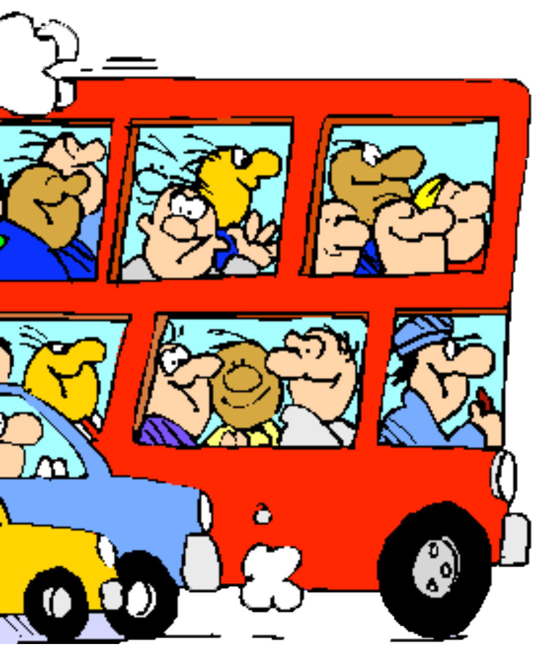
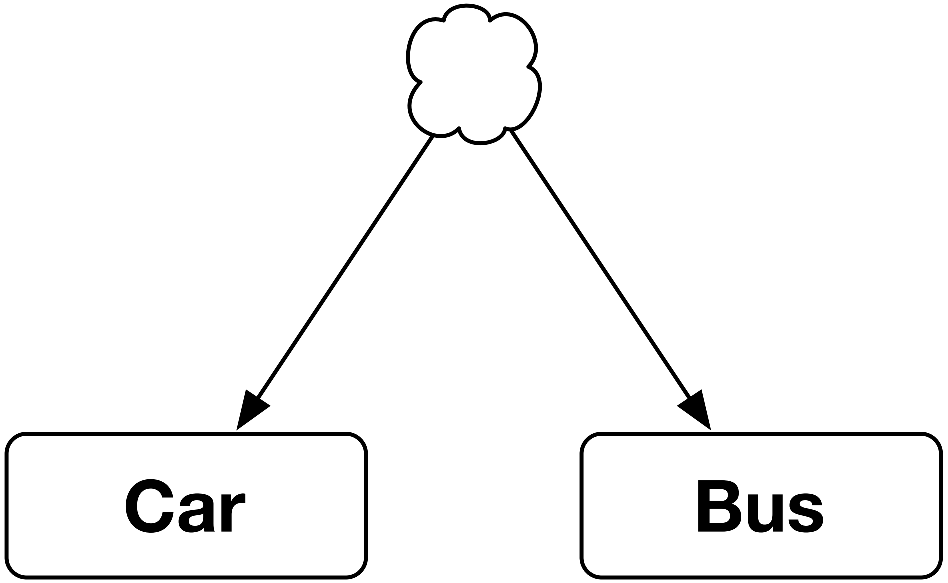
Pieces of Choices: Alternative Allocations in Network-GEV Models

