Experimental analysis of the implicit choice set generation using the Constrained Multinomial Logit

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Motivation

- Choice set mis-specification $\Rightarrow$ biased model
- Standard choice models: choice set is characterized by deterministic rules
  - Explicit (un)availability of the alternative
  - Explicit restrictions
- Some choice sets are not deterministic
  - Fuzzy rules
  - Depending on unobserved attributes
  - Complex interaction between decision maker and the environment
- Methods to model choice set generation process are usually complex: solutions?
Outline

1. Motivation
2. Probabilistic choice set generation
3. Constrained multinomial logit
4. Comparison of approaches
   - A simple example
   - Synthetic data
5. Conclusions
Probabilistic Choice Set (PCS)

- Manski (1977):

\[ P_n(i) = \sum_{C_m \subseteq C} P_n(i|C_m) \cdot P_n(C_m) \]

- Choice-set is a latent construct
- Alternative selection and choice-set generation are separate processes
- Computational complexity (combinatorial number of possible choice sets)
Constrained Multinomial Logit (CMNL)

- Martínez, Aguila and Hurtubia (2009):

$$P_n(i) = \frac{e^{V_{in} + \ln \phi_n(i)}}{\sum_{j \in C} e^{V_{jn} + \ln \phi_n(j)}}$$

where

$$\phi_n(i) = \frac{1}{1 + \exp(\omega(Y_{in} - a))} = \begin{cases} 1 & \text{if } Y_{in} - a \to -\infty \\ 0 & \text{if } Y_{in} - a \to +\infty. \end{cases}$$
Constrained Multinomial Logit (CMNL)

- CMNL:
  - Does not require enumeration of choice sets
  - Simulates the construction of the individual’s choice set
  - Heuristic based on assumptions over the utility’s functional form:
    \[ V'_{in} = V_{in} + \phi_n (i) \]
    Compensatory part
    Non-Compensatory part (penalty)
  - CMNL is an approximation to the choice-set generation procedure

How good is this approximation?
Comparison of approaches

\[ P_n(i) = \sum_{C_m \in C} e^{V_{in}} \cdot \left( \frac{e^{V_{jn}} \cdot P_n(C_m)}{\sum_{j \in C_m} e^{V_{jn}}} \right) = \frac{e^{V_{in} + \ln \phi_n(i)}}{\sum_{j \in C} e^{V_{jn} + \ln \phi_n(j)}} \]
A simple example

Binary logit

\[
P(\{1\}) = 1 - \phi(2)
\]

\[
P(\{2\}) = 0
\]

\[
P(\{1,2\}) = \phi(2)
\]
A simple example

- Probability of choosing alternative 1?

- CMNL: \[ P(1) = \frac{e^{V_1}}{e^{V_1} + e^{V_1+\ln \phi(2)}}. \]

- PCS: \[ P(1) = \left(1 - \phi(2)\right) \cdot 1 + \phi(2) \cdot \frac{e^{V_1}}{e^{V_1} + e^{V_2}}. \]
A simple example

- Probability of choosing alternative 1 ($V_1 = V_2$)
A simple example

- Probability of choosing alternative 1 ($V_1 > V_2$)
A simple example

- Probability of choosing alternative 1 ($V_1 < V_2$)
Synthetic data

- Simulated choices according to PCS approach
- Alternatives
  - Car (not always available)
  - Train (always available)
  - Swissmetro (always available)

\[
V_n(\text{CAR}) = \text{ASC}_{\text{CAR}} + \beta_{\text{cost}} \cdot \text{COST}_{\text{CAR}} + \beta_{\text{tt}} \cdot \text{TT}_{\text{CAR}}
\]
\[
V_n(\text{TRAIN}) = \beta_{\text{cost}} \cdot \text{COST}_{\text{TRAIN}} + \beta_{\text{tt}} \cdot \text{TT}_{\text{TRAIN}} + \beta_{\text{he}} \cdot \text{HE}_{\text{TRAIN}}
\]
\[
V_n(\text{SM}) = \text{ASC}_{\text{SM}} + \beta_{\text{cost}} \cdot \text{COST}_{\text{SM}} + \beta_{\text{tt}} \cdot \text{TT}_{\text{SM}} + \beta_{\text{he}} \cdot \text{HE}_{\text{SM}}
\]
Synthetic data

- Car availability:

\[
\phi(\text{CAR}) = \frac{1}{1 + \exp(\omega(TT_{\text{CAR}}/60 - a))}
\]
Synthetic data

- 2 possible choice sets:
  - Car, Train, Swissmetro
    \[ P(CAR, TRAIN, SM) = \phi(CAR) \]
  - Train, Swissmetro
    \[ P(TRAIN, SM) = 1 - \phi(CAR) \]
Synthetic data

- Results for CMNL:

<table>
<thead>
<tr>
<th>real $\omega$ value</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameter</td>
<td>real value</td>
<td>estimate</td>
<td>t-test</td>
<td>estimate</td>
<td>t-test</td>
</tr>
<tr>
<td>$ASC_{\text{CAR}}$</td>
<td>0.3</td>
<td>0.503</td>
<td>0.950</td>
<td>0.421</td>
<td>1.153</td>
</tr>
<tr>
<td>$ASC_{\text{SM}}$</td>
<td>0.4</td>
<td>0.565</td>
<td>2.013 *</td>
<td>0.550</td>
<td>2.375 *</td>
</tr>
<tr>
<td>$\beta_{\text{cost}}$</td>
<td>-0.01</td>
<td>-0.008</td>
<td>4.825 *</td>
<td>-0.008</td>
<td>3.580 *</td>
</tr>
<tr>
<td>$\beta_{\text{nc}}$</td>
<td>-0.005</td>
<td>-0.005</td>
<td>0.202</td>
<td>-0.005</td>
<td>0.151</td>
</tr>
<tr>
<td>$\beta_{\text{time}}$</td>
<td>-0.01</td>
<td>-0.007</td>
<td>3.929 *</td>
<td>-0.008</td>
<td>3.645 *</td>
</tr>
<tr>
<td>$a$</td>
<td>3</td>
<td>2.186</td>
<td>1.753</td>
<td>2.656</td>
<td>3.073 *</td>
</tr>
<tr>
<td>$\omega$</td>
<td>see top</td>
<td>1.043</td>
<td>0.239</td>
<td>2.094</td>
<td>0.403</td>
</tr>
</tbody>
</table>

- The quality of the estimates improves when the dispersion decreases.
Synthetic data

- t-test over dispersion
Conclusions

- The CMNL is a valid approximation when the constraints tend to be deterministic
- Still, is convenient to use for big choice-set problems

- Further work
  - Identify more specifically when is recommendable to use the CMNL
  - Justification from the behavioral approach?
  - Possible correction to the model?
Thank you