Bid-auction framework for microsimulation of location choice with endogenous real estate prices

Ricardo Hurtubia
Michel Bierlaire
Francisco Martínez

Urbanics
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Outline

1) Motivation
2) The bid-auction approach to location choice modeling
3) Estimation of bid-rent functions
4) Bid-auction framework for microsimulation of location choice
Motivation – Land use models

- Spatial distribution of agents and activities in a city affects:
  - Travel demand
  - Energy consumption, pollution
  - Social welfare

- Cities are complex systems:
  - Interaction of different markets
  - Many heterogeneous agents
  - Externalities

- Land use models allow to understand and forecast (?) the evolution of cities

- Location choice models are a fundamental element of land use models

- Microsimulation / agent based models are flexible and detailed, making possible to evaluate complex scenarios
**Motivation – Approaches to location choice modeling**

- **Choice:** agents (households and firms) select location of maximum utility as price takers
  - Most usual implemented approach in microsimulation
  - Requires prices/rents to be given (usually modeled with a hedonic price model and/or exogenous adjustments)

- **Bid-auction:** real estate goods are traded in auctions where prices and locations are determined by the best bidders
  - Usually implemented in equilibrium models (bids are adjusted so everyone is located somewhere)
  - Prices are endogenous (expected maximum bid)
Motivation – Bid-auction advantages

- Real estate goods (housing, land) are quasi-unique and usually scarce → competition between agents
- Explicit explanation of the price formation process (best bid in an auction)
- Bid prices can be sensitive to scenarios of demand or supply surplus
- Estimation: no price endogeneity (spatial autocorrelation)

But:
- Estimates of bid function must reproduce both prices and location distribution
- Bid-auction is not straightforward to implement in microsimulation framework
- Detailed data is usually not available
Bid-auction approach to location choice

- $B_{hi}$: willingness to pay of agent $h$ for location $i$.

\[ B_{hi} = f(x_h, z_i, \beta) \]

- $x_h$: characteristics of agent $h$ (household, firm, ...)
- $z_i$: attributes of location $i$ (housing unit, parcel of land, ...)

- Probability of agent $h$ being the best bidder for a location $i$ (Ellickson, 1981):

\[ P_{h/i} = \frac{\exp(\mu B_{hi})}{\sum_{g \in H} \exp(\mu B_{gi})} \]

$H$: set of bidding agents
Bid-auction approach to location choice

- Price or rent for one location:
  - Deterministic: bid of the winner of the auction
  - Stochastic: expected maximum bid

- $r_i$: rent/price of $i = \text{expected value of the maximum bid}$:

$$r_i = \frac{1}{\mu} \ln \left( \sum_{g \in H} \exp(\mu B_{gi}) \right) + C$$

$H$: set of bidding agents
$C$: unknown constant
Estimation of bid-rent functions
Estimation of bid-rent functions

- Rosen (1974): Prices as a function of location attributes (hedonic rent model)
- Ellickson (1981): stochastic bid approach, undetermined model ➔ relative prices
- Lerman & Kern (1983): bid approach + observed price is maximum bid ➔ absolute prices
  - Very detailed data is required (individual transaction prices)
  - Assumption: groups of homogeneous bidding agents
  - Validation only regarding rent and marginal willingness to pay for location attributes, not agent location distribution or price forecasting
  (Gross, 1988; Gross et al 1990; Gin and Sonstelie, 1992; McMillen 1996; Chattopadhyay 1998; Muto, 2006)
Estimation of bid-rent functions

- **Idea:**
  - Assume structural relationship between expected outcome of the auction and observed (average) prices
  - Estimate location choice model and price model simultaneously, using observed prices as indicators

- **Assumptions:**
  - Auction price is a latent variable (the auction itself is a latent process)
  - All agents are potential bidders for all locations
Model with price indicator

Explanatory variables \((x_h, z_i)\) → Bid function \((B_{hi})\) → Observed locations (choices) → (latent) auction prices \((r_i)\) → Observed prices \((R_i)\) → Auction price measurement model