Jeffrey Newman

One Bus, One Car

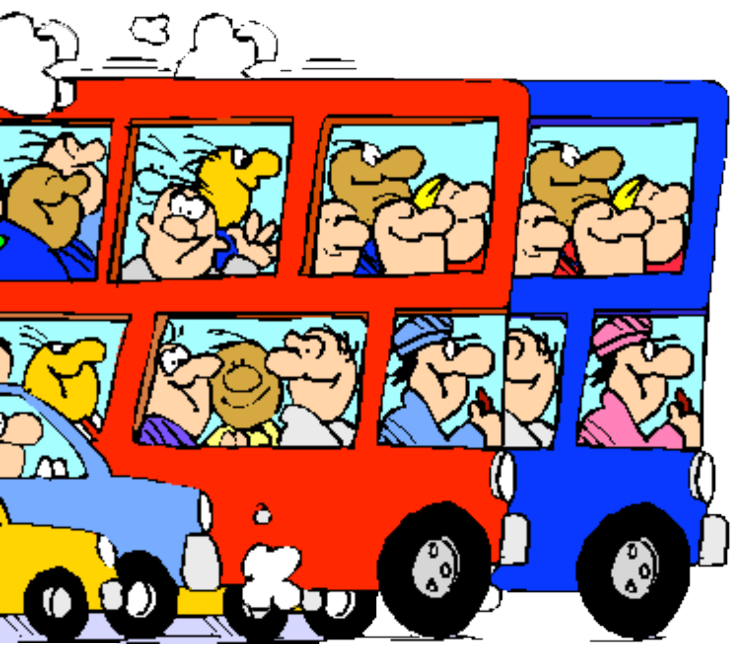


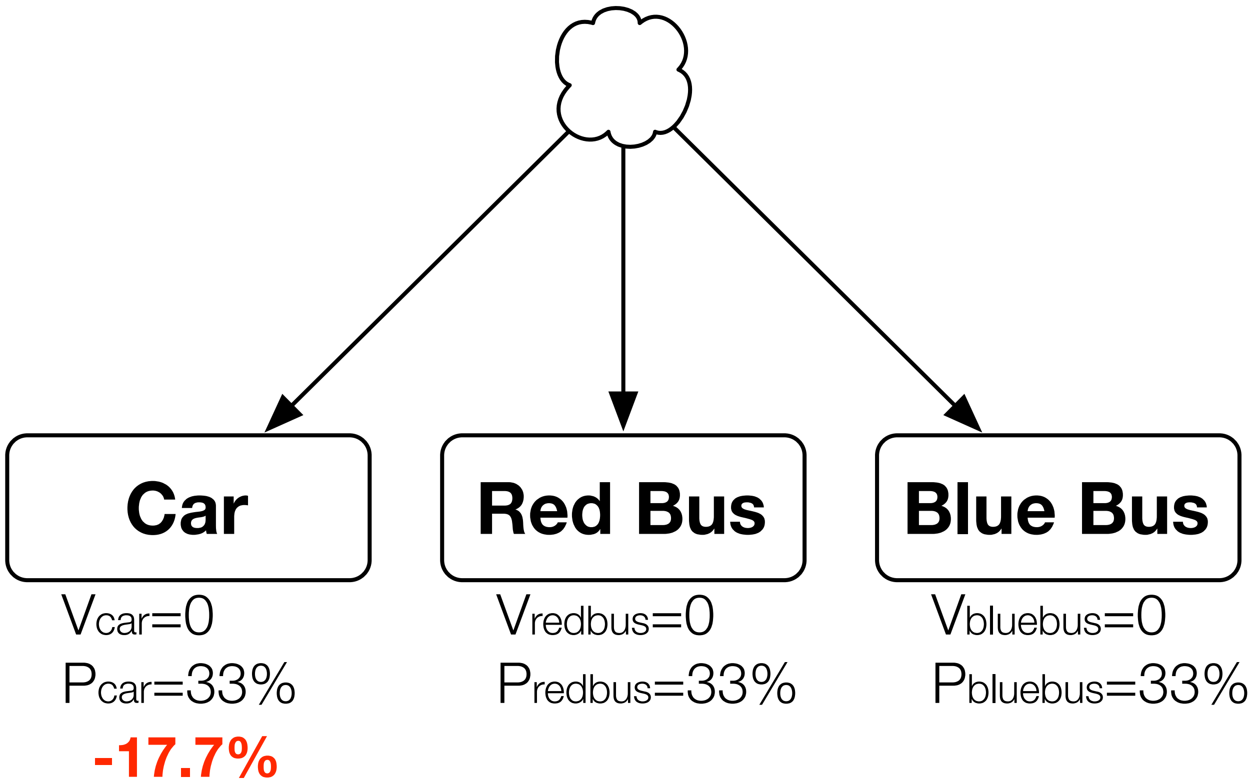


$V_{car}=0$
$P_{car}=50\%$

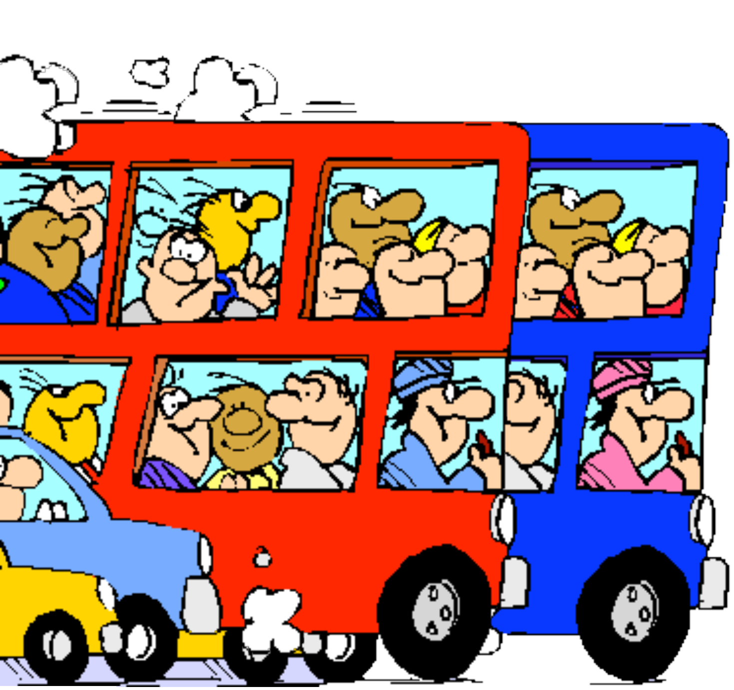
$V_{bus}=0$
$P_{bus}=50\%$

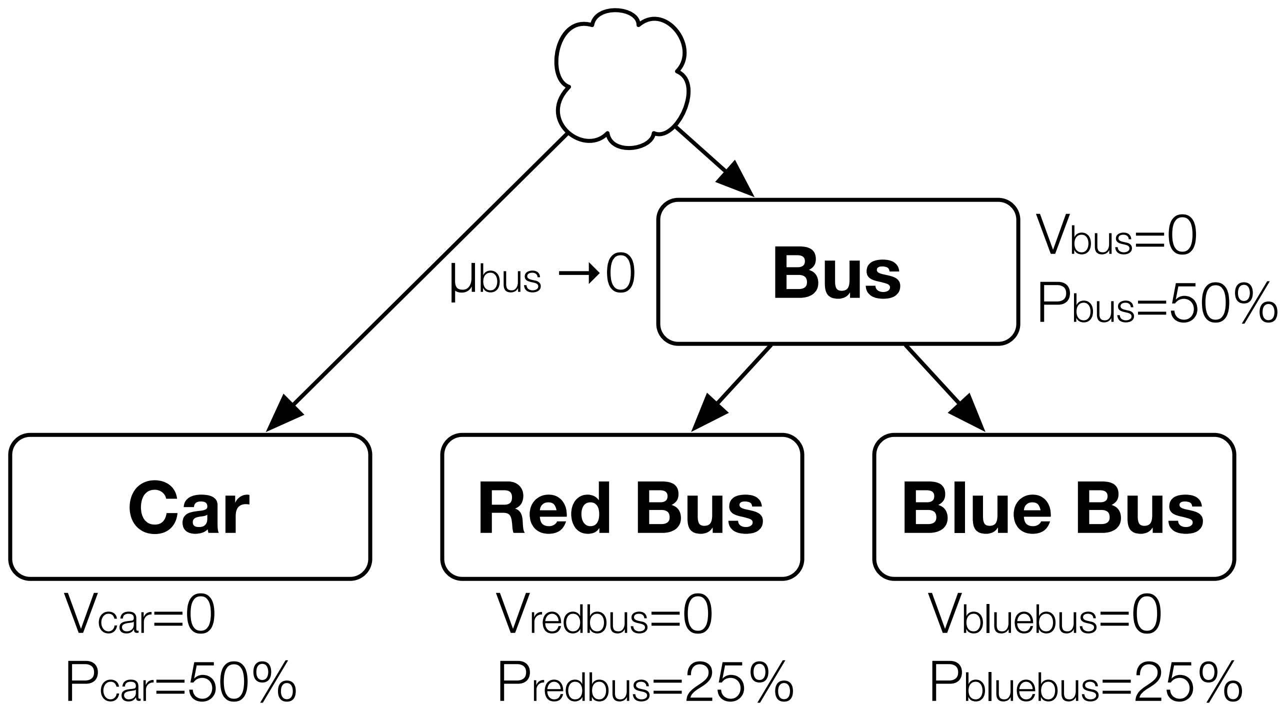
One Bus, Two Bus, Red Bus, Blue Bus



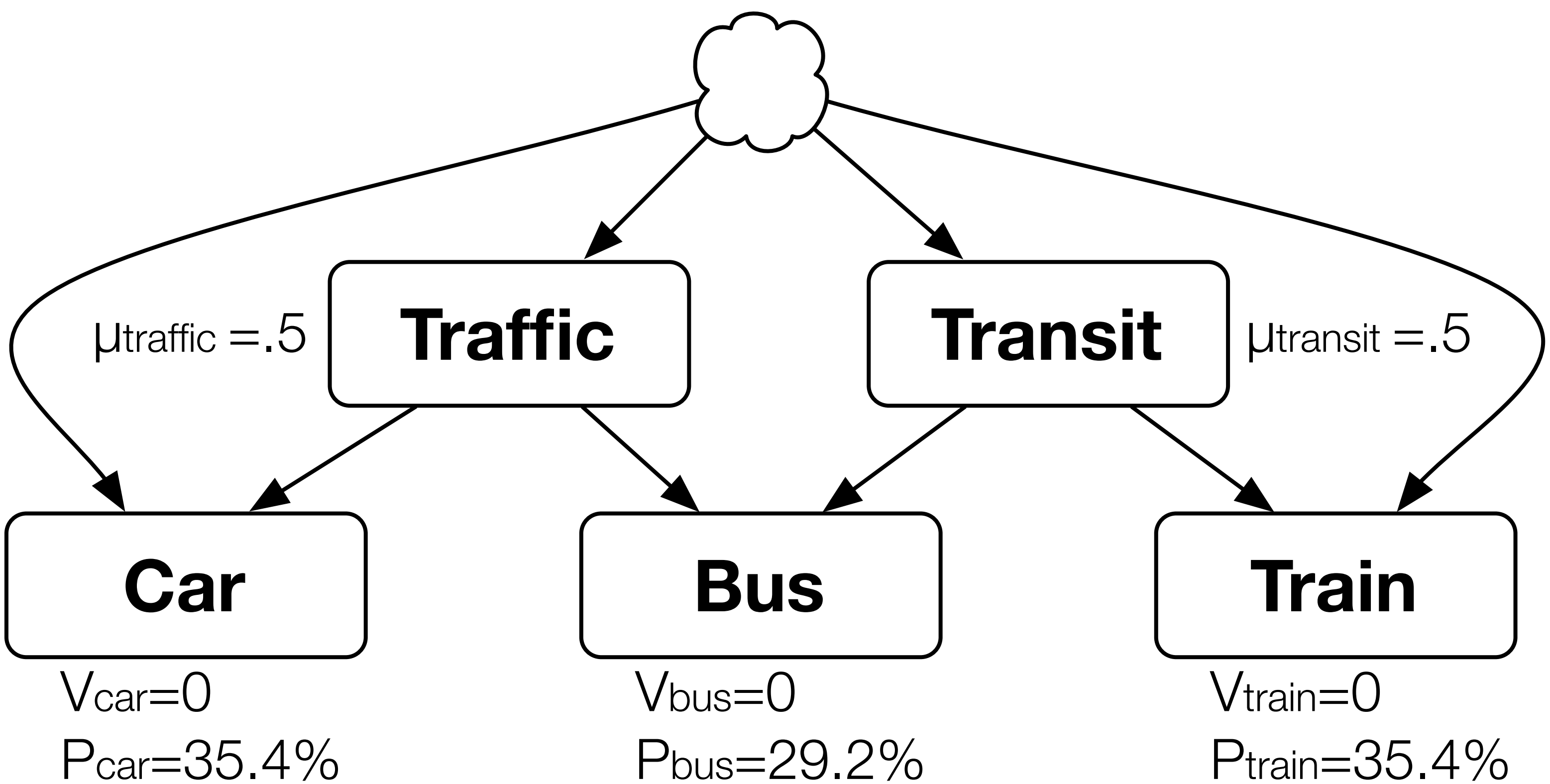


One Bus, Two Bus, Red Bus, Blue Bus

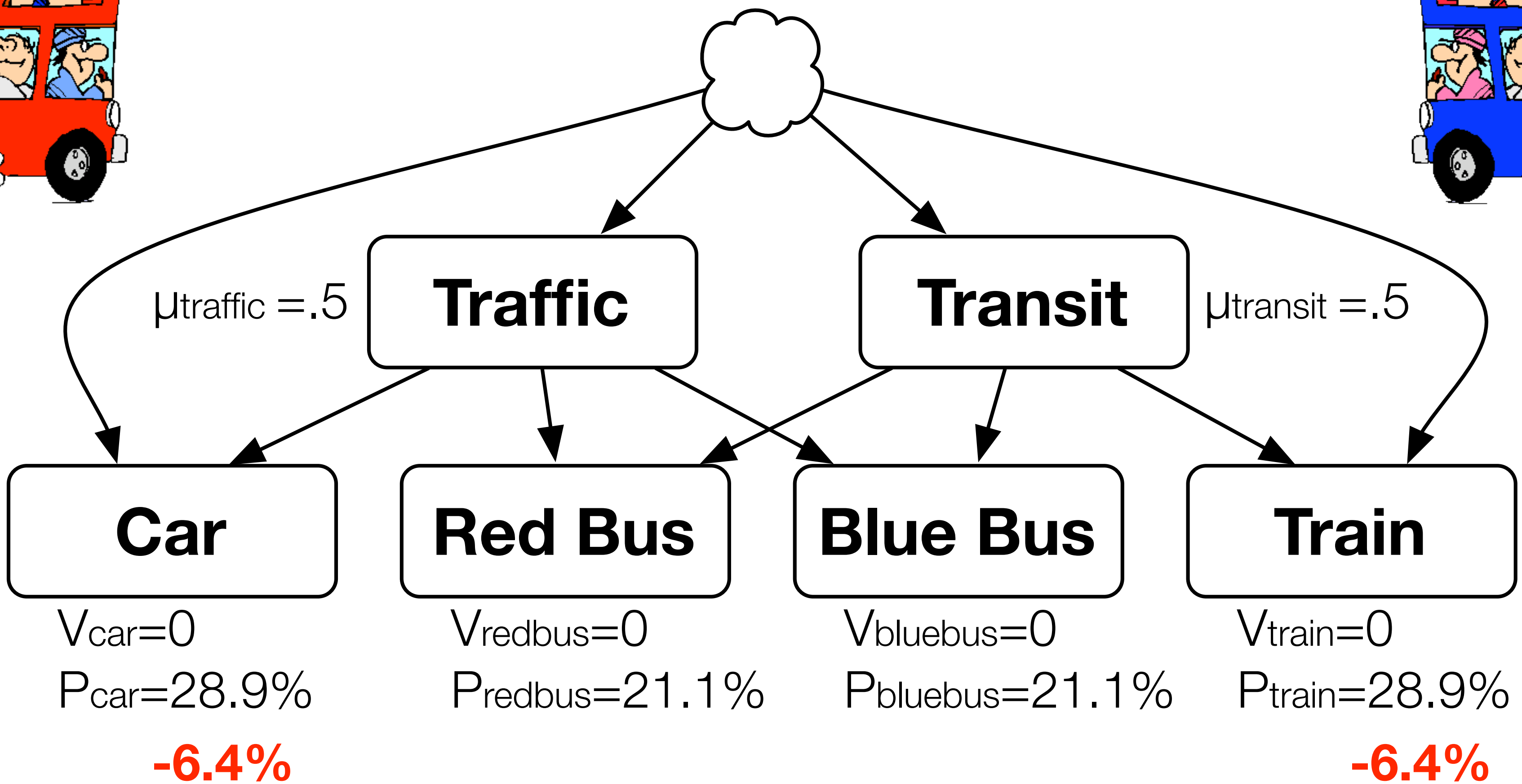




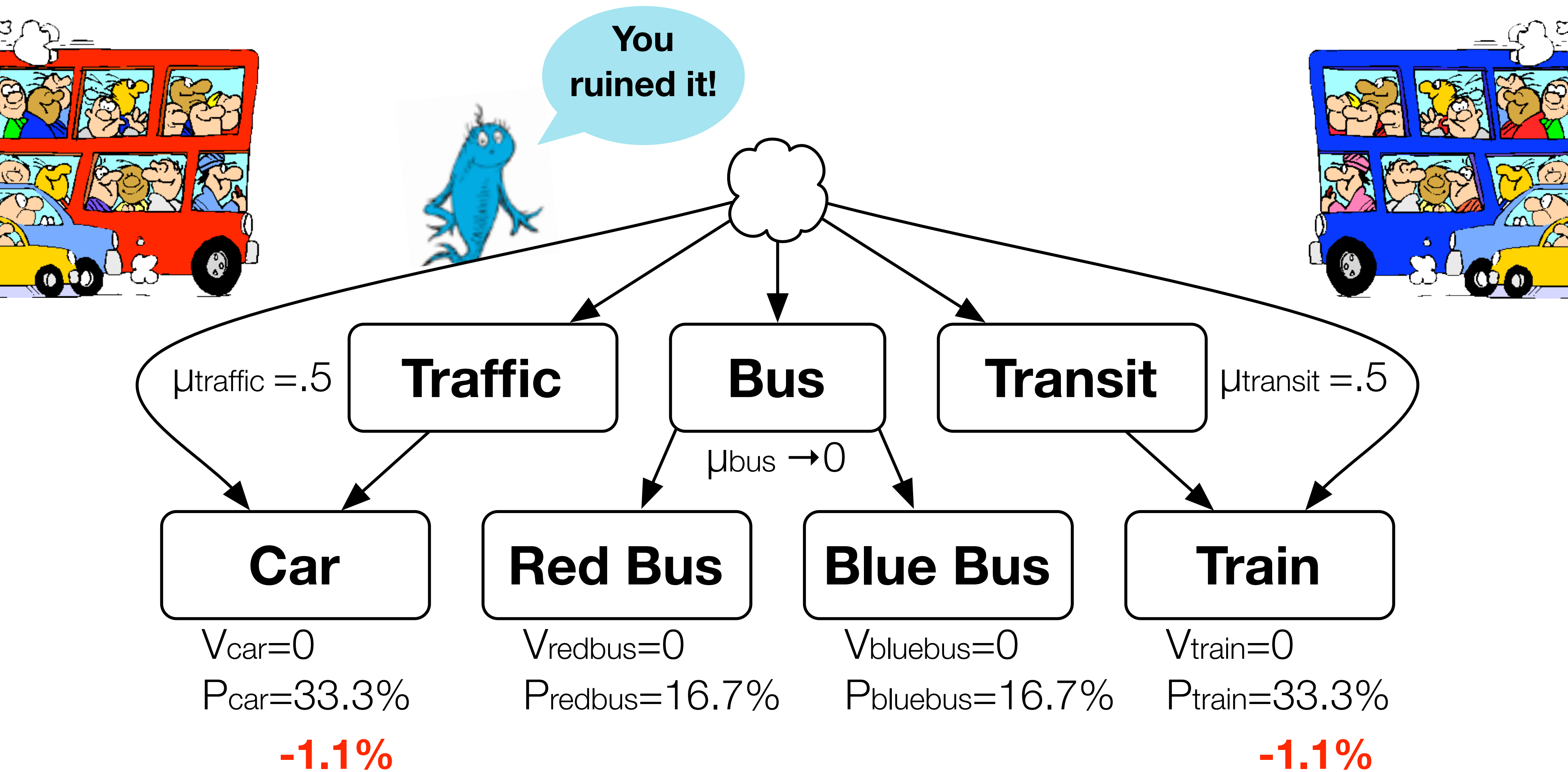
Generalized Nested Logit allows overlapping nests



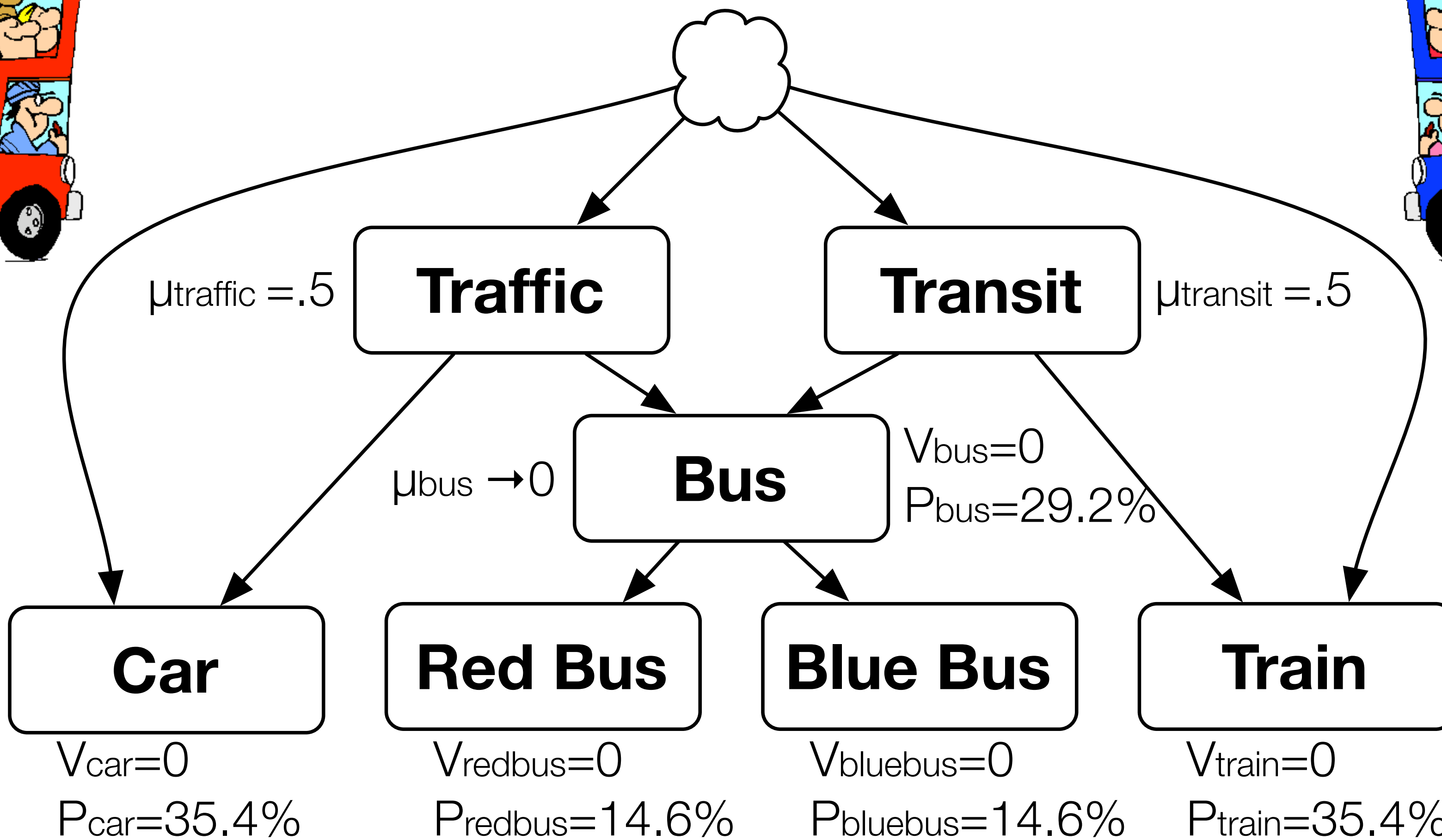
That darn blue bus operator just won’t quit!



Adding a Bus nest to a GNL ruins the model



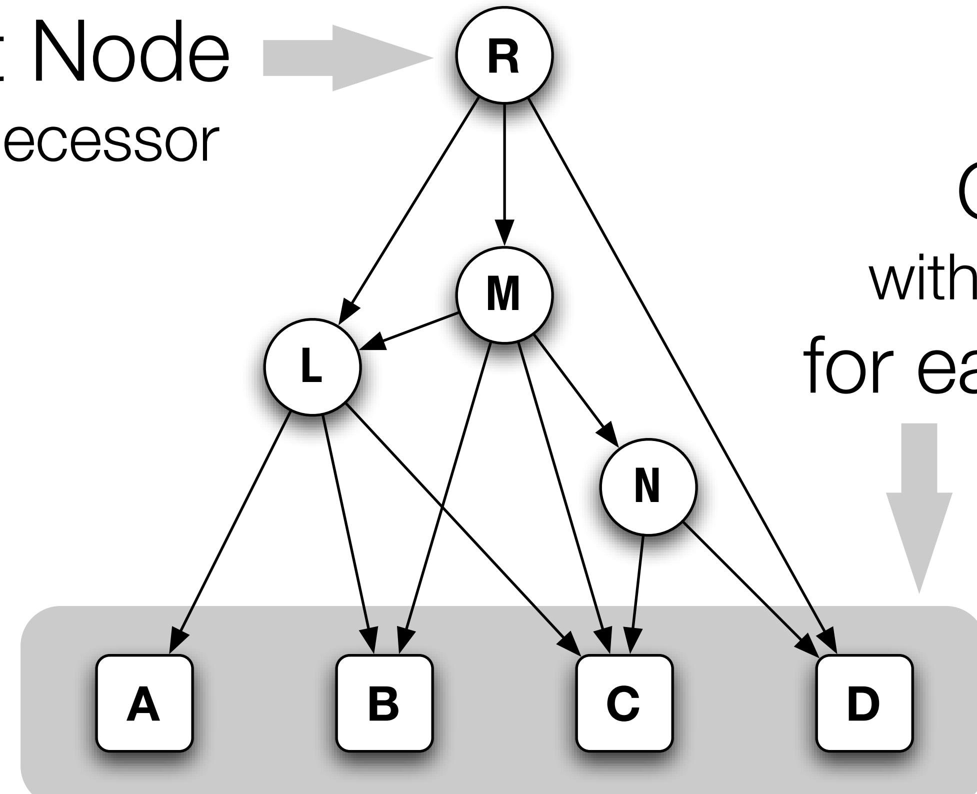
Network GEV to the rescue!



