Bid-auction framework for microsimulation of location choice with endogenous real estate prices

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Outline

1) Motivation
2) The bid-auction approach to location choice modeling
3) Estimation of bid-rent functions
4) Bid-auction framework for microsimulation of location choice

Motivation – Land use models

- Spatial distribution of agents and activities in a city affects:
  - Travel demand
  - Energy consumption, pollution
  - Social welfare
- Cities are complex systems:
  - Interaction of different markets
  - Many heterogeneous agents
  - Externalities
- Land use models allow to understand and forecast (?) the evolution of cities
- Location choice models are a fundamental element of land use models
- Microsimulation / agent based models are flexible and detailed, making possible to evaluate complex scenarios

Motivation – Approaches to location choice modeling

- **Choice:** agents (households and firms) select location of maximum utility as price takers
  - Most usual implemented approach in microsimulation
  - Requires prices/rents to be given (usually modeled with a hedonic price model and/or exogenous adjustments)
- **Bid-auction:** real estate goods are traded in auctions where prices and locations are determined by the best bidders
  - Usually implemented in equilibrium models (bids are adjusted so everyone is located somewhere)
  - Prices are endogenous (expected maximum bid)

Motivation – Bid-auction advantages

- Real estate goods (housing, land) are quasi-unique and usually scarce ➔ competition between agents
- Explicit explanation of the price formation process (best bid in an auction)
- Bid prices can be sensitive to scenarios of demand or supply surplus
- Estimation: no price endogeneity (spatial autocorrelation)
- But:
  - Estimates of bid function must reproduce both prices and location distribution
  - Bid-auction is not straightforward to implement in microsimulation framework
  - Detailed data is usually not available

Bid-auction approach to location choice

- B_{hi} : willingness to pay of agent *h* for location *i*.

$$B_{hi} = f(x_h, z_i, \beta)$$

x_h : characteristics of agent h (household, firm, …)
z_i : attributes of location i (housing unit, parcel of land, …)

- Probability of agent *h* being the best bidder for a location *i* (Ellickson, 1981):

$$P_{h/i} = \frac{\exp(\mu B_{hi})}{\sum_{g \in H} \exp(\mu B_{gi})}$$

H: set of bidding agents

Bid-auction approach to location choice

- Price or rent for one location:
  - Deterministic: bid of the winner of the auction
  - Stochastic: expected maximum bid

- r_i : rent/price of i = expected value of the maximum bid:

$$r_i = \frac{1}{\mu} \ln\left(\sum_{g \in H} \exp(\mu B_{gi}) \right) + C$$

H: set of bidding agents
C: unknown constant

Estimation of bid-rent functions

Estimation of bid-rent functions

- Rosen (1974): Prices as a function of location attributes (hedonic rent model)
- Ellickson (1981): stochastic bid approach, undetermined model ➔ relative prices
- Lerman & Kern (1983): bid approach + observed price is maximum bid ➔absolute prices
  - Very detailed data is required (individual transaction prices)
  - Assumption: groups of homogeneous bidding agents
  - Validation only regarding rent and marginal willingness to pay for location attributes, not agent location distribution or price forecasting

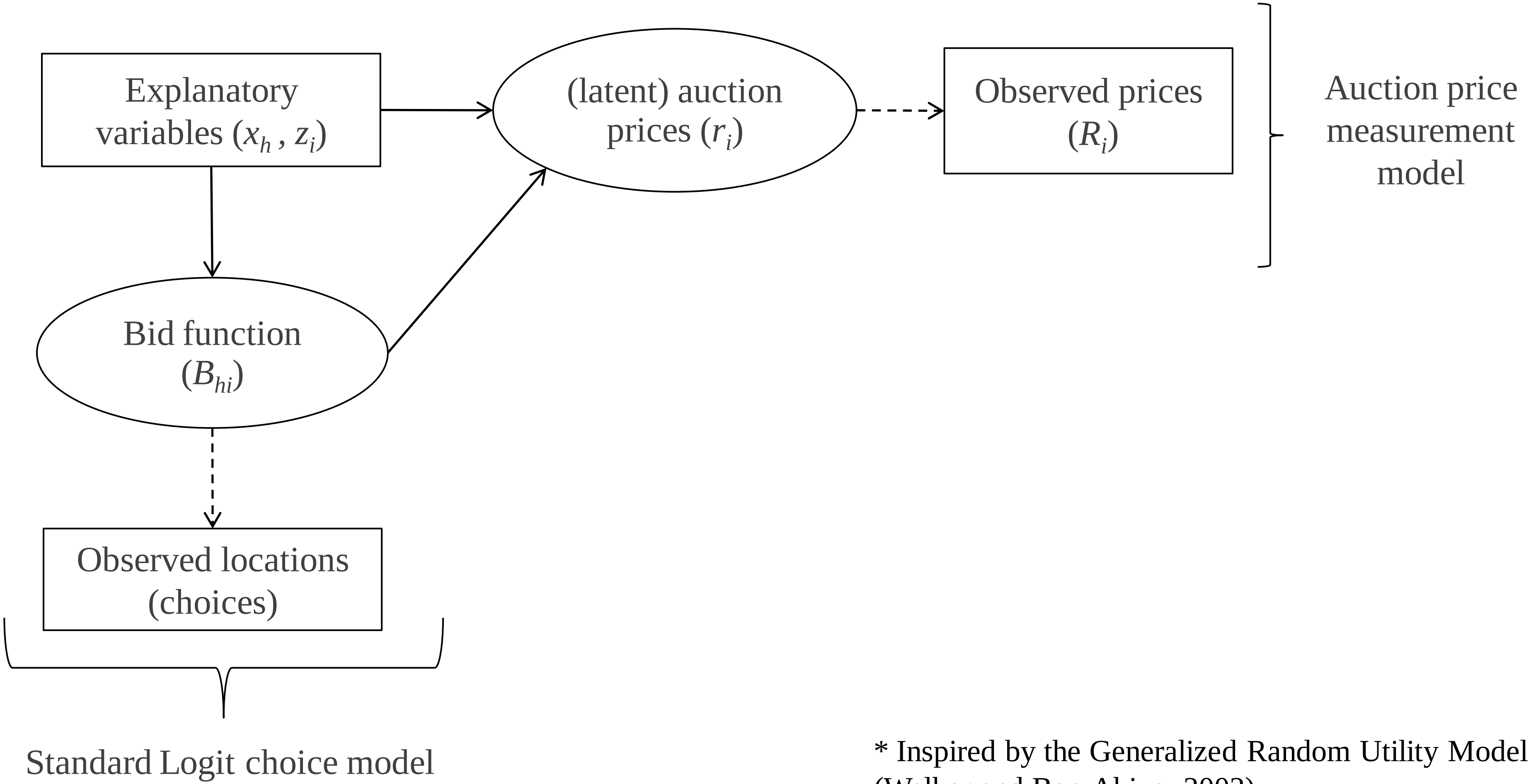
    (Gross, 1988; Gross et al 1990; Gin and Sonstelie, 1992; McMillen 1996; Chattopadhyay 1998; Muto, 2006)

Estimation of bid-rent functions

- Idea:
  - Assume structural relationship between expected outcome of the auction and observed (average) prices
  - Estimate location choice model and price model simultaneously, using observed prices as indicators

- Assumptions:
  - Auction price is a latent variable (the auction itself is a latent process)
  - All agents are potential bidders for all locations

Model with price indicator



* Inspired by the Generalized Random Utility Model (Walker and Ben-Akiva, 2002)

Model with price indicator

- Structural equation for prices:

$$r_i = \frac{1}{\mu} \ln\left(\sum_{g \in H} \exp(\mu B_{gi}) \right)$$

- Measurement equation for prices:

$$R_i = a + \gamma \cdot r_i$$

$$\sim N(0,\sigma) \Rightarrow \quad f(R_i \mid r_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{R_i - a - \gamma \cdot r_i}{2\sigma^2} \right)$$

- Likelihood:

$$L = \prod_i \left(\prod_h \left(P_{h/i} \cdot f(R_i \mid r_i) \right)^{y_{hi}} \right)$$

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Case study: Brussels

- Data collected for a FP7 European Union project (SustainCity)
  - Census 2001 (aggregated information by zone)
  - Household survey 1999 (~1300 observations), no detail on housing attributes
  - Average transaction prices by commune and 2 types of dwelling (house or apartment) from 1985 to 2008
  - Other geographical, land use databases

