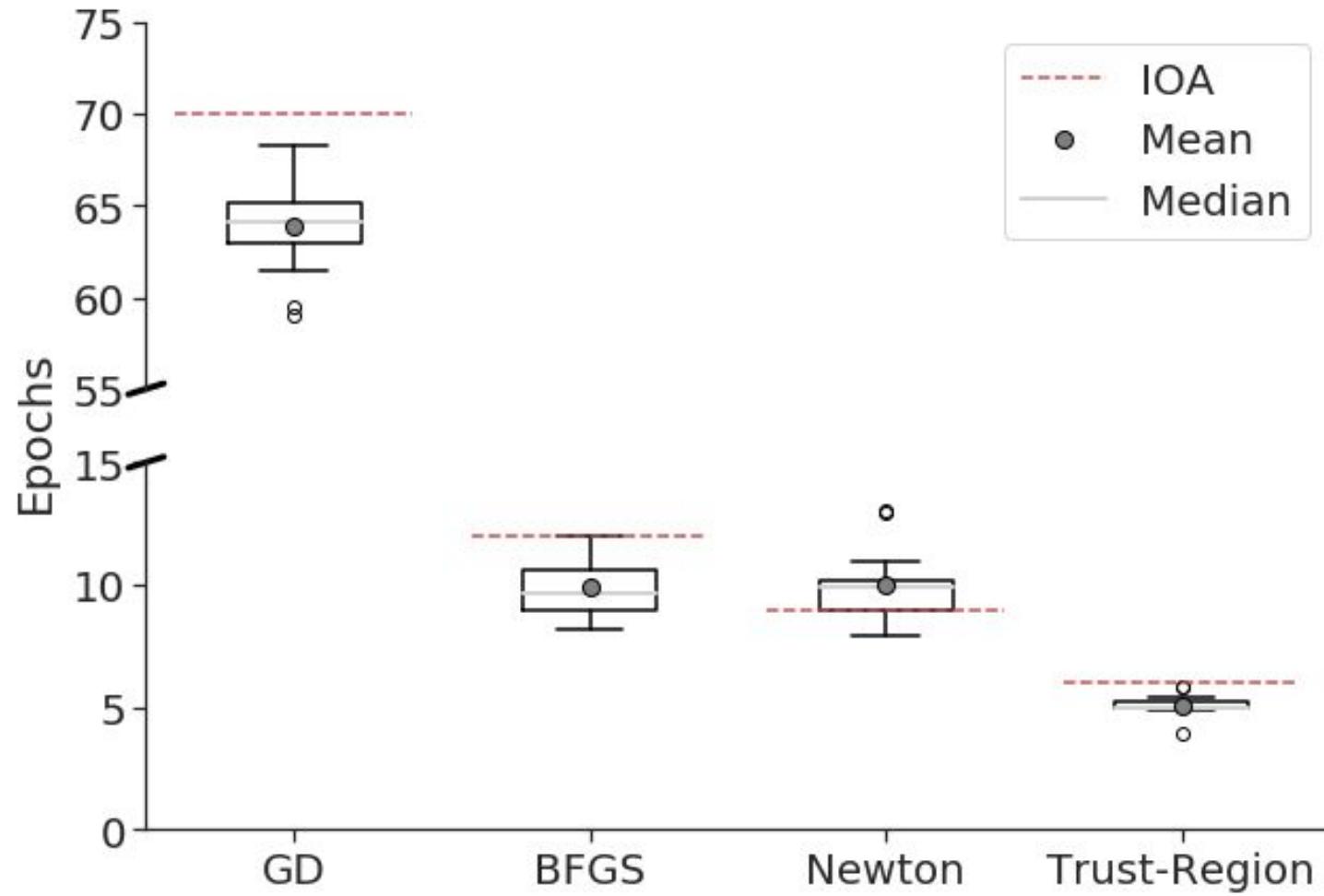
# Improvements using WMA-ABS



Optimization of model MNL-SM with SNM and WMA-ABS.

# Results of WMA-ABS

(The more interesting ones!)



Optimization of model MNL-SM with WMA-ABS. (20 runs)

# Results of WMA-ABS - Summary

- Stochastic IOA + WMA-ABS < standard IOA: **15/16**
- Improvements:
  - GD: 2-10%
  - BFGS: 10-20%
  - NM: 0-25%
  - TR: 20-54%**
- bigger/complex the model => better improvements!

# Parameter study

- Optimized parameters different from suggested parameters (expect for Count) ...
- Often suggesting higher threshold and higher factor.
- Suggested parameters seem good though!

# Conclusion

# Conclusion

- WMA-ABS works with many stochastic IOAs
- Bigger improvements on complex models (**up to 55%**)
- No need to optimize the hyper parameters

# Future work

- Write a paper (in progress...)
- Works on a heuristic to choose between stochastic and standard IOAs
- Implement the WMA-ABS + IOAs in -Biogeme!



