Integrating advanced discrete choice models into (mixed) integer linear optimization

Michel Bierlaire   Meritxell Pacheco

Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
Ecole Polytechnique Fédérale de Lausanne

January 3, 2016

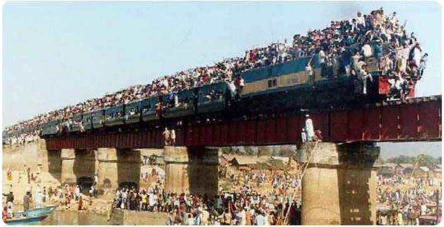
Outline

1. Demand and supply
2. Disaggregate demand models
3. Choice-based optimization
   - Applications
4. A generic framework
5. A simple example
   - Example: one theater
   - Example: two theaters
   - Example: two theaters with capac
6. Parking management
7. Conclusion

Demand models

- Supply = infrastructure
- Demand = behavior, choices
- Congestion = mismatch

Demand models



- Usually in OR:
- optimization of the supply
- for a given (fixed) demand

Aggregate demand

- Homogeneous population
- Identical behavior
- Price (P) and quantity (Q)
- Demand functions: $P = f(Q)$
- Inverse demand: $Q = f^{-1}(P)$

Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.

Demand-supply interactions

Operations Research

- Given the demand...
- configure the system

Johnson City Enterprise.

Published Every Saturday,

$1. per year—Advance Payment.

SATURDAY, APRIL 7, 1883.

TIME TABLE

E. T., V. & G. R. R.

PASSENGER,	ARRIVES,
No. 1, West,	6:37, a. m.
No. 2, East,	9:45, p. m.
No. 3, West,	11:51, p.m.
No. 4, East,	3:56, a. m.
LOCAL FREIGHT,	ARRIVES,
No. 5,	7:20, a. m.
No. 8,	6:20, p. m.

JNO. W. EAKIN, Agent.

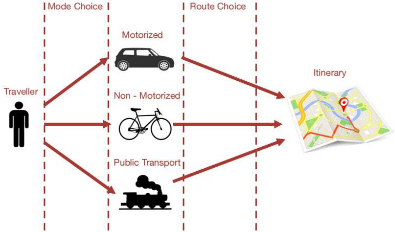
E. T. & W. N. C. R. R.

Passenger, leaves,	7, a. m.
" arrives,	6, p. m.

J. C. HARDIN, Agent.

Behavioral models

- Given the configuration of the system...
- predict the demand



Demand-supply interactions

Multi-objective optimization

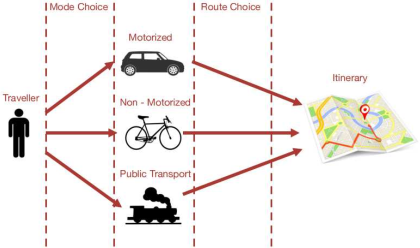
Minimize costs

Maximize satisfaction

Outline

1. Demand and supply
2. Disaggregate demand models
3. Choice-based optimization
   - Applications
4. A generic framework
5. A simple example
   - Example: one theater
   - Example: two theaters
   - Example: two theaters with capa
6. Parking management
7. Conclusion

Choice models



Behavioral models

- Demand = sequence of choices
- Choosing means trade-offs
- In practice: derive trade-offs from choice models

Choice models

Theoretical foundations

- Random utility theory
- Choice set: $\mathcal{C}_n$
- $y_{in} = 1$ if $i \in \mathcal{C}_n$, 0 if not
- Logit model:

$$P(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}$$

2000

TRANSP-OR

Logit model

Utility

$$U_{in} = V_{in} + \varepsilon_{in}$$

Choice probability

$$P_n(i|\mathcal{C}_n) = \frac{y_{in}e^{V_{in}}}{\sum_{j\in\mathcal{C}} y_{jn}e^{V_{jn}}}.$$

- Decision-maker n
- Alternative $i \in \mathcal{C}_n$

Variables: $x_{in} = (z_{in}, s_n)$

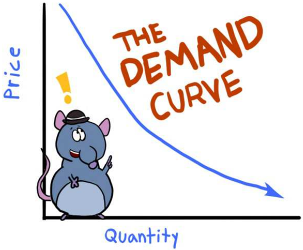
Attributes of alternative i: z_{in}

- Cost / price
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Characteristics of decision-maker n: s_n

- Income
- Age
- Sex
- Trip purpose
- Car ownership
- Education
- Profession
- etc.