* Inspired by the Generalized Random Utility Model (Walker and Ben-Akiva, 2002)
Model with price indicator

- Structural equation for prices:

\[ r_i = \frac{1}{\mu} \ln \left( \sum_{g \in H} \exp(\mu B_{gi}) \right) \]

- Measurement equation for prices:

\[ R_i = a + \gamma \cdot r_i \]

\[ \sim N(0, \sigma) \Rightarrow f(R_i \mid r_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( -\frac{R_i - a - \gamma \cdot r_i}{2\sigma^2} \right) \]

- Likelihood:

\[ L = \prod_i \left( \prod_h (P_{h/i} \cdot f(R_i \mid r_i))^{y_{hi}} \right) \]
Case study: Brussels

- Data collected for a FP7 European Union project (SustainCity)
  - Census 2001 (aggregated information by zone)
  - Household survey 1999 (~1300 observations), no detail on housing attributes
  - Average transaction prices by commune and 2 types of dwelling (house or apartment) from 1985 to 2008
  - Other geographical, land use databases

- 1267997 households, 1274701 dwellings
- 157 communes
- 4975 zones
- 4 types of dwelling (with average attributes per zone)
  - Isolated house
  - Semi-isolated house
  - Joint house
  - Apartment
Case study: Brussels

Bid function specification for location (bid) choice model (Ellickson):

\[ B_{hvi} = \beta_{surf} \cdot surf_{vi} \cdot \ln(N_h) + \beta_{sup} \cdot Q_{i}^{sup} \cdot N_{h}^{sup} + \beta_{house} \cdot \lambda_{vi}^{house} \cdot N_{h} + \]
\[ \beta_{mid\_inc} \cdot I_{i} \cdot \gamma_{h}^{mid\_inc} + \beta_{high\_inc} \cdot I_{i} \cdot \gamma_{h}^{high\_inc} + \beta_{trans} \cdot \gamma_{i}^{trans} \cdot \gamma_{h}^{cars=0} + \]
\[ \beta_{trans2} \cdot \gamma_{i}^{trans} \cdot \gamma_{h}^{cars>1} + \beta_{comm} \cdot \gamma_{i}^{comm} \cdot \ln(N_h) + \beta_{off} \cdot \gamma_{i}^{off} \cdot W_h + \beta_{green} \cdot \gamma_{i}^{green} \cdot W_h \]

- \( surf_{vi} \) is the average surface of a residential unit in buildings type \( v \) in zone \( i \). The building types consider 3 types of house (fully-detached, semi-detached and attached) and apartments.
- \( N_h \) is the size (number of individuals) of a household.
- \( W_h \) is number of active individuals (workers) in a household
- \( N_{h}^{sup} \) is number of persons in the household who achieved a university degree as their maximum education level.
- \( Q_{i}^{sup} \) is percentage of the population in zone \( i \) with a superior level education-degree.
- \( I_{i} \) is the average income of zone \( i \) (calculated from tax declarations)
- \( \gamma_{i}^{trans} \) is a measurement of the quality of public transport for zone \( i \) (accessibility)
- \( \gamma_{i}^{comm}, \gamma_{i}^{off}, \gamma_{i}^{green} \) are measurement of the presence of commerce, offices and public green areas respectively.
Case study: Brussels

Table 1: Estimation results for Brussels

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Logit</th>
<th>Logit with price indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Std err</td>
</tr>
<tr>
<td>( \beta_{surf} )</td>
<td>0.00636</td>
<td>0.00261</td>
</tr>
<tr>
<td>( \beta_{mid_inc} )</td>
<td>0.0439</td>
<td>0.0111</td>
</tr>
<tr>
<td>( \beta_{high_inc} )</td>
<td>0.0574</td>
<td>0.0153</td>
</tr>
<tr>
<td>( \beta_{sup} )</td>
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<td>0.108</td>
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</tr>
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</tr>
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<td>( \beta_{green} )</td>
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<td>( \beta_{off} )</td>
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Final Log-Likelihood: -7011.03 (-7091.13**)
Likelihood ratio-test: 232.44 (1478.97 (72.23**)

