

hEART 2015

# **A dynamic network loading model for anisotropic and congested pedestrian flows**

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# Unsteady, anisotropic and congested flow

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**Figure:** Passageway in Central Station (MTR), Hong Kong

# Aggregate pedestrian flow models

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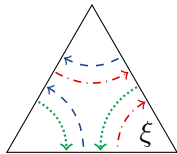
- graph-based models [CS94, Løv94]
  - interaction between streams entirely neglected
- cell transmission models [ASKT07, GHW11, HBFM14]
  - inherent assumption of isotropy
- continuum models [Hug02, HWZ<sup>+</sup>09, HvWKDD14]
  - expensive, particularly for multi-class applications

Scope: 'cheap' anisotropic macroscopic loading model

# Decomposition of pedestrian flow into streams

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- contiguous area  $\xi$  of size  $A_\xi$
- each stream  $\lambda \in \Lambda_\xi$  characterized by
  - exogenous direction
  - accumulation  $M_\lambda$
  - uni-directional speed  $V_\lambda$



e.g. triangular area

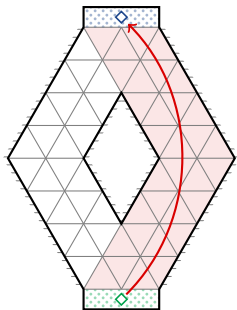
stream-based fundamental diagram  $\mathbf{f}(\mathbf{M})$  [WLC<sup>+</sup>10, XW15, FL15]

- accumulation and speed vectors:  $\mathbf{M}_\xi = [M_\lambda]$ ,  $\mathbf{V}_\xi = [V_\lambda]$
- bounded velocity:  $0 \leq V_\lambda \leq V_f, \forall \lambda \in \Lambda_\xi$
- monotonic density-speed relation:  $\partial V_\lambda / \partial M_{\lambda'} \leq 0, \forall \lambda, \lambda' \in \Lambda_\xi$

$$\mathbf{V}_\xi = V_f \mathbf{f}_\xi(\mathbf{M}_\xi; A_\xi)$$

# Time, space and demand

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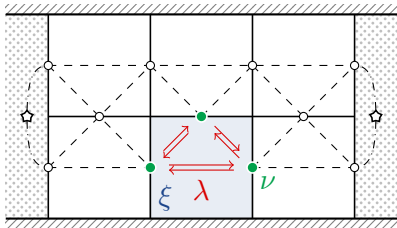


- time interval  $\tau \in \mathcal{T}$ 
  - choice of  $\Delta T = |\tau|$  crucial
- area  $\xi \in \mathcal{X}$ 
  - no assumption regarding shape and size
- route  $\rho \in \mathcal{R}$ 
  - origin/destination area:  $\xi_\rho^o, \xi_\rho^d$
  - accessible network:  $\mathcal{X}_\rho \subset \mathcal{X}$
- pedestrian group  $\ell \in \mathcal{L}$ 
  - departure time interval  $\tau_\ell$
  - group size  $x_\ell$
  - route  $\rho_\ell$

# Pedestrian walking network

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- $\mathcal{X}$ : set of areas  $\xi \in \mathcal{X}$
- $\mathcal{N}$ : set of nodes  $\nu \in \mathcal{N}$
- $\Lambda$ : set of streams  $\lambda \in \Lambda$ ,  $\lambda : \nu_\lambda^o \rightarrow \nu_\lambda^d$ 
  - $L_\lambda > 0$ : length of stream  $\lambda$ ,  $L_{\min} = \min_{\lambda \in \Lambda} L_\lambda$
  - $\Lambda_\xi$ : set of streams associated with area  $\xi$



- **area**: range of interaction
- **node**: flow valve/splitter
- **stream**: uni-directional flow