Demand curve



Disaggregate model

$$P_n(i|c_{in}, z_{in}, s_n)$$

Total demand

$$D(i) = \sum_n P_n(i|c_{in}, z_{in}, s_n)$$

Difficulty

Non linear and non convex in c_{in} and z_{in}

Outline

1. Demand and supply
2. Disaggregate demand models
3. Choice-based optimization
   - Applications
4. A generic framework
5. A simple example
   - Example: one theater
   - Example: two theaters
   - Example: two theaters with capa
6. Parking management
7. Conclusion

Choice-Based Optimization Models

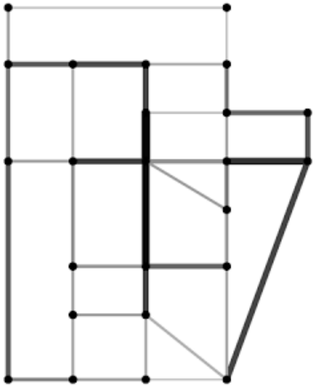
Benefits

- Merging supply and demand aspect of planning
- Accounting for the heterogeneity of demand
- Dealing with complex substitution patterns
- Investigation of demand elasticity against its main driver (e.g. price)

Challenges

- Nonlinearity and nonconvexity
- Assumptions for simple models (logit) may be inappropriate
- Advanced demand models have no closed-form
- Endogeneity: same variable(s) both in the demand function and the cost function

Stochastic traffic assignment



Features

- Nash equilibrium
- Flow problem
- Demand: path choice
- Supply: capacity

Selected literature

- [Dial, 1971]: logit
- [Daganzo and Sheffi, 1977]: probit
- [Fisk, 1980]: logit
- [Bekhor and Prashker, 2001]: cross-nested logit
- and many others...

Revenue management

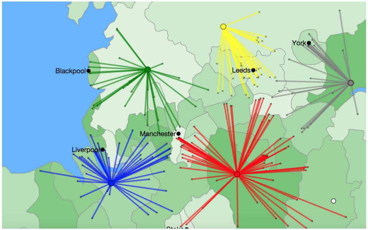
Features

- Stackelberg game
- Bi-level optimization
- Demand: purchase
- Supply: price and capacity

Selected literature

- [Labbé et al., 1998]: bi-level programming
- [Andersson, 1998]: choice-based RM
- [Talluri and Van Ryzin, 2004]: choice-based RM
- [Gilbert et al., 2014a]: logit
- [Gilbert et al., 2014b]: mixed logit
- [Azadeh et al., 2015]: global optimization
- and many others...

Facility location problem



Features

- Competitive market
- Opening a facility impact the costs
- Opening a facility impact the demand
- Decision variables: availability of the alternatives

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}.$$

Selected literature

- [Hakimi, 1990]: competitive location (heuristics)
- [Benati, 1999]: competitive location (B & B, Lagrangian relaxation, submodularity)
- [Serra and Colomé, 2001]: competitive location (heuristics)
- [Marianov et al., 2008]: competitive location (heuristic)
- [Haase and Müller, 2013]: school location (simulation-based)

Outline

1. Demand and supply
2. Disaggregate demand models
3. Choice-based optimization
   - Applications
4. A generic framework
5. A simple example
   - Example: one theater
   - Example: two theaters
   - Example: two theaters with capa
6. Parking management
7. Conclusion

The main idea

Linearization

Hopeless to linearize the logit formula (we tried...)

