
MODELING CAR OWNERSHIP AND USAGE: A DYNAMIC DISCRETE-CONTINUOUS CHOICE MODELING APPROACH

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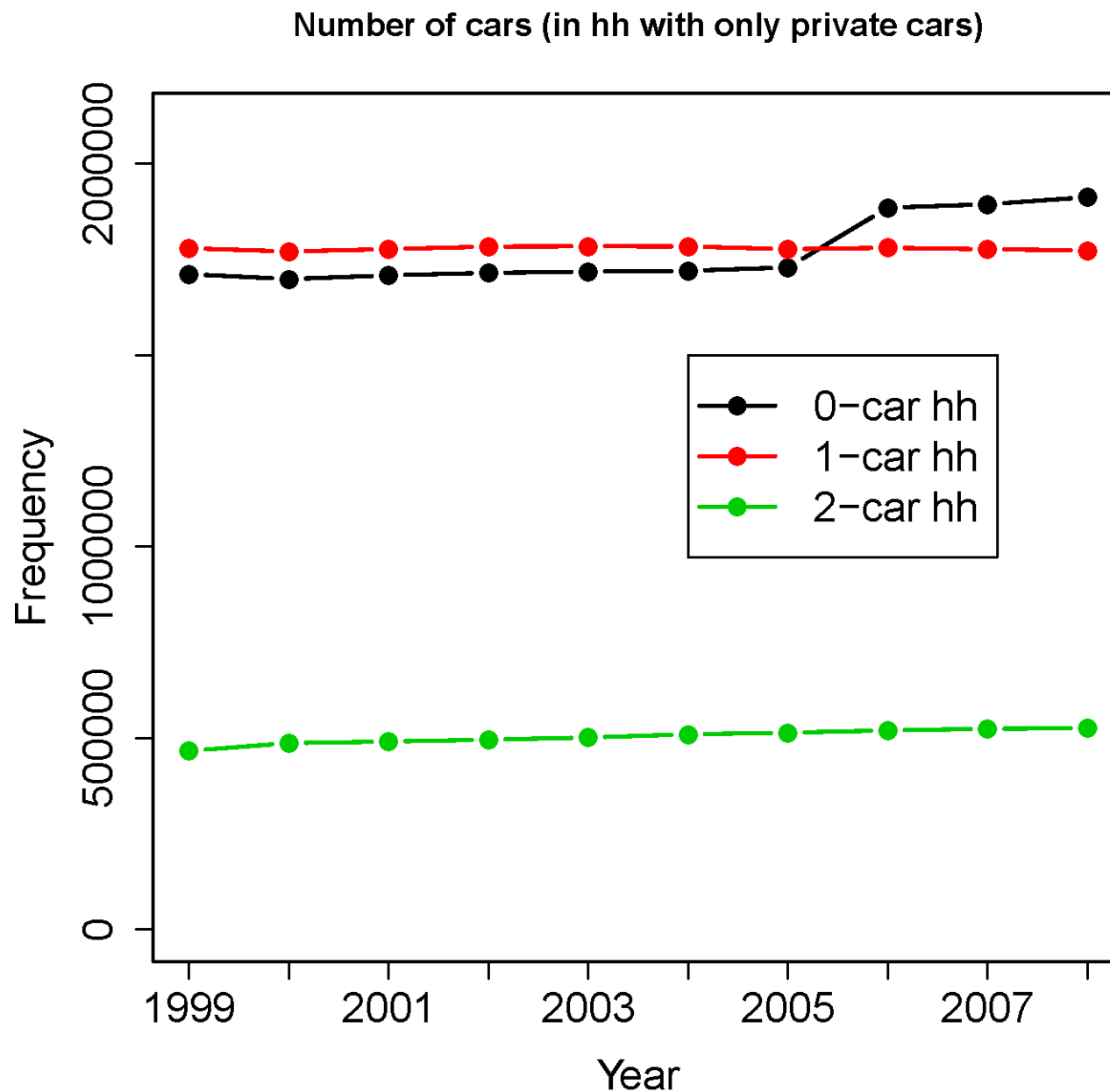
- Introduction
- Background and data
- The dynamic discrete-continuous choice modeling framework
- Illustration of model application
- Conclusion and future works

Aim of the research:

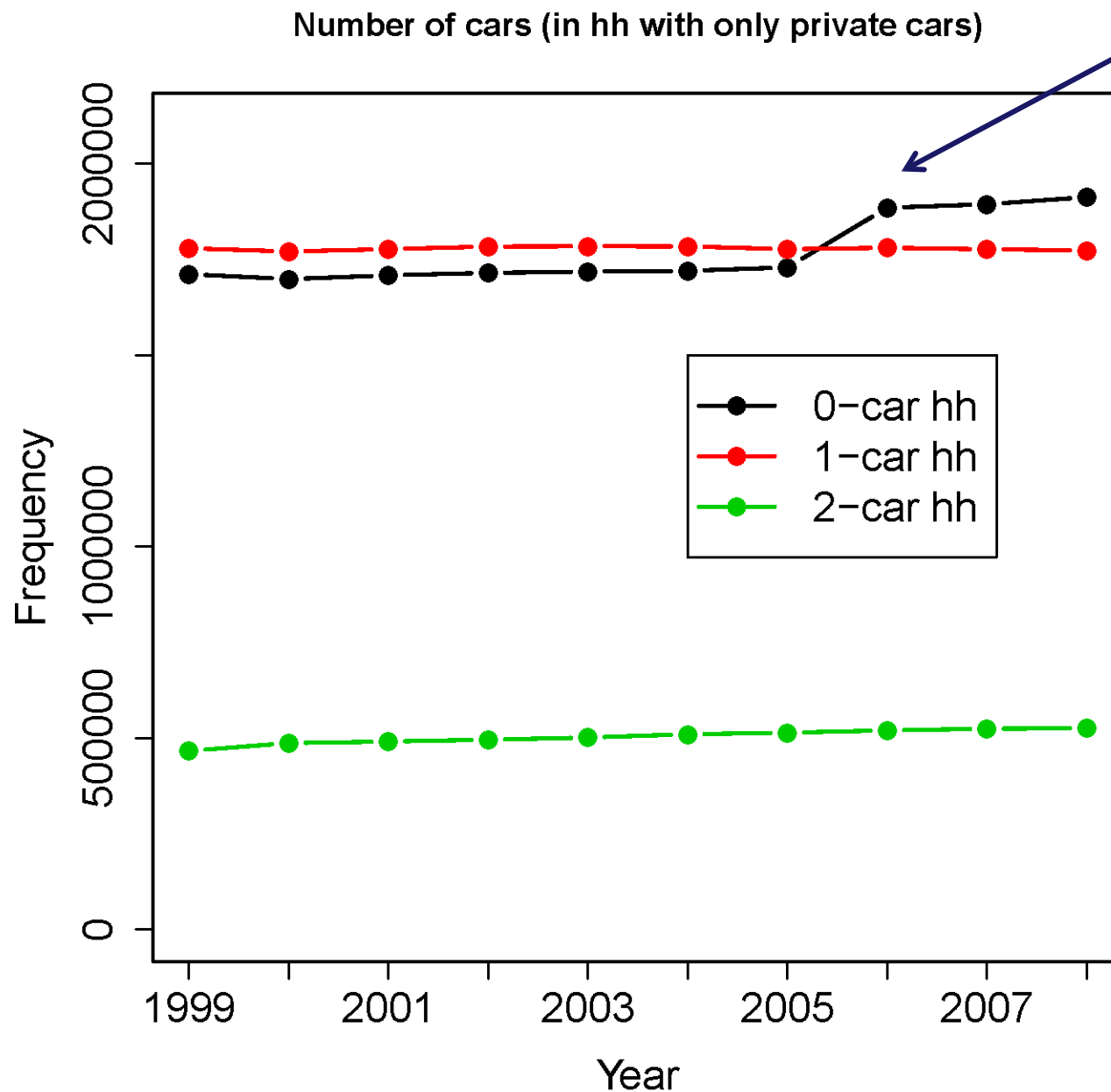
- Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet
 - Motivations
 - Governmental policies to reduce carbon emissions / car usage:
 - Stockholm congestion tax
 - Independence of fossil fuels
 - Technology changes:
 - Increase of alternative-fuel vehicles
 - Economical features:
 - Financial crisis
 - Fuel price changes
- ⇒ Car ownership and usage vary importantly over time.
- ⇒ Model needed to analyze and predict impact of policies on ownership

Register data of Swedish population and car fleet:

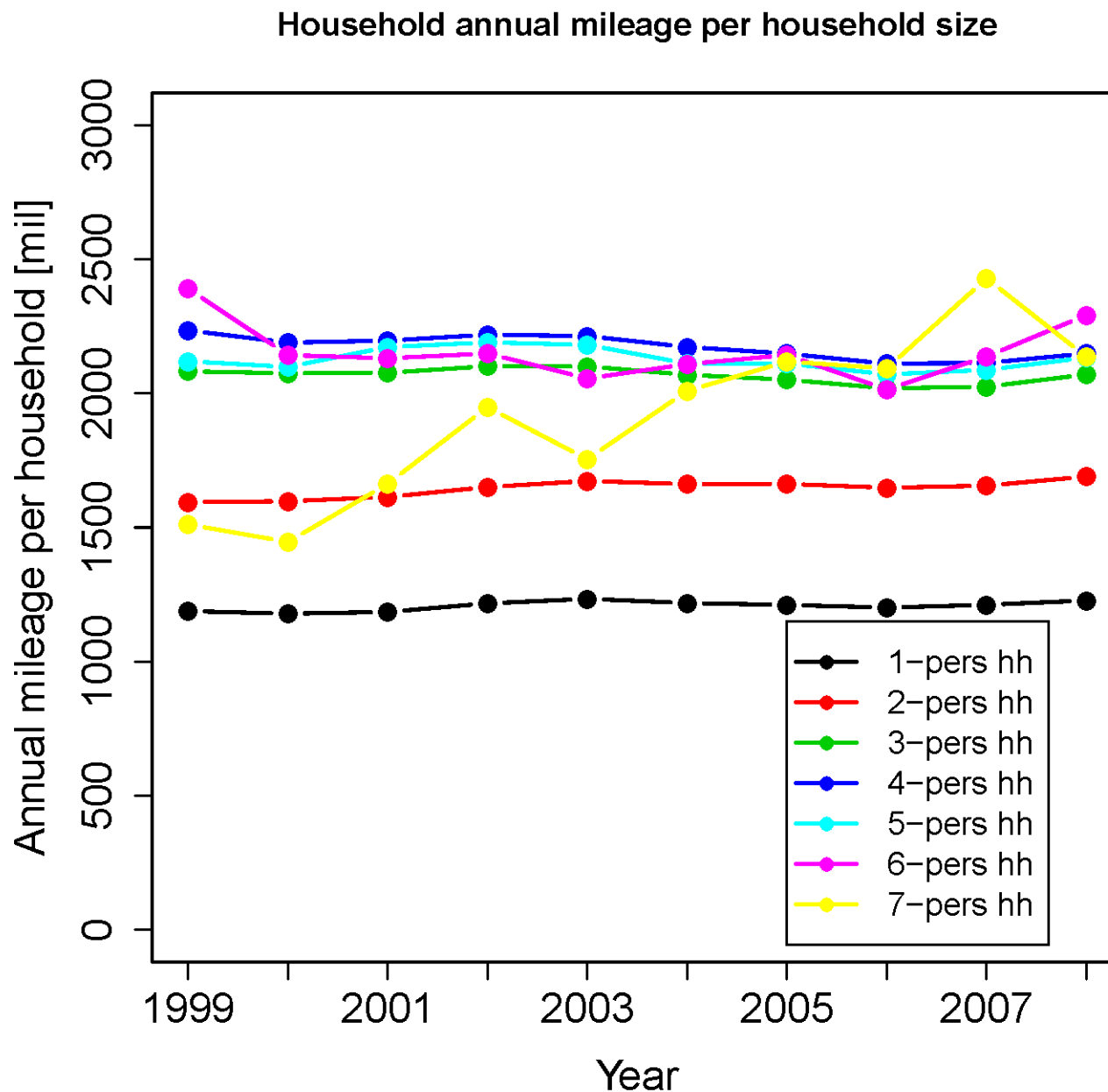
- Data from 1998 to 2008
- All individuals
 - **Individual information:** socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
 - **Household information:** composition (families with children and married couples)
- All vehicles
 - Privately-owned cars, cars from privately-owned company and **company cars**
 - Vehicle **characteristics** (make, model, fuel consumption, fuel type, age)
 - **Annual mileage** from odometer readings
 - Car bought **new or second-hand**



BACKGROUND AND DATA



OWNERSHIP
Congestion tax
Stockholm



LITERATURE

- **Car ownership models in transportation literature:**
 - Discrete choice models (DCM) widely used, but mostly **static models**.
 - Main drawback: do not account for forward-looking behavior
 - Important aspect to account for since car is a durable good
 - **Econometric literature: dynamic programming (DP)** models + DCM
 - Recently, **dynamic discrete choice models (DDCM)** starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)
- **Joint models of car ownership and usage:**
 - **Duration models** and regression techniques for car holding duration and usage (De Jong, 1996)
 - Vehicle type, usage and replacement decisions using **dynamic programming, discrete-continuous**, mixed logit (Schjerning, 2008, and Munk-Nielsen, 2012)
 - **Discrete-continuous model** of vehicle choice and usage based on register data (Gillingham, 2012)
- **Wide literature on car ownership and usage models**

- Car are durable goods \Rightarrow Need to account for **forward-looking behavior** of agents
- Difficulty of modeling a **discrete-continuous choice** when jointly modeling car ownership and usage
- Many models focus on individual decisions, but choices regarding car ownership and usage made at **household level**

- Car are durable goods \Rightarrow Need to account for **forward-looking behavior** of agents
- Difficulty of modeling a **discrete-continuous choice** when jointly modeling car ownership and usage
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Proposed methodology:

- Attempt to address these issues by applying **dynamic discrete-continuous choice model (DDCCM)**

Large **register data** of all **individuals** and **cars** in Sweden



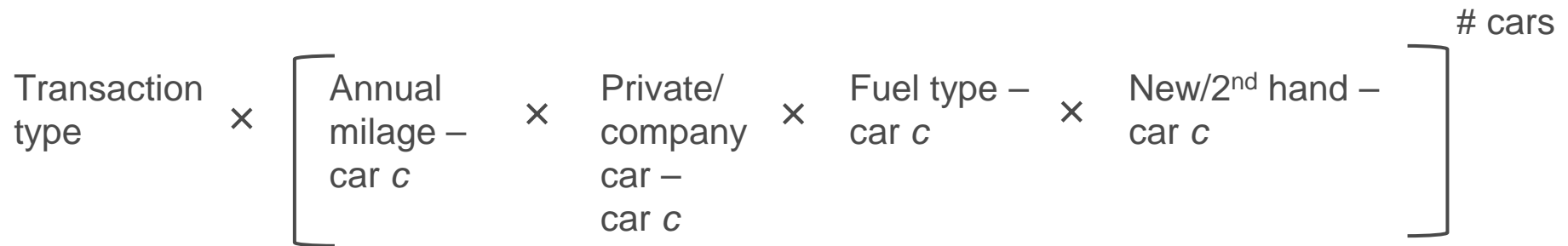
MAIN FEATURES

- In the area of **dynamic choice modeling**
 - Choices modeled at **household level**
 - **Up to two cars** allowed
- **Constant elasticity of substitution (CES) utility** to model annual driving distance for 2-car households
- Several **choices modeled simultaneously**

Objective

Model simultaneously car ownership, usage and fuel type.

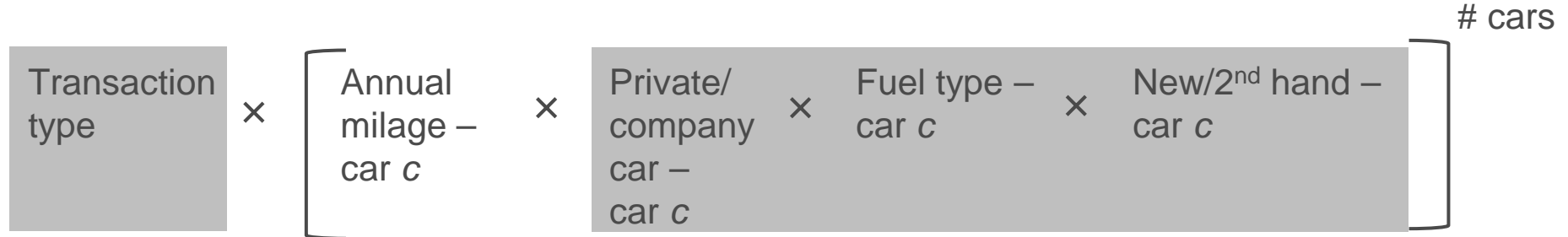
In details: model **simultaneous choice** of



Objective

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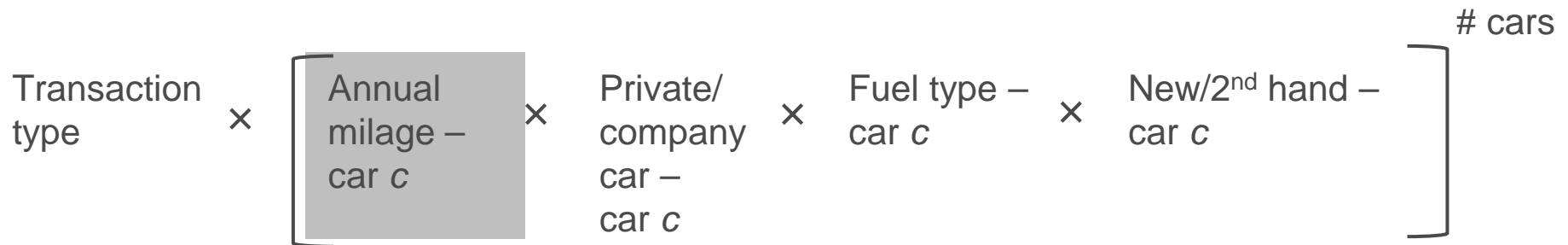


Discrete variables

Objective

Model simultaneously car ownership, usage and fuel type.

In details: model **simultaneous choice** of



Continuous variables

1. **Choice at household level:** up to 2 cars in household
2. **Strategic choice of:**
 - Transaction
 - Type(s) of ownership (company vs private car)
 - Fuel type(s)
 - Car state(s) (new vs 2nd-hand)

⇒ Account for forward-looking behavior of households
3. **Myopic choice of:**
 - Annual mileage(s)
4. **Choice of mileage conditional** on choice of discrete variables

Myopic choice (static case)

$$P(\text{action}) = \frac{\exp\{\text{instantaneous utility}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities}\}}$$

Strategic choice (dynamic case)

$$P(\text{action}) = \frac{\exp\{\text{instantaneous utility} + \text{expected discounted utility of future choices}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities} + \text{expected discounted utilities of future choices}\}}$$

Myopic choice (static case)

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Strategic choice (dynamic case)