- 1267997 households, 1274701 dwellings
- 157 communes
- 4975 zones
- 4 types of dwelling (with average attributes per zone)
  - Isolated house
  - Semi-isolated house
  - Joint house
  - Apartment

Case study: Brussels

Bid function specification for location (bid) choice model (Ellickson):

$$B_{hvi} = \beta_{surf} \cdot surf_{vi} \cdot \ln(N_h) + \beta_{sup} \cdot Q_i^{sup} \cdot N_h^{sup} + \beta_{house} \cdot \lambda_{vi}^{house} \cdot N_h +$$
$$\beta_{mid\_inc} \cdot I_i \cdot \gamma_h^{mid\_inc} + \beta_{high\_inc} \cdot I_i \cdot \gamma_h^{high\_inc} + \beta_{trans} \cdot Y_i^{trans} \cdot \gamma_h^{cars=0} +$$
$$\beta_{trans2} \cdot Y_i^{trans} \cdot \gamma_h^{cars>1} + \beta_{comm} \cdot Y_i^{comm} \cdot \ln(N_h) + \beta_{off} \cdot Y_i^{off} \cdot W_h + \beta_{green} \cdot Y_i^{green} \cdot W_h$$

- $surf_{vi}$ is the average surface of a residential unit in buildings type v in zone i. The building types consider 3 types of house (fully-detached, semi-detached and attached) and apartments.
- N_h is the size (number of individuals) of a household.
- W_h is number of active individuals (workers) in a household
- N_h^{sup} is number of persons in the household who achieved a university degree as their maximum education level.
- Q_i^{sup} is percentage of the population in zone i with a superior level education-degree.
- I_i is the average income of zone i (calculated from tax declarations)
- Y_i^{trans} is a measurement of the quality of public transport for zone i (accessibility)
- Y_i^{comm}, Y_i^{off}, Y_i^{green} are measurement of the presence of commerce, offices and public green areas respectively

Case study: Brussels

Table 1: Estimation results for Brussels

Parameter	Standard Logit			Logit with price indicator		
	Value	Std err	t-test	Value	Std err	t-test
β_{surf}	0.00636	0.00261	2.43	0.000311	0.000225	1.38*
β_{mid_inc}	0.0439	0.0111	3.94	-0.00317	0.00717	-0.44*
β_{high_inc}	0.0574	0.0153	3.76	0.0161	0.00998	1.61*
β_{sup}	0.403	0.108	3.73	0.728	0.0739	9.84
β_{trans0}	0.408	0.136	3	0.599	0.0849	7.06
β_{trans2}	-0.532	0.153	-3.48	-0.31	0.0791	-3.91
β_{house}	0.461	0.0615	7.5	0.0563	0.00702	8.03
β_{comm}	-1.34	0.278	-4.83	-0.0366	0.031	-1.18*
β_{green}	-0.349	0.0717	-4.86	0.136	0.0201	6.74
β_{off}	-0.295	0.0931	-3.16	0.0896	0.0413	2.17
α	-	-	-	-16.4	3.23	-5.08
γ	-	-	-	1.92	0.229	8.39
σ	-	-	-	-1.92	0.0225	-85.48
Final Log-Likelihood	-7011.03			-6387.76 (-7091.13**)		
Likelihood ratio-test	232.44			1478.97 (72.23**)		

*parameters not significant at the 95% level

** log-likelihood considering only the choice probabilities

Estimation performed with PythonBiogeme (Bierlaire and Fetiarison ,2010)

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Case study: Brussels

Table 2: Estimation results for Brussels

Parameter	Standard Logit			L&K		
	Value	**Std err**	**t-test**	**Value**	**Std err**	**t-test**
β_{surf}	0.00636	0.00261	2.43	-0.00136	0.000855	-1.59*
β_{mid_inc}	0.0439	0.0111	3.94	0.0194	0.00608	3.19
β_{high_inc}	0.0574	0.0153	3.76	0.0474	0.00796	5.95
β_{sup}	0.403	0.108	3.73	0.416	0.0669	6.22
β_{trans0}	0.408	0.136	3	-1.01	0.0716	-14.1
β_{trans2}	-0.532	0.153	-3.48	-0.226	0.0887	-2.54
β_{house}	0.461	0.0615	7.5	0.0167	0.0182	0.92*
β_{comm}	-1.34	0.278	-4.83	-0.768	0.0977	-7.85
β_{green}	-0.349	0.0717	-4.86	0.286	0.0367	7.78
β_{off}	-0.295	0.0931	-3.16	-0.767	0.0533	-14.38
μ	-	-	-	1.66	0.0173	95.74
Final Log-Likelihood	-7011.03			-7569.645 (-11813.1**)		
Likelihood ratio-test	232.44			1478.97 (72.23**)		

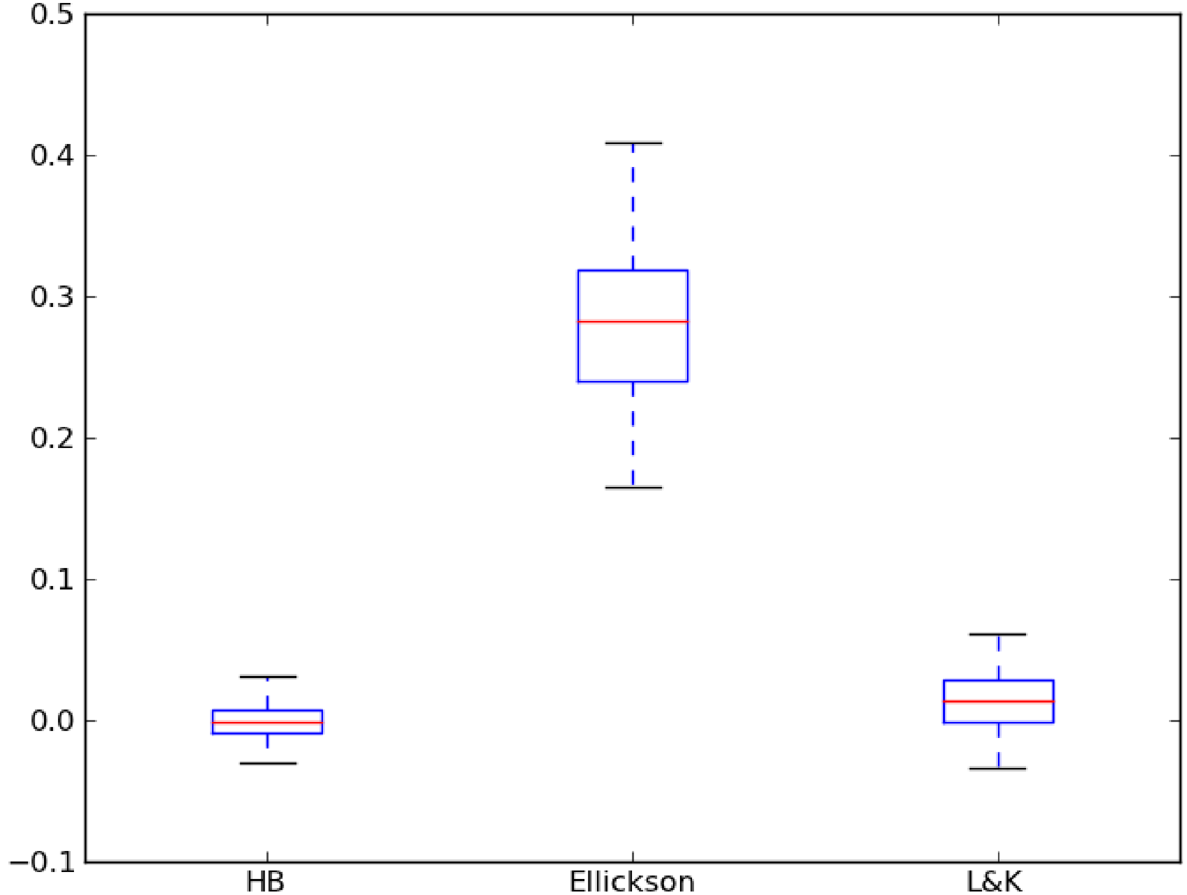
*parameters not significant at the 95% level

** log-likelihood considering only the choice probabilities

Estimation performed with PythonBiogeme (Bierlaire and Fetiarison ,2010)

