A DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL OF CAR OWNERSHIP AND USAGE

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OUTLINE

- Introduction
- Background and data
- The dynamic discrete-continuous choice modeling framework
    - Assumptions
    - Definition of the components
    - Solving the dynamic programming problem
    - Model estimation
- Illustration of model application
- Conclusion and future works

INTRODUCTION

Aim of the research project:

- Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet

- Motivations
  - Governmental policies:
    - Goals of reducing carbon emissions
  - Technology changes:
    - Increase of alternative-fuel vehicles
    - Changes in the supply
  - Company cars: represent important share of new car sales

INTRODUCTION

Current literature on car ownership and usage modeling:

- Car ownership models in transportation literature:
  - Mostly **static models**:
    - Main drawback: do not account for forward-looking behavior
    - Important aspect to account for since car is a durable good
  - **Econometric literature**: **dynamic programming (DP)** models + discrete choice models (DCM)
  - Recently, **dynamic discrete choice models (DDCM)** starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)

- Joint models of car ownership and usage:
  - Early references: e.g. **duration models** and regression techniques for car holding duration and usage (De Jong, 1996)
  - **Dynamic programming mixed logit (DPMXL)** (Schjerning, 2007) used to model car ownership, type of car and usage (Munk-Nielsen, 2012)
  - **Discrete-continuous model** of vehicle choice and usage based on register data (Gillingham, 2012)

INTRODUCTION

Research issues:

- Car are durable goods $\Longrightarrow$ Need to account for **forward-looking behavior** of individuals
- Difficulty of modeling a **discrete-continuous choice**
- Many models focus on individual decisions, but choices regarding car ownership and usage made at **household level**

INTRODUCTION

Research issues:

- Car are durable goods ⟹ Need to account for **forward-looking behavior** of individuals
- Difficulty of modeling a **discrete-continuous choice**
- Many models focus on individual decisions, but choices regarding car ownership and usage made at **household level**

Proposed methodology:

- Attempt to address these issues by applying **dynamic discrete-continuous choice model (DDCCM)**
- Large **register data** of all **individuals** and **cars** in Sweden

BACKGROUND AND DATA

Register data of Swedish population and car fleet:

- Data from 1998 to 2008
- All individuals
  - **Individual information**: socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
  - **Household information**: composition (families with children and married couples)
- All vehicles
  - Privately-owned cars, cars from privately-owned company and **company** cars
  - Vehicle **characteristics** (make, model, fuel consumption, fuel type, age)
  - **Annual mileage** from odometer readings
  - Car bought **new or second-hand**

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

Aim of the project:

- Model simultaneously car ownership, usage and fuel type.
  In details: model simultaneous choice of

Transaction type × [Annual milage – car *c* × Private/company car – car *c* × Fuel type – car *c* × New/2nd hand – car *c*] ^# cars

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Discrete variables

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Continuous variables

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

Motivations for discrete-continous vs discrete model

- Mileage variable(s) are continuous: lose information by discretizing it.
- In a discrete-continuous approach:

If choice of mileage **conditional** on the discrete choice

⇩

Reduction of size of the discrete action space

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DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

ASSUMPTIONS

- Decisions at household level: up to 2 cars in household
- **Strategic choice** of:
  - Transaction
  - Type(s) of ownership (company vs private car)
  - Fuel type(s)
  - Car state(s) (new vs 2nd-hand)

⟹ Account for forward-looking behavior of households

- **Myopic choice** of:
  - Annual mileage(s)
- **Choice** of **mileage conditional** on choice of discrete variables

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Agent: household
- Time step t: year
- State space S

$$s_t = (\underbrace{y_{1,t}}_{\text{Age – 1st car}}, \underbrace{I_{1,t}}_{\text{Private/ company car – 1st car}}, \underbrace{f_{1,t}}_{\text{Fuel type – 1st car}}, \underbrace{y_{2,t}}_{\text{Age – 2nd car}}, \underbrace{I_{2,t}}_{\text{Private/ company car – 2nd car}}, \underbrace{f_{2,t}}_{\text{Fuel type – 2nd car}})$$

$$\begin{aligned} |S| \quad &= \quad (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \\ &+ \quad (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) \\ &+ \quad 1. \end{aligned}$$