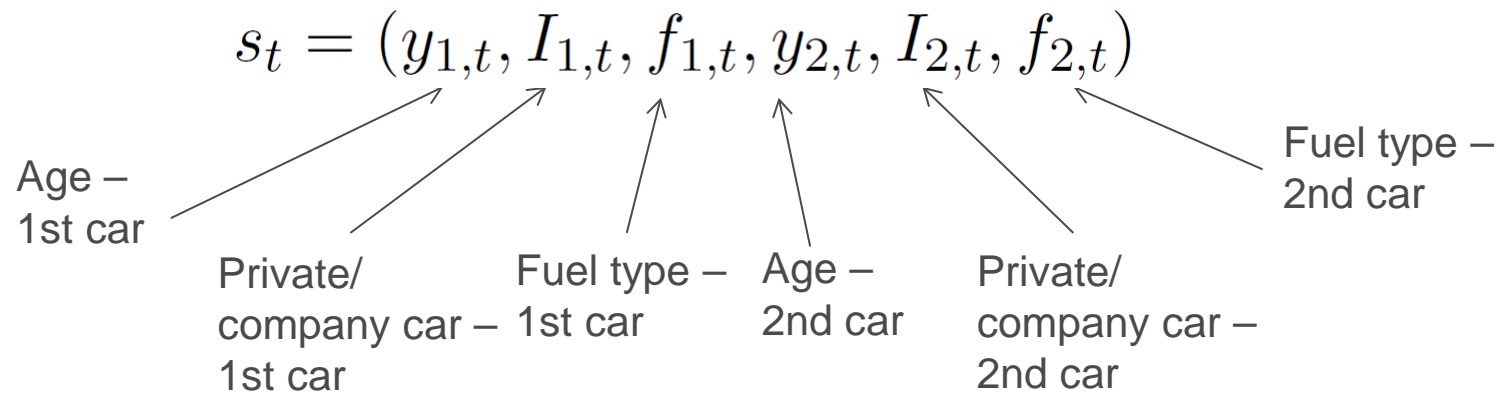
$$P(\text{action}) = \frac{\exp\{\text{instantaneous utility} + \text{expected discounted utility of future choices}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities} + \text{expected discounted utilities of future choices}\}}$$

Embeds a **choice model** into a **dynamic programming framework**

Components of the DDCCM:

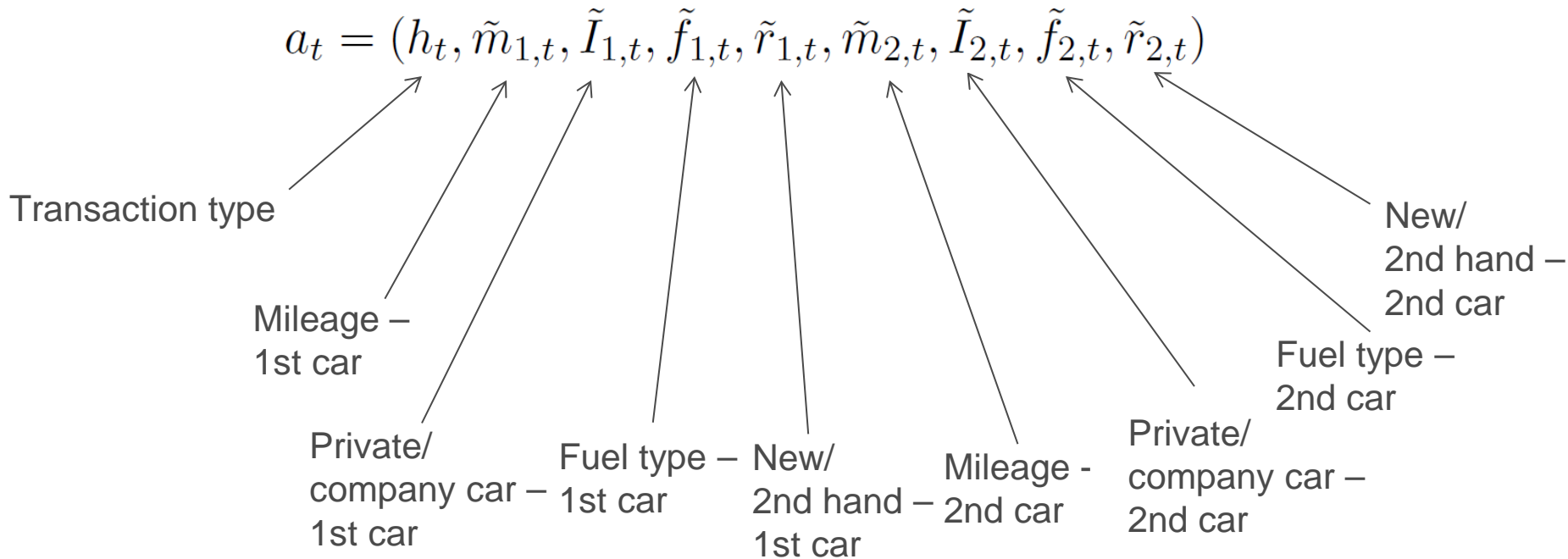
- Agent
- Time step
- State space
- Action space
- Transition rule
- Instantaneous utility function

- **Agent:** household
- **Time step** t : year
- **State space** S



DEFINITION OF THE COMPONENTS

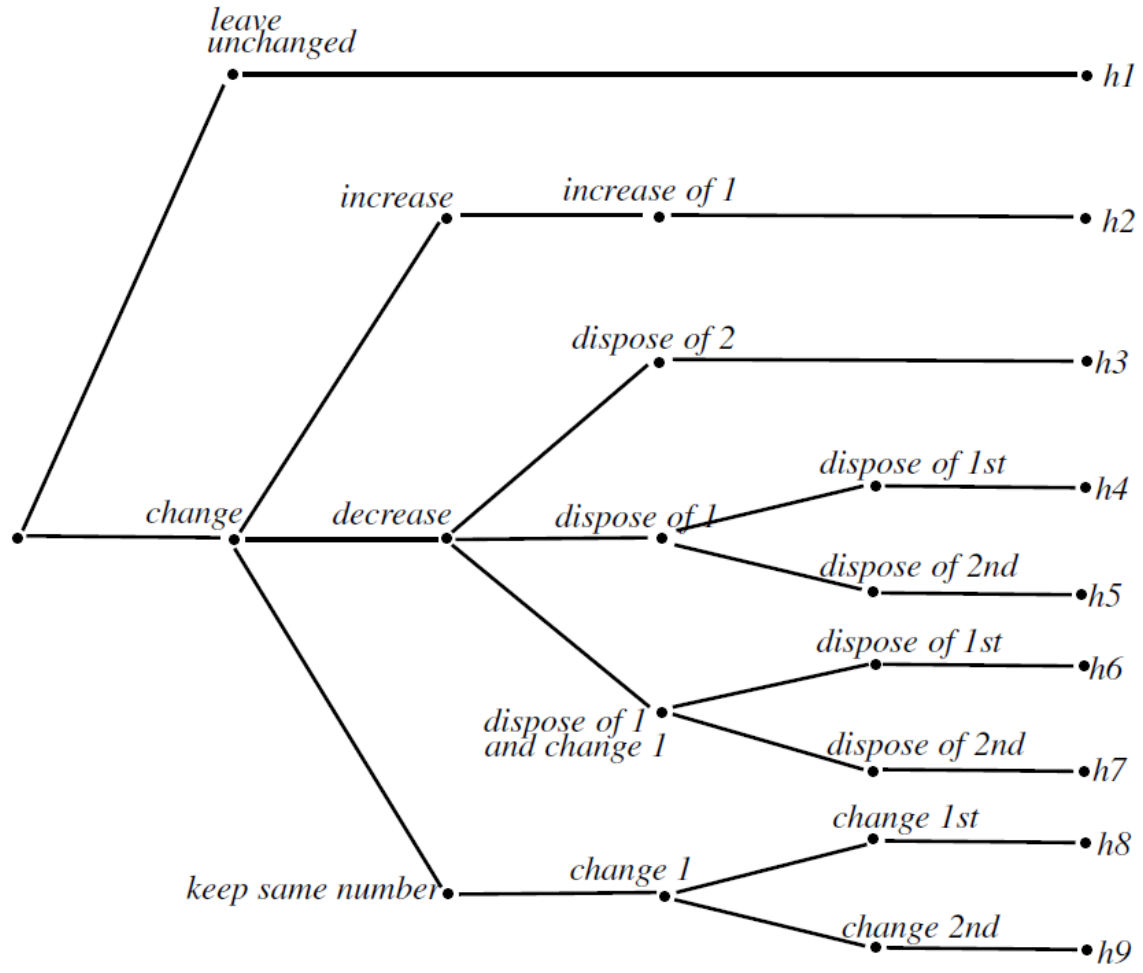
- Action space A



DEFINITION OF THE COMPONENTS

- Action space A

Transaction types



- **Transition rule:** deterministic rule: each state s_{t+1} can be inferred exactly once s_t and a_t are known.

DEFINITION OF THE COMPONENTS

- **Transition rule**: deterministic rule: each state s_{t+1} can be inferred exactly once s_t and a_t are known.
- **Instantaneous utility function**:

$$u(s_t, a_t^C, a_t^D, x_t, \theta) = \underbrace{v(s_t, a_t^C, a_t^D, x_t, \theta)}_{\text{Deterministic term}} + \underbrace{\varepsilon_D(a_t^D)}_{\text{Random term for discrete choices}}$$

Assume additive **deterministic utility** for simplicity (see also Munk-Nielsen, 2012):

$$v(s_t, a_t^C, a_t^D, x_t, \theta) = \underbrace{v_t^D(s_t, a_t^D, x_t, \theta)}_{\text{Utility for discrete actions}} + \underbrace{v_t^C(s_t, a_t^D, a_t^C, x_t, \theta)}_{\text{Utility for continuous actions}}$$

- Instantaneous utility function

- Utility for continuous actions:

Constant elasticity of substitution (CES) utility function (e.g. Zabalza, 1983):

$$v_t^C(s_t, a_t^D, a_t^C, x_t, \theta) = (m_{1,t}^{-\rho} + \alpha \cdot m_{2,t}^{-\rho})^{-1/\rho}$$

- Captures substitution patterns between the choice of both annual driving distances
 - ρ = elasticity of substitution
 - α = share parameter
- Formulation could be extended to introduce randomness in α .