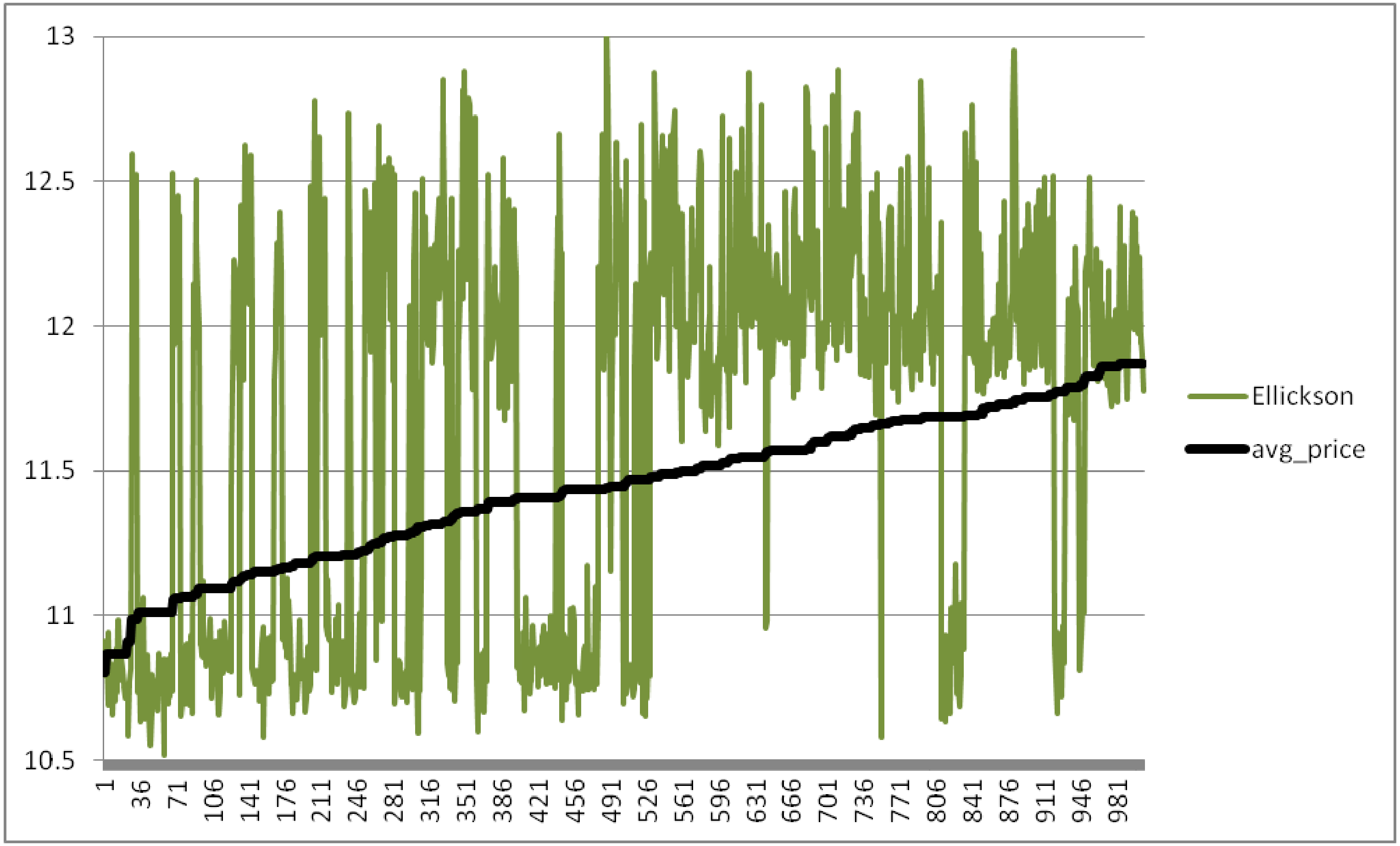
Case study: Brussels

- Prices per commune and type (% error) (over estimation dataset)



Case study: Brussels

- Prices (over estimation dataset)



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Case study: Brussels

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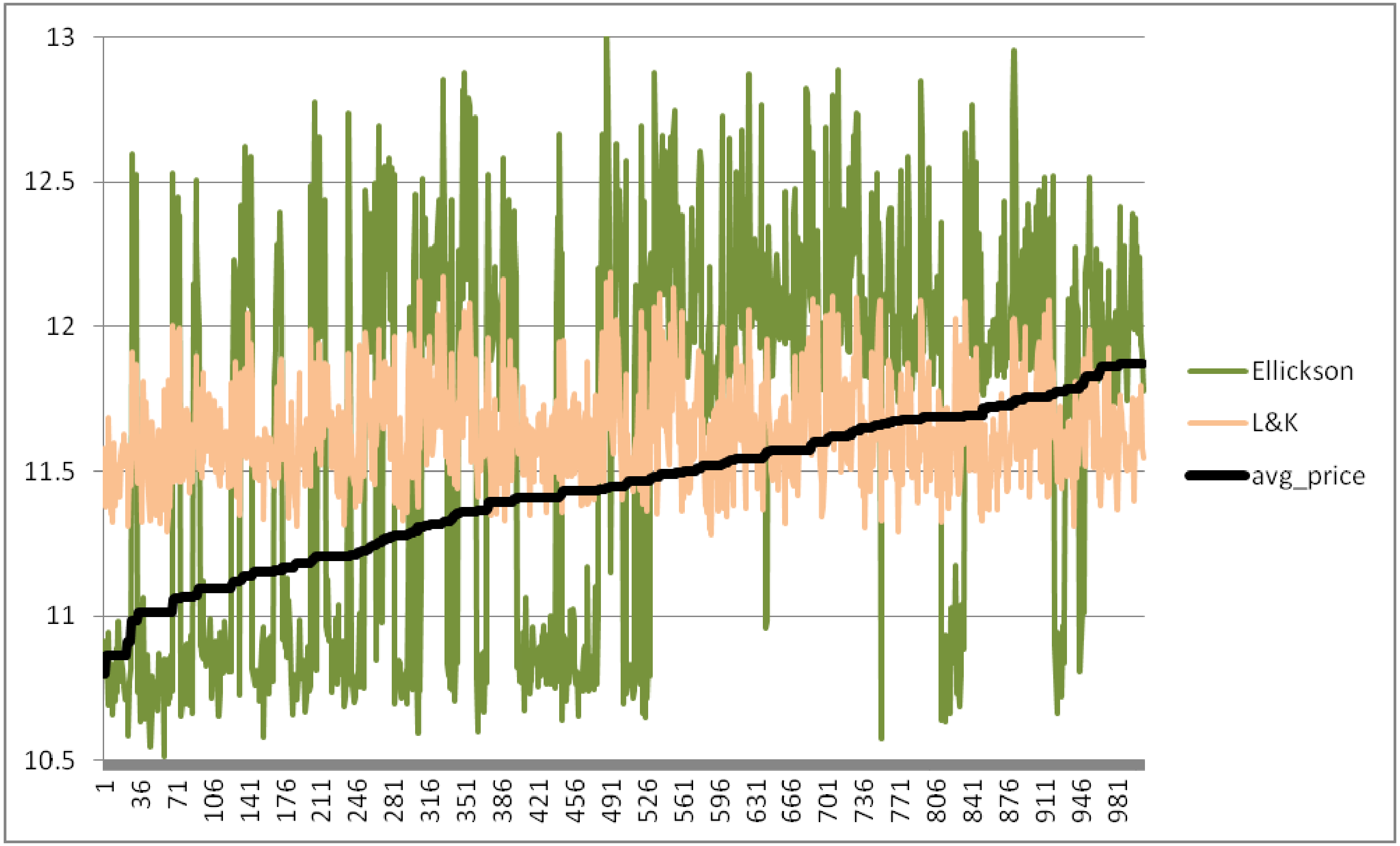
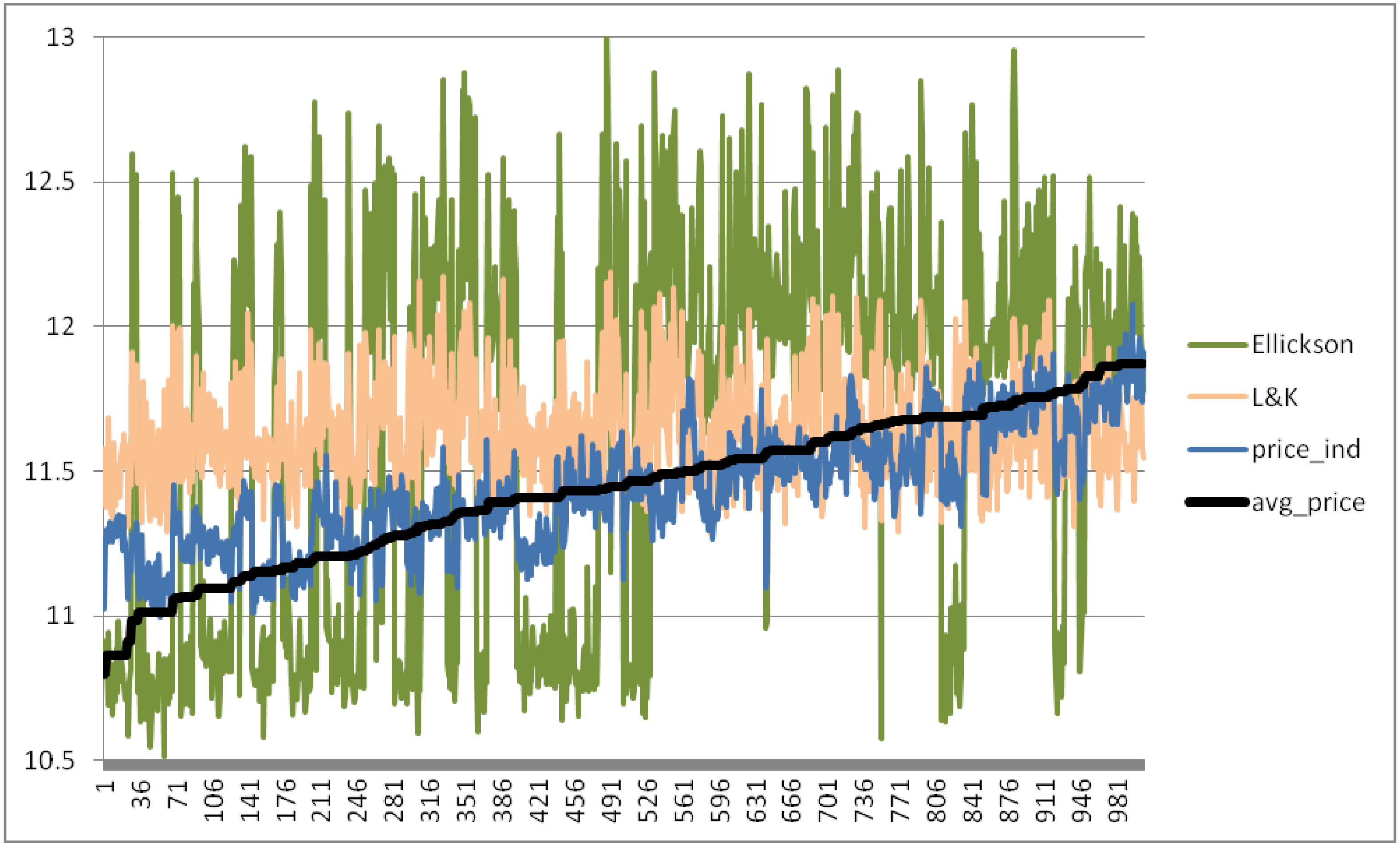