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- For each scenario r, we can identify the largest utility.
- It corresponds to the chosen alternative.

Variables

Availability

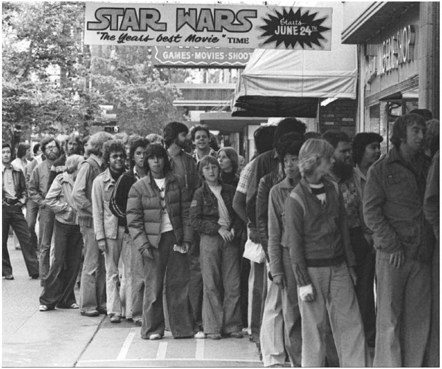
$$y_{in} = \begin{cases} 1 & \text{if alt. } i \text{ available for } n, \\ 0 & \text{otherwise.} \end{cases}$$

Choice

$$w_{inr} = \begin{cases} 1 & \text{if } y_{in} = 1 \text{ and } U_{inr} = \max_{j|y_{jn}=1} U_{jnr}, \\ 0 & \text{if } y_{in} = 0 \text{ or } U_{inr} < \max_{j|y_{jn}=1} U_{jnr}. \end{cases}$$

Capacities

- Demand may exceed supply
- Each alternative i can be chosen by maximum c_i individuals.
- An exogenous priority list is available.
- The numbering of individuals is consistent with their priority.



Priority list

Application dependent

- First in, first out
- Frequent travelers
- Subscribers
- ...

In this framework

The list of customers must be sorted

Capacities

Variables

- y_{in}: decision of the operator
- y_{inr}: availability

Constraints

$$\sum_{i \in \mathcal{C}} w_{inr} = 1 \qquad \forall n, r.$$

$$\sum_{n=1}^{N} w_{inr} \leq c_i \qquad \forall i, n, r.$$

$$w_{inr} \leq y_{inr} \qquad \forall i, n, r.$$

$$y_{inr} \leq y_{in} \qquad \forall i, n, r.$$

$$y_{i(n+1)r} \leq y_{inr} \qquad \forall i, n, r.$$

Demand and revenues

Demand

$$D_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} w_{inr}.$$

Revenues

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}.$$

Revenues

Non linear specification

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}.$$

Linearization

Binary basis

$$p_{in} = \frac{1}{10^d} \left(\ell_{in} + \sum_{\ell=0}^{L_{in}-1} 2^{\ell} \lambda_{in\ell} \right).$$

New decision variables

$\lambda_{in\ell} \in \{0, 1\}$

References

- Technical report: [Bierlaire and Azadeh, 2016]
- Conference proceeding: [Pacheco et al., 2016]

Outline

1. Demand and supply
2. Disaggregate demand models
3. Choice-based optimization
   - Applications
4. A generic framework
5. A simple example
   - Example: one theater
   - Example: two theaters
   - Example: two theaters with capa
6. Parking management
7. Conclusion

A simple example

Data

- $\mathcal{C}$: set of movies
- Population of N individuals
- Utility function:
  $U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}$