One ~~Ring~~ Model to Rule them All

One Root Node
with no predecessor

One Node
with no successors
for each Alternative



Directed | Connected | Finite | Circuit Free

Daly & Bierlaire, 2006

Generalized Extreme Value Models

There are four requirements for a GEV generating function:

1. $G(y) \geq 0$, for all y in $\mathbb{R}_+^J$

2. G is homogeneous of degree $\mu > 0$

3. $\lim_{y_i \to +\infty} G(y) = +\infty, \ \forall i \in \{1,2,\ldots,J\}$

4. The mixed partial derivatives of G with respect to elements of y exist, are continuous, and alternate in sign

Model Structure and Mathematics are Localized

$$G^i : \mathbb{R}_+^{\dim_{\mathbb{R}}(y)} \rightarrow \mathbb{R}_+ : G^i\left(y\right) = \left(\sum_{j \in i^{\downarrow}} \left[\left(a_{ij} G^j(y)\right)^{1/\mu_i}\right]\right)^{\mu_i}$$

$$G^c\left(y\right) = y$$

G^i depends only on:

- G^j of direct successor nodes
- a_{ij} allocation function on outbound edges
- μ_i logsum parameter of node i

Network GEV Rules

- Each network edge ij has an allocation parameter $a_{ij} > 0$

- Each network node i has a logsum parameter μ_i, and $\mu_i \leq \mu_j$ for all immediate predecessor nodes j

If the parameters obey these rules, then G for every node is an asymptotic GEV generating function, and G^R is a complete GEV generating function.

Normalization of Allocation

- GEV models are invariant to scale, and thus require normalization
- Allocation parameters also require normalization

$$G^i : \mathbb{R}_+^{\dim_{\mathbb{R}}(y)} \rightarrow \mathbb{R}_+ : G^i\left(y\right) = \left(\sum_{j \in i^{\downarrow}} \left[\left(a_{ij} G^j(y)\right)^{1/\mu_i}\right]\right)^{\mu_i}$$

Normalization of Allocation

- We don't want the model to artificially prefer one alternative over another as a result of the structure of the network alone.

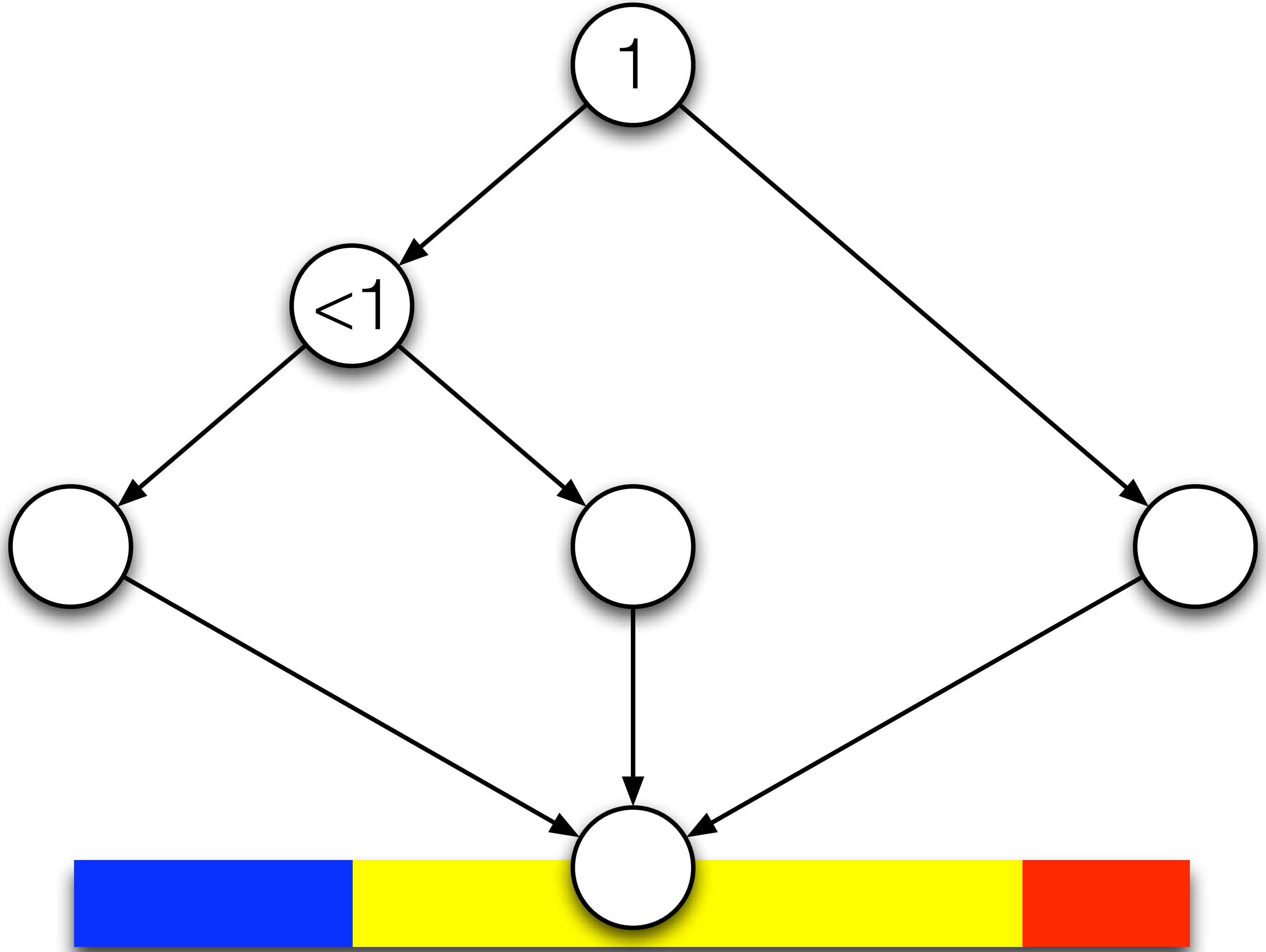
- Instead, we want:

$$\bar{U}_{\mathbf{i}} = V_{\mathbf{i}} + \bar{\varepsilon}_{\mathbf{i}} = V_{\mathbf{i}} + \kappa, \ \forall \mathbf{i}$$

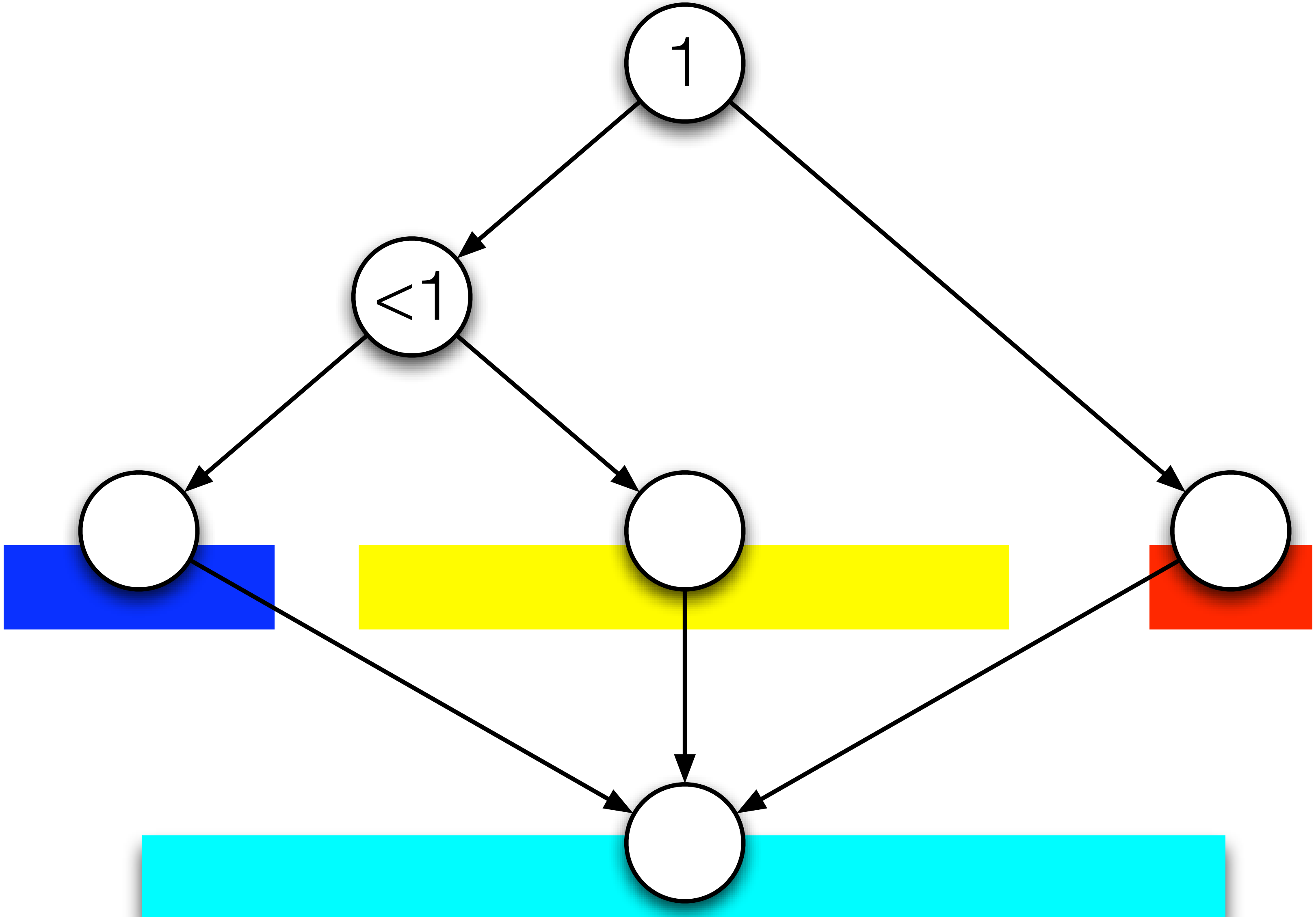
- Or, we can have a complete set of alternative specific constants, which conflate 'real' preference bias and model bias correction factors. It is difficult (or impossible?) to separately identify these two factors when the model is normalized in this manner.

- However, getting the unbiased model can be tricky...