1. Finding the optimal value(s) of annual mileage conditional on the discrete choices
2. Solving the Bellman equation to find expected utility of future choices (value function)

Finding the optimal value(s) of mileage

- Maximization of the continuous utility: $\max_{m_{1,t}, m_{2,t}} v_t^C$
 s.t. $p_{1,t}m_{1,t} + p_{2,t}m_{2,t} = \text{Inc}_t$

- Find analytical solutions: $m_{1,t}^*$ and $m_{2,t}^*$

$$m_{2,t}^* = \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))} \alpha^{(-1/(\rho+1))}}$$

$$m_{1,t}^* = \frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} m_{2,t}^*$$

- Optimal continuous utility $v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)$

Solving the Bellman equation

- **Logsum** formula used in the completely discrete case (DDCM) (Aguirregabiria and Mira, 2010; Cirillo and Xu, 2011)
- Logsum can be applied here given the **key assumptions**:
 - Choice of mileage(s) is conditional on discrete actions
 - Choice of mileage(s) is myopic

$$\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t^D} \left\{ \exp\{v_t^D(s_t, a_t^D, x_t, \theta) + v_t^{C^*}(s_t, a_t^D, a_t^{C^*}, x_t, \theta)\} + \beta \sum_{s_{t+1} \in \mathcal{S}} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1} | s_t, a_t) \right\}$$

- Iterate on **Bellman equation** to find integrated value function \bar{V}

- Parameters obtained by maximizing likelihood:

$$\mathcal{L} = \prod_{n=1}^N \prod_{t=1}^{T_n} P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta)$$

- Optimization algorithm is Rust's **nested fixed point algorithm (NFXP)** (Rust, 1987):
 - Outer optimization algorithm:** search algorithm to obtain parameters maximizing likelihood
 - Inner value iteration algorithm:** solves the DP problem for each parameter trial
- Plan to investigate variants of NFXP to speed up computational time (e.g. swapped algorithm from Aguirregabiria and Mira, 2002)

ILLUSTRATIVE EXAMPLE

Assumptions for the example:

- Deterministic utility function

$$v_t^D(s_t, a_t^D, x_t, \theta) = C(s_t) + \tau(a_t^D) + \beta_{\text{Age}}(a_t^D, s_t) \cdot \max(\text{Age1}_t, \text{Age2}_t)$$

Constant for
households with
at least one car

Transaction
costs

Transaction-dependant
parameters relative to age
of oldest car

- Chose arbitrary values for parameters

ILLUSTRATIVE EXAMPLE

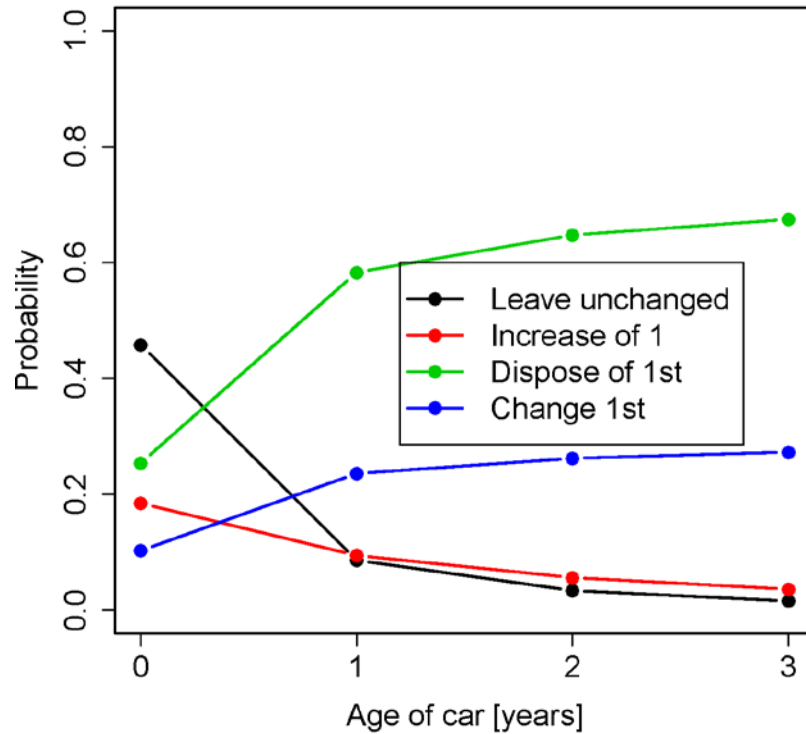
Transaction name	Case	β_{Age}			τ
		0 car	1 car	2 cars	all households
h_1 : leave unchanged		0	-1	-1	0
h_2 : increase 1		0	0	-	-3
h_3 : dispose 2		-	-	1	0
h_4 : dispose 1st	1st car is oldest	-	1.5	1.5	0
	2nd car is oldest	-	-	0	0
h_5 : dispose 2nd	1st car is oldest	-	-	0	0
	2nd car is oldest	-	-	1.5	0
h_6 : dispose 1st and change 2nd		-	-	0	-4
h_7 : dispose 2nd and change 1st		-	-	0	-4
h_8 : change 1st	1st car is oldest	-	1.5	1.5	-4
	2nd car is oldest	-	-	0	-4
h_9 : change 2nd	1st car is oldest	-	-	0	-4
	2nd car is oldest	-	-	1.5	-4

ILLUSTRATIVE EXAMPLE

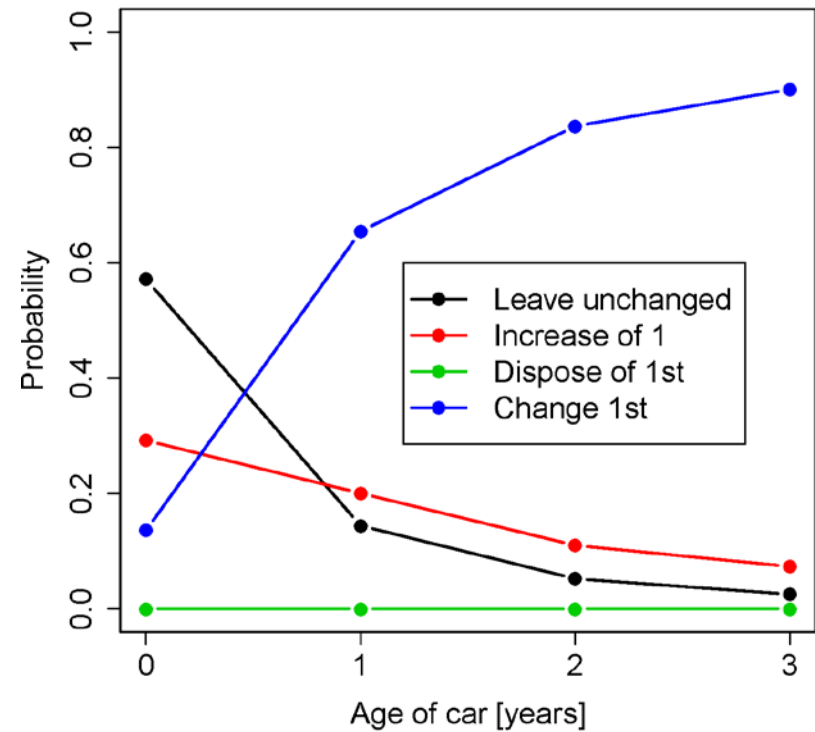
Assumptions for the example:

- Visualize choice probabilities for one observation:
 - 1-car household
 - Annual income = 530'000 SEK (\approx 58300 €, 84800 AUD)
 - 8% expenses on fuel