Decision variables

- What movies to propose? y_i
- What price? p_{in}

Back to the example: pricing

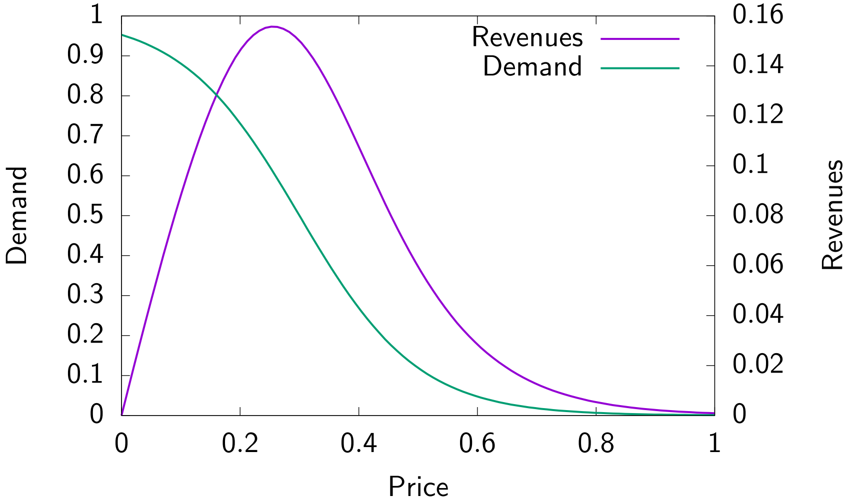
Data

- Two alternatives: my theater (m) and the competition (c)
- We assume an homogeneous population of N individuals

$$U_c = 0 + \varepsilon_c$$
$$U_m = \beta_c p_m + \varepsilon_m$$

- $\beta_c < 0$
- Logit model: ε_m i.i.d. EV

Demand and revenues



Optimization (with GLPK)

Data

- $N = 1$
- $R = 100$
- $U_m = -10p_m + 3$
- Prices: 0.10, 0.20, 0.30, 0.40, 0.50

Results

- Optimum price: 0.3
- Demand: 56%
- Revenues: 0.168

Heterogeneous population

Two groups in the population

$$U_{in} = -\beta_n p_i + c_n$$

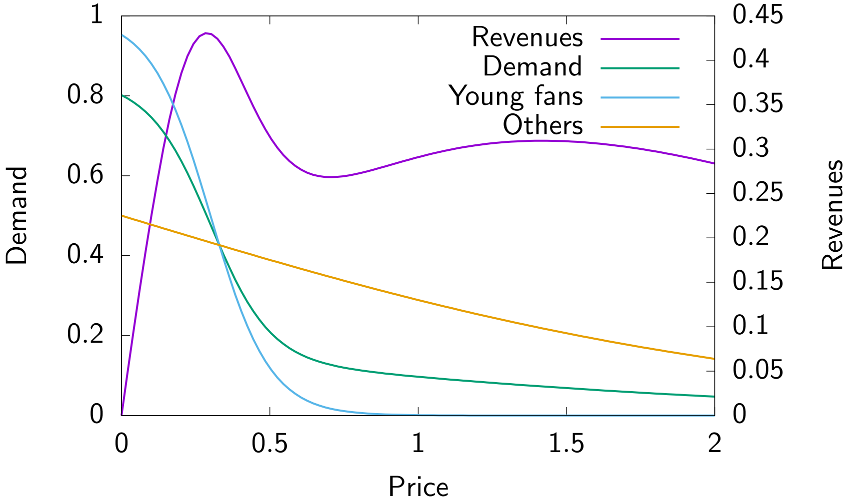
Young fans: 2/3

$\beta_1 = -10,\ c_1 = 3$

Others: 1/3

$\beta_1 = -0.9,\ c_1 = 0$

Demand and revenues



Optimization

Data

- $N = 3$
- $R = 100$
- $U_{m1} = -10p_m + 3$
- $U_{m2} = -0.9p_m$
- Prices: 0.3, 0.7, 1.1, 1.5, 1.9

Results

- Optimum price: 0.3
- Customer 1 (fan): 60% [theory: 50 %]
- Customer 2 (fan) : 49% [theory: 50 %]
- Customer 3 (other) : 45% [theory: 43 %]
- Demand: 1.54 (51%)
- Revenues: 0.48

Two theaters, different types of films

Two theaters, different types of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Tinker Tailor Soldier Spy

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, different types of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + \boxed{4}$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + \boxed{0}$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.6
- 4 young customers: 0
- 2 old customers: 0.5
- Demand: 0.5 (8.3%)
- Revenues: 0.8

Theater k

- Optimum price m: 0.5
- Young customers: 0.8
- Old customers: 1.5
- Demand: 2.3 (38%)
- Revenues: 1.15