# State variables and hydrodynamic flow

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- fragment size
  - $M_{\lambda,\tau}^{\ell}$ : accumulation of group  $\ell$  on stream  $\lambda$  during interval  $\tau$
- aggregated variables
  - stream accumulation:  $M_{\lambda,\tau} = \sum_{\ell \in \mathcal{L}} M_{\lambda,\tau}^{\ell}$
  - area accumulation:  $M_{\xi,\tau} = \sum_{\lambda \in \Lambda_{\xi}} M_{\lambda,\tau}$
- 'hydrodynamic flow' on stream  $\lambda \in \Lambda$  during interval  $\tau$ 
  - for uni-directional flow: flux = density  $\times$  velocity
  - $\Delta Q_{\lambda,\tau} = L_{\min}/L_{\lambda} M_{\lambda,\tau} f_{\lambda}(\mathbf{M}_{\xi,\tau})$  if  $\Delta T = \Delta L_{\min}/V_f$  (CFL)
  - reaches maximum  $\Delta Q_{\lambda,\tau}^{\text{opt}}$  at  $M_{\lambda,\tau}^{\text{opt}}$

# Hydrodynamic flow capacities

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- hydrodynamic inflow capacity

$$\Delta Q_{\lambda,\tau}^{\text{in}} = \begin{cases} \Delta Q_{\lambda,\tau}^{\text{opt}} & \text{if } M_{\lambda,\tau} \leq M_{\lambda,\tau}^{\text{opt}} \\ \Delta Q_{\lambda,\tau} & \text{otherwise} \end{cases}$$

- hydrodynamic outflow capacity

$$\Delta Q_{\lambda,\tau}^{\text{out}} = \begin{cases} \Delta Q_{\lambda,\tau} & \text{if } M_{\lambda,\tau} \leq M_{\lambda,\tau}^{\text{opt}} \\ \Delta Q_{\lambda,\tau}^{\text{opt}} & \text{otherwise} \end{cases}$$



# Sending capacity

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- receiving capacity of stream  $\lambda$  during interval  $\tau$

$$R_{\lambda,\tau} = \Delta Q_{\lambda,\tau}^{\text{in}}$$

- sending capacity of group  $\ell$  on stream  $\lambda$  during interval  $\tau$

$$S_{\lambda \rightarrow \lambda', \tau}^{\ell} = \delta_{\lambda \rightarrow \lambda', \tau}^{\rho \ell} \min \left\{ M_{\lambda, \tau}^{\ell}, \frac{M_{\lambda, \tau}^{\ell}}{M_{\lambda, \tau}} \Delta Q_{\lambda, \tau}^{\text{out}} \right\}$$

- $\delta_{\lambda \rightarrow \lambda', \tau}^{\rho}$ : turning proportion
- free-flow: full local group proceeds
- congestion: demand-proportional supply distribution

# Actual transition flow

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- candidate inflow to stream  $\lambda$  during interval  $\tau$

$$S_{\lambda,\tau} = \sum_{\lambda' \in \Phi_{\lambda}^{\rho}} \sum_{\ell \in \mathcal{L}} S_{\lambda' \rightarrow \lambda, \tau}^{\ell}$$

- $\Phi_{\lambda}^{\rho}, \Theta_{\lambda}^{\rho}$ : set of up-/downstream adjacent streams on route  $\rho$

- actual transition flow

$$G_{\lambda \rightarrow \lambda', \tau}^{\ell} = \begin{cases} S_{\lambda \rightarrow \lambda', \tau}^{\ell} & \text{if } S_{\lambda', \tau} \leq R_{\lambda', \tau} \\ \zeta_{\lambda \rightarrow \lambda', \tau}^{\ell} R_{\lambda', \tau} & \text{otherwise} \end{cases}$$

- congestion: demand-proportional supply

$$\zeta_{\lambda \rightarrow \lambda', \tau}^{\ell} = \frac{S_{\lambda \rightarrow \lambda', \tau}^{\ell}}{S_{\lambda', \tau}}$$

# Propagation model

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- continuity equation  $\forall \tau \in \mathcal{T}, \forall \lambda \in \Lambda, \forall \ell \in \mathcal{L}$

$$M_{\lambda, \tau+1}^{\ell} = M_{\lambda, \tau}^{\ell} + \sum_{\lambda' \in \Phi_{\lambda}^{\rho \ell}} G_{\lambda' \rightarrow \lambda, \tau}^{\ell} - \sum_{\lambda'' \in \Theta_{\lambda}^{\rho \ell}} G_{\lambda \rightarrow \lambda'', \tau}^{\ell} + W_{\lambda, \tau}^{\ell}$$