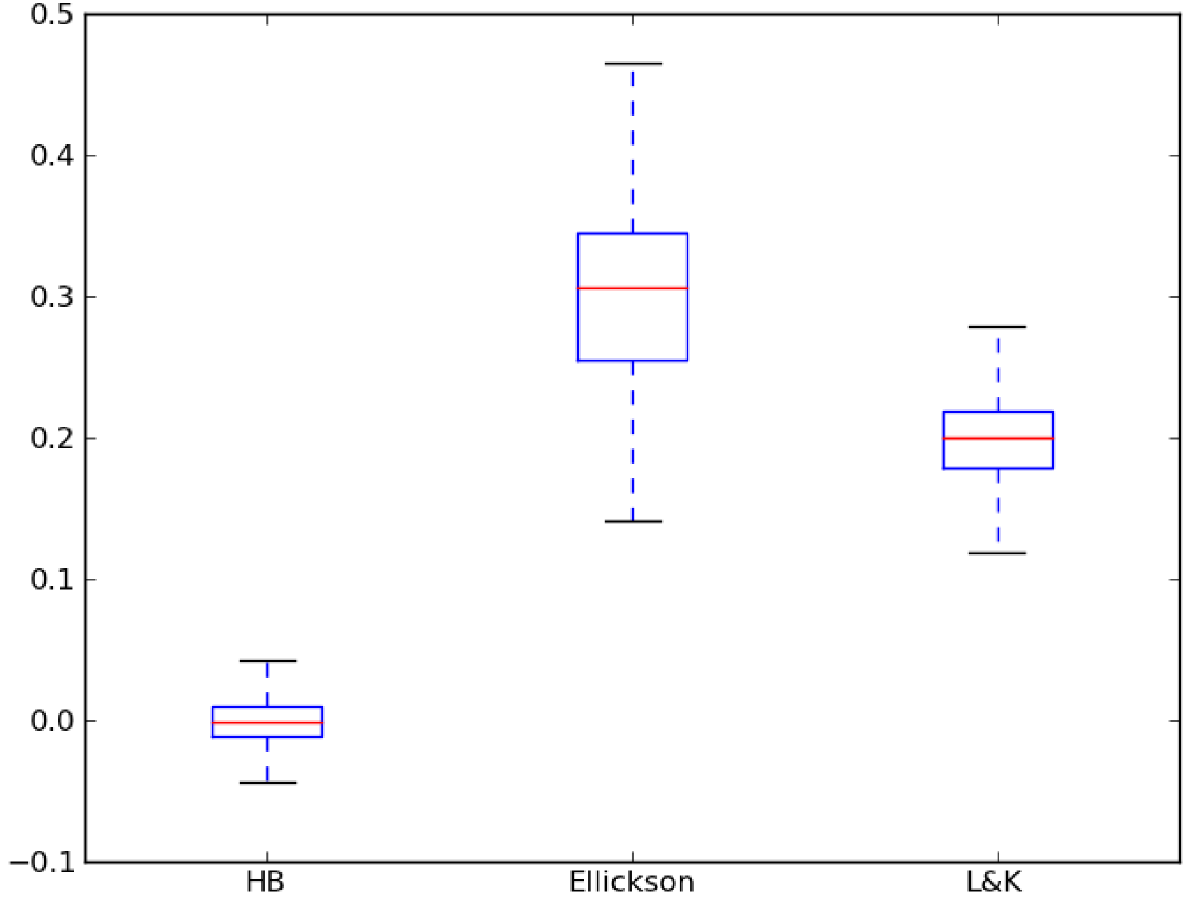
Case study: Brussels

- Prices (over estimation dataset)



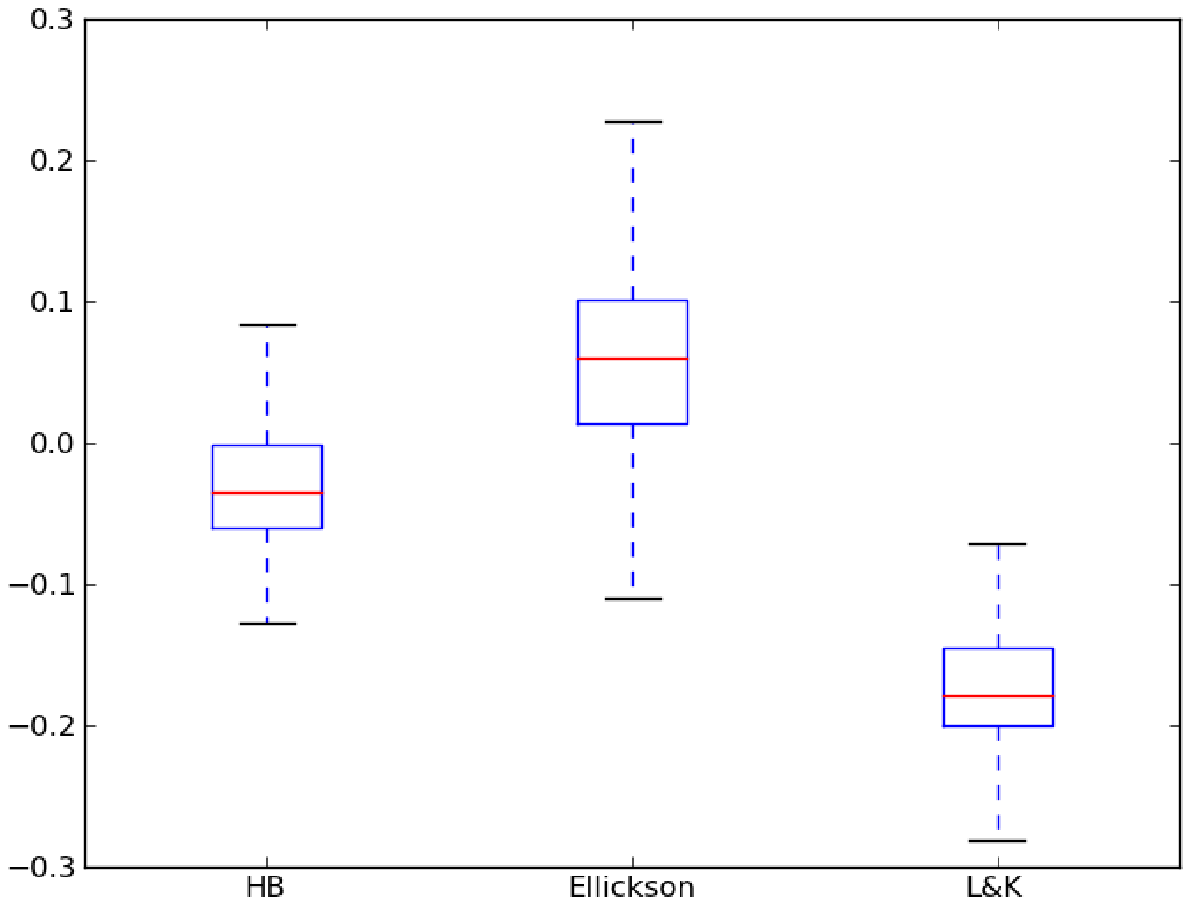
Case study: Brussels (forecasting/validation)

