MODELING CAR OWNERSHIP AND USAGE:
A DYNAMIC DISCRETE-CONTINUOUS CHOICE MODELING APPROACH

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• Introduction
• Background and data
• The dynamic discrete-continuous choice modeling framework
• Illustration of model application
• Conclusion and future works
INTRODUCTION

Aim of the research:

• Model dynamics of car transactions, usage and choice of fuel type in the Swedish car fleet

• Motivations
  • Governmental policies to reduce carbon emissions / car usage:
    • Stockholm congestion tax
    • Independence of fossil fuels
  • Technology changes:
    • Increase of alternative-fuel vehicles
  • Economical features:
    • Financial crisis
    • Fuel price changes

Car ownership and usage vary importantly over time.
Model needed to analyze and predict impact of policies on ownership
Register data of Swedish population and car fleet:

- **Data from 1998 to 2008**

- **All individuals**
  - Individual information: socio-economic information on car holder (age, gender, income, home/work location, employment status/sector, etc.)
  - Household information: composition (families with children and married couples)

- **All vehicles**
  - Privately-owned cars, cars from privately-owned company and **company** cars
  - Vehicle characteristics (make, model, fuel consumption, fuel type, age)
  - Annual mileage from odometer readings
  - Car bought **new or second-hand**
Number of cars (in hh with only private cars)

- **0-car hh**
- **1-car hh**
- **2-car hh**

<table>
<thead>
<tr>
<th>Year</th>
<th>1999</th>
<th>2001</th>
<th>2003</th>
<th>2005</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-car hh</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-car hh</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-car hh</td>
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</tr>
</tbody>
</table>
Number of cars (in hh with only private cars)

- 0-car hh
- 1-car hh
- 2-car hh


Congestion tax
Stockholm
BACKGROUND AND DATA

Household annual mileage per household size

- 1-pers hh
- 2-pers hh
- 3-pers hh
- 4-pers hh
- 5-pers hh
- 6-pers hh
- 7-pers hh

Year

1999 2001 2003 2005 2007

Annual mileage per household [mil]

0 500 1000 1500 2000 2500 3000
BACKGROUND AND DATA

- Car ownership models in transportation literature:
  - Discrete choice models (DCM) widely used, but mostly static models.
    - Main drawback: do not account for forward-looking behavior
    - Important aspect to account for since car is a durable good
  - Econometric literature: dynamic programming (DP) models + DCM
    - Recently, dynamic discrete choice models (DDCM) starting to be applied in transportation field (Cirillo and Xu, 2011; Schiraldi, 2011)

- Joint models of car ownership and usage:
  - Duration models and regression techniques for car holding duration and usage (De Jong, 1996)
  - Vehicle type, usage and replacement decisions using dynamic programming, discrete-continuous, mixed logit (Schjerning, 2008, and Munk-Nielsen, 2012)
  - Discrete-continuous model of vehicle choice and usage based on register data (Gillingham, 2012)

- Wide literature on car ownership and usage models
• Car are durable goods \(\rightarrow\) Need to account for \textit{forward-looking behavior} of agents

• Difficulty of modeling a \textit{discrete-continuous choice} when jointly modeling car ownership and usage

• Many models focus on individual decisions, but choices regarding car ownership and usage made at \textit{household level}
BACKGROUND AND DATA

RESEARCH ISSUES

• Car are durable goods $\Rightarrow$ Need to account for forward-looking behavior of agents

• Difficulty of modeling a discrete-continuous choice when jointly modeling car ownership and usage

• Many models focus on individual decisions, but choices regarding car ownership and usage made at household level

Proposed methodology:

• Attempt to address these issues by applying dynamic discrete-continuous choice model (DDCCM)

Large register data of all individuals and cars in Sweden
In the area of dynamic choice modeling
  - Choices modeled at household level
  - Up to two cars allowed

Constant elasticity of substitution (CES) utility to model annual driving distance for 2-car households

Several choices modeled simultaneously
Objective

Model simultaneously car ownership, usage and fuel type. In details: model simultaneous choice of

Transaction type \times \left[ \text{Annual milage – car } c \right] \times \left[ \text{Private/company car – car } c \right] \times \left[ \text{Fuel type – car } c \right] \times \left[ \text{New/2\textsuperscript{nd} hand – car } c \right] \times \text{# cars}
Objective

Model simultaneously car ownership, usage and fuel type.