[https://knowyourmeme.com/memes/  
improvise-adapt-overcome](https://knowyourmeme.com/memes/improvise-adapt-overcome)



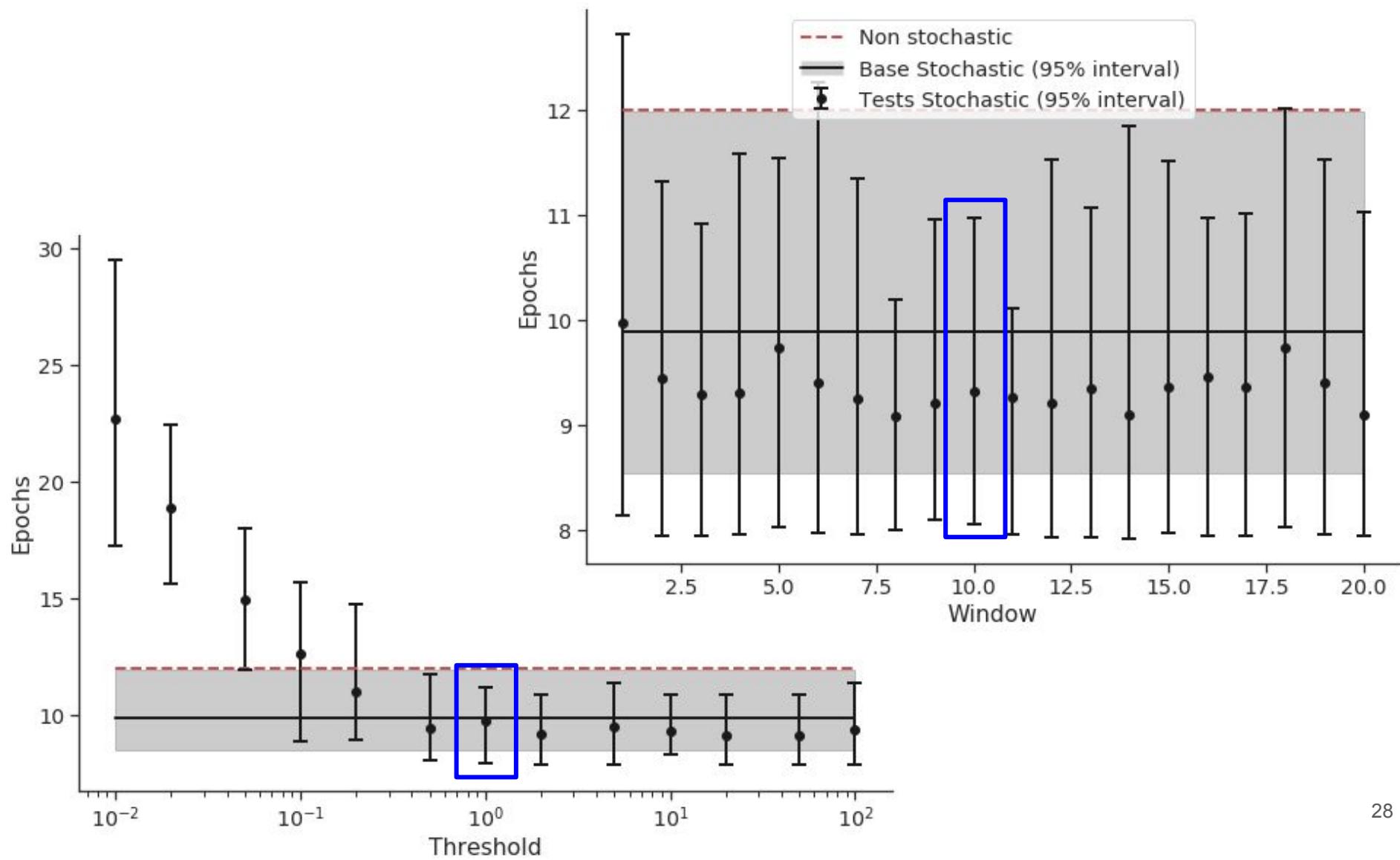
**Make a step. Adapt. Optimize!**

# References

- **Ortelli, Rodrigues, Pereira & Bierlaire**, *Automatic Utility Specific in Discrete Choice Models*, 2019 (Master thesis; Not published yet)
- **Duchi, Hazan & Singer**, *Adaptive Subgradient Methods for Online Learning and Stochastic Optimization*, 2011
- **Reddi, Kale & Kumar**, *On the Convergence of Adam and Beyond*, 2019
- **Keskar & Berahas**, *adaQN: An Adaptive Quasi-Newton Algorithm for Training RNNs*, 2016
- **Bordes, Bottou, Gallinari, Chang & Smith**, *SGD-QN: Careful Quasi-Newton Stochastic Gradient Descent*, 2009
- **Byrd, Chin, Neveitt & Nocedal**, *On the Use of Stochastic Hessian Information in Optimization Methods for Machine Learning*, 2011
- **Balles, Romero & Hennig**, *Coupling Adaptive Batch Sizes with Learning Rates*, 2016
- **Devarakonda, Naumov & Garland**, *AdaBatch: Adaptive Batch Sizes for Training Deep Neural Networks*, 2017

# Backup slides

# Effects of the parameters (1/2)



# Effects of the parameters (2/2)