- Prices per commune and type (% error) (over full supply for 2001)



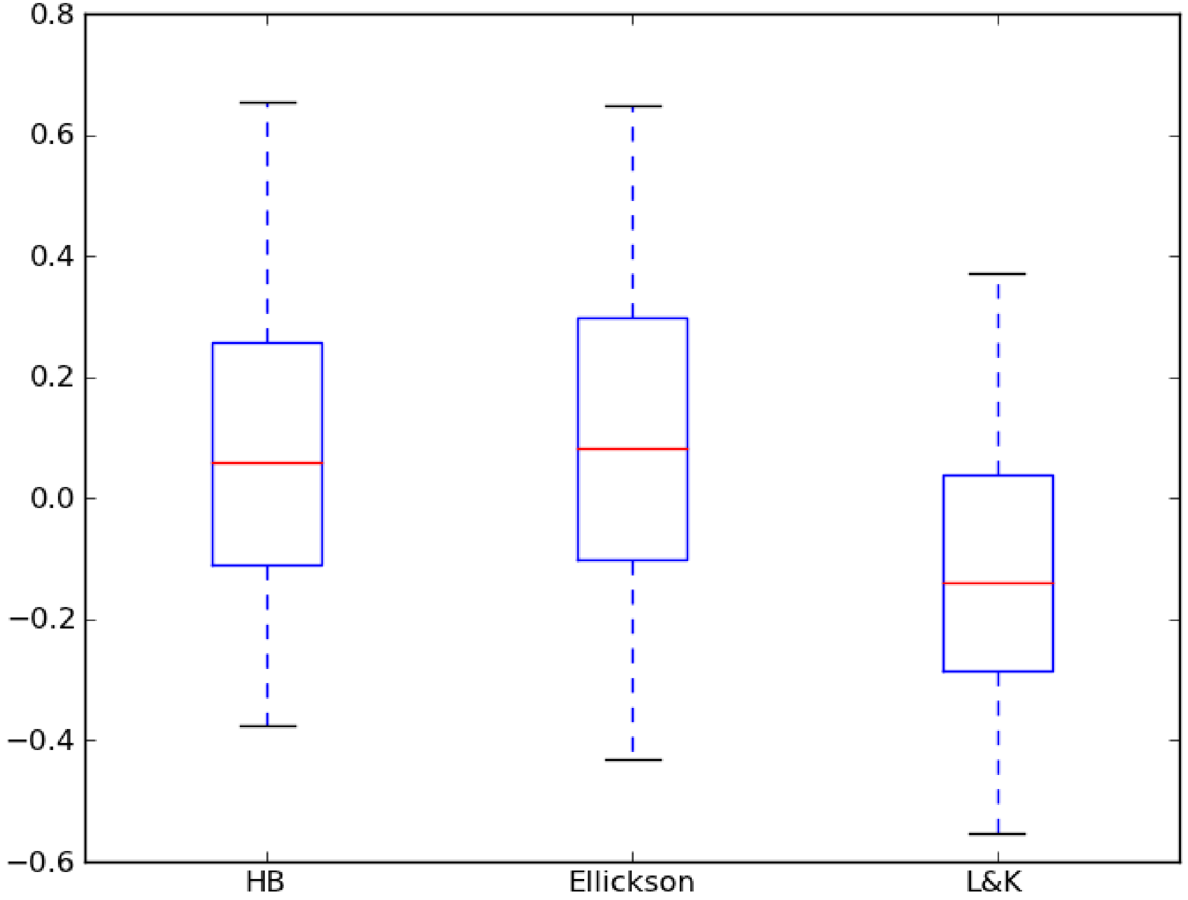
Case study: Brussels (forecasting/validation)

- Number of people per commune (% error)



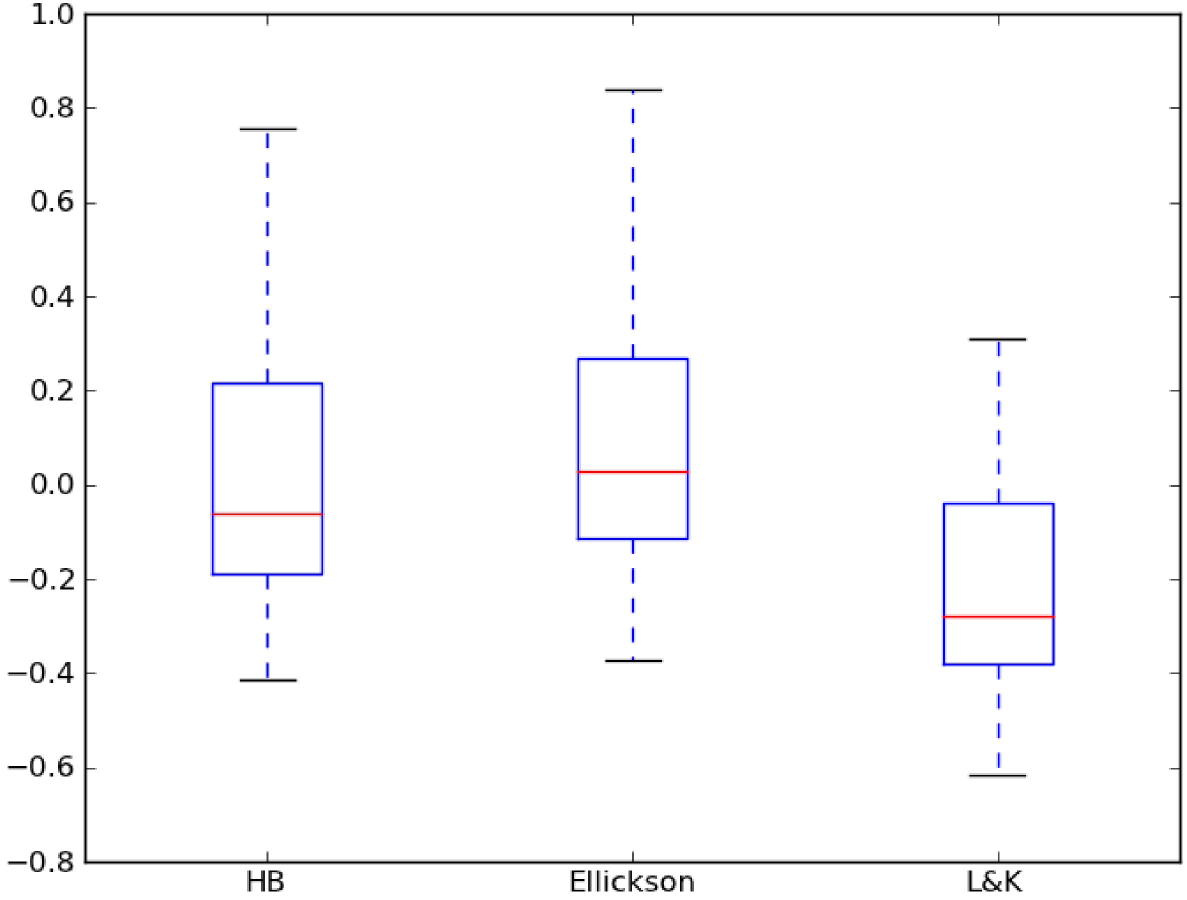
Case study: Brussels (forecasting/validation)