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Agent: household
- Time step *t*: year
- State space *S*

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

Age – 1st car

Private/ company car – 1st car

Fuel type – 1st car

Age – 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

$$\begin{aligned} |S| &= \boxed{(|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2} \quad \text{2-car households} \\ &+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) \\ &+ 1. \end{aligned}$$

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Agent: household
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$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

Age – 1st car

Private/ company car – 1st car

Fuel type – 1st car

Age – 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

$$\begin{aligned} |S| &= (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \\ &+ \boxed{(|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)} \\ &+ 1. \end{aligned}$$

1-car households

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DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

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$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

Age – 1st car

Private/ company car – 1st car

Fuel type – 1st car

Age – 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

$$\begin{aligned} |S| &= (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \\ &+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) \\ &+ \boxed{1}. \end{aligned}$$

0-car households

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Agent: household
- Time step *t*: year
- State space *S*

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

Age – 1st car

Private/ company car – 1st car

Fuel type – 1st car

Age – 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

$$\begin{aligned} |S| &= \left(\overset{=6}{|Y|} \times (\overset{=4}{|I_C| - 2}) \times (\overset{=4}{|F|} - 1) + 1\right)^2 \\ &+ \left(|Y| \times (|I_C| - 2) \times (|F| - 1) + 1\right) \\ &+ 1. \end{aligned}$$

= 1407 ➔ relatively small state space

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Action space *A*

$$a_t = (h_t, \tilde{m}_{1,t}, \tilde{I}_{1,t}, \tilde{f}_{1,t}, \tilde{r}_{1,t}, \tilde{m}_{2,t}, \tilde{I}_{2,t}, \tilde{f}_{2,t}, \tilde{r}_{2,t})$$