– source/sink term



# Specification: Pedestrian fundamental diagram

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- specification inspired by research at HKU [WLC<sup>+</sup>10, XW15]
- stream-based fundamental diagram (SbFD)

$$V_\lambda = V_f \cdot \exp \left\{ -\vartheta \left( \frac{M_\xi}{A_\xi} \right)^2 \right\} \prod_{\lambda' \in \Lambda_\xi} \exp \left( -\beta (1 - \cos \varphi_{\lambda, \lambda'}) \frac{M_{\lambda'}}{A_\xi} \right)$$

- isotropic reduction (Drake, 1967)
- reduction due to pair-wise interaction of streams  
 $\varphi_{\lambda, \lambda'}$ : intersection angle between streams  $\lambda, \lambda'$

- state-of-the-practice: Weidmann, 1992 [Wei92]

$$V_\lambda = V_f \left\{ 1 - \exp \left[ -\gamma \left( \frac{A_\xi}{M_\xi} - \frac{1}{k_{\text{jam}}} \right) \right] \right\}$$

# Specification: Turning proportions

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Potential field-based model [GHW11, HBFM14]

- route-specific potential  $P_{\nu,\tau}^{\rho}$ 
  - e.g.  $P_{\nu,\tau}^{\rho} \sim$  shortest path distance from node  $\nu$  to area  $\xi_{\rho}^d$  along route  $\rho$  for traffic conditions prevalent during interval  $\tau$
- turning proportions ( $\lambda' \in \Theta_{\lambda}^{\rho}$ )
  - logit-type model with weight  $\mu$

$$\delta_{\lambda \rightarrow \lambda', \tau}^{\rho} = \frac{\exp\{-\mu P_{\nu_{\lambda'}, \tau}^{\rho}\}}{\sum_{\lambda'' \in \Theta_{\lambda}^{\rho}} \exp\{-\mu P_{\nu_{\lambda''}, \tau}^{\rho}\}}$$

# Calibration

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- maximum likelihood estimation
  - $\theta$ : unknown parameter vector
  - pedestrian  $i = \{1, \dots, N\}$ 
    - ▶  $tt_i^{\text{obs}}$ : observed travel time
    - ▶  $f_i^{\text{est}}(tt|\mathbf{X}, \theta)$ : estimated travel time probability density

$$\hat{\theta} = \arg \max \tilde{\mathcal{L}}(\mathbf{tt}_{\text{obs}}|\mathbf{X}, \theta)$$

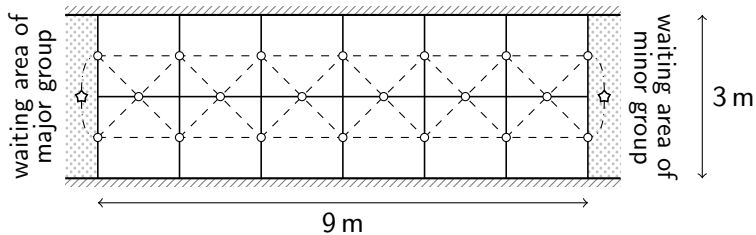
with

$$\tilde{\mathcal{L}}(\mathbf{tt}_{\text{obs}}|\mathbf{X}, \theta) = \sum_{i=1}^N \log \left( f_i^{\text{est}}(tt_i^{\text{obs}}|\mathbf{X}, \theta) \right)$$

- optimization algorithm: derivative-free trust-region method with random sampling of initial parameters [Pow09]

# Counter-flow experiment (Wong et al., 2010)

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## Counter-flow experiment: Observed speeds

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Exp.	major group		minor group	
#84	87 ped	$1.08 \pm 0.15$ m/s	–	–
#85	79	$1.19 \pm 0.13$	9 ped	$0.80 \pm 0.14$ m/s
#86	68	$0.90 \pm 0.10$	18	$0.74 \pm 0.15$
#87	61	$0.82 \pm 0.06$	26	$0.67 \pm 0.10$
#88	53	$0.83 \pm 0.09$	30	$0.79 \pm 0.15$
#89	44	$0.79 \pm 0.10$	44	$0.79 \pm 0.18$

Extracted from Wong et al., 2010 [WLC<sup>+</sup>10]