- Number of people with univ degree per commune (% error)



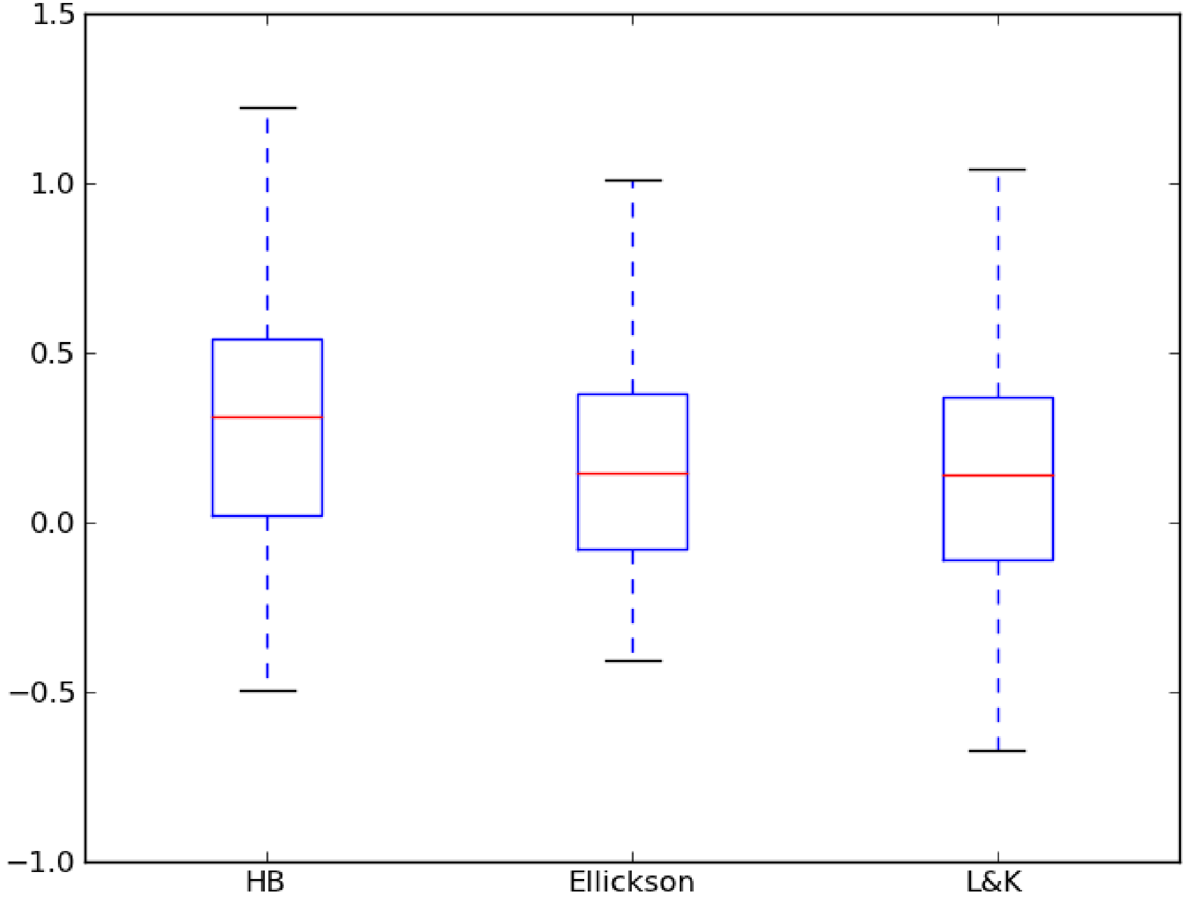
Case study: Brussels (forecasting/validation)

- Number of households with 2+ cars (% error)



Case study: Brussels (forecasting/validation)

- Number of households with 0 cars (% error)



Discussion

- The proposed estimation method finds estimates that reproduce the location distribution of agents and the average market prices of dwellings better than other methods

- Proposed method requires less detailed data ➔ more suitable for extensive land use models

- Well estimated bid functions (willingness to pay) allow to generate a good forecast of the transaction prices, without the need of hedonic price models ➔ this helps if we want to microsimulate using a bid approach

Bid-auction framework for microsimulation of location choice

Microsimulation with a bid approach

- When bids are simulated and we get:
  - Spatial distribution of agents
  - Real estate prices
- But, in order to account for competition between agents for scarce goods, we need market clearing
  - Through hedonic price models (UrbanSim)
    - Simple but not real market clearing
  - Individual auctions (ILUTE)
    - Expensive in computational terms
  - Equilibrium (MUSSA)
    - Aggregated approach

The market clearing problem

Joint probability of household *h* occupying location *i*:

$$P(i,h) = P(i \mid h)P(h) = P(h \mid i)P(i)$$

$P(h \mid i)$ Maximum bid probability

$P(i \mid h)$ Maximum surplus (utility) probability

$P(i)$ Selling probability

$P(h)$ Locating probability

Re-visiting Equilibrium

- In equilibrium models it's usually assumed that supply (*S*) equals demand (*H*)

$$P(h) = P(i) = 1 \quad \forall h, i \quad \Rightarrow H = S$$

- Possible equilibrium conditions:

$$\sum_h P(i,h) \Rightarrow \sum_h P(i \mid h) P(h) = P(i) = 1 \quad \forall i$$ **(everything is sold)**

$$\sum_i P(i,h) \Rightarrow \sum_i P(h \mid i) P(i) = P(h) = 1 \quad \forall h$$ **(everyone is located)**

Re-visiting Equilibrium

- Market clearing can be achieved by imposing one of the equilibrium conditions and finding prices/bids that produce them

$$\exists r_i : \sum_h P(i \mid h) = 1 \quad \forall i$$ **(prices clear the market)**

$$\exists b_h : \sum_i P(h \mid i) = 1 \quad \forall h$$ **(bids clear the market)**

Due to interdependence, these are usually fixed point problems

Re-visiting Equilibrium

- If we have an auction market and the best bidder rule is observed, adjusting prices or bids is equivalent in equilibrium
- When market conditions change (supply, demand, etc) utility levels of the decision makers have to be adjusted, this is reflected in the level of the prices or bids

→ idea: quasi-equilibrium

Quasi-equilibrium

- Periodical location of new and re-locating agents, given exogenous supply
- Assumption: all households looking for a location are located somewhere $P(h)=1 \quad \forall h$
  - Total supply must be greater or equal than total demand $\Rightarrow H \leq S$
  - Not all locations are necessarily used $P(i)\leq 1 \quad \forall i$

Quasi-equilibrium

- No equilibrium ➔
  - no perfect information (aggregate supply, previous prices)
  - No iterative negotiation/bidding
  - No absolute adjustment of bids/prices
- Instead, adjustment of "perception" of agents that goes in the direction of an equilibrium but does not solve it.