Transaction type

Mileage – 1st car

Private/ company car – 1st car

Fuel type – 1st car

New/ 2nd hand – 1st car

Mileage - 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

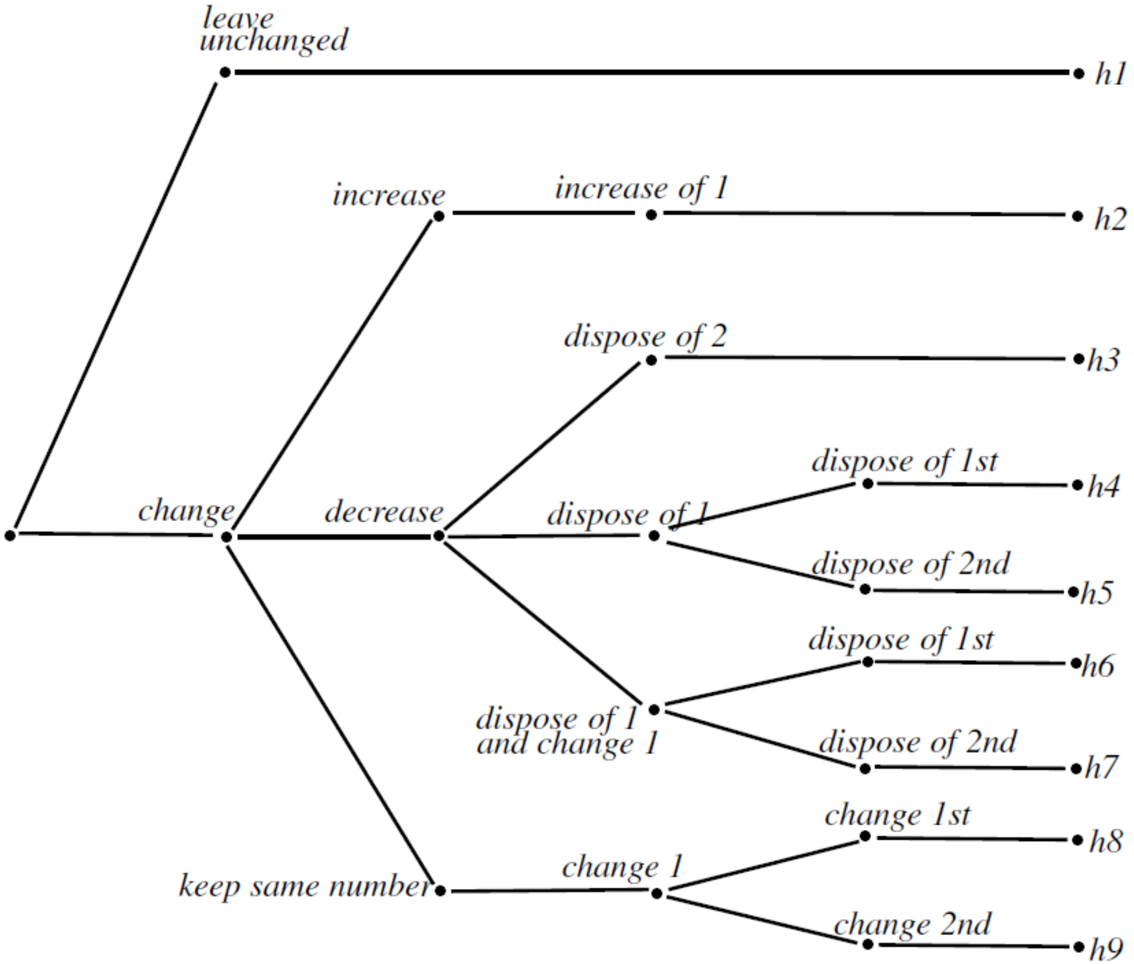
New/ 2nd hand – 2nd car

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Action space *A*

Transaction type: details



DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Action space *A*

Transaction name	0 car	1 car	2 cars
$h1$: leave unchanged	1	1	1
$h2$: increase 1	18	18	-
$h3$: dispose 2	-	-	1
$h4$: dispose 1st	-	1	1
$h5$: dispose 2nd	-	-	1
$h6$: dispose 1st and change 2nd	-	-	18
$h7$: dispose 2nd and change 1st	-	-	18
$h8$: change 1st	-	18	18
$h9$: change 2nd	-	-	18
Sum	19	38	76

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DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Action space *A*

Size of the **discrete part** of the action space assuming:

- 3 levels company car
- 3 levels fuel type
- 2 levels new/2nd hand

Transaction name	0 car	1 car	2 cars
$h1$: leave unchanged	1	1	1
$h2$: increase 1	18	18	-
$h3$: dispose 2	-	-	1
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$h6$: dispose 1st and change 2nd	-	-	18
$h7$: dispose 2nd and change 1st	-	-	18
$h8$: change 1st	-	18	18
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Sum	19	38	$\boxed{76}$

Maximum size of action space << for DDCM

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Transition rule: deterministic rule: each state s_{t+1} can be inferred exactly once s_t and a_t are known.

Example:

2 years private car diesel increase 1

If $s_t = [2,1,2,0,0,0]$ and $a_t = [1,12'000,0,0,0,0,3,0,0]$

mileage 1st car company car

then $s_{t+1} = [3,1,2,0,3,0]$.