Two theaters, same type of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Star Wars Episode VIII

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, same type of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + \textcircled{4}$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + \textcircled{4}$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.8
- Young customers: 0
- Old customers: 1.9
- Demand: 1.9 (31.7%)
- Revenues: 3.42

Theater k

Closed

Two theaters with capacity, different types of films

Data

- Theaters m and k
- Capacity: 2
- $N = 6$
- $R = 5$
- $U_{mn} = -10p_m + 4, \ n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m, \ n = 3, 6$
- $U_{kn} = -10p_k + 0, \ n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k, \ n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.8
- Demand: 0.2 (3.3%)
- Revenues: 0.36

Theater k

- Optimum price m: 0.5
- Demand: 2 (33.3%)
- Revenues: 1.15

Example of two scenarios

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	0	2	2
3	k	2	1
4	0	2	1
5	0	2	1
6	k	2	0

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	k	2	1
3	0	2	1
4	k	2	0
5	0	2	0
6	0	2	0

Outline

1 Demand and supply
2 Disaggregate demand models
3 Choice-based optimization
  - Applications
4 A generic framework
5 A simple example
  - Example: one theater
  - Example: two theaters
  - Example: two theaters with capac
6 Parking management
7 Conclusion

Parking management

Alternatives

- paid on-street parking (PSP)
- paid parking in an underground car park (PUP)
- free on-street parking (FSP)

Demand model

[Ibeas et al., 2014]

Scenario

- 50 customers
- Optimize revenues

Number of draws

Unlimited capacity

		Prices		Demand			
R	Solution time	PSP	PUP	PSP	PUP	FSP	Revenue
5	2.91 s	0.54	0.79	27.000	15.000	8.000	26.430
10	6.35 s	0.53	0.74	26.000	17.000	7.000	26.360
25	28.6 s (*)	0.54	0.79	28.040	14.880	7.080	26.897
50	3.70 min	0.54	0.75	25.160	17.840	7.000	26.966
100	17.0 min	0.54	0.74	24.440	18.520	7.040	26.902
250	11.7 h (*)	0.54	0.74	24.768	18.204	7.028	26.846

(*) Instances not solved to optimality, gap of 0.01% for the MIP best bound found

Number of draws

Capacity of PSP and PUP: 20

R	Solution time	Prices		Demand			Revenue
		PSP	PUP	PSP	PUP	FSP	
5	14.95 s	0.63	0.84	18.200	17.200	14.600	25.914
10	96.45s	0.57	0.78	19.900	17.900	12.200	25.305
25	15.9 min (*)	0.59	0.80	19.480	18.080	12.440	25.957
50	2.76 h	0.59	0.80	19.540	18.200	12.260	26.089
100	8.31 h (*)	0.59	0.79	19.130	18.660	12.210	26.028
250	6.94 days	0.60	0.80	19.044	18.128	12.828	25.929

(*) Instances not solved to optimality, gap of 0.01% for the MIP best bound found

Heterogenous demand

Residents

- Subsidy from the city
- Residents pay less
- Operator receives the same revenues

Subsidy

Subsidy (%)	Prices res PSP	Prices res PUP	Demand res PSP	Demand res PUP	Demand res FSP	Prices non res PSP	Prices non res PUP	Demand non res PSP	Demand non res PUP	Demand non res FSP	Revenue
20	0.54	0.77	11.8	9.40	5.78	0.68	0.92	7.46	8.60	6.94	29.7
25	0.54	0.77	12.2	10.2	4.64	0.68	0.92	7.34	8.72	6.94	30.7
30	0.50	0.67	12.7	10.4	3.86	0.72	0.96	6.16	8.50	8.34	31.8
40	0.48	0.65	13.7	10.7	2.6	0.80	1.08	4.88	7.20	10.9	34.2
50	0.46	0.64	15.0	10.4	1.62	0.92	1.28	3.74	5.32	13.94	37.3

