DYNAMIC DISCRETE CHOICE MODELING

FOR CAR USE, OWNERSHIP AND FUEL TYPE BASED ON REGISTER DATA

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OUTLINE

- Introduction
- The data
- Possible approaches
- The dynamic discrete choice model
- Value iteration results
- Conclusion and future works

INTRODUCTION

Aim of the research project:

- Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet

- Motivations
  - Governmental policies (Source: Swedish parliament):
    - Goal of fossil-independent vehicle fleet by 2030
    - No emissions by 2050
  - Technology changes:
    - Increase of alternative-fuel vehicles
    - Changes in the supply
  - Company cars: represent important share of new car sales

- Difficulties
  - Car are durable goods ⟹ modeling transactions not straightforward
  - Need to account for forward-looking behavior of individuals

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INTRODUCTION

Current literature on car ownership and usage modeling:

- Car ownership models in transportation literature:
  - Mostly **static models**:
    - Main drawback: do not account for forward-looking behavior
    - Important aspect to account for since car is a durable good
  - **Econometric literature**: **dynamic programming (DP)** models
  - Recently, **dynamic discrete choice models (DDCM)** starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)

- Other types of joint models of car ownership and usage:
  - **Discrete-continuous model** of vehicle choice and usage based on register data (Gillingham, 2012)
  - **Duration models** and regression techniques for car holding duration and usage (De Jong, 1996)

THE DATA

Register data of Swedish population and car fleet:

- Data from 1998 to 2008
- All individuals
  - **Individual information**: socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
  - **Household information**: composition (families with children and married couples)
- All vehicles
  - Privately-owned cars, cars from privately-owned company and **company** cars
  - Vehicle **characteristics** (make, model, fuel consumption, fuel type, age)
  - **Annual mileage** from odometer readings
  - Car bought **new or second-hand**

THE DATA

Advantage of such detailed data:

- Can observe and analyze **demand shifts** that occurred as a response to changes in policies:
  - Changes in vehicle circulation taxes
  - Changes in fuel prices

    } at a national/regional level
  - Introduction of congestion pricing } at a local/regional level

- Can test the impact of **planned policies** on the demand for vehicles and car usage (e.g. fossil-independent fleet by 2030)

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MOTIVATIONS

Aim of the project:

- Model simultaneously car ownership, usage and fuel type.
  In details: model simultaneous choice of

$$\text{Transaction type} \times \left[\text{Annual milage – car } c \times \text{Private/company car – car } c \times \text{Fuel type – car } c \times \text{New/2}^{\text{nd}}\text{ hand – car } c \right]^{\#\text{ cars}}$$

- No more than 2 cars in household

- Account for forward-looking behavior of households

POSSIBLE APPROACHES

How can car ownership and usage be modeled?

Studied 2 approaches:

1. Discrete-continuous dynamic programming:
   mileage(s) considered as continuous and other variables as discrete.

2. Dynamic discrete choice modeling:
   all components of a choice variable are discretized.

POSSIBLE APPROACHES

How can car ownership and usage be modeled?

1. Discrete-continuous DP

   - **Endogeneous grid method (EGM)** to solve continuous choice problems (Carroll, 2006) in a fast way: avoids a numerical rootfinding procedure when the Euler equation is solved

   - EGM **generalized for discrete-continuous** choices (Iskhakov et al, 2012). Valid when one continuous variable is considered.

POSSIBLE APPROACHES

How can car ownership and usage be modeled?

2. DDCM

- All variables considered as discrete ⟹ mileage(s) discretized
- In transportation research, DP methods using random utility theory developed for discrete actions
  - Account for random term in utility function
  - Assumption that choices are affected by unobserved attributes, taste variations, etc.
- These DP methods lead to simple closed-form formula for the Bellman equation (e.g. Aguirregabiria and Mira, 2010)

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POSSIBLE APPROACHES

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Approach selected

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POSSIBLE APPROACHES

Motivations for DDCM

- In transportation engineering, usually work with random term in utility
- Though treating mileage as continuous is more adapted, no work on discrete-continuous DP models accommodating random utility theory
- Current DDCM were developed for a different setting:
  - Rust (1987) (and other applications) uses discrete actions and discretizes a continuous state space.
  - Here: large and discrete-continuous action space (while the state space is discrete and small)