Quasi-equilibrium

- Algorithm (in each period):
  - All agents (H) observe the market: prices and supply $\left(r_i^{t-1}, z_i^{t-1}, S_i\right)$
  - All gents (simultaneously) adjust their bids, attempting to make their expected number of winning auctions equal to one:

$$\sum_{i \in S} q(h \mid i) = 1 \quad \forall h$$

q(h|i): perceived probability of being the best bidder for *i*

  - All agents bid at the same time for all locations ➔ prices and location distributions are defined
  - The assignment mechanism is an auction ➔ for each location a best bidder and a price is determined

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Quasi-equilibrium

Bid function: $B_{hi} = I_h - U_h + V_h(z_i) = V_h(z_i) - b_h$

- Perceived probability:

$$q(h|i) = \frac{\exp\left(V_h(z_i^t) - b_h^t\right)}{\sum_{g \in H} \exp(B_{gi}^t)} \approx \exp\left(V_h(z_i^t) - b_h^t - r_i^{t-1}\right)$$

$$\sum_{i \in S} q(h|i) = 1 \quad \Rightarrow \widehat{b_h^t} = \ln\left(\sum_{i \in S} \exp\left(V_h(z_i^t) - r_i^{t-1}\right)\right)$$

Advantage: no fixed point, just evaluation of equation ➔ it is possible to apply to large populations without excessive computational cost

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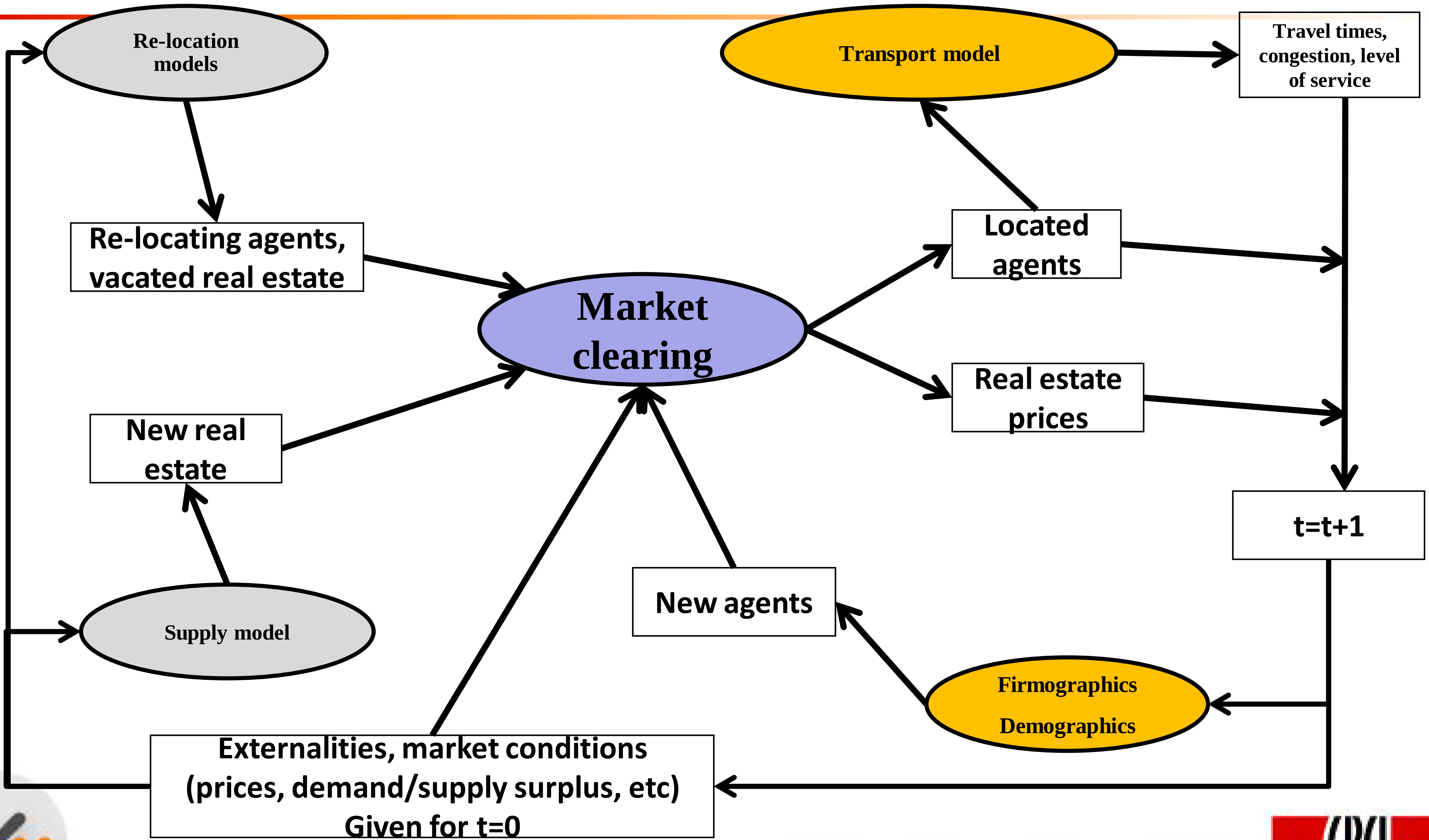
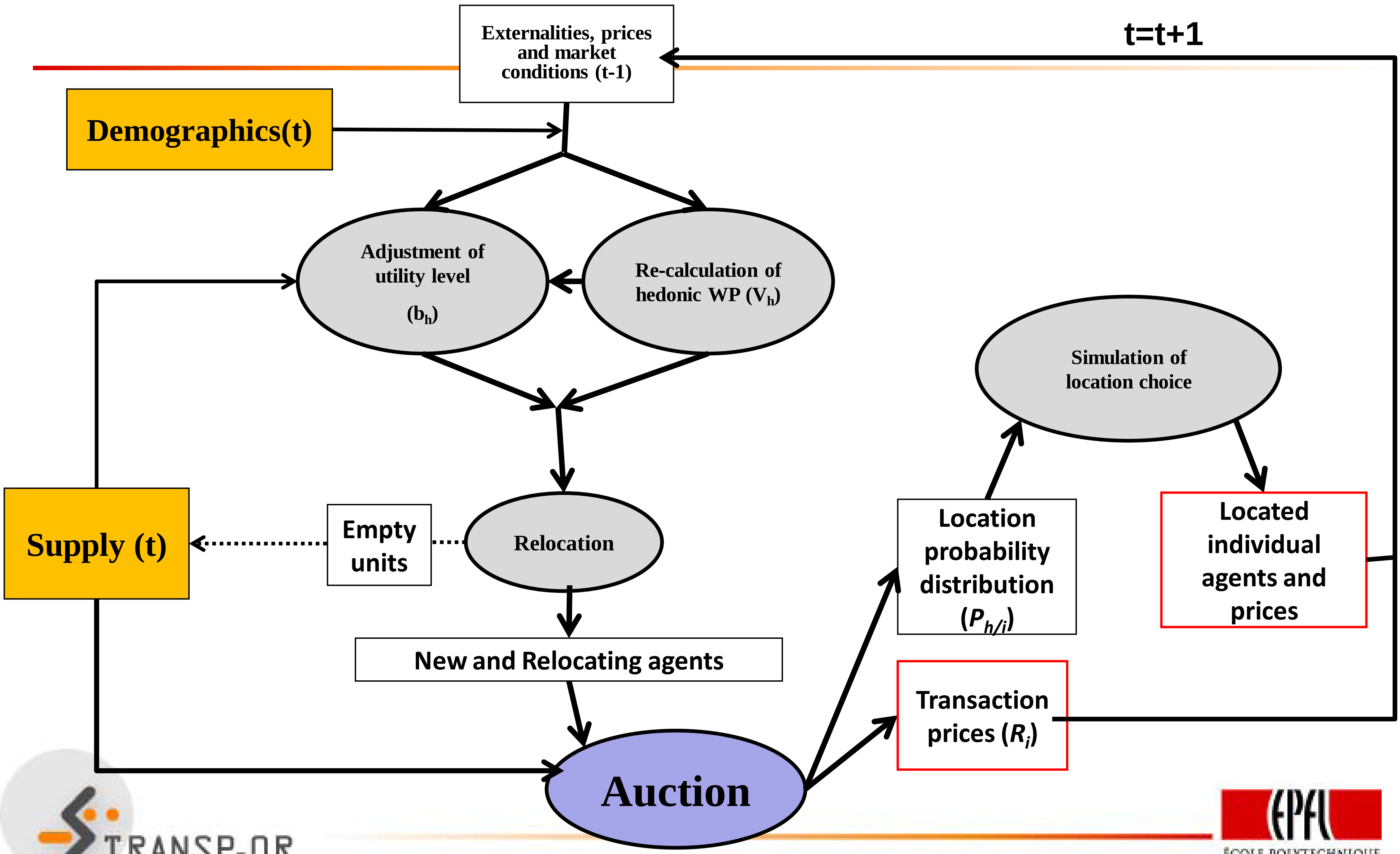
General framework