In details: model simultaneous choice of

Transaction type \times \begin{bmatrix}
\text{Annual milage – car } c \\
\text{Private/ company car – car } c \\
\text{Fuel type – car } c \\
\text{New/2}^{nd} \text{ hand – car } c
\end{bmatrix} \times \# \text{ cars}
Objective

Model simultaneously car ownership, usage and fuel type.

In details: model simultaneous choice of

- Transaction type
- Annual milage – car \( c \)
- Private/company car – car \( c \)
- Fuel type – car \( c \)
- New/2\(^{nd}\) hand – car \( c \)

Continuous variables

# cars
1. Choice at household level: up to 2 cars in household

2. Strategic choice of:
   - Transaction
   - Type(s) of ownership (company vs private car)
   - Fuel type(s)
   - Car state(s) (new vs 2nd-hand)

Account for forward-looking behavior of households

3. Myopic choice of:
   - Annual mileage(s)

4. Choice of mileage conditional on choice of discrete variables
Myopic choice (static case)

\[ P(\text{action}) = \frac{\exp\{\text{instantaneous utility}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities}\}} \]

Strategic choice (dynamic case)

\[ P(\text{action}) = \frac{\exp\{\text{instantaneous utility + expected discounted utility of future choices}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities + expected discounted utilities of future choices}\}} \]
Myopic choice (static case)

\[ P(\text{action}) = \frac{\exp\{\text{instantaneous utility}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities}\}} \]

Strategic choice (dynamic case)

\[ P(\text{action}) = \frac{\exp\{\text{instantaneous utility} + \text{expected discounted utility of future choices}\}}{\sum_{\text{all poss. actions}} \exp\{\text{instantaneous utilities} + \text{expected discounted utilities of future choices}\}} \]

Embeds a choice model into a dynamic programming framework
Components of the DDCCM:

- Agent
- Time step
- State space
- Action space
- Transition rule
- Instantaneous utility function
• Agent: household

• Time step $t$: year

• State space $S$

$$s_t = (y_{1,t}, I_{1,t}, f_{1,t}, y_{2,t}, I_{2,t}, f_{2,t})$$

- Age – 1st car
- Private/Company car – 1st car
- Fuel type – 1st car
- Age – 2nd car
- Fuel type – 2nd car
- Private/Company car – 2nd car
- Fuel type – 2nd car
DYNAMIC DISCRETE-CONTINUOUS CHOICE MODEL

DEFINITION OF THE COMPONENTS

- Action space $A$

\[
a_t = (h_t, \tilde{m}_{1,t}, \tilde{I}_{1,t}, \tilde{f}_{1,t}, \tilde{r}_{1,t}, \tilde{m}_{2,t}, \tilde{I}_{2,t}, \tilde{f}_{2,t}, \tilde{r}_{2,t})
\]
• Action space $A$

Transaction types
• **Transition rule:** deterministic rule: each state $s_{t+1}$ can be inferred exactly once $s_t$ and $a_t$ are known.
Transition rule: deterministic rule: each state $s_{t+1}$ can be inferred exactly once $s_t$ and $a_t$ are known.