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Instantaneous utility function:

$$u(s_t, a_t^C, a_t^D, x_t, \theta) = v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) + \varepsilon_D(a_t^D)$$

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Assume additive **deterministic utility** for simplicity (see also Munk-Nielsen, 2012):

$$v(s_t, a_t^C, a_t^D, x_t, \varepsilon_C(a_t^C), \theta) = v_t^D(s_t, a_t^D, x_t, \theta) + v_t^C(s_t, a_t^D, a_t^C, x_t, \varepsilon_C(a_t^C), \theta)$$

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DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

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DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Instantaneous utility function
  - Utility for continuous actions:
    Constant elasticity of substitution (CES) utility function:
    - Captures substitution patterns between the choice of both annual driving distances
    - ρ is elasticity of substitution

$$v_t^C(s_t, a_t^D, a_t^C, x_t, \varepsilon_C(a_t^C), \theta) = (m_{1,t}^{-\rho} + \alpha \cdot m_{2,t}^{-\rho})^{-1/\rho}$$

  - Randomness introduced in $\alpha := \exp\{\gamma x_t - \varepsilon_C(a_t^C)\}$
  - x_t contains price of fuel, car consumption and other socio-economic characteristics

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

SOLVING THE DYNAMIC PROGRAMMING PROBLEM

1. Finding the optimal value(s) of annual mileage conditional on the discrete choices

2. Solving the Bellman equation

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

SOLVING THE DYNAMIC PROGRAMMING PROBLEM

Finding the optimal value(s) of mileage

- Maximization of the continuous utility: $\max_{m_{1,t}, m_{2,t}} v_t^C$

  s.t. $p_{1,t} m_{1,t} + p_{2,t} m_{2,t} = \text{Inc}_t$

- Find analytical solutions: $m^*_{1,t}$ and $m^*_{2,t}$

$$m^*_{2,t} = \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))} \alpha^{(-1/(\rho+1))}}$$

$$m^*_{1,t} = \frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} m^*_{2,t}$$

- Optimal continuous utility $v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)$

DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

SOLVING THE DYNAMIC PROGRAMMING PROBLEM

Solving the Bellman equation

- Logsum formula used in the completely discrete case (DDCM)
  (Aguirregabiria and Mira, 2010; Cirillo and Xu, 2011)

- Logsum can be applied here given the key assumptions:
  - Choice of mileage(s) is conditional on discrete actions
  - Choice of mileage(s) is myopic

$$\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t^D} \left\{ \exp\{v_t^D(s_t, a_t^D, x_t, \theta) + v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)\} + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1} | s_t, a_t) \right\}$$

- Iterate on Bellman equation to find integrated value function $\bar{V}$

MODEL ESTIMATION

- Parameters obtained by maximizing likelihood:

$$\mathcal{L} = \prod_{n=1}^{N} \prod_{t=1}^{T_n} P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta)$$

- Optimization algorithm is Rust's **nested fixed point algorithm (NFXP)** (Rust, 1987):
  - **Outer optimization algorithm:** search algorithm to obtain parameters maximizing likelihood
  - **Inner value iteration algorithm:** solves the DP problem for each parameter trial
- Plan to investigate variants of NFXP to speed up computational time

ILLUSTRATION OF MODEL APPLICATION

Assumptions for the example:

- Size state space = 651
  - Max age = 3
  - Company car levels = 3
  - Number of fuel types = 3
- Size action space = max 745
  - Number of transaction types = 9
  - Number of state levels (new/old) = 2
- Utility function contains:
  - Transaction cost τ
  - Transaction-dependent parameters for age of oldest car

$$\left.\begin{array}{l}\text{Transaction cost } \tau \\ \text{Transaction-dependent parameters for age of oldest car}\end{array}\right\} \textbf{postulated}$$

$$v_t^D(s_t, a_t^D, x_t, \theta) = \tau(a_t^D) + \beta_{\mathrm{Age}}(a_t^D, s_t) \cdot \max(\mathrm{Age1}_t, \mathrm{Age2}_t)$$

- Parameters of DP problem:
  - Discount factor $\beta = 0.7$
  - Stopping criterion $\varepsilon = 0.01$

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ILLUSTRATION OF MODEL APPLICATION

Program:

- Code in C++
- 2 minutes on 20-core server

Graph:

- $\bar{V}$ vs age of oldest of cars in household fleet

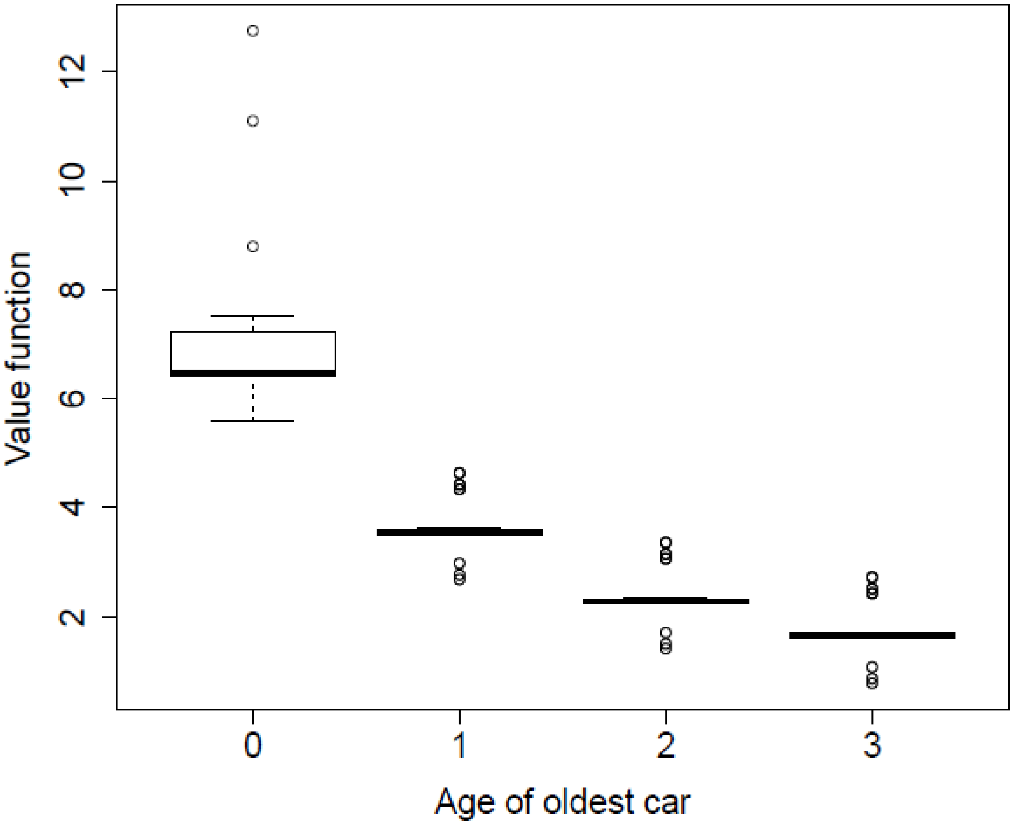
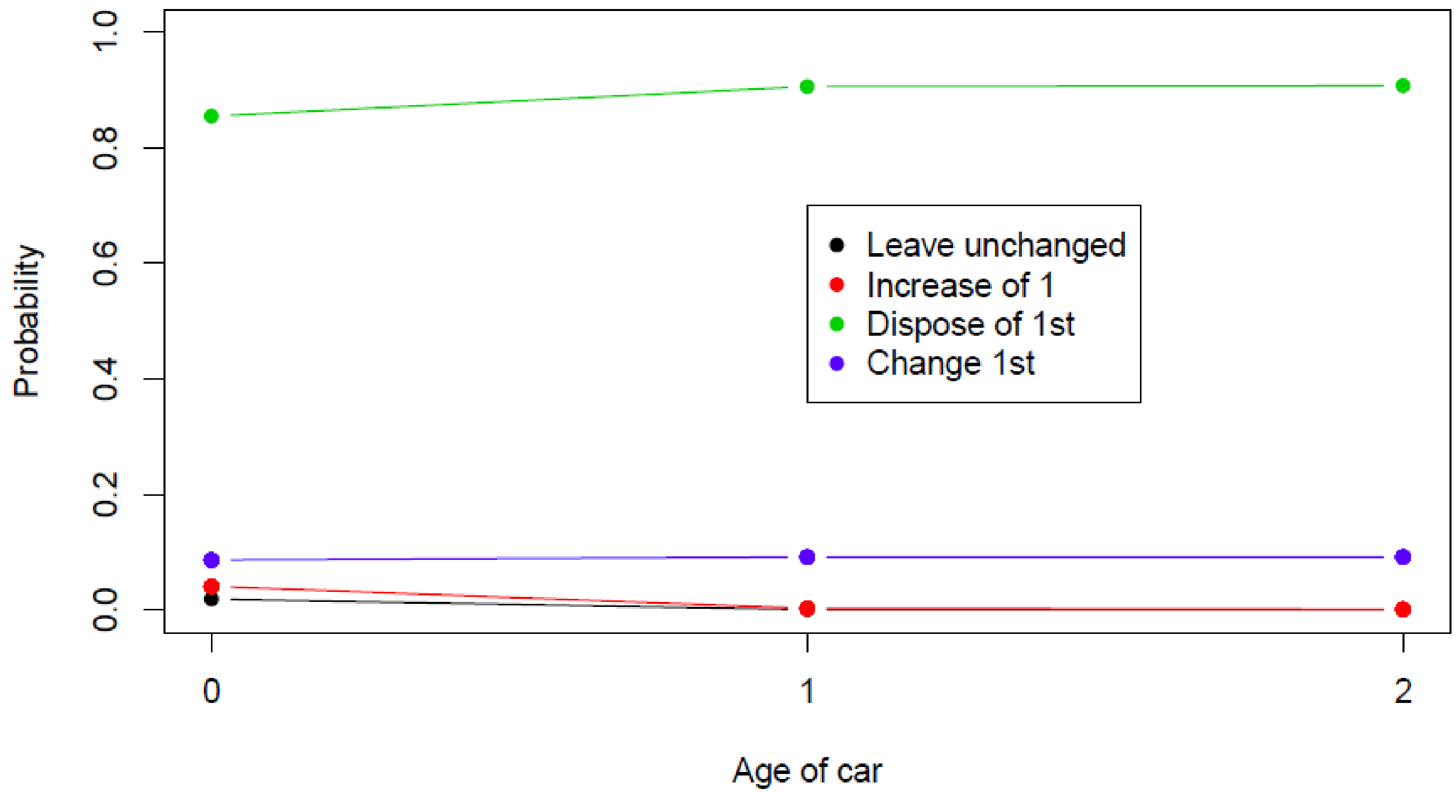