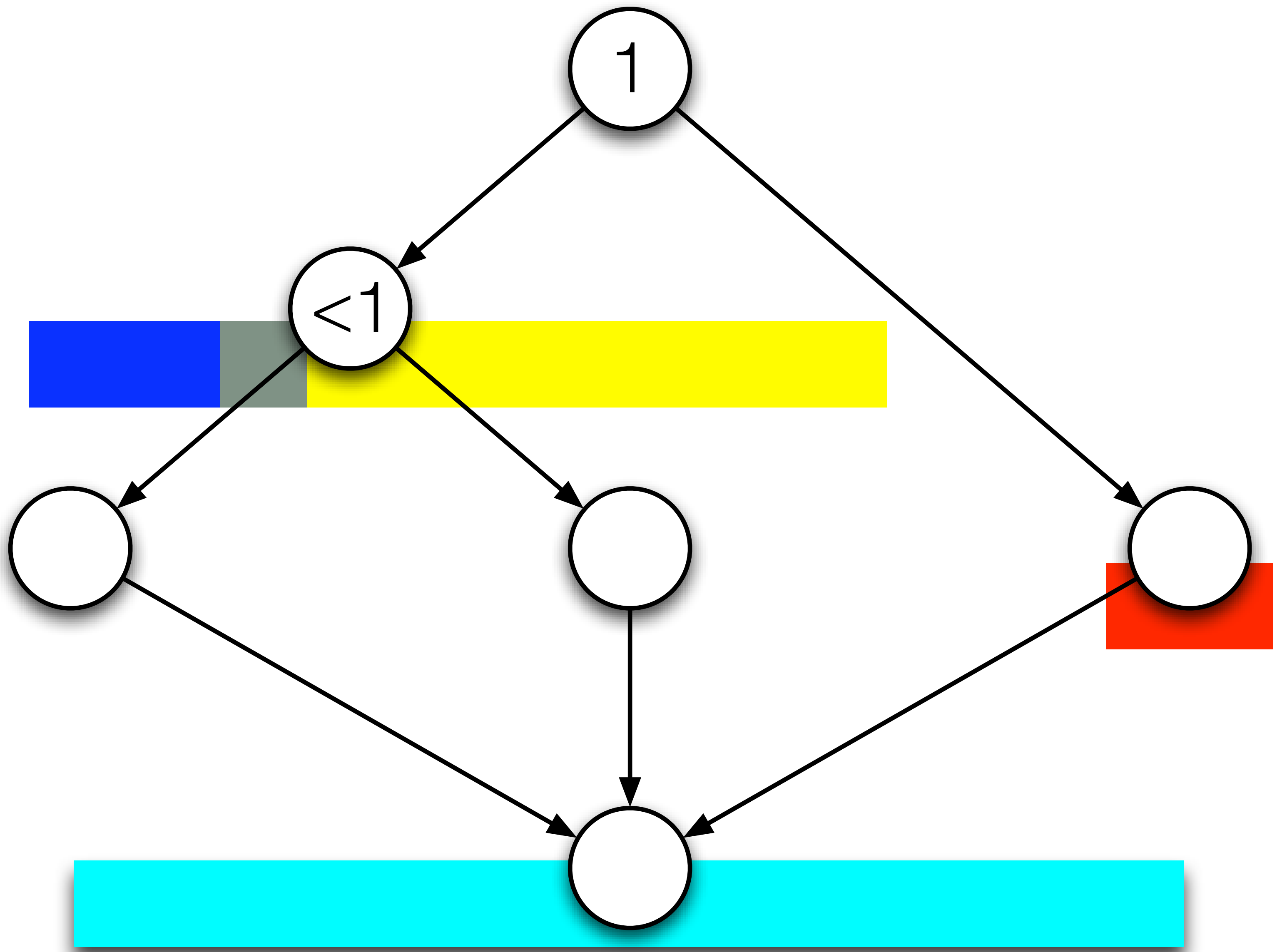
Error Recomposition Crash



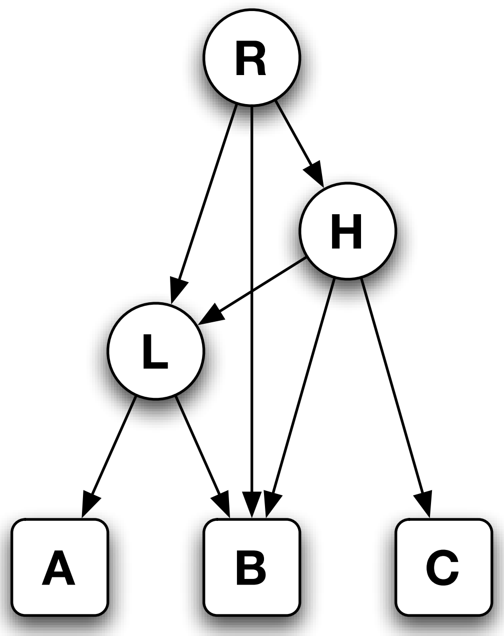
Error Recomposition Crash



Error Recomposition Crash

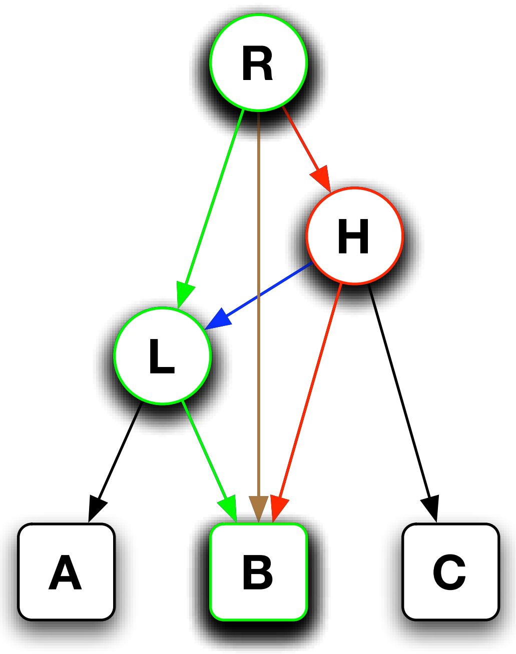


Crash Free Networks: Single Path Divergence

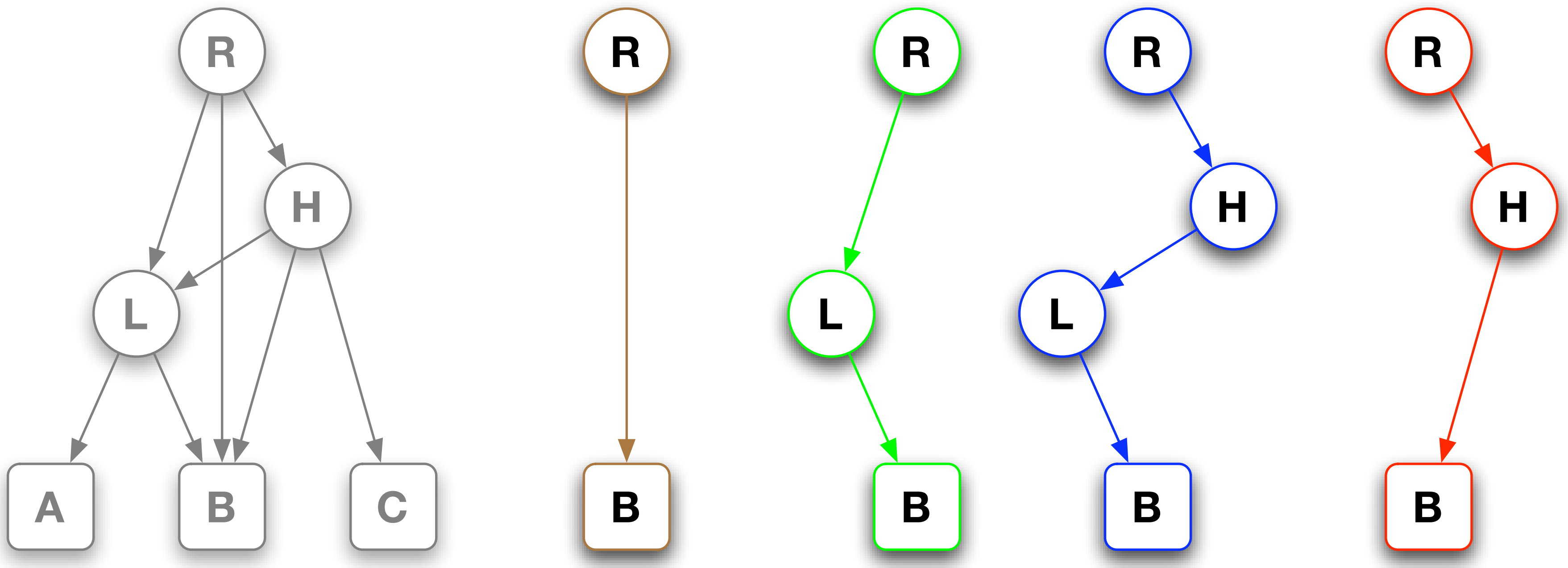


- All paths from root to any given elemental alternative must diverge at the root
- No two such paths may share an outbound edge from the root node
- If two such paths converge prior to their terminus, they may not subsequently diverge.
- Prevents the crash

Crash Free Networks



Crash Free Networks

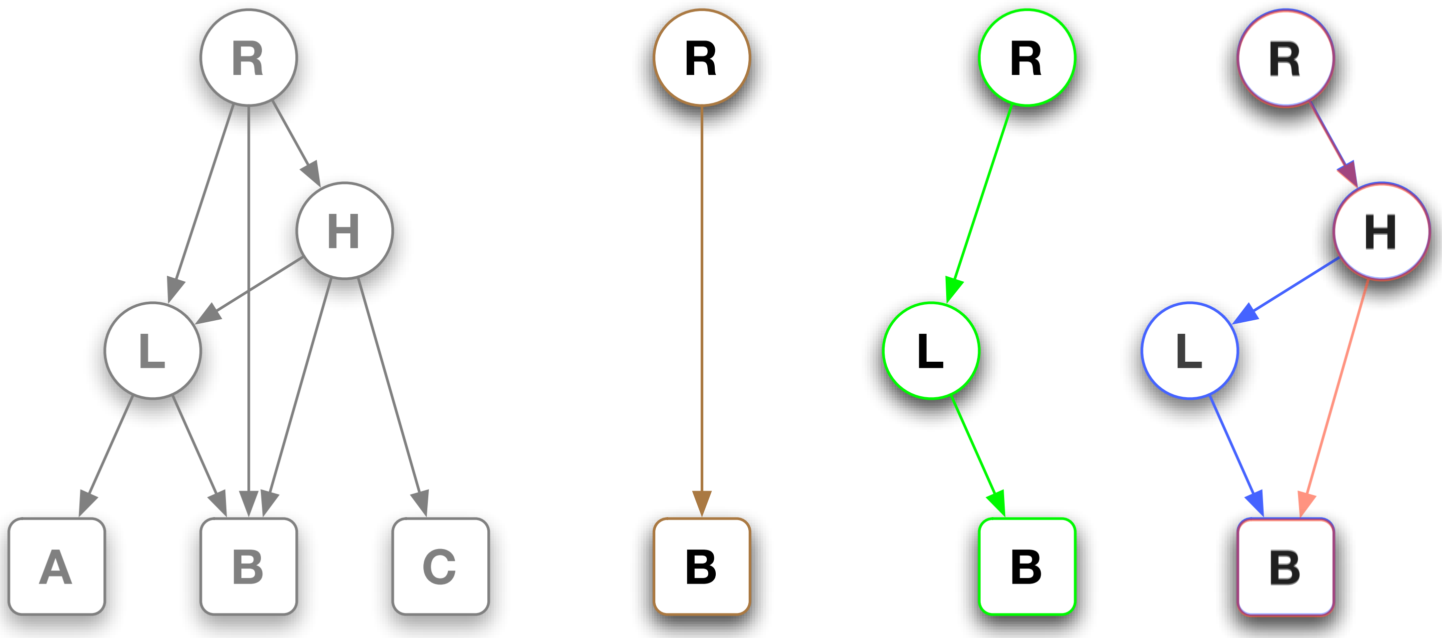


There are 4 paths from R to B

The red and blue paths share the link R→H

This network is *not* crash free

Crash Free Networks



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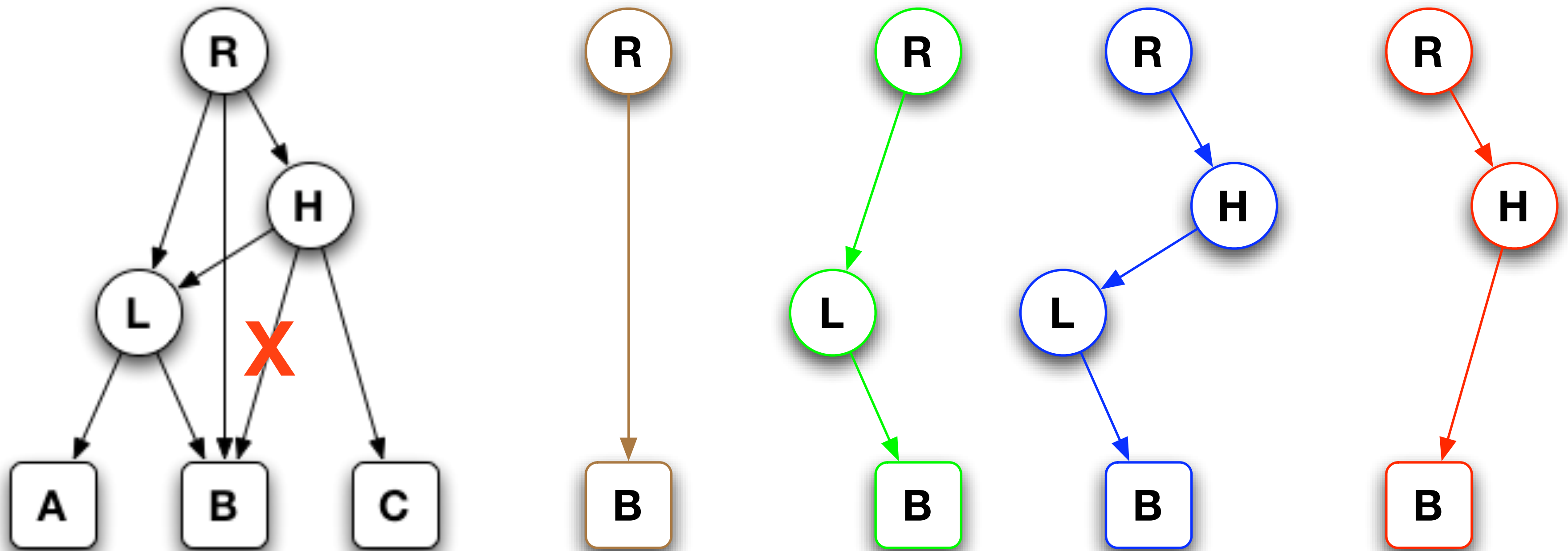
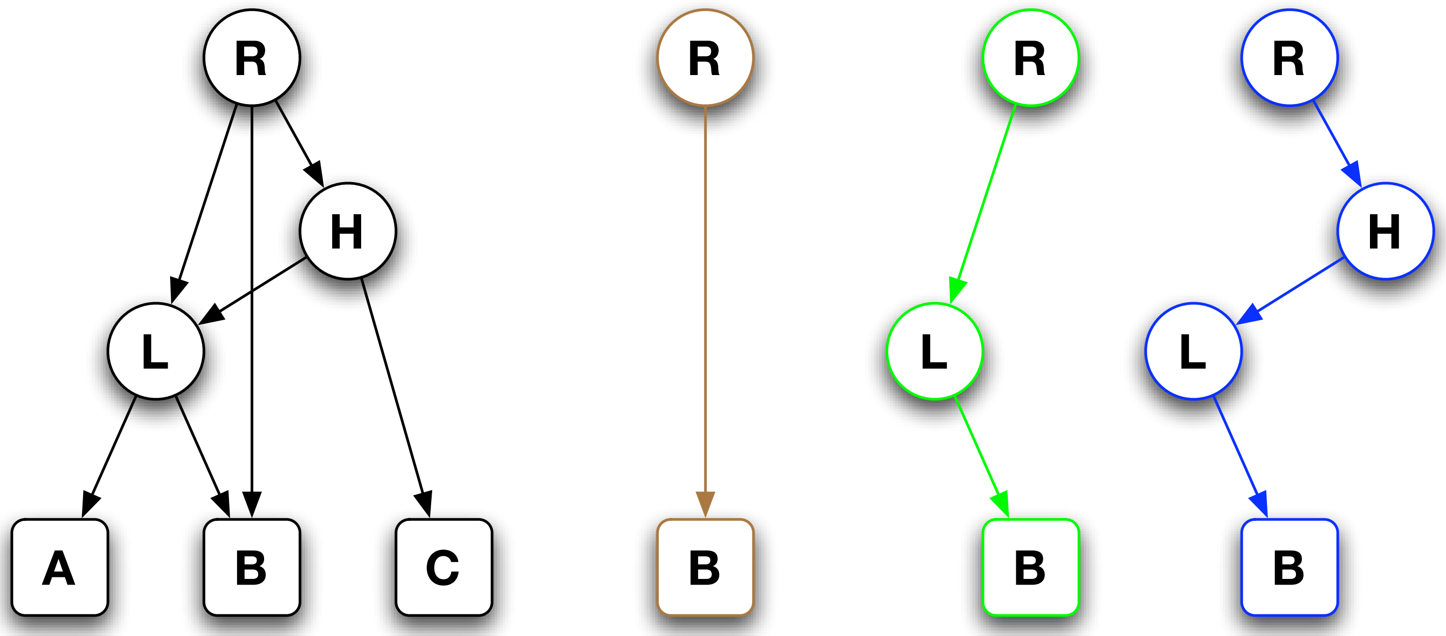
Crash Free Networks