Market clearing



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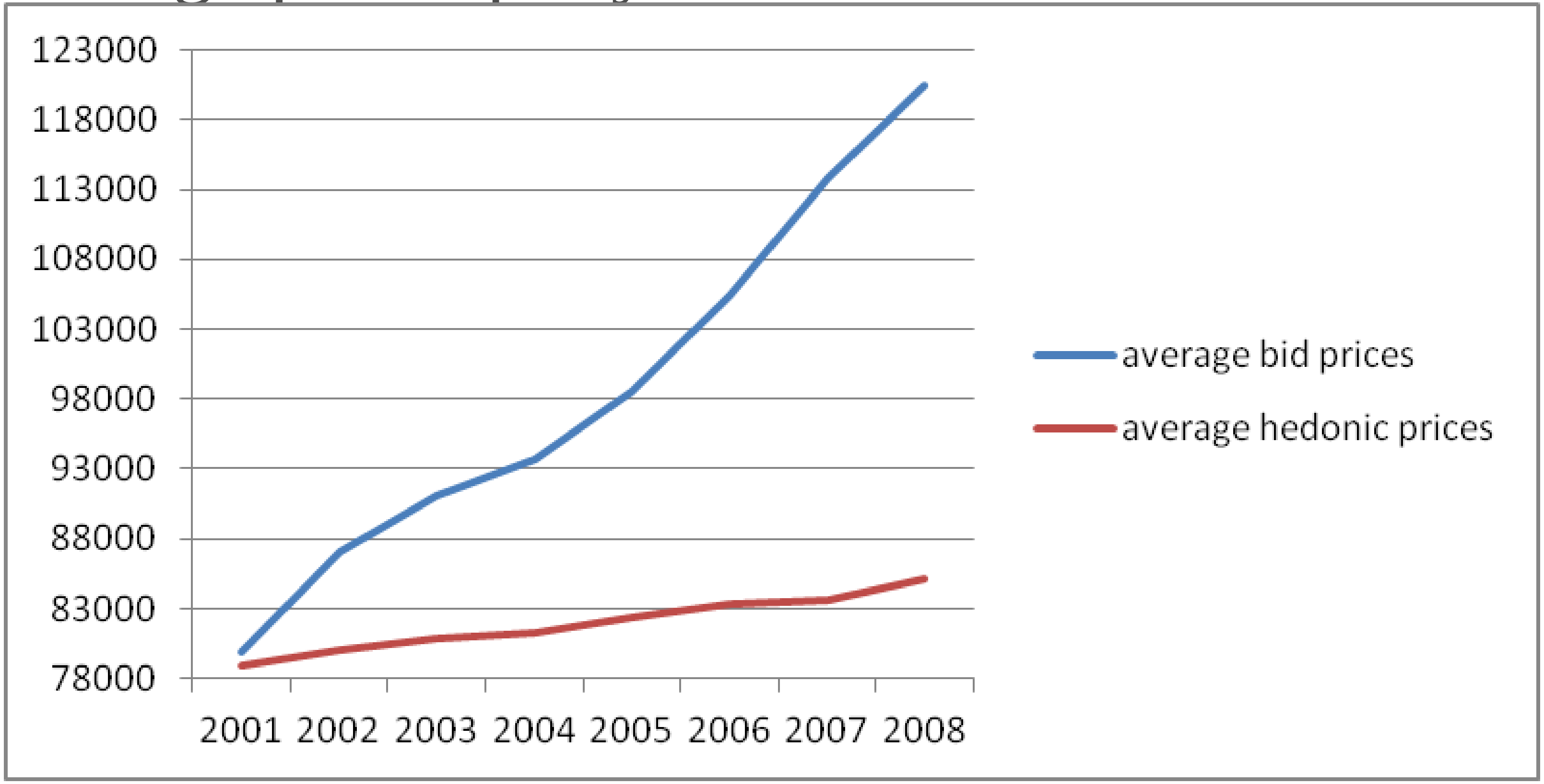
Some preliminary results

- Average prices per year



  - Average price growth: BID: 50%, HEDONIC: 7%

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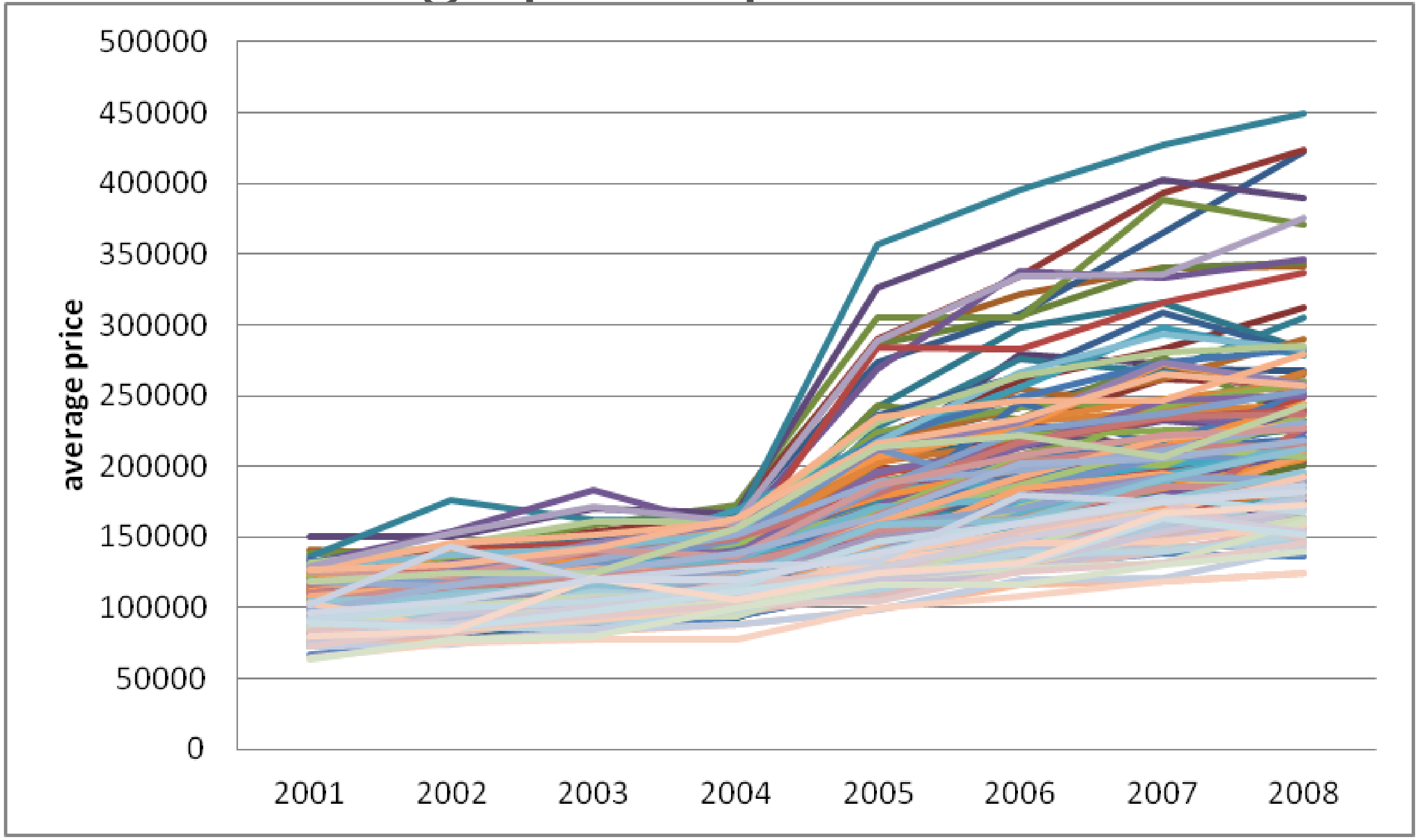
- Observed average prices per commune



- Average price growth :108%

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Advantages

- Agents have an individual behavior but they relate to a “higher level” market mechanism through the utility level adjustment and the simultaneous auction.
- Quasi-equilibrium :
  - Demand is not cleared: utility adjustment does NOT assure allocation
  - Supply is not cleared
  - System tends to equilibrium but does not clear
- Adjustment of utility levels instead of prices allow to
  - Explain price formation (no need for hedonic price models)
  - Detect all agents utility levels, including those not active in the market, triggering future re-location

Thank you

Main assumptions of the general framework

- Auction market
- Agents adjust their utility level (individually in each period)
  - to ensure location (ex-ante expectations)
  - given market conditions: previous period rents, current supply
- Time lag:
  - In production of real estate goods:
  - In perception of attributes of locations (non-instantaneous)
- Simultaneous (macro level) bid of all agents for all locations
  - Location (best bidder) distributions and expected rents (R_i).
  - No iterative transactions.
  - Computationally simpler than transaction-specific price clearing
- Microsimulation:
  - Actual allocation following macro distributions (simulation of auctions)
  - Rents at micro level (r_i)