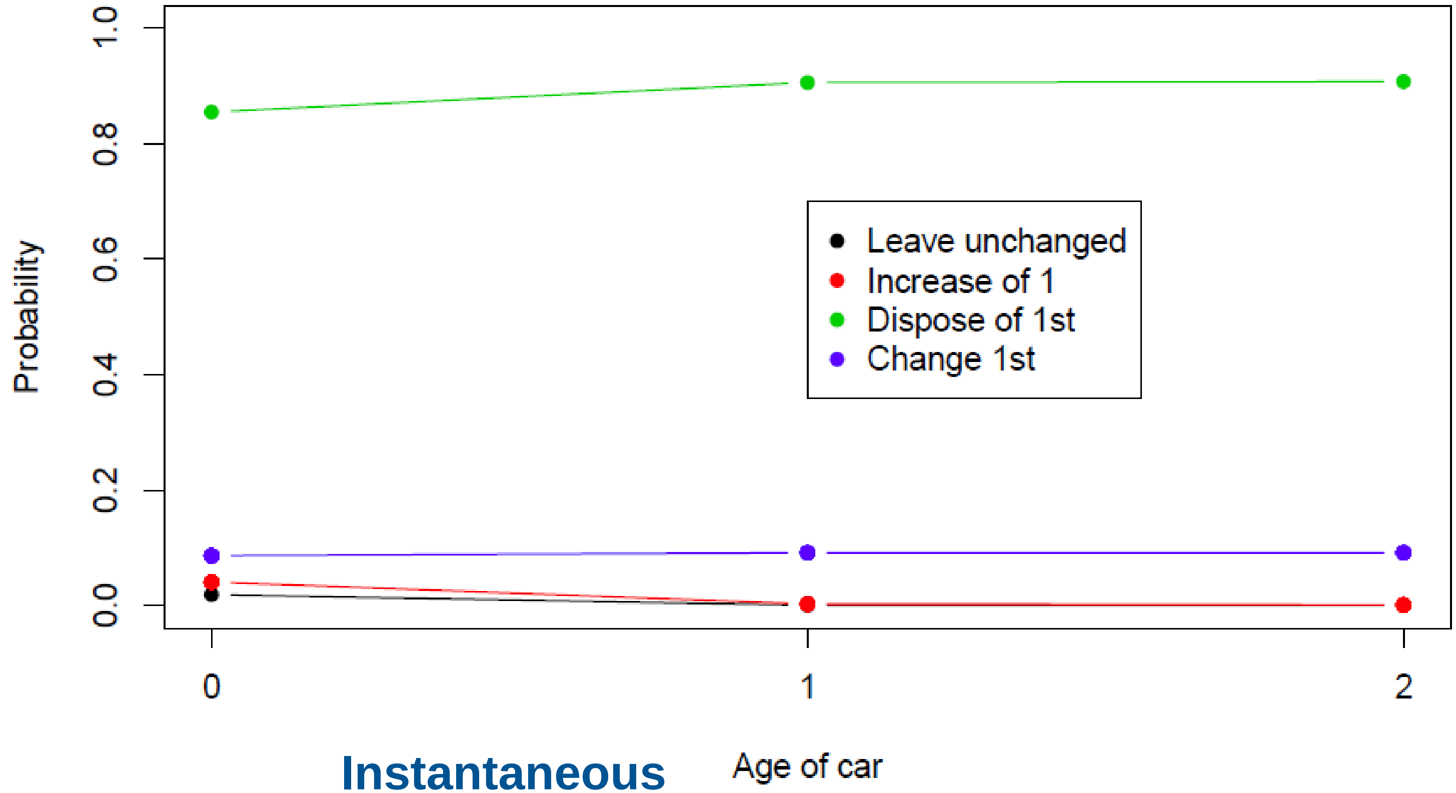