THE DYNAMIC DISCRETE CHOICE MODEL

Definition of the components of the dynamic programming model:

- Agent: household
- Time step *t*: year
- State space *S*

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

- Age – 1st car
- Private/ company car – 1st car
- Fuel type – 1st car
- Age – 2nd car
- Private/ company car – 2nd car
- Fuel type – 2nd car

$$\begin{aligned} |S| &= (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \\ &+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) \\ &+ 1. \end{aligned}$$

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Private/ company car – 1st car

Fuel type – 1st car

Age – 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

$$\begin{aligned} |S| &= \boxed{(|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2} \\ &+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) \\ &+ 1. \end{aligned}$$

2-car households

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Age – 1st car

Private/ company car – 1st car

Fuel type – 1st car

Age – 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

$$\begin{aligned} |S| &= (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \\ &+ \boxed{(|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)} \\ &+ 1. \end{aligned}$$

1-car households

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$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

- $y_{1,t}$: Age – 1st car
- $I_{1,t}$: Private/ company car – 1st car
- $f_{1,t}$: Fuel type – 1st car
- $y_{2,t}$: Age – 2nd car
- $I_{2,t}$: Private/ company car – 2nd car
- $f_{2,t}$: Fuel type – 2nd car

$$\begin{aligned} |S| &= (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \\ &+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) \\ &+ \boxed{1.} \end{aligned}$$

0-car households

THE DYNAMIC DISCRETE CHOICE MODEL

Definition of the components of the dynamic programming model:

- Agent: household
- Time step *t*: year
- State space *S*

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

Age – 1st car

Private/ company car – 1st car

Fuel type – 1st car

Age – 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

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$$\begin{aligned} |S| &= (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1)^2 \\ &+ (|Y| \times (|I_C| - 2) \times (|F| - 1) + 1) \\ &+ 1. \end{aligned}$$

= 1407 ➔ relatively small state space

THE DYNAMIC DISCRETE CHOICE MODEL

Definition of the components of the dynamic programming model:

- Action space *A*

$$a_t = (h_t, \tilde{m}_{1,t}, \tilde{I}_{1,t}, \tilde{f}_{1,t}, \tilde{r}_{1,t}, \tilde{m}_{2,t}, \tilde{I}_{2,t}, \tilde{f}_{2,t}, \tilde{r}_{2,t})$$

Transaction type

Mileage – 1st car

Private/ company car – 1st car

Fuel type – 1st car

New/ 2nd hand – 1st car

Mileage - 2nd car

Private/ company car – 2nd car

Fuel type – 2nd car

New/ 2nd hand – 2nd car

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Definition of the components of the dynamic programming model:

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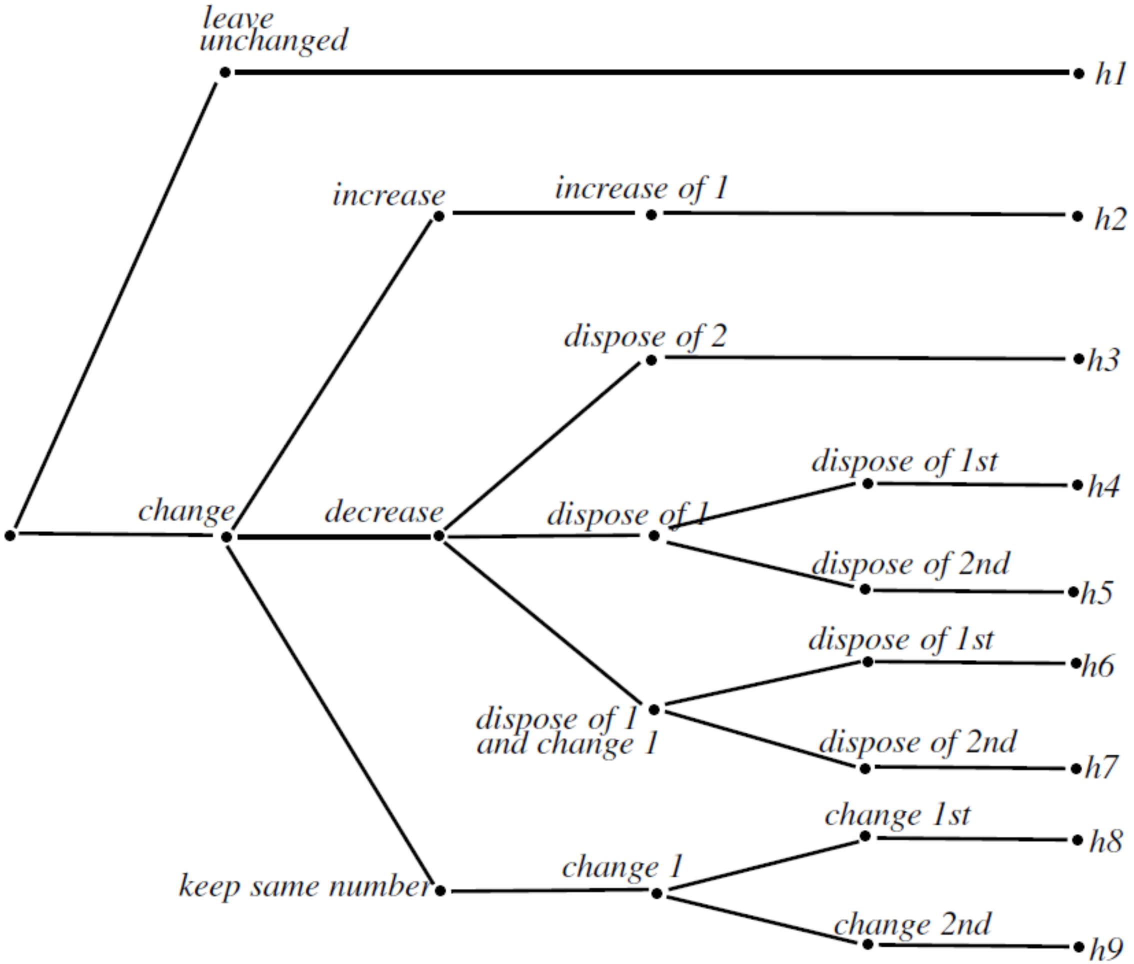
Transaction type: details



THE DYNAMIC DISCRETE CHOICE MODEL

Definition of the components of the dynamic programming model:

- Action space *A*

Transaction name	0 car	1 car	2 cars
$h1$: leave unchanged	1	4	16
$h2$: increase 1	72	288	-
$h3$: dispose 2	-	-	1
$h4$: dispose 1st	-	1	4
$h5$: dispose 2nd	-	-	4
$h6$: dispose 1st and change 2nd	-	-	72
$h7$: dispose 2nd and change 1st	-	-	72
$h8$: change 1st	-	72	288
$h9$: change 2nd	-	-	288
Sum	73	365	745

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THE DYNAMIC DISCRETE CHOICE MODEL

Definition of the components of the dynamic programming model:

- Action space *A*

Size of the action space assuming:

- 4 levels mileage
- 3 levels company car
- 3 levels fuel type
- 2 levels new/2nd hand

Transaction name	0 car	1 car	2 cars
$h1$: leave unchanged	1	4	16
$h2$: increase 1	72	288	-
$h3$: dispose 2	-	-	1
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Maximum size of action space

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Definition of the components of the dynamic programming model:

- Instantaneous utility: $u(s_t, a_t, x_t, \theta) = v(s_t, a_t, x_t, \theta) + \varepsilon(a_t)$
- Transition rule: deterministic rule: each state s_{t+1} can be inferred exactly once s_t and a_t are known.

Example:

2 ans | private car | diesel | increase 1 | mileage level 1

If $s_t = [2,1,2,0,0,0]$ and $a_t = [1,2,0,0,0,1,3,0,0]$

mileage level 2 | company car

then $s_{t+1} = [3,1,2,0,3,0]$.