Outline

1. Demand and supply
2. Disaggregate demand models
3. Choice-based optimization
   - Applications
4. A generic framework
5. A simple example
   - Example: one theater
   - Example: two theaters
   - Example: two theaters with capa
6. Parking management
7. Conclusion

Summary

Demand and supply

- Supply: prices and capacity
- Demand: choice of customers
- Interaction between the two

Discrete choice models

- Rich family of behavioral models
- Strong theoretical foundations
- Great deal of concrete applications
- Capture the heterogeneity of behavior
- Probabilistic models

Optimization

Discrete choice models

- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation

- Linear in the decision variables
- Large scale
- Fairly general

Ongoing research

- Decomposition methods
- Scenarios are (almost) independent from each other (except objective function)
- Individuals are also loosely coupled (except for capacity constraints)

Thank you!

Bibliography I

Andersson, S.-E. (1998).
Passenger choice analysis for seat capacity control: A pilot project in scandinavian airlines.
International Transactions in Operational Research, 5(6):471–486.

Azadeh, S. S., Marcotte, P., and Savard, G. (2015).
A non-parametric approach to demand forecasting in revenue management.
Computers & Operations Research, 63:23–31.

Bekhor, S. and Prashker, J. (2001).
Stochastic user equilibrium formulation for generalized nested logit model.
Transportation Research Record: Journal of the Transportation Research Board, (1752):84–90.

Bibliography II

Benati, S. (1999).
The maximum capture problem with heterogeneous customers.
Computers & operations research, 26(14):1351–1367.

Bierlaire, M. and Azadeh, S. S. (2016).
Demand-based discrete optimization.
Technical Report 160209, Transport and Mobility Laboratory, Ecole Polytechnique Fédérale de Lausanne.

Daganzo, C. F. and Sheffi, Y. (1977).
On stochastic models of traffic assignment.
Transportation science, 11(3):253–274.

Bibliography III

Dial, R. B. (1971).
A probabilistic multipath traffic assignment model which obviates path enumeration.
Transportation research, 5(2):83–111.

Fisk, C. (1980).
Some developments in equilibrium traffic assignment.
Transportation Research Part B: Methodological, 14(3):243–255.

Gilbert, F., Marcotte, P., and Savard, G. (2014a).
Logit network pricing.
Computers & Operations Research, 41:291–298.

Gilbert, F., Marcotte, P., and Savard, G. (2014b).
Mixed-logit network pricing.
Computational Optimization and Applications, 57(1):105–127.

Bibliography IV

Haase, K. and Müller, S. (2013).
Management of school locations allowing for free school choice.
Omega, 41(5):847–855.

Hakimi, S. L. (1990).
Locations with spatial interactions: competitive locations and games.
Discrete location theory, pages 439–478.

Ibeas, A., dell'Olio, L., Bordagaray, M., and de D. Ortúzar, J. (2014).
Modelling parking choices considering user heterogeneity.
Transportation Research Part A: Policy and Practice, 70:41 – 49.

Labbé, M., Marcotte, P., and Savard, G. (1998).
A bilevel model of taxation and its application to optimal highway pricing.
Management Science, 44(12-part-1):1608–1622.

TRANSP-OR

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Bibliography V

Marianov, V., Ríos, M., and Icaza, M. J. (2008).
Facility location for market capture when users rank facilities by shorter travel and waiting times.
European Journal of Operational Research, 191(1):32–44.

Pacheco, M., Azadeh, S. S., and Bierlaire, M. (2016).
A new mathematical representation of demand using choice-based optimization method.
In *Proceedings of the 16th Swiss Transport Research Conference*, Ascona, Switzerland.

Serra, D. and Colomé, R. (2001).
Consumer choice and optimal locations models: formulations and heuristics.
Papers in Regional Science, 80(4):439–464.

Bibliography VI

Talluri, K. and Van Ryzin, G. (2004).
Revenue management under a general discrete choice model of consumer behavior.
Management Science, 50(1):15–33.