ILLUSTRATION OF MODEL APPLICATION



$$P(a^D_{n,t}|s_{n,t},x_{n,t},\theta) = \frac{v^D_{n,t} + v^{C*}_{n,t} + \beta \sum_{s_{n,t+1}\in S} \bar{V} f}{\sum_{\tilde{a}^D_{n,t}} \left\{ v^D_{n,t} + v^{C*}_{n,t} + \beta \sum_{s_{n,t+1}\in S} \bar{V} f \right\}}$$

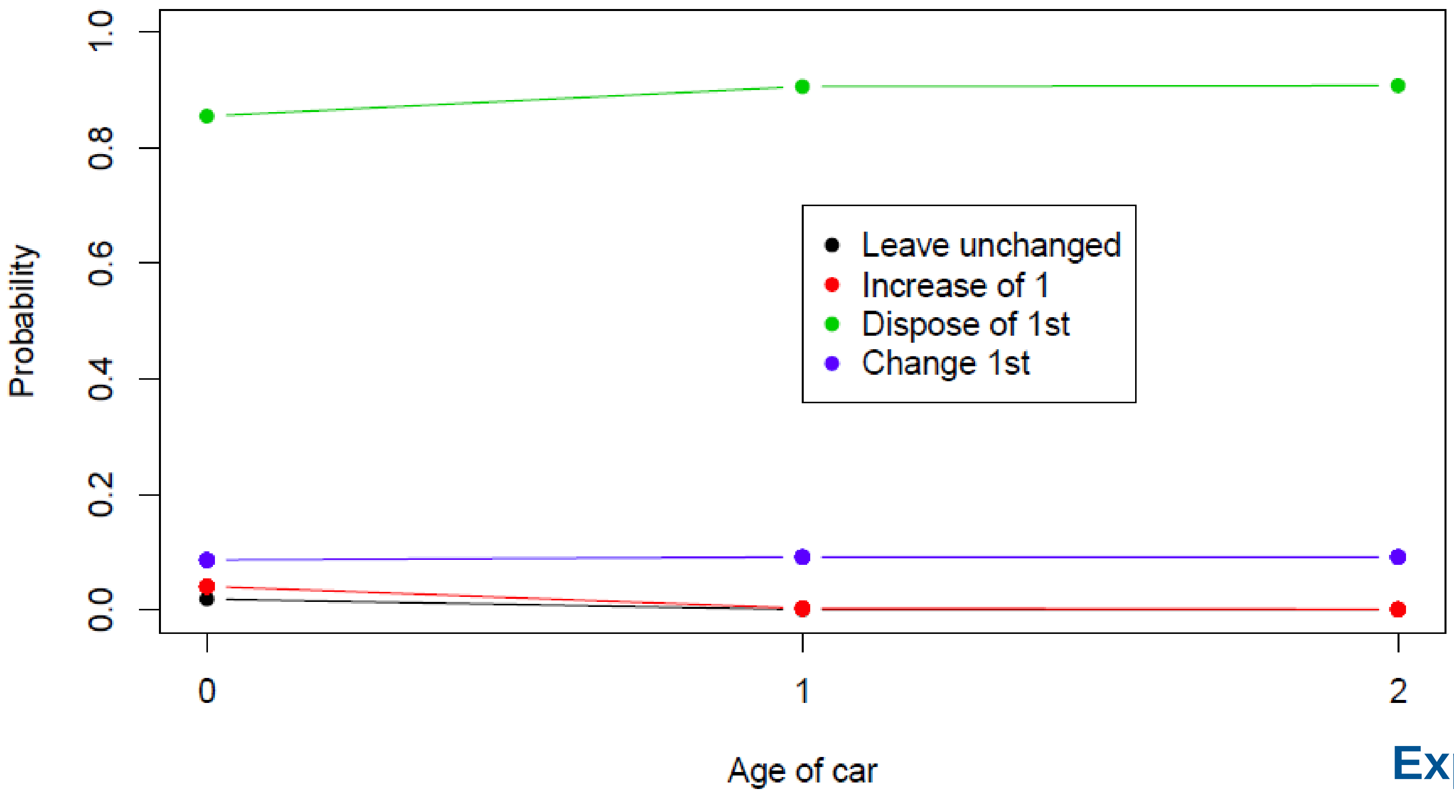
ILLUSTRATION OF MODEL APPLICATION



Instantaneous utility

$$P(a_{n,t}^{D} | s_{n,t}, x_{n,t}, \theta) = \frac{\boxed{v_{n,t}^{D} + v_{n,t}^{C*}} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f}{\sum_{\tilde{a}_{n,t}^{D}} \left\{ v_{n,t}^{D} + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f \right\}}$$

ILLUSTRATION OF MODEL APPLICATION



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Expected discounted utility

CONCLUSION AND FUTURE WORKS

Conclusion:

- Methodology to model choice of car ownership and usage dynamically
- Example of application shows feasibility of approach

Next steps:

- Exploratory analysis to specify instantaneous utility
- Model estimation on small sample of synthetic data
- Model estimation on register data
- Scenario testing:
  - Validation of policy measures taken during the years available in the data
  - Test policy measures that are planned to be applied in future years

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Thanks!