There are 4 paths from R to B

Crash Free Networks



There are 3 paths from R to B

This network *is* crash free

Crash Free Networks

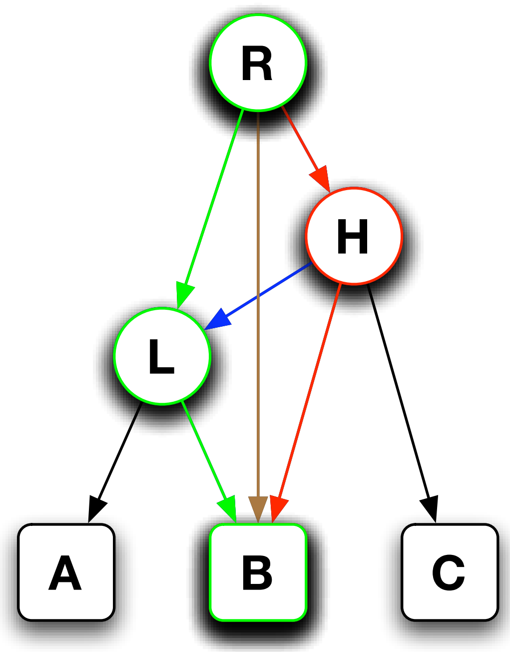
If Single Path Divergence applies, redefine the allocation parameters:

$$a_{ij} = \alpha_{ij}^{\mu_R}$$

and enforce

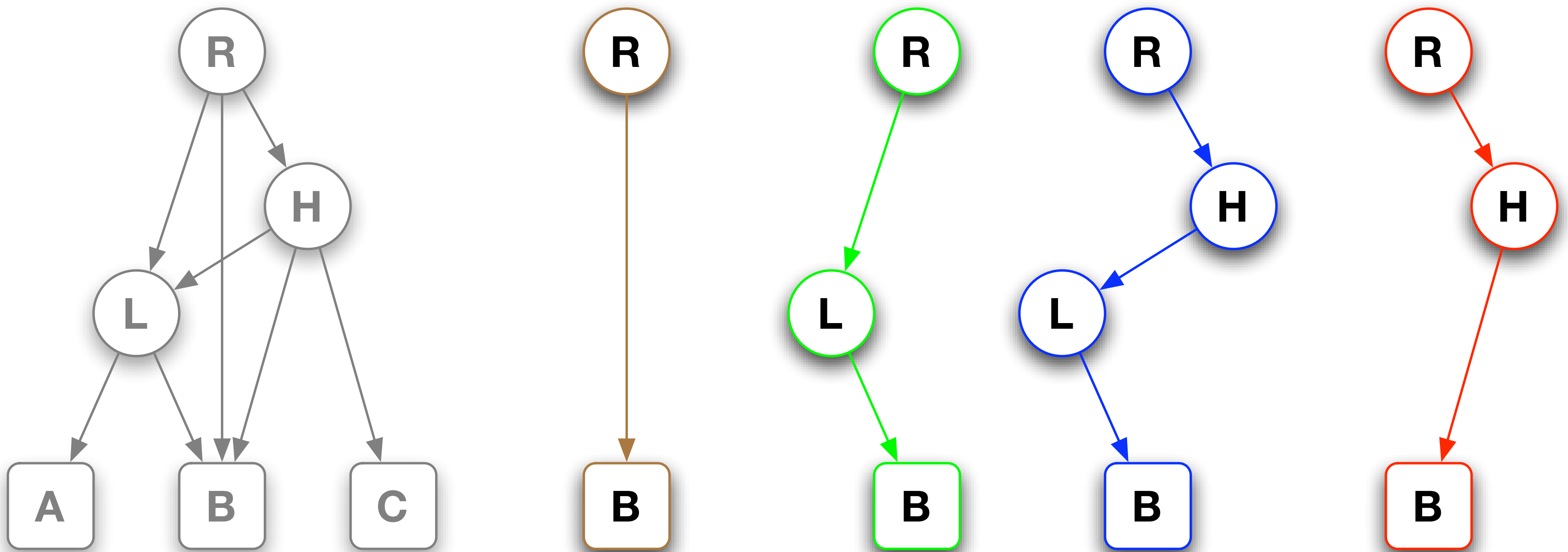
$$\sum_{i \in j^{\uparrow}} \alpha_{ij} = 1$$

Crash Safe Networks: Single Path Convergence



- All paths from root to any given elemental alternative must converge at the elemental alternative
- No two such paths may share an inbound edge to the elemental alternative
- If two such paths diverge subsequent from their origin, they may not subsequently converge until reaching the elemental alternative

Crash Safe Networks

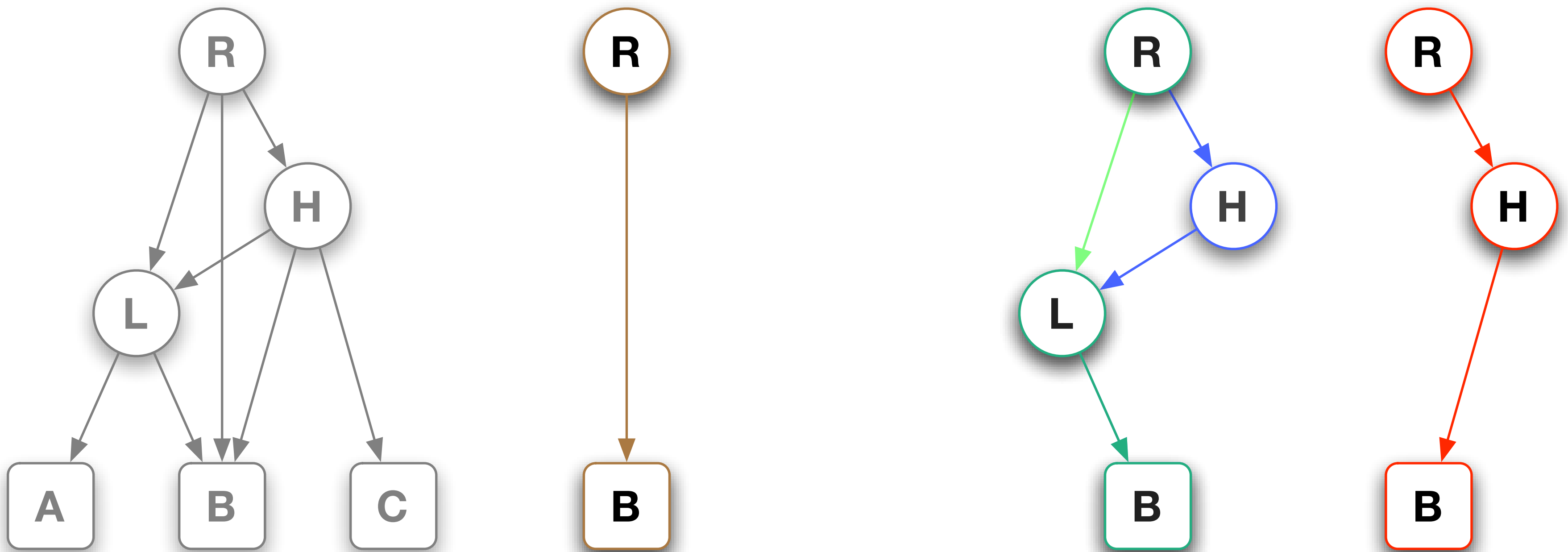


There are 4 paths from R to B

The blue and green paths share the link L→B

This network is *not* consistent with Single Path Convergence

Crash Safe Networks



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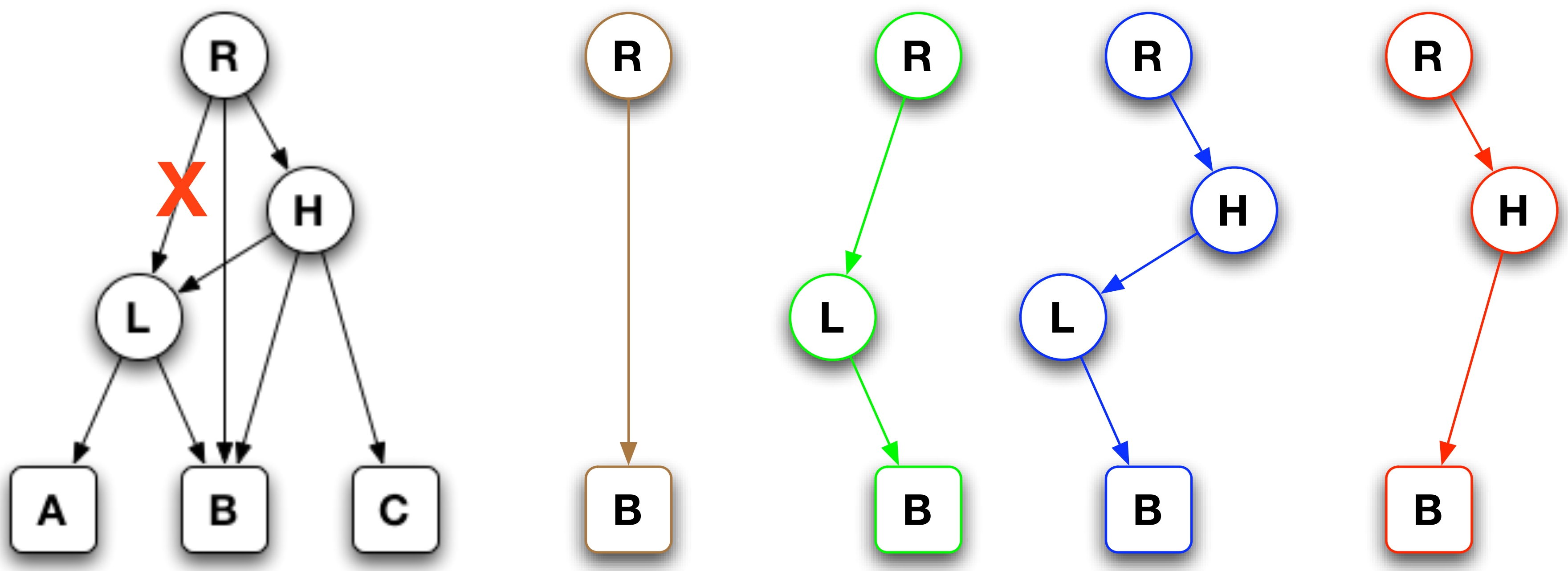
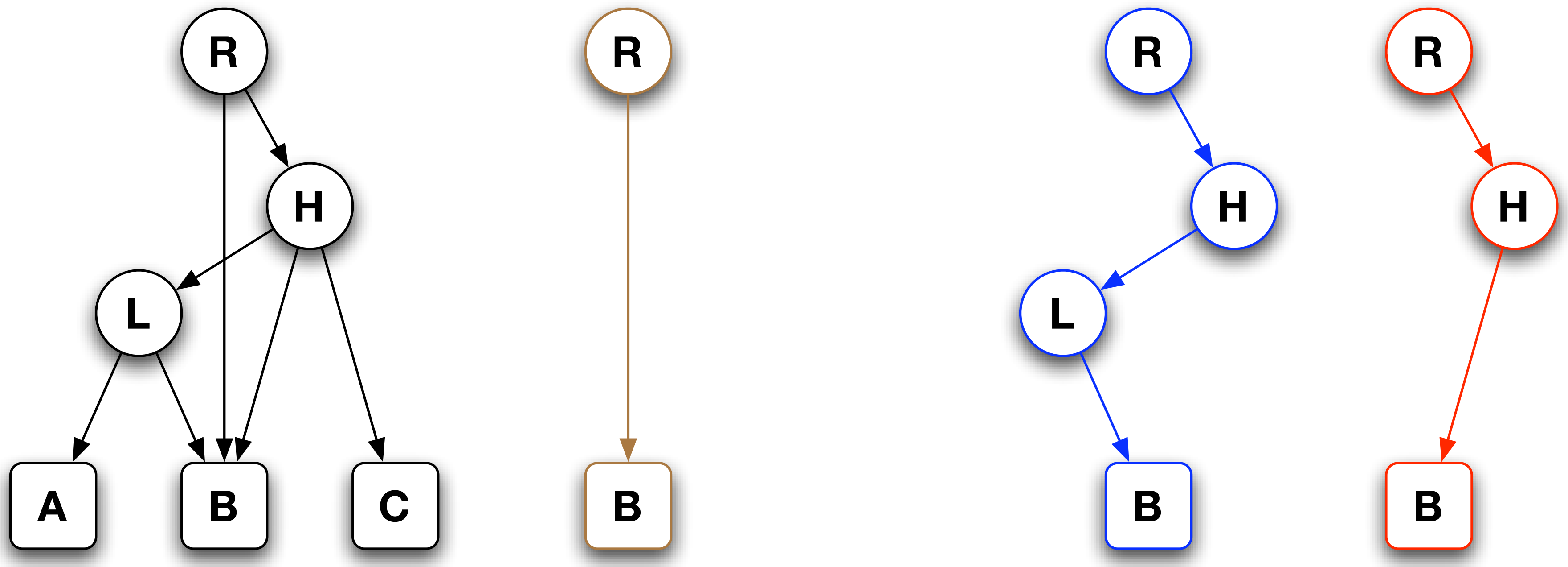
Crash Safe Networks