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THE DYNAMIC DISCRETE CHOICE MODEL

Resolution of the dynamic programming problem:

Value iteration

- Value function:

$$V(s_t, x_t, \theta) = \max_{a_t \in A}\{u(s_t, a_t, x_t, \theta) + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1}|s_t, a_t)\}$$

- Using the integrated value function $\bar{V}(s_t, x_t, \theta) = \int V(s_t, x_t, \varepsilon_t) dG_\varepsilon(\varepsilon_t)$
  - Another update rule that has a closed-form expression can be defined
  - Obtained from the expected maximum utility

$$\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t \in A} \exp\{v(s_t, a_t, x_t, \theta) + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1}|s_t, a_t)\}$$

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THE DYNAMIC DISCRETE CHOICE MODEL

Resolution of the DDCM problem:

- Parameters obtained by maximizing likelihood:

$$\mathcal{L} = \prod_{n=1}^{N} \prod_{t=1}^{T} P(a_{n,t} | s_{n,t}, x_{n,t}, \theta) f(s_{n,t} | s_{n,t-1}, a_{n,t-1})$$

- Optimization algorithm is Rust's **nested fixed point algorithm** (Rust, 1987):
  - **Outer optimization algorithm:** search algorithm to obtain parameters maximizing likelihood
  - **Inner value iteration algorithm:** solves the DP problem for each parameter trial

VALUE ITERATION RESULTS

Preliminary results for the value iteration algorithm:

Inputs:

- Size state space = 1407
  - Max age = 5
  - Company car levels = 3
  - Number of fuel types = 3
- Size action space = max 745
  - Number of transaction types = 9
  - Number of mileage levels = 4
  - Number of levels of new/old = 2
- Utility function contains:
  - Transaction-dependent parameters for fuel type
  - Transaction-dependent parameters for age of oldest car

  } arbitrarily defined
- Parameters of DP problem:
  - Discount factor $\beta = 0.7$
  - Stopping criterion $\varepsilon = 0.01$

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VALUE ITERATION RESULTS

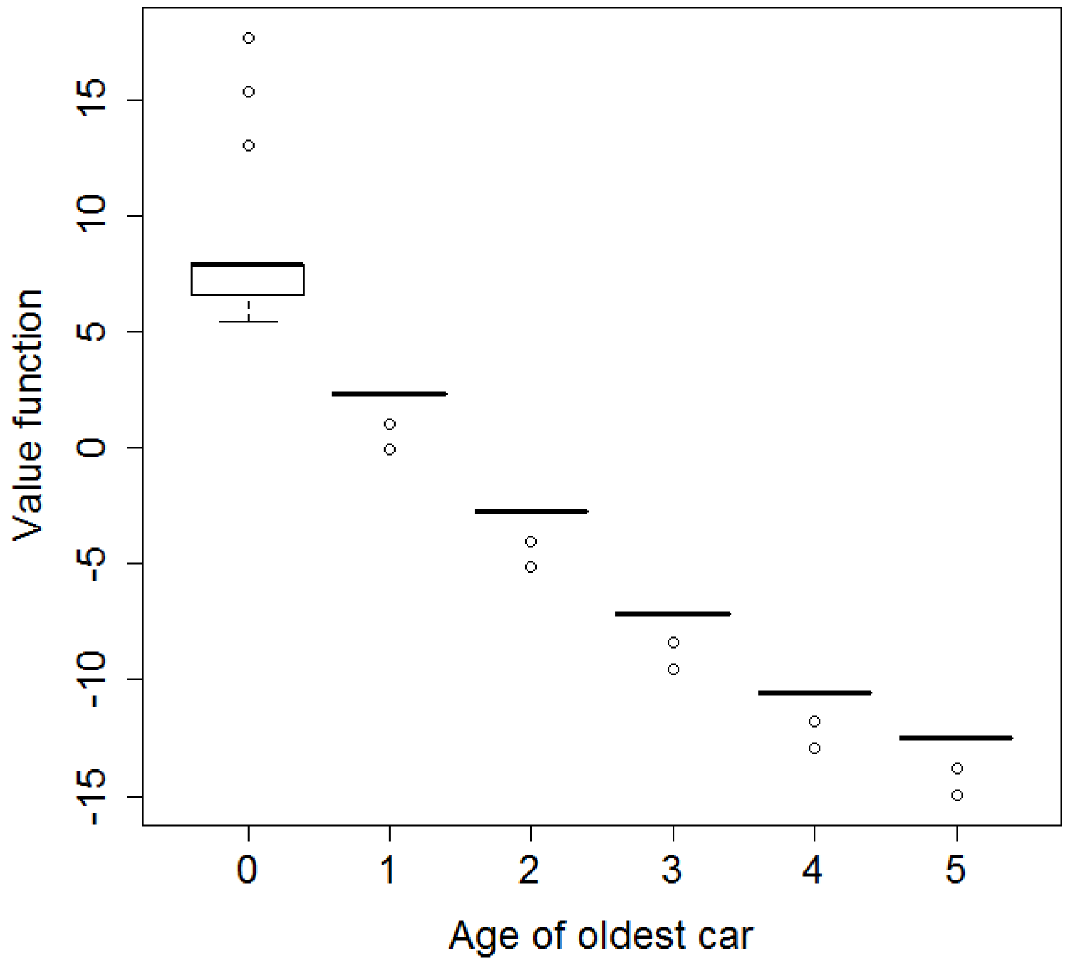
Preliminary results for the value iteration algorithm:

Program:

- Prototype in Matlab
- 15 hours on 8-core server

Graph:

- $\bar{V}$ vs age of oldest of 2 cars



VALUE ITERATION RESULTS

Preliminary results for the value iteration algorithm:

Program:

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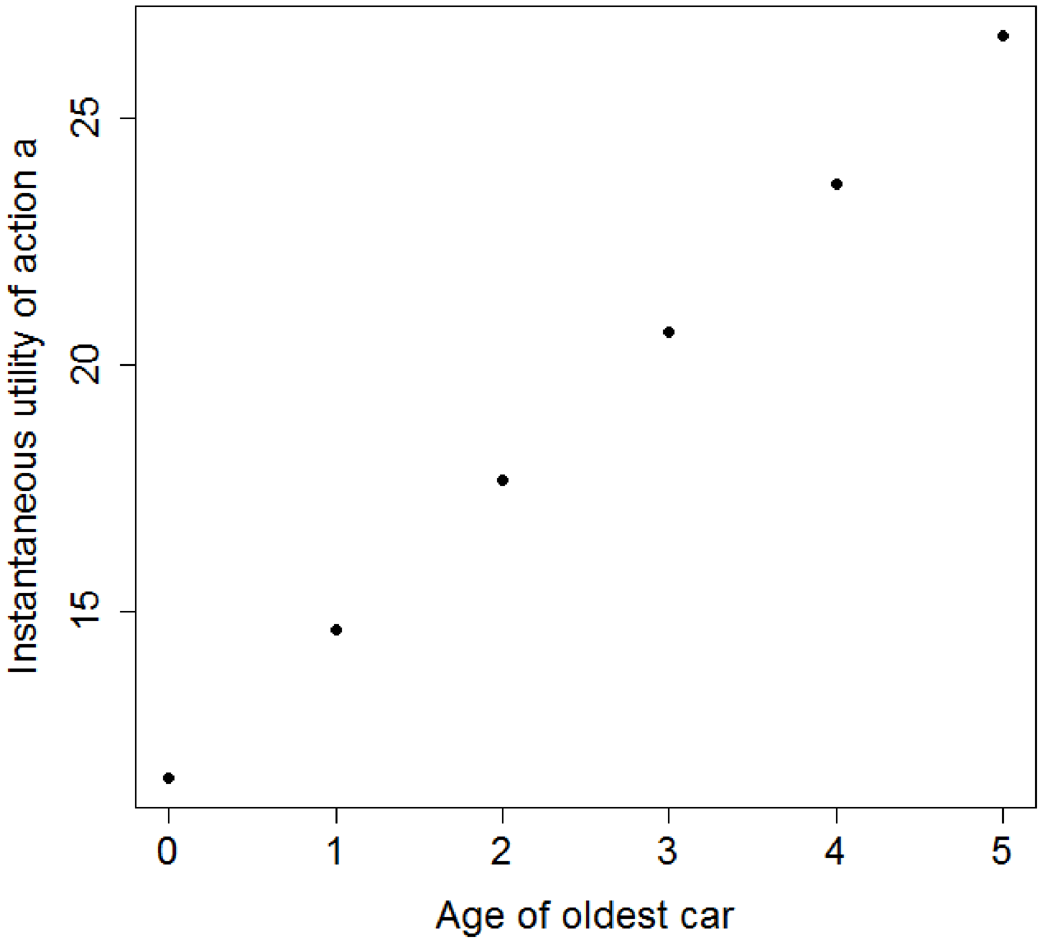
Graph:

- Assume an initial action $a = [3, 1, 0, 0, 0, 0, 0, 0, 0]$ $u(a)$ vs age of oldest car