Instantaneous utility function:

$$u(s_t, a^C_t, a^D_t, x_t, \theta) = v(s_t, a^C_t, a^D_t, x_t, \theta) + \varepsilon_D(a^D_t)$$

- **Deterministic term**
- **Random term for discrete choices**

Assume additive **deterministic utility** for simplicity (see also Munk-Nielsen, 2012):

$$v(s_t, a^C_t, a^D_t, x_t, \theta) = v_D^t(s_t, a^D_t, x_t, \theta) + v_C^t(s_t, a^D_t, a^C_t, x_t, \theta)$$

- **Utility for discrete actions**
- **Utility for continuous actions**
• Instantaneous utility function

• Utility for continuous actions:

\[ v^C_t(s_t, a^D_t, a^C_t, x_t, \theta) = \left( m_{1,t}^{-\rho} + \alpha \cdot m_{2,t}^{-\rho} \right)^{-1/\rho} \]

• Captures substitution patterns between the choice of both annual driving distances
• \( \rho \) = elasticity of substitution
• \( \alpha \) = share parameter

• Formulation could be extended to introduce randomness in \( \alpha \).
1. Finding the optimal value(s) of annual mileage conditional on the discrete choices

2. Solving the Bellman equation to find expected utility of future choices (value function)
Finding the optimal value(s) of mileage

- Maximization of the continuous utility:
  \[ \max_{m_{1,t}, m_{2,t}} v_t^C \]
  \[ \text{s.t.} \quad p_{1,t}m_{1,t} + p_{2,t}m_{2,t} = \text{Inc}_t \]

- Find analytical solutions: \( m_{1,t}^* \) and \( m_{2,t}^* \)

\[
m_{2,t}^* = \frac{\text{Inc}_t \cdot p_{2,t}^{(-1/(\rho+1))}}{p_{2,t}^{(\rho/(\rho+1))} + p_{1,t}^{(\rho/(1+\rho))} \alpha^{(-1/(\rho+1))}}
\]

\[
m_{1,t}^* = \frac{\text{Inc}_t}{p_{1,t}} - \frac{p_{2,t}}{p_{1,t}} m_{2,t}^*
\]

- Optimal continuous utility
  \[ v_t^C(s_t, a_t^D, a_t^{C*}, x_t, \theta) \]
Solving the Bellman equation

- **Logsum** formula used in the completely discrete case (DDCM) (Aguirregabiria and Mira, 2010; Cirillo and Xu, 2011)

- Logsum can be applied here given the **key assumptions**:  
  - Choice of mileage(s) is conditional on discrete actions  
  - Choice of mileage(s) is myopic

\[
\bar{V}(s_t, x_t, \theta) = \log \sum_{a_t^D} \left\{ \exp\{v_t^D(s_t, a_t^D, x_t, \theta) + v_t^{C*}(s_t, a_t^D, a_t^{C*}, x_t, \theta)\} \right\} + \beta \sum_{s_{t+1} \in S} \bar{V}(s_{t+1}, x_{t+1}, \theta) f(s_{t+1}|s_t, a_t)
\]

- Iterate on **Bellman equation** to find integrated value function \(\bar{V}\)
Parameters obtained by maximizing likelihood:

\[
\mathcal{L} = \prod_{n=1}^{N} \prod_{t=1}^{T_n} P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta)
\]

Optimization algorithm is Rust’s nested fixed point algorithm (NFXP) (Rust, 1987):

- **Outer optimization algorithm**: search algorithm to obtain parameters maximizing likelihood
- **Inner value iteration algorithm**: solves the DP problem for each parameter trial
- Plan to investigate variants of NFXP to speed up computational time (e.g. swapped algorithm from Aguirregabiria and Mira, 2002)
Assumptions for the example:

- Deterministic utility function

\[ v^D_t(s_t, a^D_t, x_t, \theta) = C(s_t) + \tau(a^D_t) + \beta_{\text{Age}}(a^D_t, s_t) \cdot \max(\text{Age1}_t, \text{Age2}_t) \]