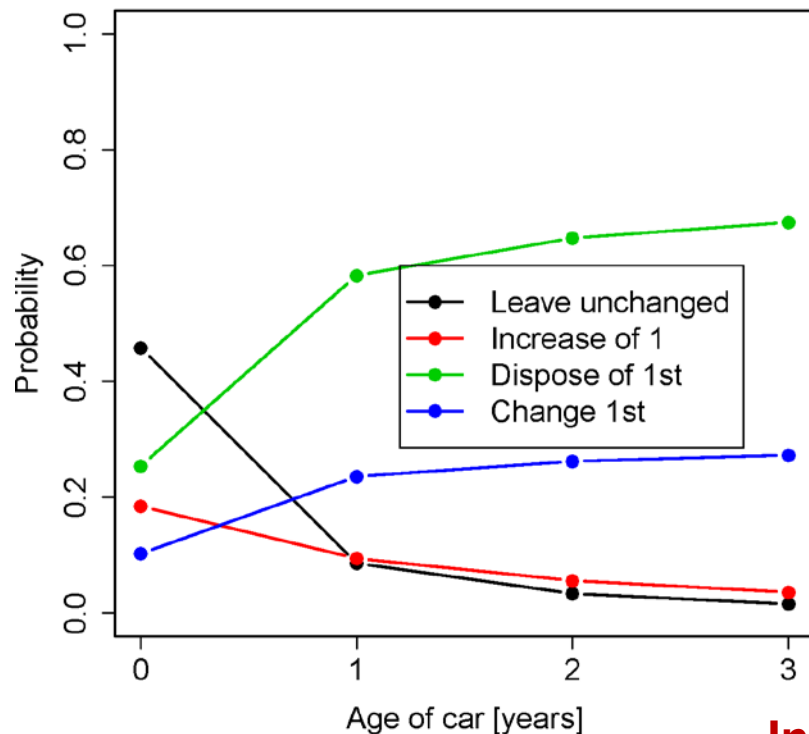
Static



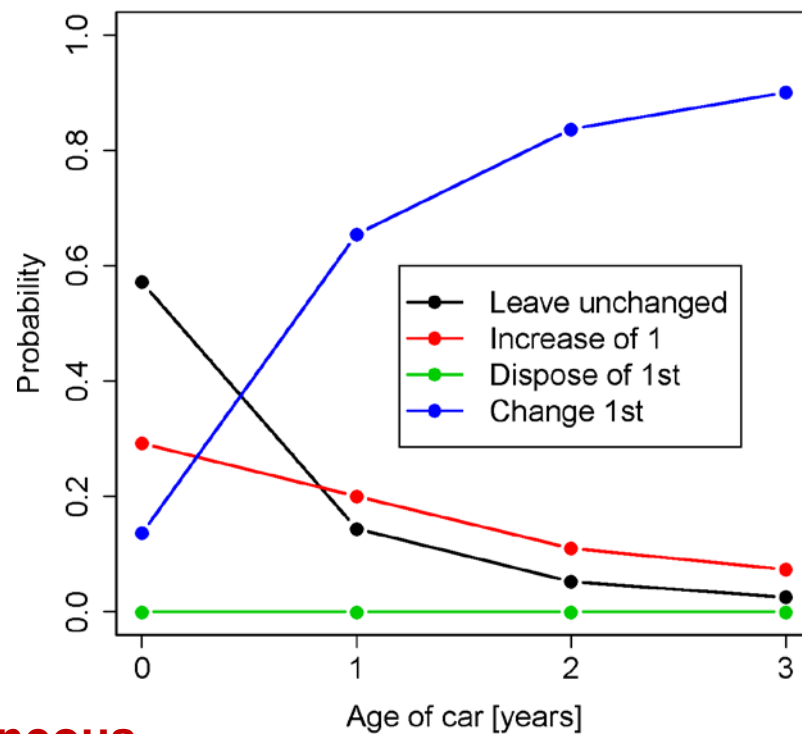
FROM MODEL APPLICATION Dynamic



Static



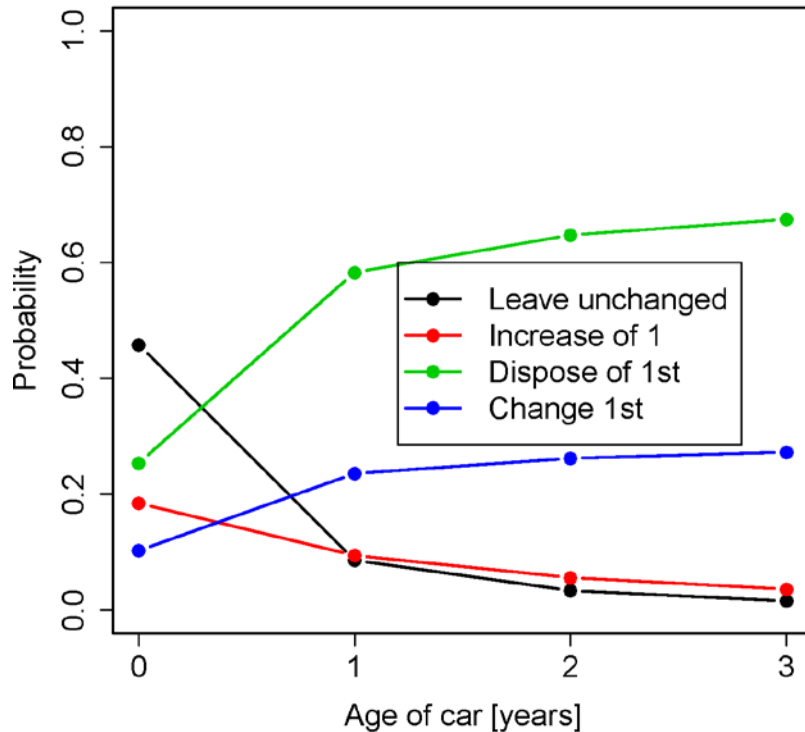
FROM MODEL APPLICATION Dynamic



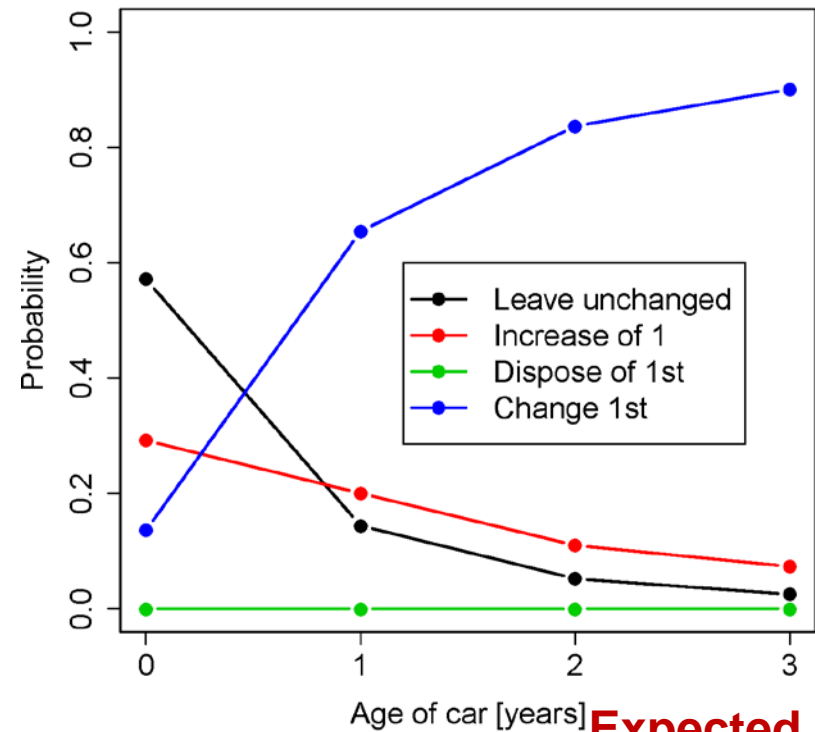
Instantaneous utility

$$P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f}{\sum_{a_{n,t}^{\tilde{D}}} \left\{ v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f \right\}}$$

Static



FROM MODEL APPLICATION Dynamic

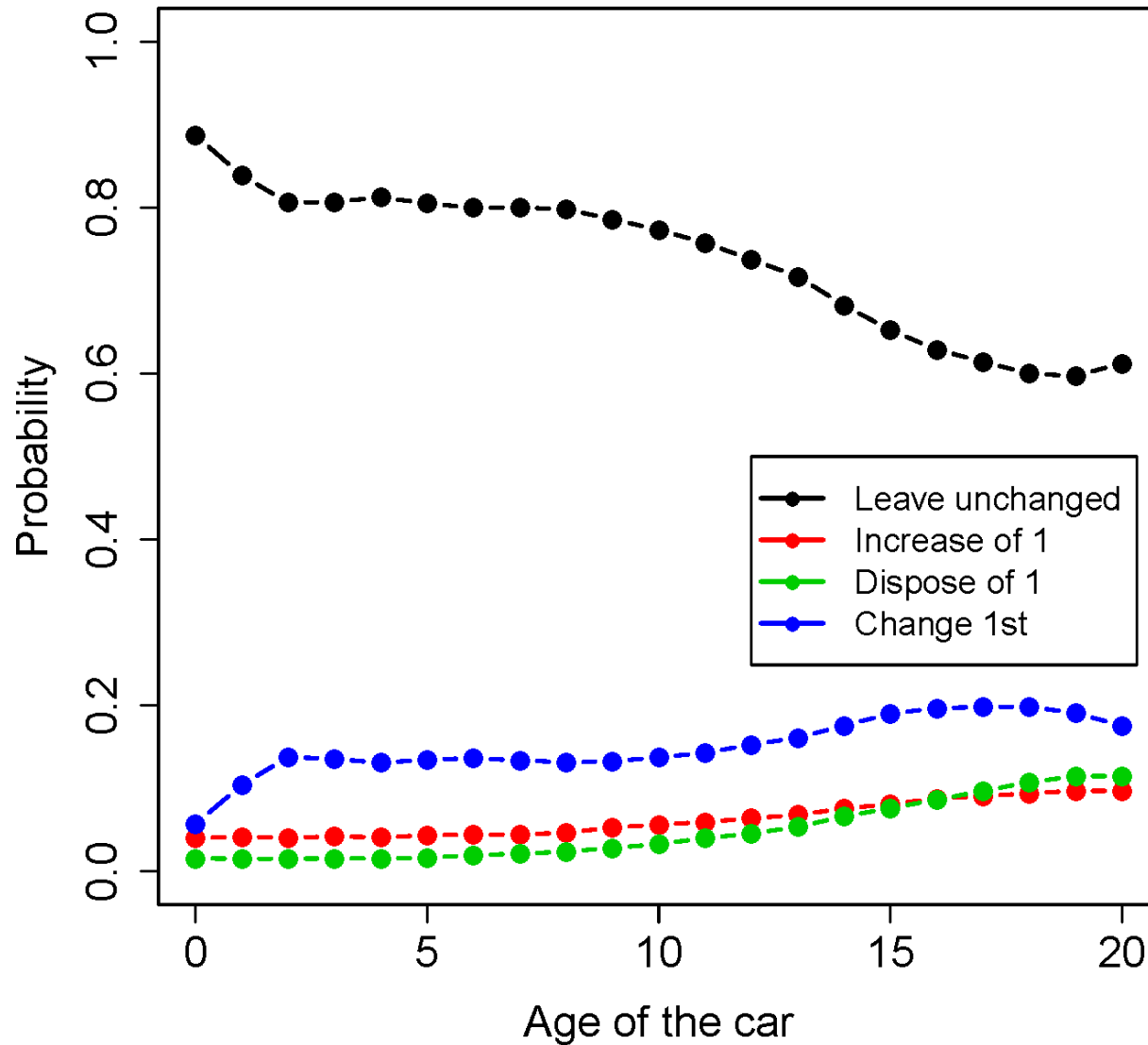


**Expected
discounted
utility**

$$P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f}{\sum_{a_{n,t}} \left\{ v_{n,t}^D + v_{n,t}^{C*} + \beta \sum_{s_{n,t+1} \in S} \bar{V} f \right\}}$$