There are 4 paths from R to B

Crash Safe Networks



There are 3 paths from R to B

This network *is* consistent with Single Path Divergence

Crash Safe Networks

If Single Path Convergence applies, define each node's combined through-path allocation

$$\tilde{\alpha}_{ji} = \sum_{p \in \mathrm{T}(R,j,i)} \vec{\alpha}_{p_{Ri}}$$

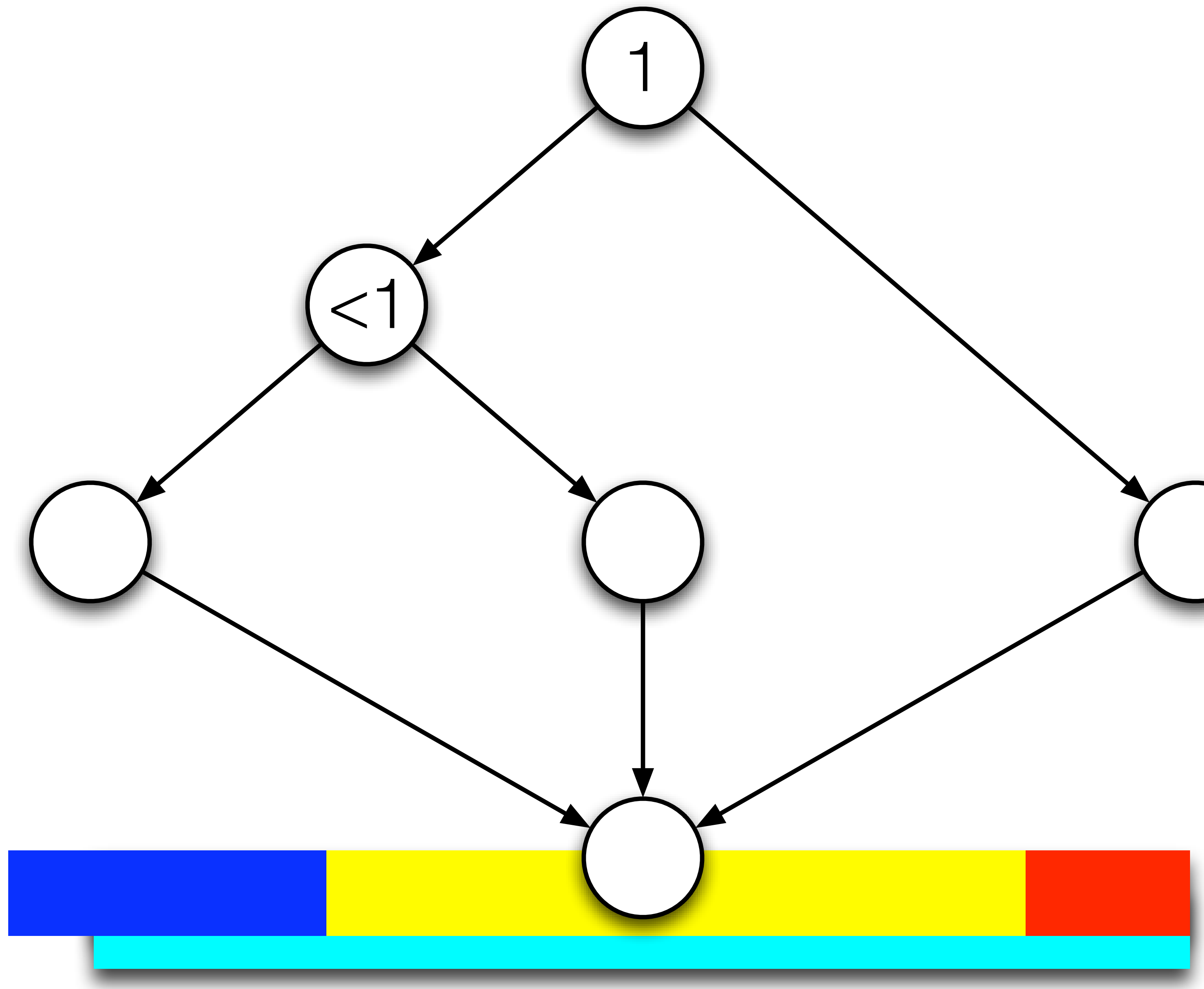
redefine the allocation parameters:

$$a_{ni} = \left(\frac{\alpha_{ni}}{\tilde{\alpha}_{ni}}\right)^{\mu_n} \left(\frac{\tilde{\alpha}_{ni}}{\tilde{\alpha}_{\dot{n}i}}\right)^{\mu_{\dot{n}}} \left(\frac{\tilde{\alpha}_{\dot{n}i}}{\tilde{\alpha}_{\ddot{n}i}}\right)^{\mu_{\ddot{n}}} \left(\frac{\tilde{\alpha}_{\ddot{n}i}}{\tilde{\alpha}_{\dddot{n}i}}\right)^{\mu_{\dddot{n}}} \cdots \left(\frac{\tilde{\alpha}_{\tilde{n}i}}{\tilde{\alpha}_{Ri}}\right)^{\mu_R}$$

and enforce

$$\sum\nolimits_{i \in j^{\uparrow}} \alpha_{ij} = 1$$

Crash Safe Networks: Crash Padding



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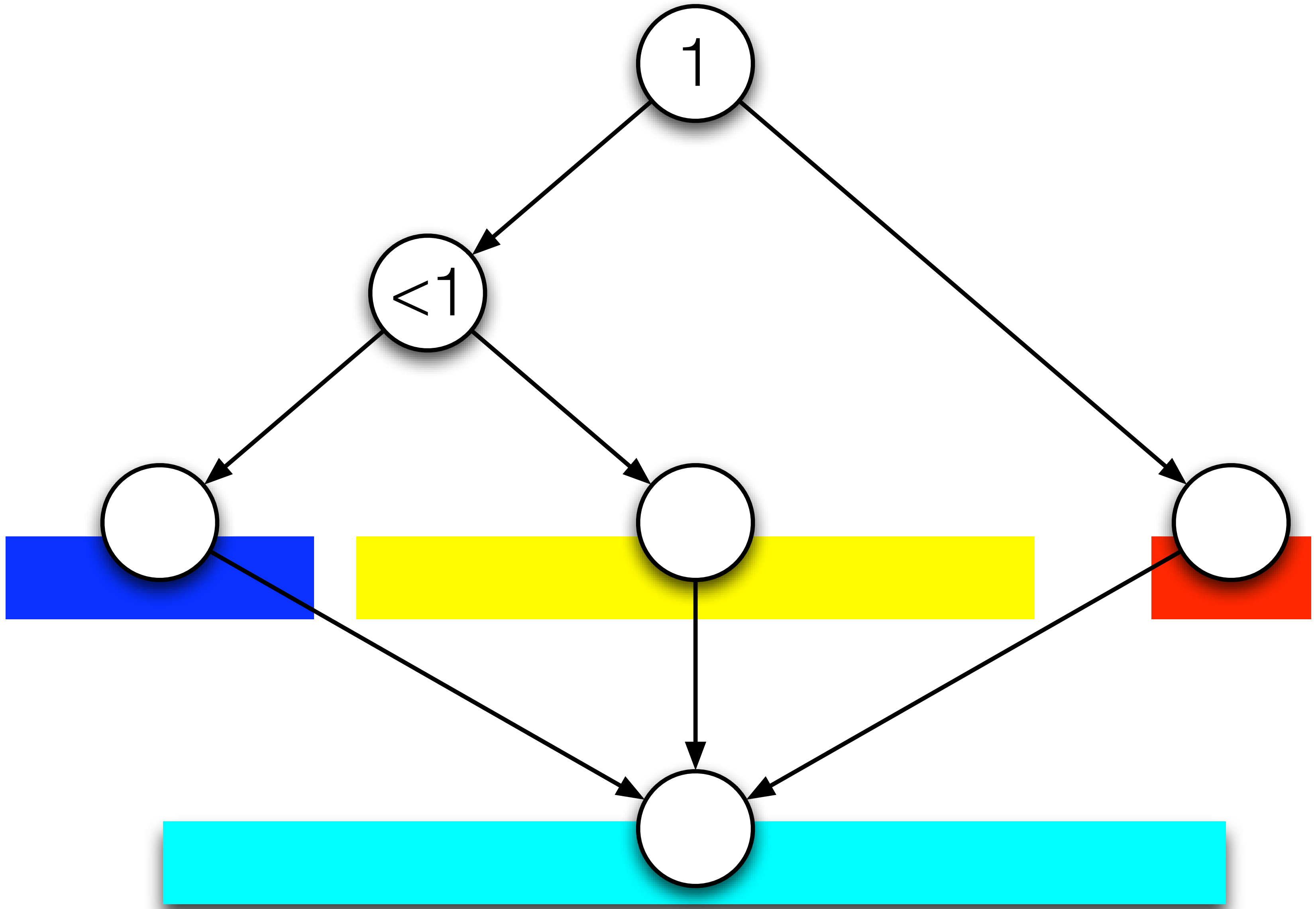
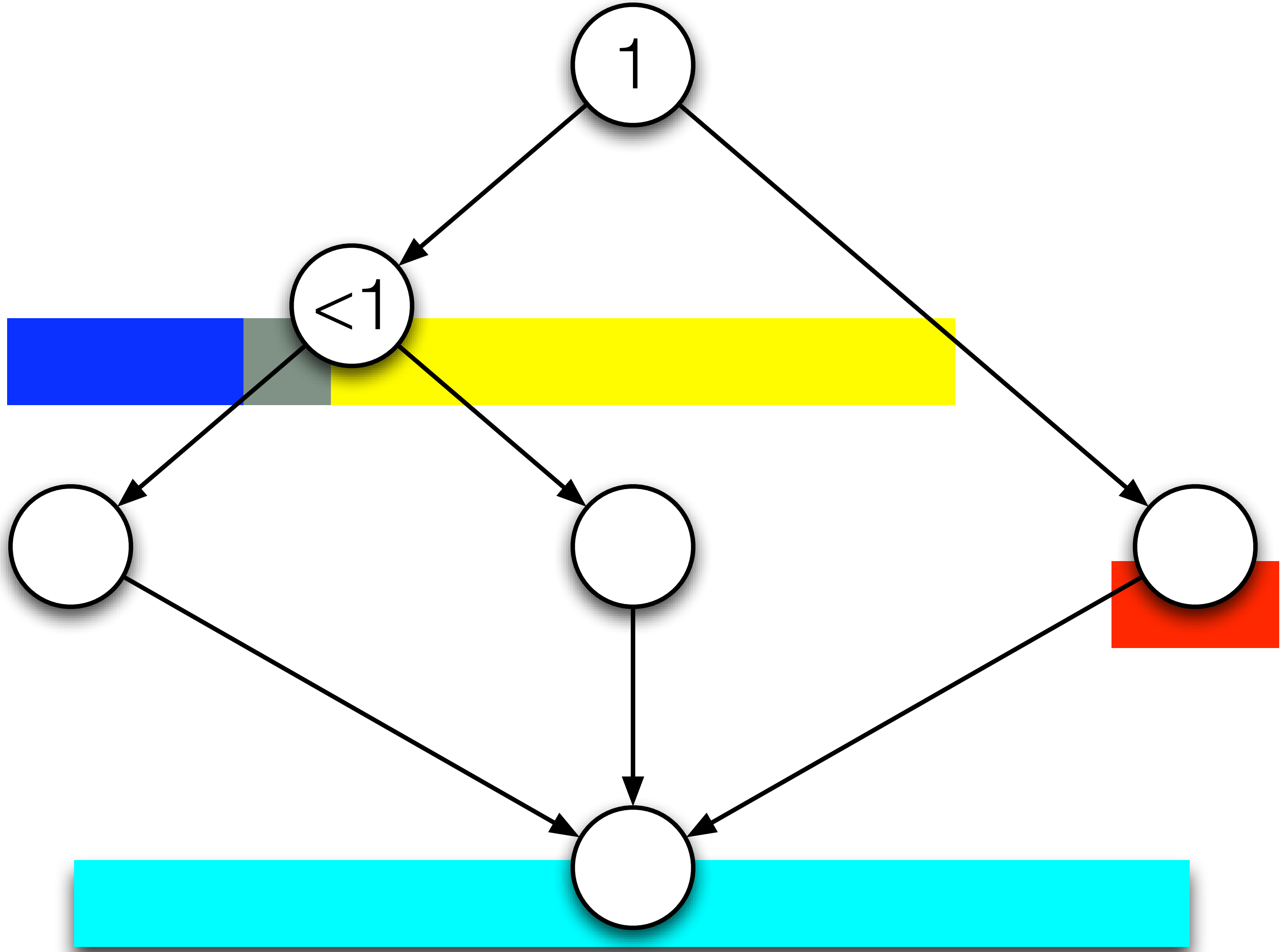
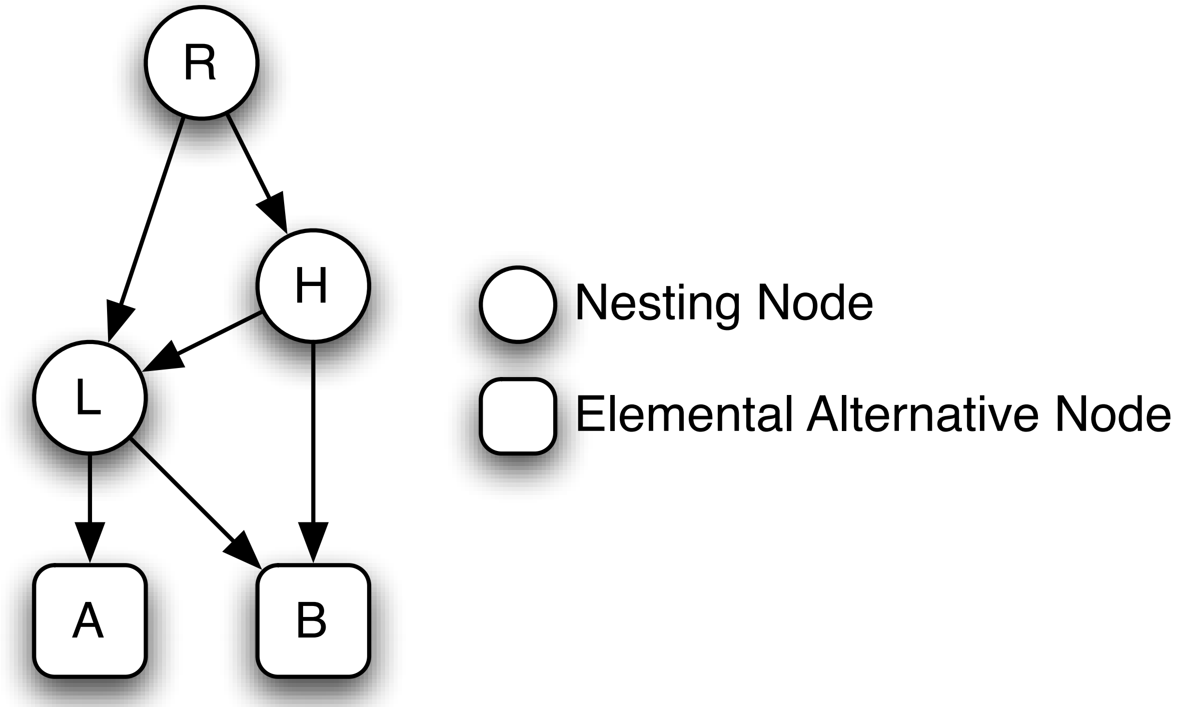