*parameters not significant at the 95% level
** log-likelihood considering only the choice probabilities

Estimation performed with PythonBiogeme (Bierlaire and Fetiarison, 2010)
Case study: Brussels

Table 2: Estimation results for Brussels

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<tr>
<td>$\mu$</td>
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Final Log-Likelihood: -7011.03 (-7569.645 (-11813.1**))
Likelihood ratio-test: 232.44 (1478.97 (72.23**))

*parameters not significant at the 95% level
** log-likelihood considering only the choice probabilities

Estimation performed with PythonBiogeme (Bierlaire and Fetiarison, 2010)
Case study: Brussels

- Prices per commune and type (% error) (over estimation dataset)
Case study: Brussels

- **Prices (over estimation dataset)**
Case study: Brussels

- Prices (over estimation dataset)
Case study: Brussels

- Prices (over estimation dataset)
Case study: Brussels (forecasting/validation)

- Prices per commune and type (% error) (over full supply for 2001)
Case study: Brussels (forecasting/validation)

- Number of people per commune (% error)
Case study: Brussels (forecasting/validation)

- Number of people with univ degree per commune (% error)
Case study: Brussels (forecasting/validation)

- Number of households with 2+ cars (% error)
Case study: Brussels (forecasting/validation)

- Number of households with 0 cars (% error)
Discussion

- The proposed estimation method finds estimates that reproduce the location distribution of agents and the average market prices of dwellings better than other methods.

- Proposed method requires less detailed data ➔ more suitable for extensive land use models.

- Well estimated bid functions (willingness to pay) allow to generate a good forecast of the transaction prices, without the need of hedonic price models ➔ this helps if we want to microsimulate using a bid approach.
Bid-auction framework for microsimulation of location choice
Microsimulation with a bid approach

- When bids are simulated and we get:
  - Spatial distribution of agents
  - Real estate prices

- But, in order to account for competition between agents for scarce goods, we need market clearing
  - Through hedonic price models (UrbanSim)
    - Simple but not real market clearing
  - Individual auctions (ILUTE)
    - Expensive in computational terms
  - Equilibrium (MUSSA)
    - Aggregated approach
The market clearing problem

Joint probability of household $h$ occupying location $i$:

$$P(i,h) = P(i|h)P(h) = P(h|i)P(i)$$

- $P(h|i)$ Maximum bid probability
- $P(i|h)$ Maximum surplus (utility) probability
- $P(i)$ Selling probability
- $P(h)$ Locating probability
Re-visiting Equilibrium

- In equilibrium models it’s usually assumed that supply ($S$) equals demand ($H$)
  \[ P(h) = P(i) = 1 \quad \forall h, i \quad \Rightarrow H = S \]

- Possible equilibrium conditions:
  \[
  \sum_h P(i, h) \Rightarrow \sum_h P(i \mid h)P(h) = P(i) = 1 \quad \forall i \quad \text{(everything is sold)}
  \]
  \[
  \sum_i P(i, h) \Rightarrow \sum_i P(h \mid i)P(i) = P(h) = 1 \quad \forall h \quad \text{(everyone is located)}
  \]
Re-visiting Equilibrium

- Market clearing can be achieved by imposing one of the equilibrium conditions and finding prices/bids that produce them

\[ \exists r_i : \sum_h P(i \mid h) = 1 \quad \forall i \]  
(prices clear the market)

\[ \exists b_h : \sum_i P(h \mid i) = 1 \quad \forall h \]  
(bids clear the market)

Due to interdependence, these are usually fixed point problems
Re-visiting Equilibrium

- If we have an auction market and the best bidder rule is observed, adjusting prices or bids is equivalent in equilibrium.
- When market conditions change (supply, demand, etc) utility levels of the decision makers have to be adjusted, this is reflected in the level of the prices or bids.