VALUE ITERATION RESULTS

Preliminary results for the value iteration algorithm:

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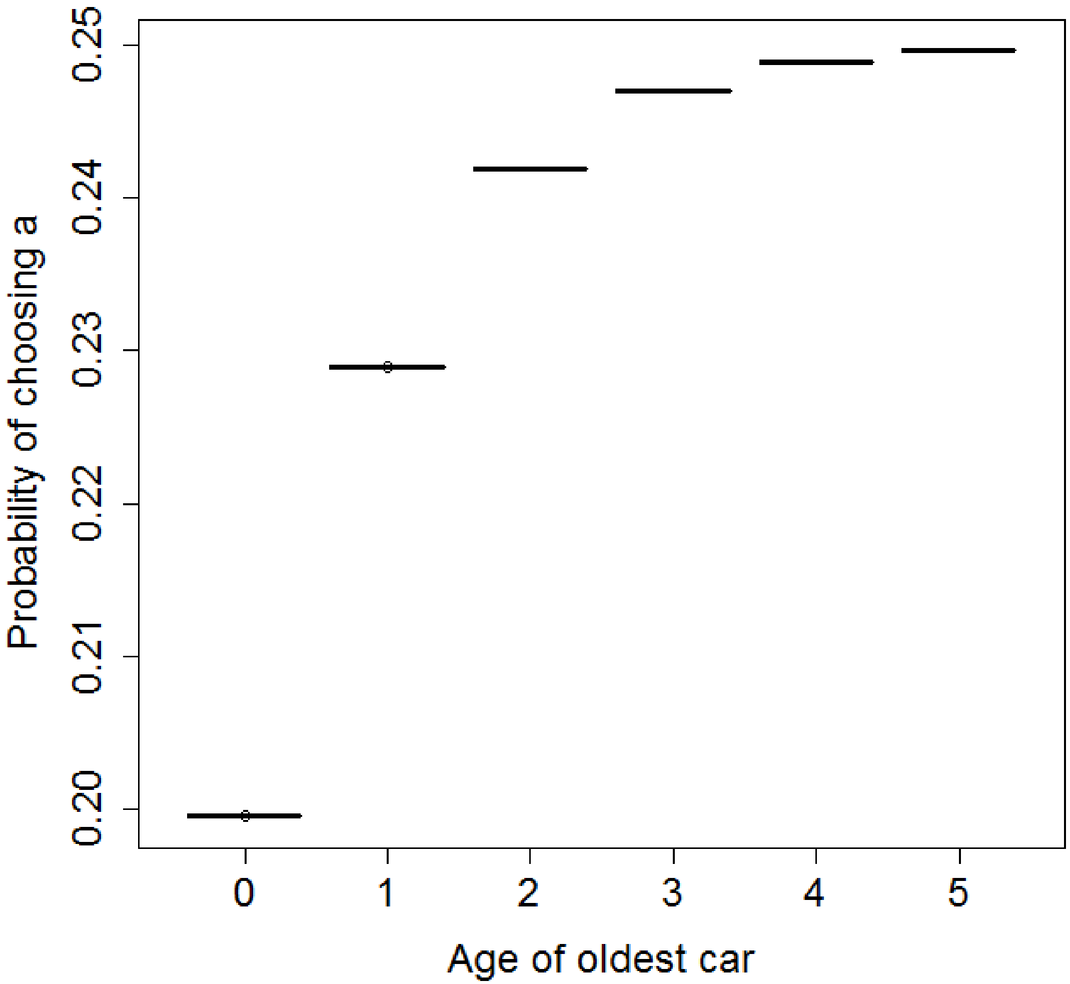
Graph:

- Assume an initial action $a = [3, 1, 0, 0, 0, 0, 0, 0, 0]$ $P(a)$ vs age of oldest car



CONCLUSION AND FUTURE WORKS

Conclusion:

- First results of value function
- Shows feasibility of problem

Future works:

- Speed up computation of integrated value function ⟹ C++
- Sequential choice of two vehicles if computational time still too large
- Exploratory analysis to specify instantaneous utility
- Implement outer loop (maximization algorithm)
- Scenario testing:
    - Validation of policy measures taken during the years available in the data
    - Test policy measures that are planned to be applied in future years

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Thanks!