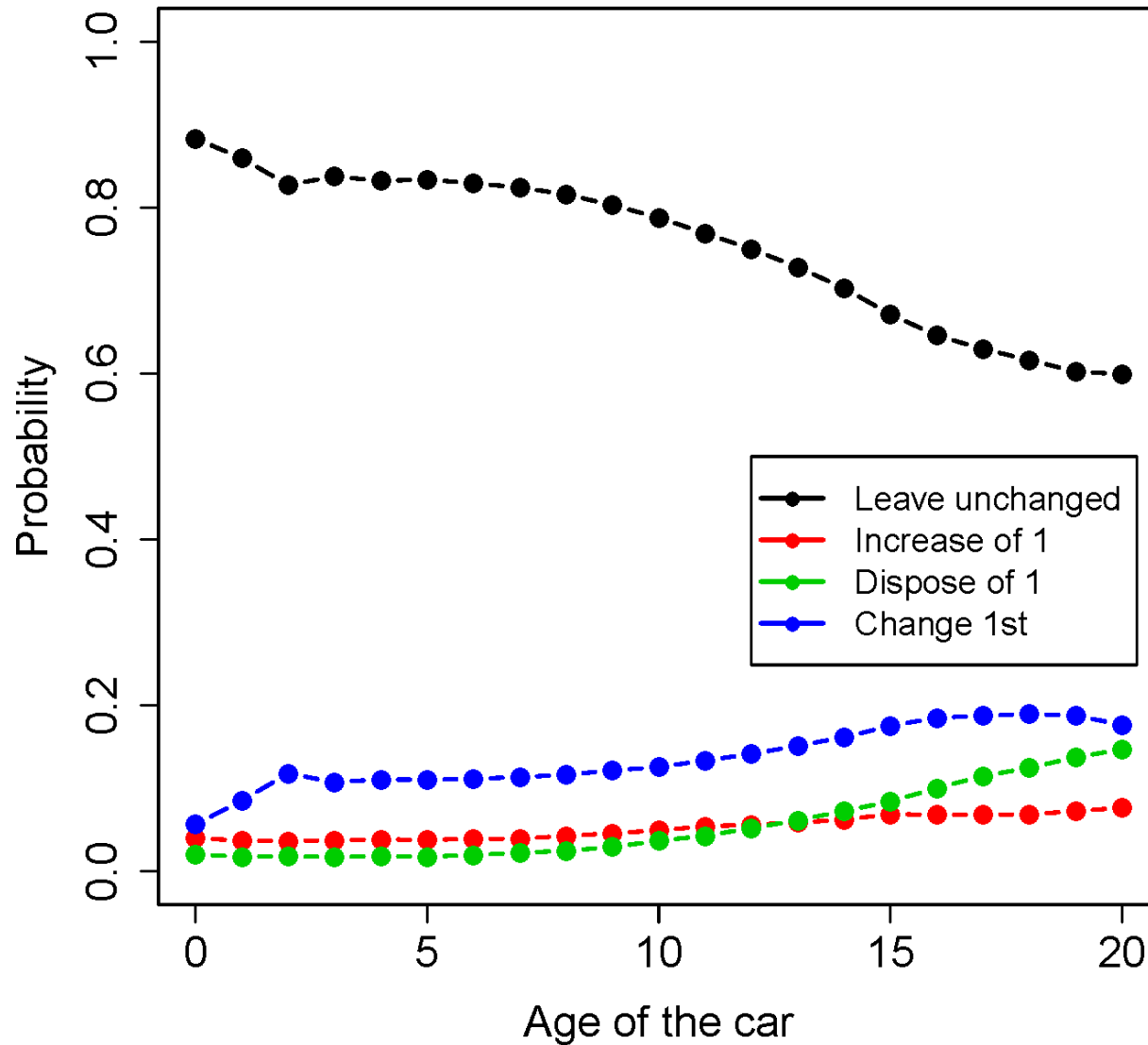
FROM REGISTER DATA

Transitions 1999–2000: 1-car households



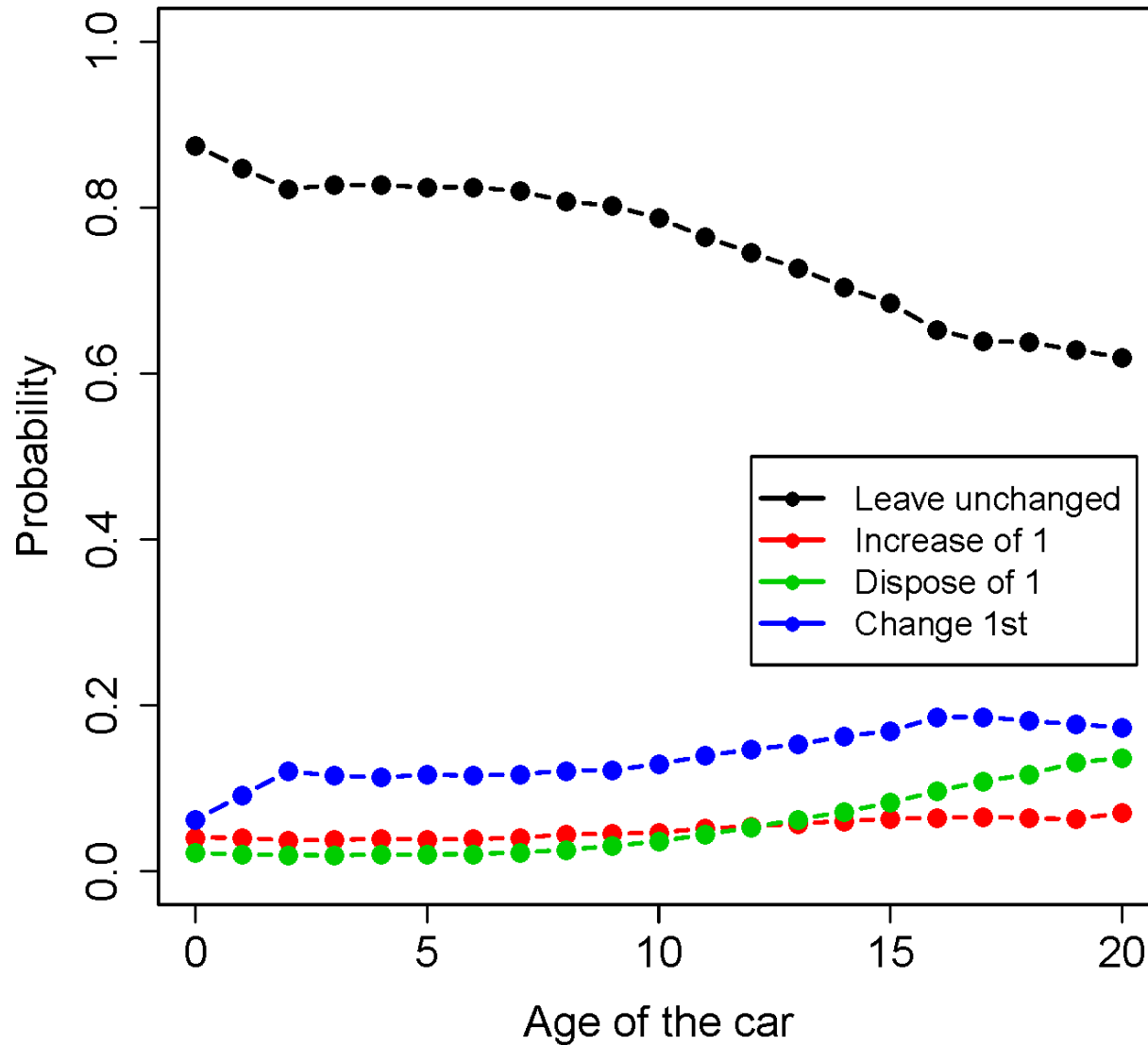
FROM REGISTER DATA

Transitions 2000–2001: 1-car households



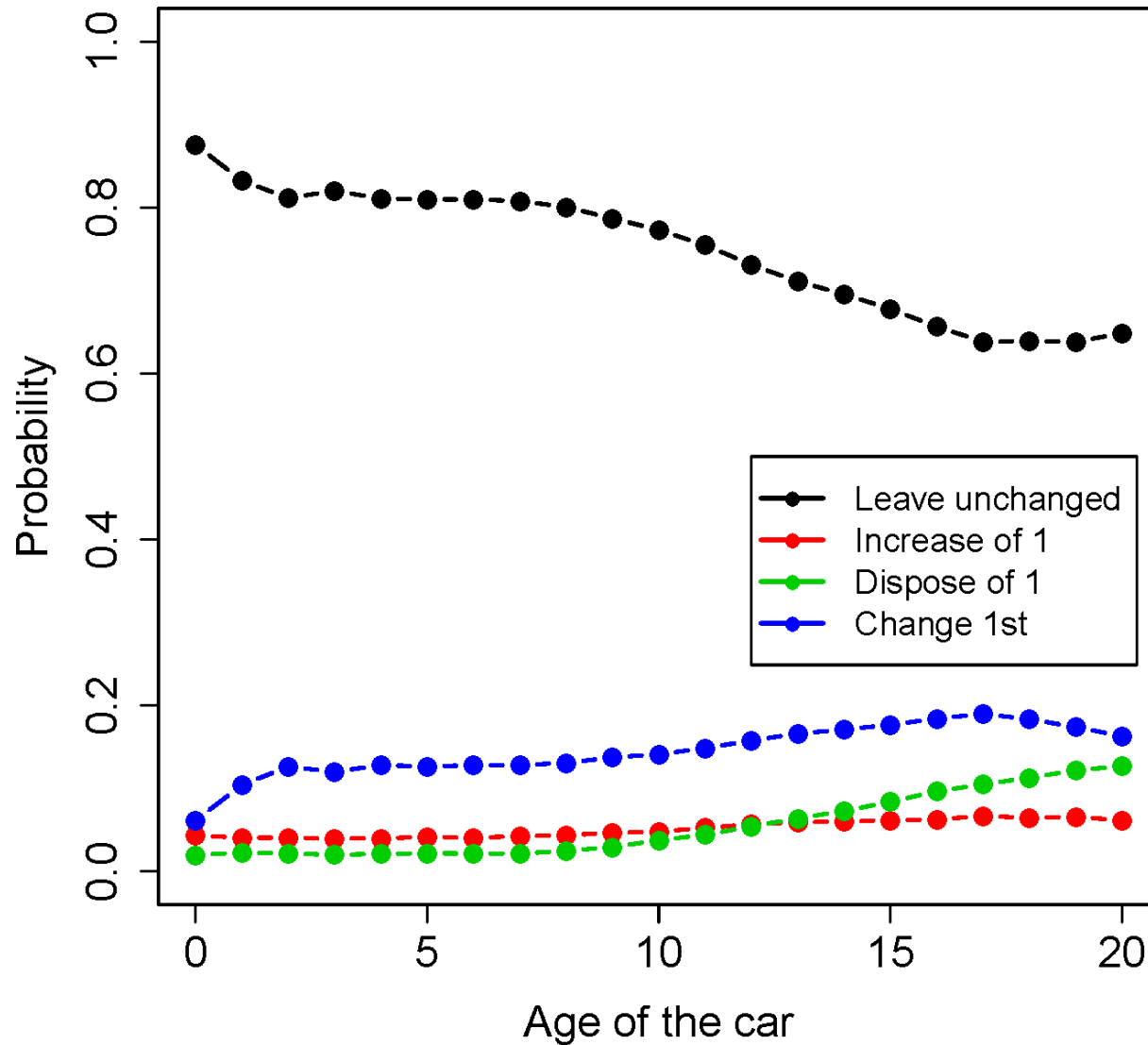
FROM REGISTER DATA

Transitions 2001–2002: 1-car households



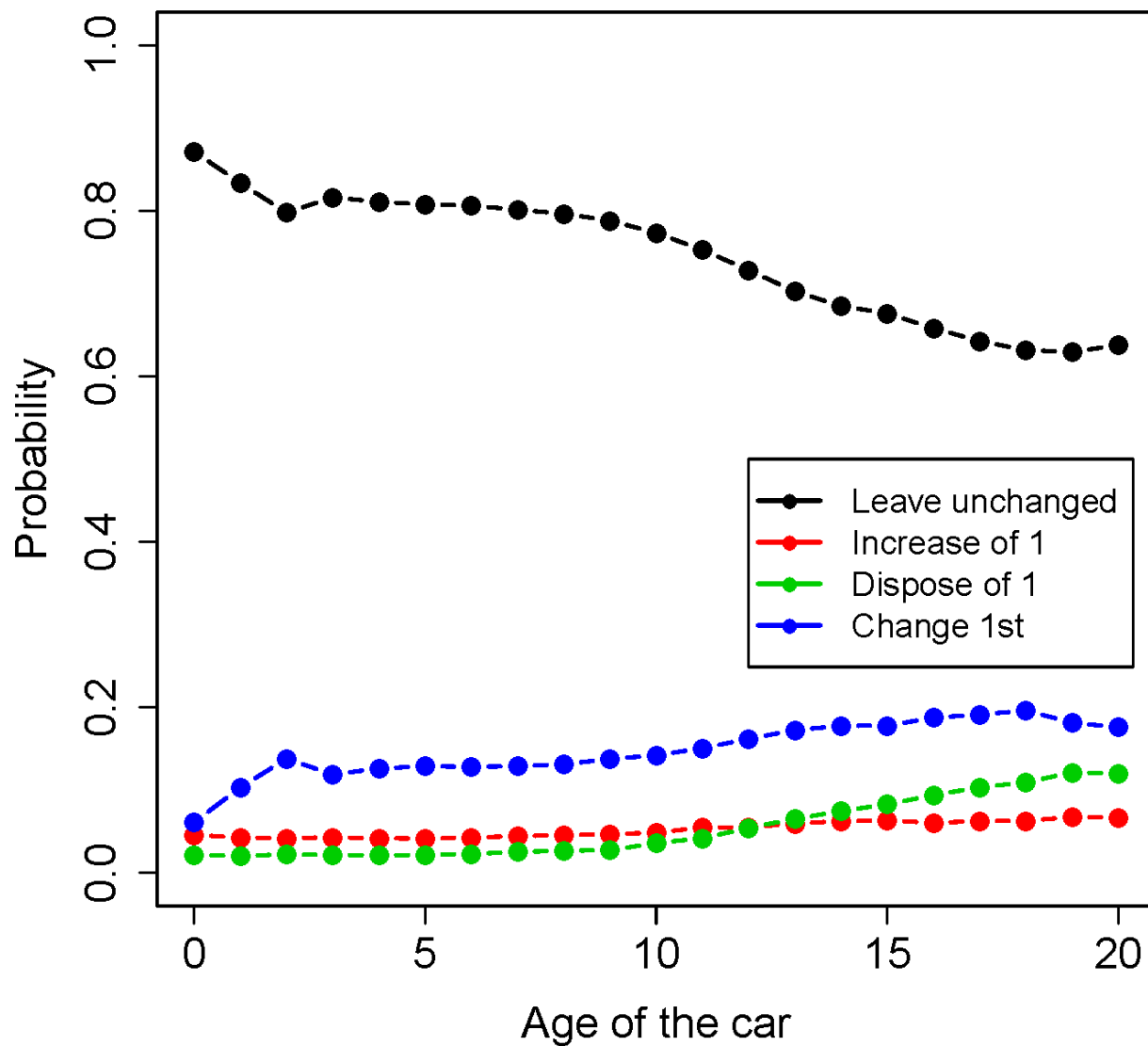
FROM REGISTER DATA

Transitions 2002–2003: 1-car households



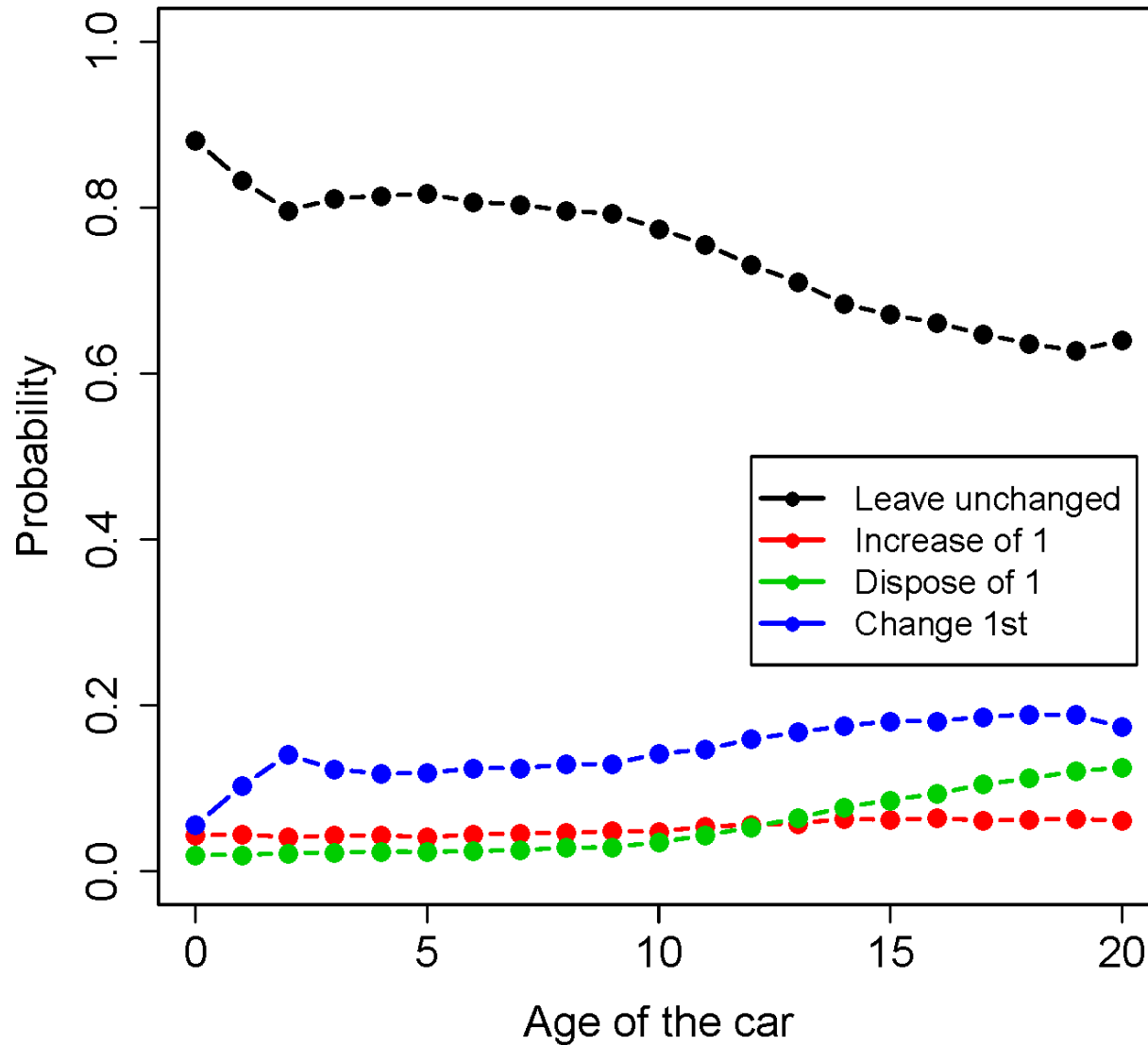