➔ idea: quasi-equilibrium
Quasi-equilibrium

- Periodical location of new and re-locating agents, given exogenous supply
- Assumption: all households looking for a location are located somewhere $P(h) = 1 \ \forall h$
  - Total supply must be greater or equal than total demand $\Rightarrow H \leq S$
  - Not all locations are necessarily used $P(i) \leq 1 \ \forall i$
Quasi-equilibrium

- No equilibrium ➔
  - no perfect information (aggregate supply, previous prices)
  - No iterative negotiation/bidding
  - No absolute adjustment of bids/prices
- Instead, adjustment of “perception” of agents that goes in the direction of an equilibrium but does not solve it.
Quasi-equilibrium

- Algorithm (in each period):
  - All agents \( (H) \) observe the market: prices and supply \( \left( r_{i}^{t-1}, z_{i}^{t-1}, S_{i} \right) \)
  - All gents (simultaneously) adjust their bids, attempting to make their expected number of winning auctions equal to one:
    \[
    \sum_{i \in S} q(h | i) = 1 \quad \forall h
    \]
    \( q(h|i) \): perceived probability of being the best bidder for \( i \)
  - All agents bid at the same time for all locations \( \Rightarrow \) prices and location distributions are defined
  - The assignment mechanism is an auction \( \Rightarrow \) for each location a best bidder and a price is determined
Quasi-equilibrium

Bid function: \( B_{hi} = I_h - U_h + V_h(z_i) = V_h(z_i) - b_h \)

- Perceived probability:
  \[
  q(h | i) = \frac{\exp(V_h(z_i^t) - b_h^t)}{\sum_{g \in H} \exp(B_{gi}^t)} \approx \exp(V_h(z_i^t) - b_h^t - r_i^{t-1})
  \]

\[
\sum_{i \in S} q(h | i) = 1 \Rightarrow \hat{b}_h^t = \ln\left(\sum_{i \in S} \exp\left(V_h(z_i^t) - r_i^{t-1}\right)\right)
\]

Advantage: no fixed point, just evaluation of equation \( \Rightarrow \) it is possible to apply to large populations without excessive computational cost
General framework

- Re-location models
- Located agents
- Real estate prices
- New agents
- Supply model
- New real estate
- Re-locating agents, vacated real estate
- Market clearing
- Transport model

Externalities, market conditions (prices, demand/supply surplus, etc) Given for t=0

Travel times, congestion, level of service

t=t+1
Market clearing

Externalities, prices and market conditions (t-1)

Demographics(t) → Adjustment of utility level (bₙ) → Auction

Supply (t) → Empty units → Relocation

Re-calculation of hedonic WP (Vₙ) → Transaction prices (Rᵢ)

Simulation of location choice

Location probability distribution (Pₜᵢ) → Located individual agents and prices

t=t+1

New and Relocating agents

Supply (t)
Some preliminary results

- **Average prices per year**

  Average price growth: BID: 50%, HEDONIC: 7%
Observed average prices per commune

Average price growth: 108%
Advantages

- Agents have an individual behavior but they relate to a “higher level” market mechanism through the utility level adjustment and the simultaneous auction.
- Quasi-equilibrium:
  - Demand is not cleared: utility adjustment does NOT assure allocation
  - Supply is not cleared
  - System tends to equilibrium but does not clear
- Adjustment of utility levels instead of prices allow to
  - Explain price formation (no need for hedonic price models)
  - Detect all agents utility levels, including those not active in the market, triggering future re-location
Thank you
Main assumptions of the general framework

- Auction market
- Agents adjust their utility level (individually in each period)
  - to ensure location (ex-ante expectations)
  - given market conditions: previous period rents, current supply
- Time lag:
  - In production of real estate goods:
  - In perception of attributes of locations (non-instantaneous)
- Simultaneous (macro level) bid of all agents for all locations
  - Location (best bidder) distributions and expected rents (Ri).
  - No iterative transactions.
  - Computationally simpler than transaction-specific price clearing
- Microsimulation:
  - Actual allocation following macro distributions (simulation of auctions)
  - Rents at micro level (ri)