- Chose arbitrary values for parameters
### Illustrative Example

<table>
<thead>
<tr>
<th>Transaction name</th>
<th>Case</th>
<th>$\beta_{\text{Age}}$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 car</td>
<td>1 car</td>
</tr>
<tr>
<td>$h_1$: leave unchanged</td>
<td></td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$h_2$: increase 1</td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$h_3$: dispose 2</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_4$: dispose 1st</td>
<td>1st car is oldest</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>2nd car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_5$: dispose 2nd</td>
<td>1st car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2nd car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_6$: dispose 1st and change 2nd</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_7$: dispose 2nd and change 1st</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_8$: change 1st</td>
<td>1st car is oldest</td>
<td>-</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>2nd car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h_9$: change 2nd</td>
<td>1st car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>2nd car is oldest</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Assumptions for the example:

- Visualize choice probabilities for one observation:
  - 1-car household
  - Annual income = 530’000 SEK (≈ 58300 €; 84800 AUD)
  - 8% expenses on fuel
ILLUSTRATIVE EXAMPLE

FROM MODEL APPLICATION

Static

Dynamic

Probability

Age of car [years]

Probability

Age of car [years]

Leave unchanged
Increase of 1
Dispose of 1st
Change 1st

Leave unchanged
Increase of 1
Dispose of 1st
Change 1st
ILLUSTRATIVE EXAMPLE

FROM MODEL APPLICATION

Static

Dynamic

Instantaneous utility

$$P(d_{n,t}^D | s_{n,t}, x_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \tilde{V}f}{\sum_{\tilde{d}_{n,t}} \left( v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \tilde{V}f \right)}$$
ILLUSTRATIVE EXAMPLE

FROM MODEL APPLICATION

Static

Dynamic

Probability

Probability

Age of car [years]

Age of car [years]

\[ P(a_{n,t}^D | s_{n,t}, x_{n,t}, \theta) = \frac{v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \bar{V}_f}{\sum_{a_{n,t}} \left( v_{n,t}^D + v_{n,t}^C + \beta \sum_{s_{n,t+1} \in S} \bar{V}_f \right)} \]
ILLUSTRATIVE EXAMPLE FROM REGISTER DATA


- Leave unchanged
- Increase of 1
- Dispose of 1
- Change 1st
ILLUSTRATIVE EXAMPLE FROM REGISTER DATA


- Leave unchanged
- Increase of 1
- Dispose of 1
- Change 1st

Age of the car

Probability
ILLUSTRATIVE EXAMPLE FROM REGISTER DATA

Transitions 2001–2002: 1–car households

- Probability
- Age of the car

Legend:
- Black solid: Leave unchanged
- Red solid: Increase of 1
- Green dashed: Dispose of 1
- Blue dashed: Change 1st

Graph shows the probability of different transitions for 1-car households as a function of the age of the car.
ILLUSTRATIVE EXAMPLE

FROM REGISTER DATA


- Leave unchanged
- Increase of 1
- Dispose of 1
- Change 1st

Age of the car

Probability
ILLUSTRATIVE EXAMPLE FROM REGISTER DATA


- Probability
- Age of the car

Legend:
- Leave unchanged
- Increase of 1
- Dispose of 1
- Change 1st
ILLUSTRATIVE EXAMPLE

FROM REGISTER DATA


- Leave unchanged
- Increase of 1
- Dispose of 1
- Change 1st

Probability

Age of the car
ILLUSTRATIVE EXAMPLE
FROM REGISTER DATA


Probability vs. Age of the car

- Black dashed: Leave unchanged
- Red: Increase of 1
- Green: Dispose of 1
- Blue dotted: Change 1st
Conclusion:

- Methodology to model choice of car ownership and usage dynamically
- Example of application shows feasibility of approach

Next steps:

- Model estimation on small sample of synthetic data
- Model estimation on register data
- Scenario testing:
  - Validation of policy measures taken during the years available in the data
  - Test policy measures that are planned to be applied in future years
Thanks!