FROM REGISTER DATA

Transitions 2003–2004: 1-car households



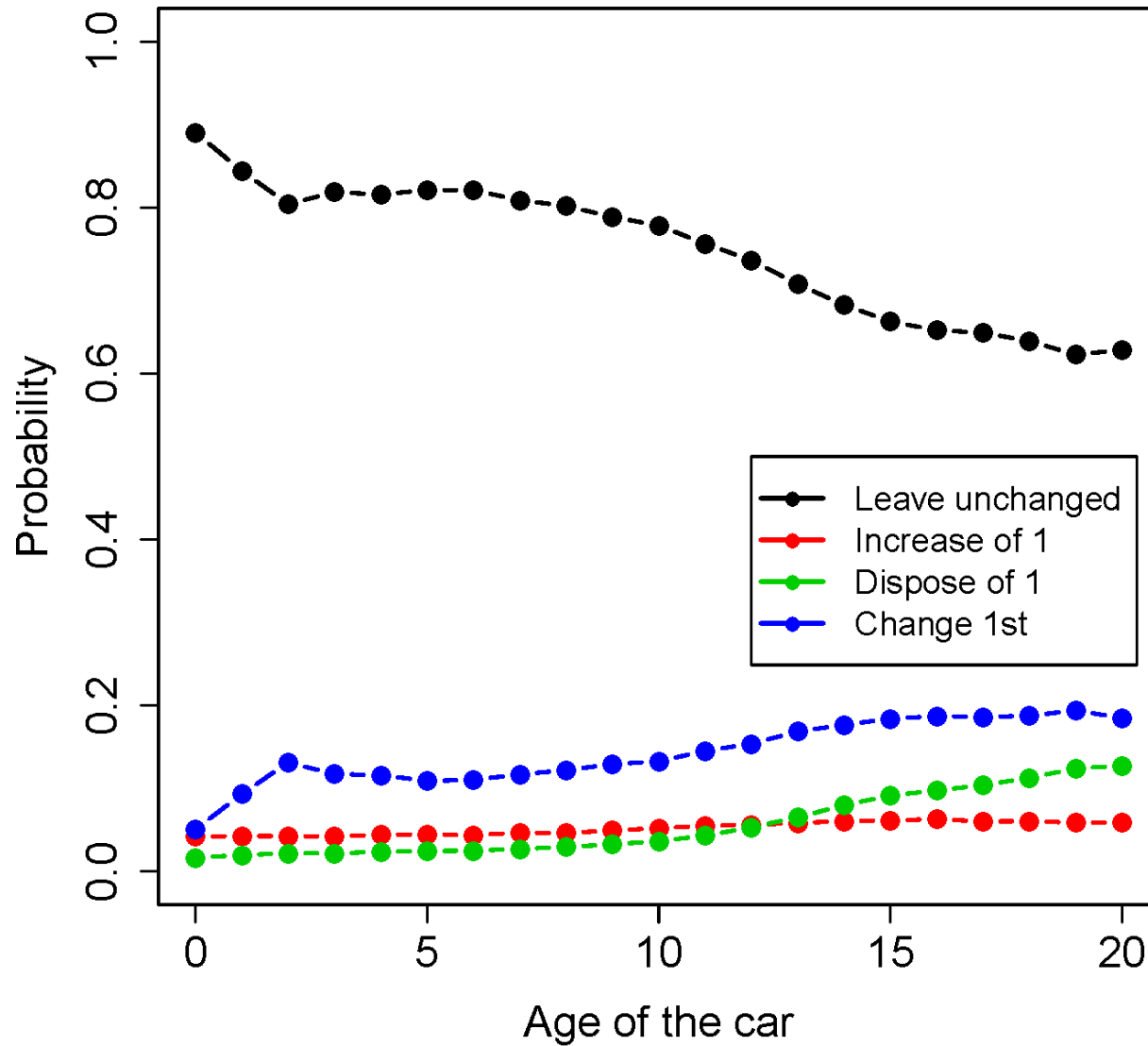
FROM REGISTER DATA

Transitions 2004–2005: 1-car households



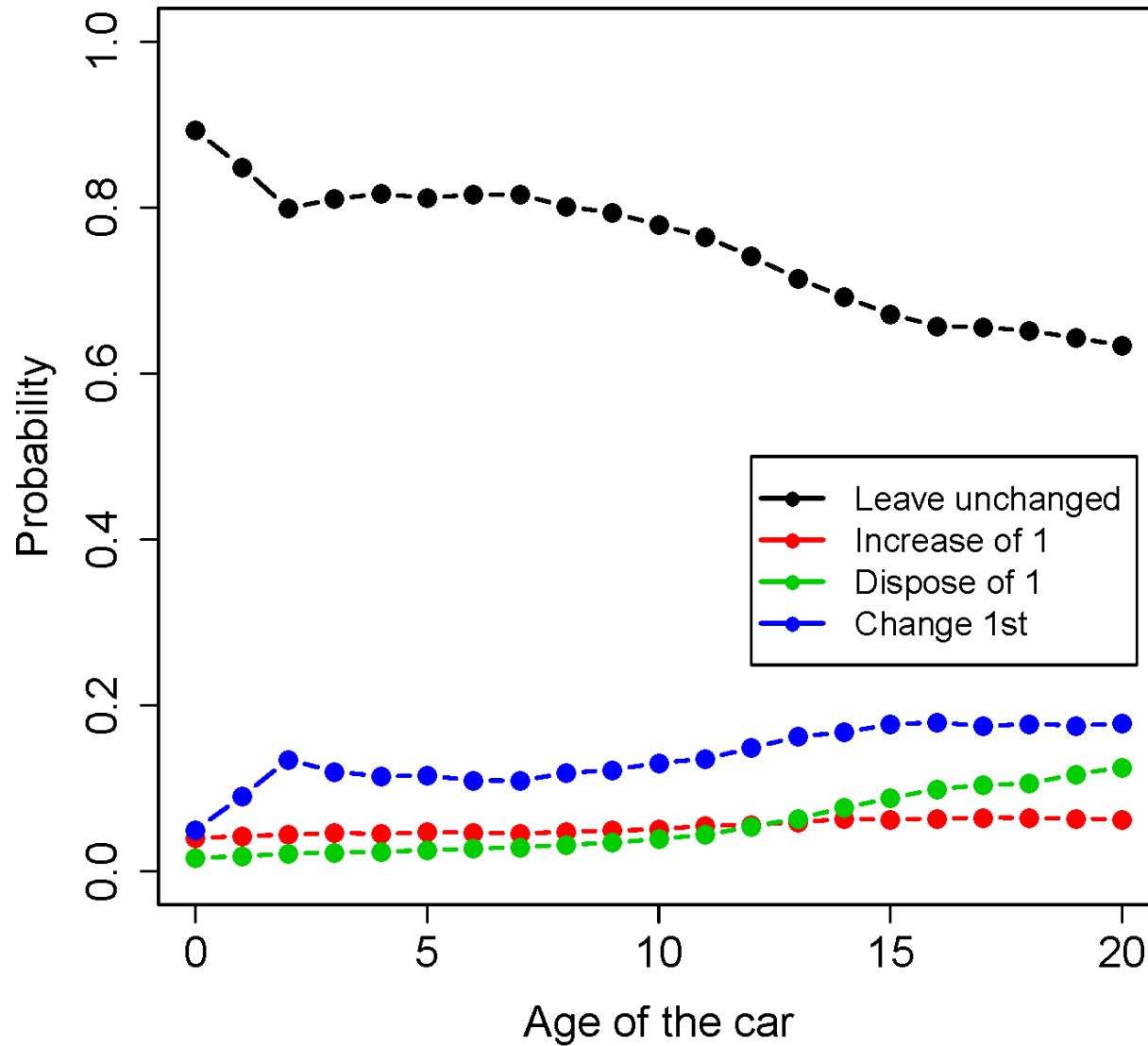
FROM REGISTER DATA

Transitions 2005–2006: 1-car households



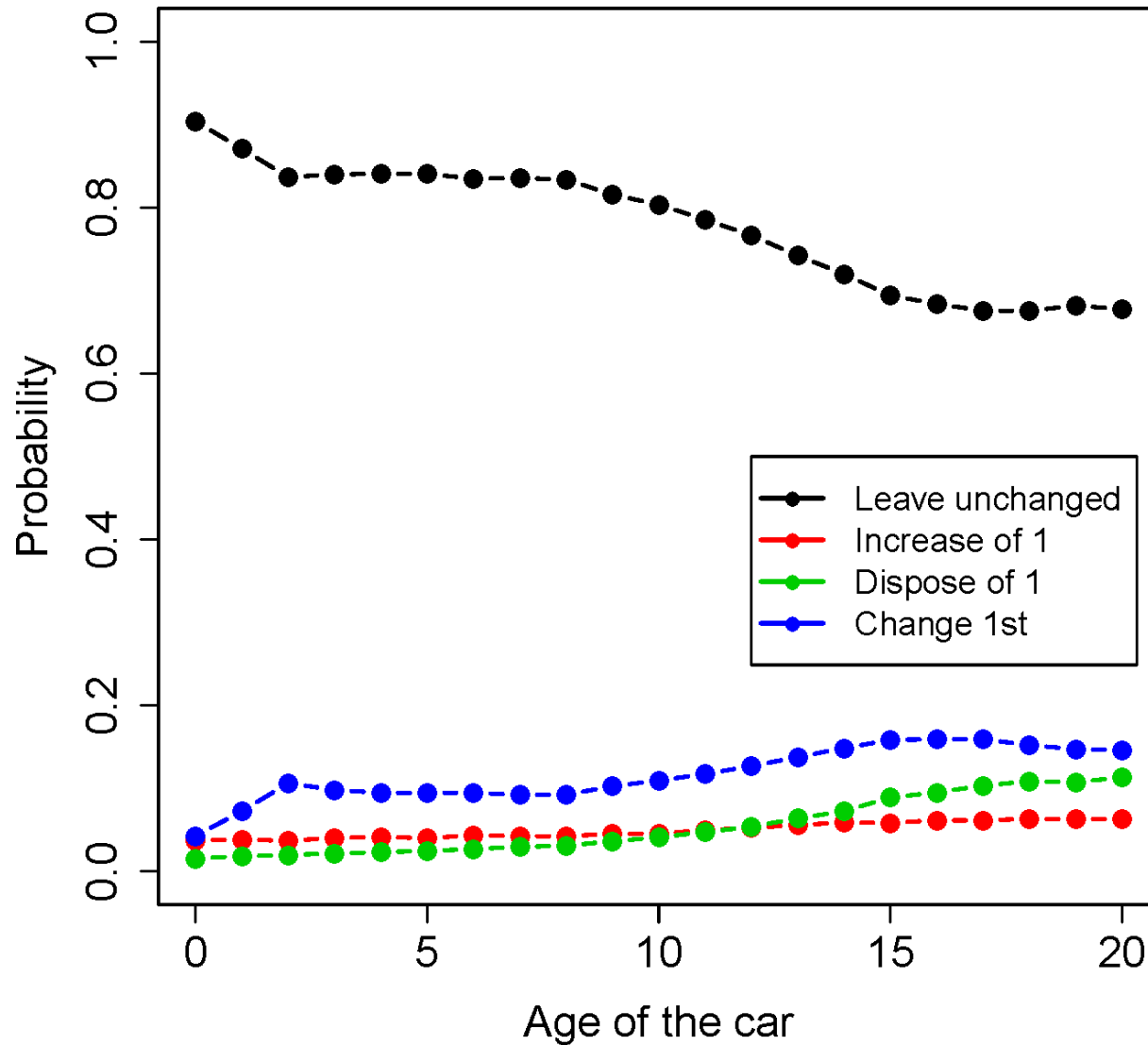
FROM REGISTER DATA

Transitions 2006–2007: 1-car households



FROM REGISTER DATA

Transitions 2007–2008: 1-car households



CONCLUSION AND FUTURE WORKS

Conclusion:

- Methodology to model choice of car ownership and usage dynamically
- Example of application shows feasibility of approach

Next steps:

- Model estimation on small sample of synthetic data
- Model estimation on register data
- Scenario testing:
 - Validation of policy measures taken during the years available in the data
 - Test policy measures that are planned to be applied in future years

Thanks!