Crash Safe Networks: Crash Padding



A Simple Crash-Happy Network



Structural Parameters

$\mu_R = 1$

μ_L

μ_H

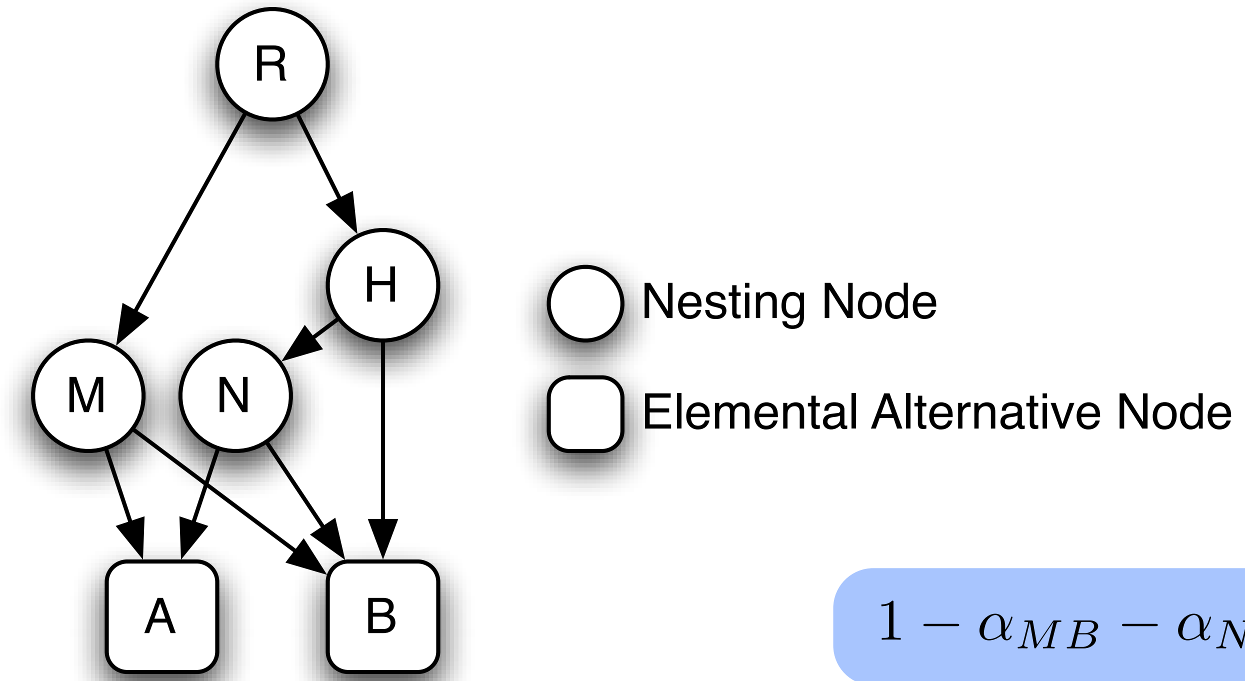
$\alpha_{LB} = 1 - \alpha_{HB}$

α_{HB}

$\alpha_{RL} = 1 - \alpha_{HL}$

α_{HL}

The Crash-Happy Network becomes Crash-Safe



Structural Parameters

$\mu_R = 1$

μ_M

$\mu_N = \mu_M$

μ_H

α_{MB}

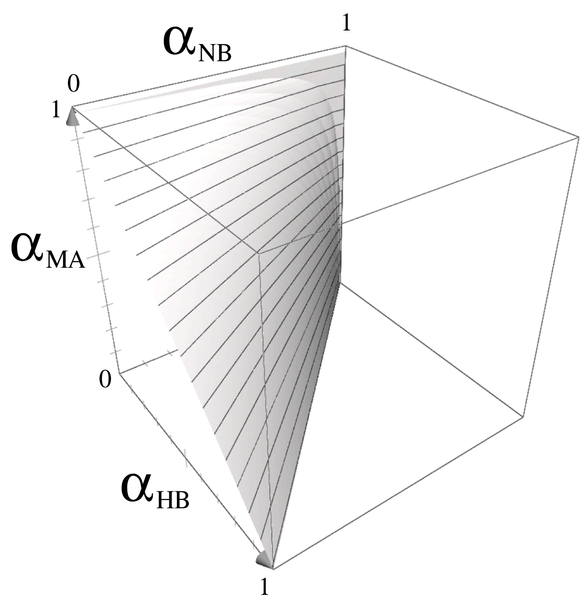
α_{NB}

$1 - \alpha_{MB} - \alpha_{NB} = \alpha_{HB}$

$\alpha_{NA} = 1 - \alpha_{MA}$

$$\left(\frac{(\alpha_{NB})^{\mu_H/\mu_R} (\alpha_{HB} + \alpha_{NB})^{1-(\mu_H/\mu_R)}}{1 - (\alpha_{HB} + \alpha_{NB})} + 1 \right)^{-1} = \alpha_{MA}$$

Oh my, grandma! What big non-linear constraints you have on your network allocation parameters!



$\frac{\mu_H}{\mu_R}=1.0$ $\frac{\mu_H}{\mu_R}=0.5$ $\frac{\mu_H}{\mu_R}=0.1$

Relaxing Parameter Constraints

$$\sum_{i \in j^{\uparrow}} \alpha_{ij} = 1$$

$$\alpha_{ij} = \frac{\exp\left(\phi_{ij}\right)}{\sum_{k \in j^{\uparrow}} \left[\exp\left(\phi_{kj}\right)\right]}$$

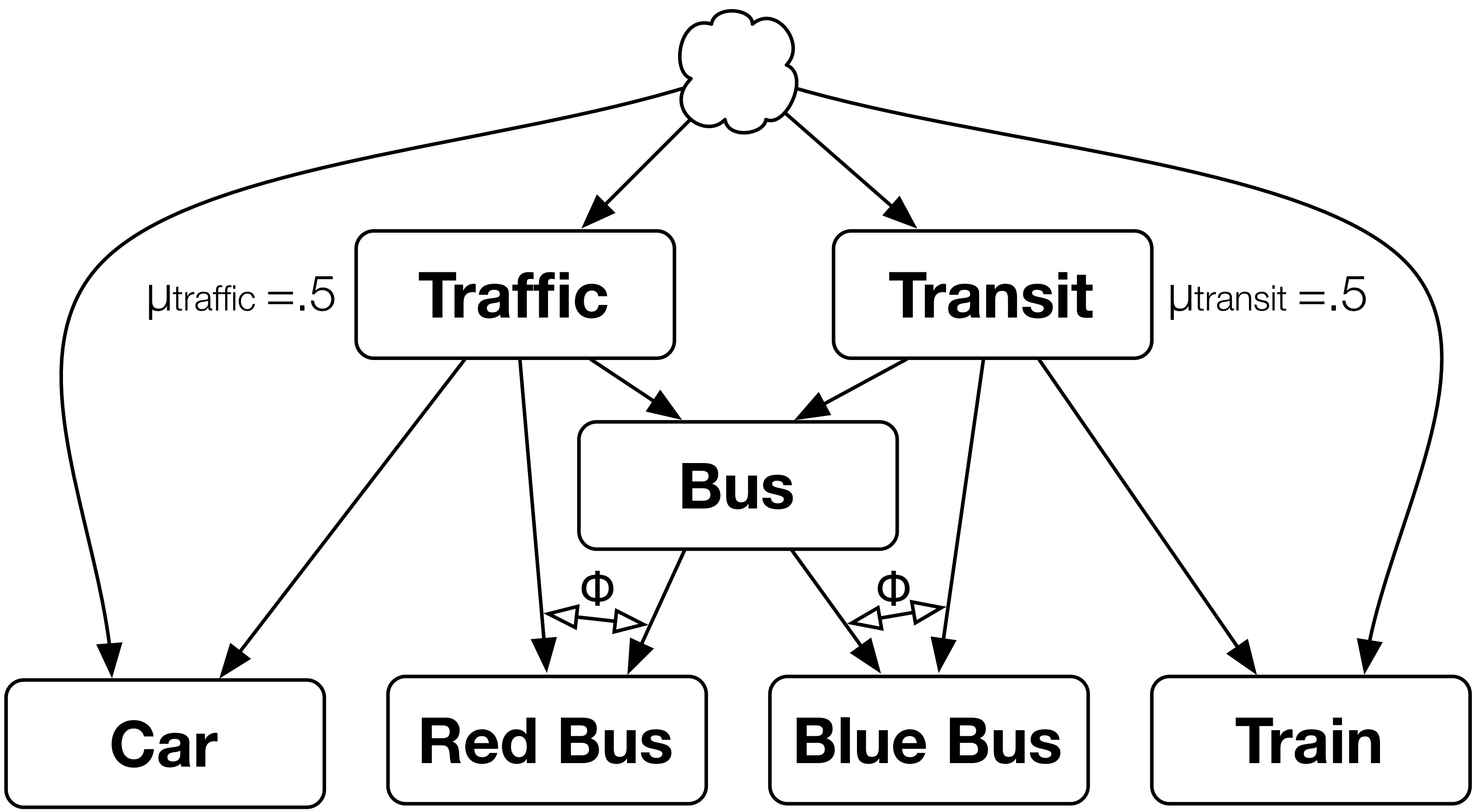
Relaxing Parameter Constraints

- Estimation of transformed phi parameters is simpler, but...
- Changes the shape of the distribution of estimators
- A non-allocation is represented by $\phi_{ij} = -\infty$ which is resistant to hypothesis testing

  The original form allowed $\alpha_{ij} = 0$ which facilitates hypothesis testing

$$\alpha_{ij} = \frac{\exp\left(\phi_{ij}\right)}{\sum_{k \in j^{\uparrow}} \left[\exp\left(\phi_{kj}\right)\right]}$$

What happens when some people care about color?



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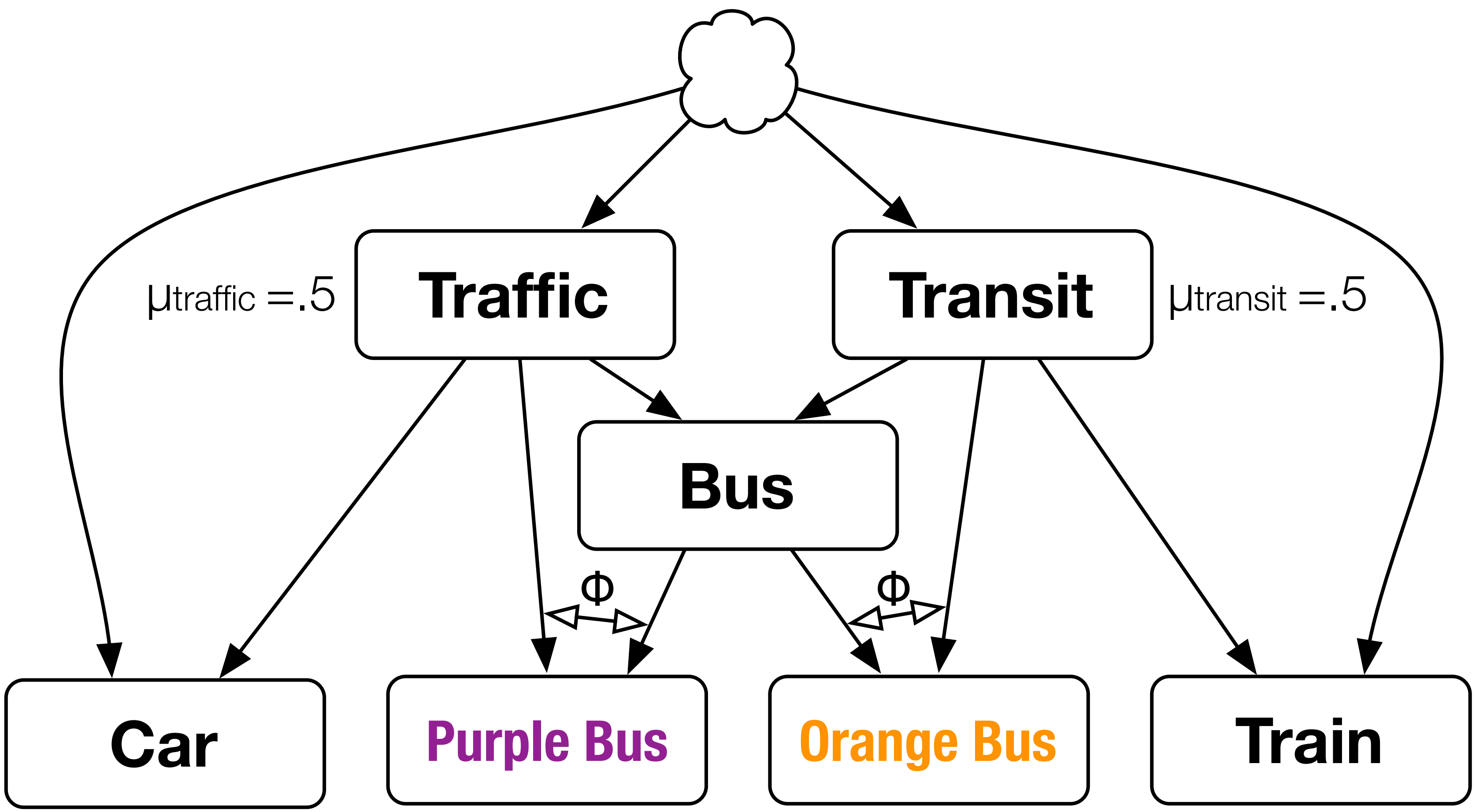


Structural Disaggregation

$$\alpha_{ij} = \frac{\exp\left(\phi_{ij}\right)}{\sum_{k \in j^{\uparrow}} \left[\exp\left(\phi_{kj}\right)\right]}$$

$$\alpha_{tij} = \frac{\exp\left(\phi^{*}_{ij} + \phi_{ij} Z_{t}\right)}{\sum_{k \in j^{\uparrow}} \left[\exp\left(\phi^{*}_{kj} + \phi_{kj} Z_{t}\right)\right]}$$

This allows decision-maker attributes to be incorporated into the structural form of the model

A New, More Basic Model?

- Nested Logit as a simplification of NetGEV:
  α is binary and fixed by the modeler
  μ is estimated

- What if we simplify the other way?
  μ is binary and fixed by the modeler
  α is estimated

At the limit, nests become deterministic blocks

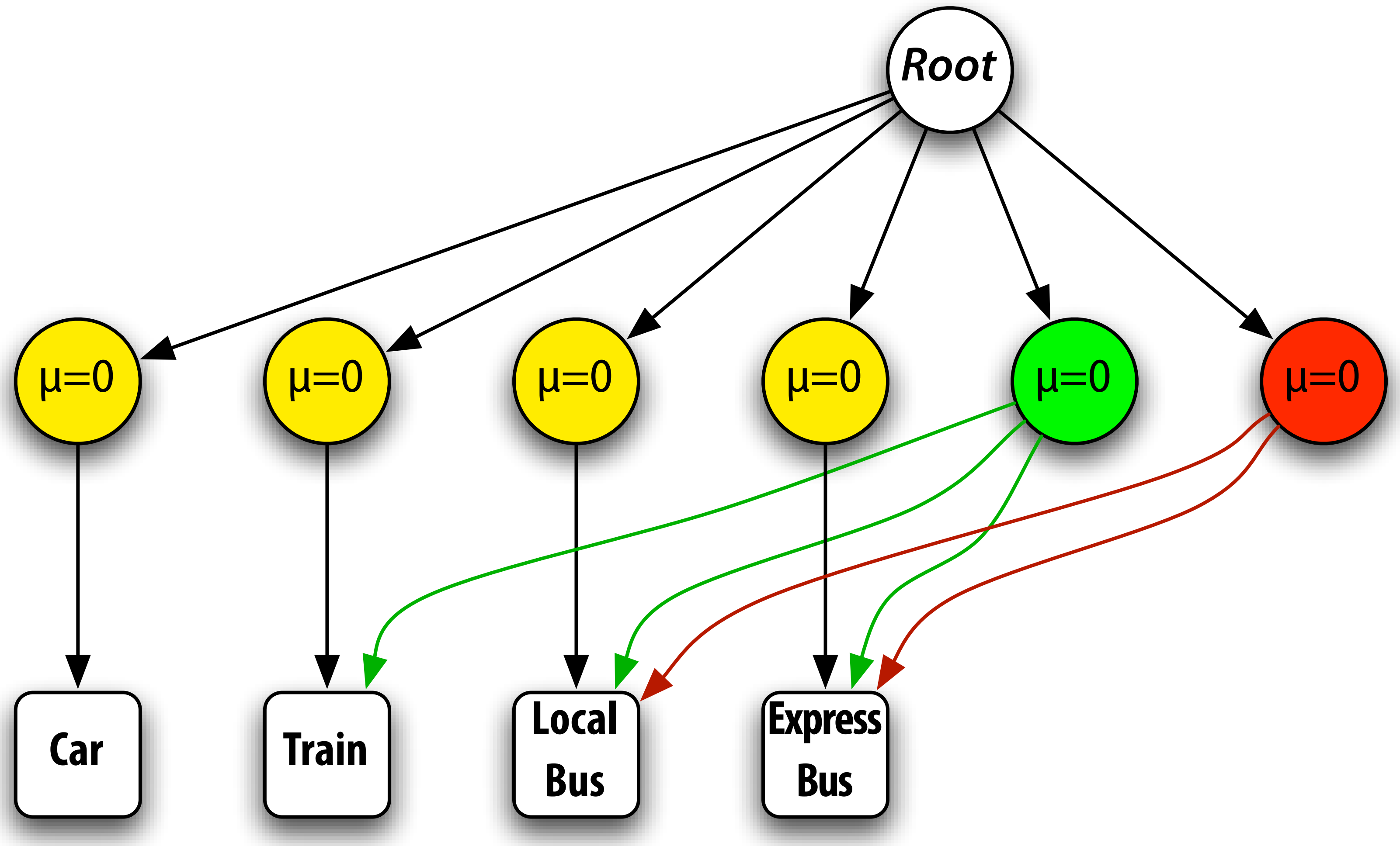
$$\lim_{\mu_i \to 0} \left[\left(\sum_{j \in i^{\downarrow}} \left[\left(a_{ij} G^j(y) \right)^{1/\mu_i} \right] \right)^{\mu_i} \right] = \max_{j \in i^{\downarrow}} \left\{ a_{ij} G^j(y) \right\}$$

Utilities can be expressed as the maximum among "utilitarian" building blocks

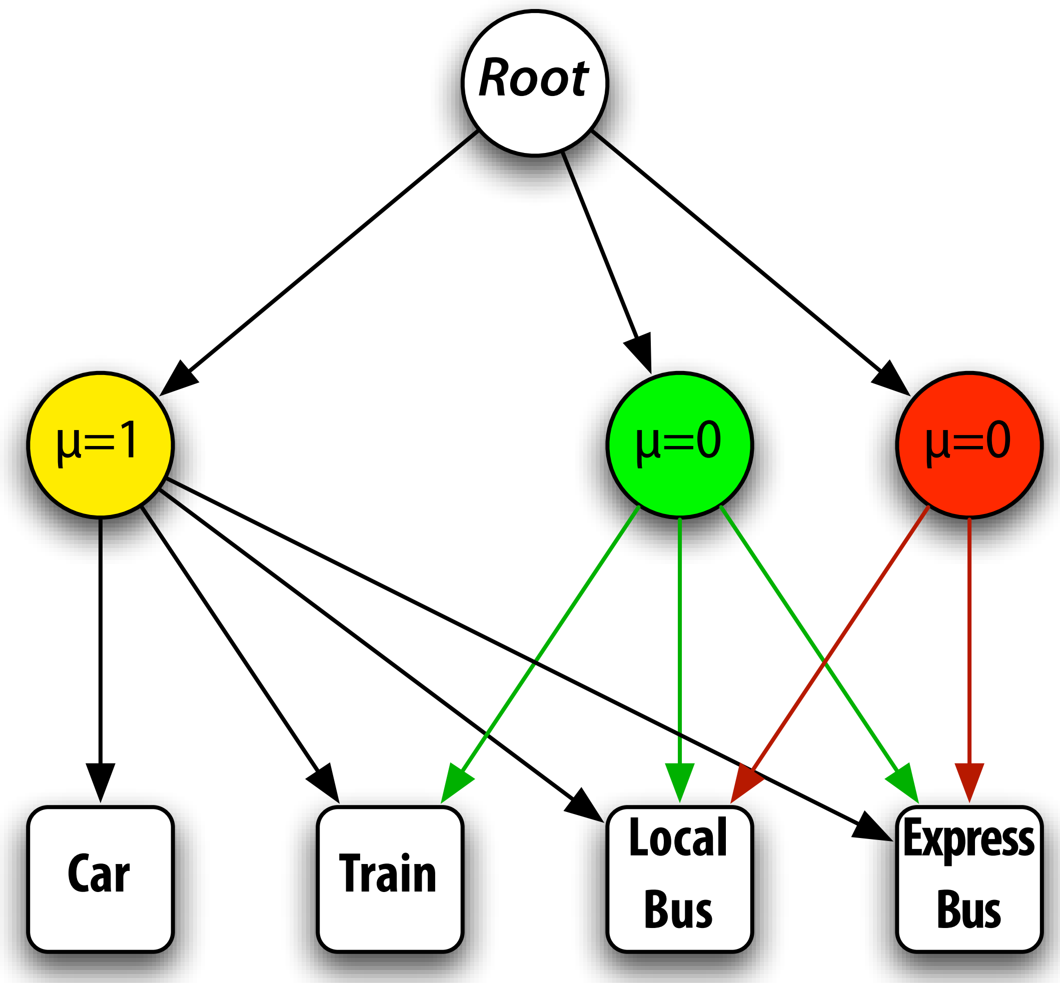
$$U_i = V_i + \max\left\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N, \varepsilon_i\right\}$$

- ε_k distributed Gumbel $(k^*, 1)$
- Individual ε_k appear in the utility functions for multiple alternatives, except for one unique error term for each alternative
- The network structure defines the set of epsilons for each alternative

Block Logit, Independent Blocking

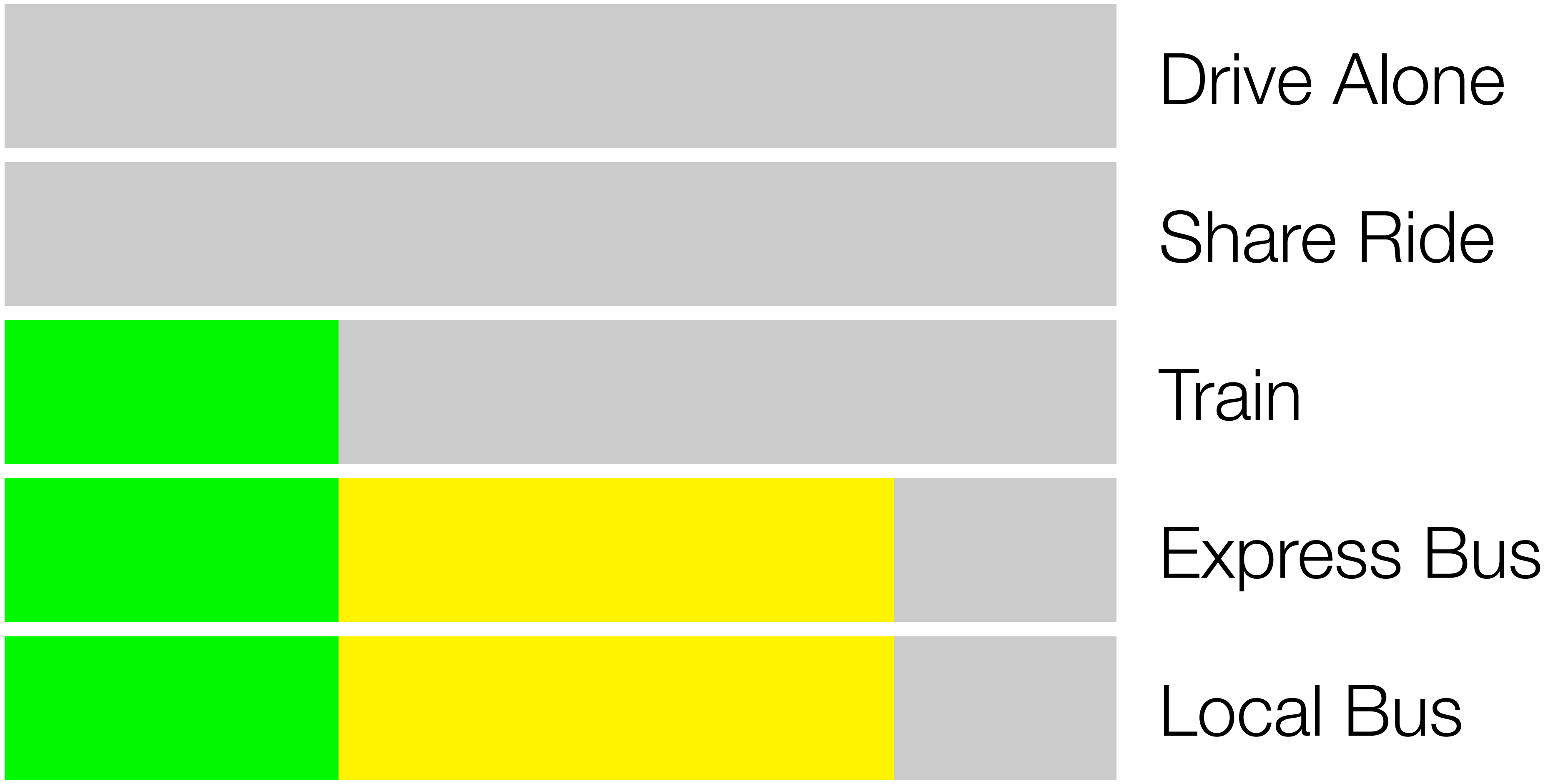


Block Logit, Competitive Blocking

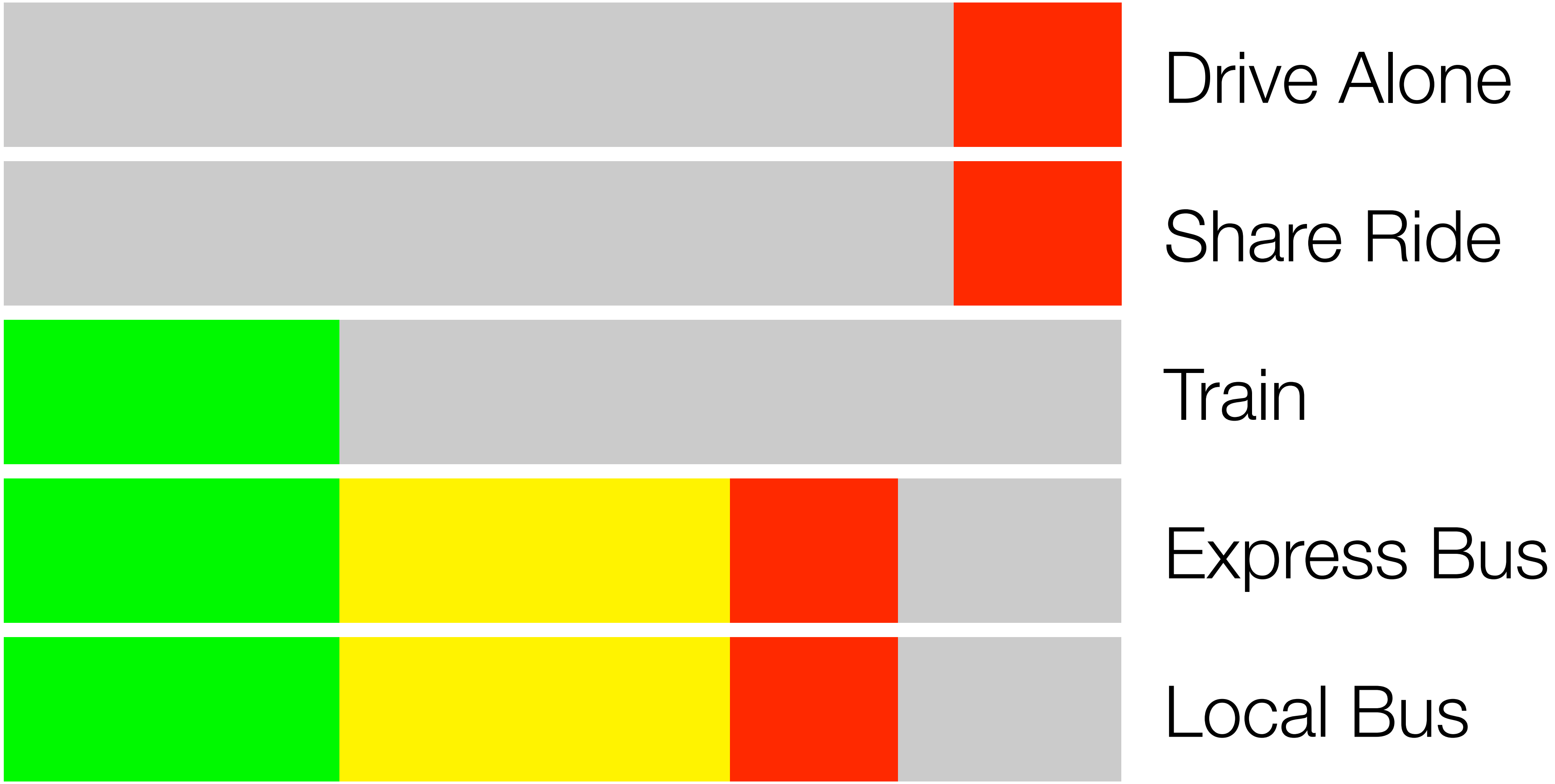


Blocking

Blocking



Blocking



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