## Counter-flow experiment: Results

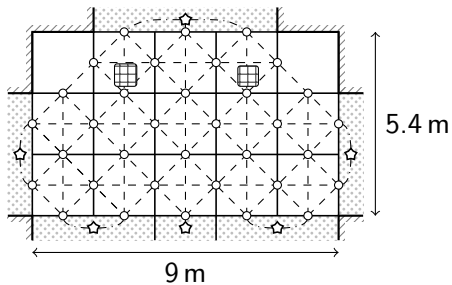
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	Zero-Model	Drake	SbFD	Weidmann
$\tilde{\mathcal{L}}_{85,87}^{\text{calib}}$	-416.9	-374.0	-348.2	-360.7
$V_f$ [m/s]	1.166	1.170	1.115	1.169
$\mu$ [-]	1.43	12.15	10.18	14.84
$\vartheta$ [m <sup>4</sup> ]		0.078	0.001	
$\beta$ [m <sup>2</sup> ]			0.210	
$\gamma$ [m <sup>-2</sup> ]				4.92
$k_j$ [m <sup>-2</sup> ]				6.58
$\tilde{\mathcal{L}}_{84}^{\text{valid}}$	-175.6	-166.2	-151.7	-170.1
$\tilde{\mathcal{L}}_{86}^{\text{valid}}$	-188.9	-182.6	-173.7	-196.7
$\tilde{\mathcal{L}}_{88}^{\text{valid}}$	-198.1	-189.3	-178.0	-213.7
$\tilde{\mathcal{L}}_{89}^{\text{valid}}$	-227.1	-201.4	-194.4	-223.3

(SbFD also significantly better at aggregate level – not shown)

# Cross-flow experiment (Plaue et al., 2014)

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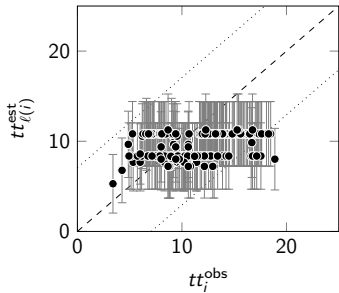
## Cross-flow experiment: Results

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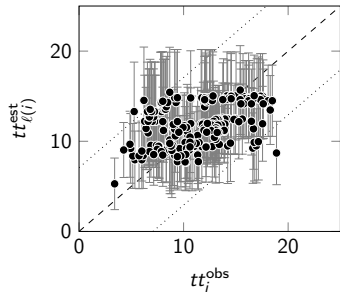
	Zero-Model	Drake	SbFD	Weidmann
$\tilde{\mathcal{L}}$	-578.0	-547.5	-527.3	-545.4
$V_f$ [m/s]	1.307	1.308	1.308	1.332
$\mu$ [-]	1.16	1.39	2.64	2.05
$\vartheta$ [m <sup>4</sup> ]		0.139	0.143	
$\beta$ [m <sup>2</sup> ]			0.300	
$\gamma$ [m <sup>-2</sup> ]				1.76
$k_j$ [m <sup>-2</sup> ]				5.99

Aggregate route travel times:

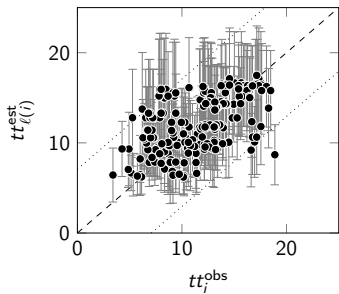
	$N_{\text{ped}}$	$tt_{\text{obs}}$	$tt_{\text{zero}}$	$tt_{\text{drake}}$	$tt_{\text{sbfd}}$	$tt_{\text{weid}}$
W $\rightarrow$ E	118	12.4	10.8	13.3	12.6	14.0
N $\rightarrow$ S	46	10.6	8.4	10.0	10.9	9.9



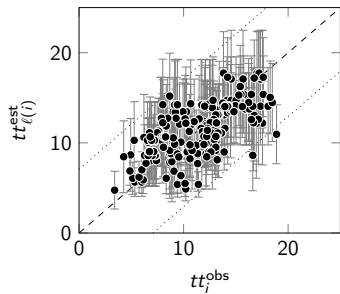
(a) Zero-Model ( $L^2$ -error: 53.3 s)



(b) Drake ( $L^2$ -error: 47.6 s)

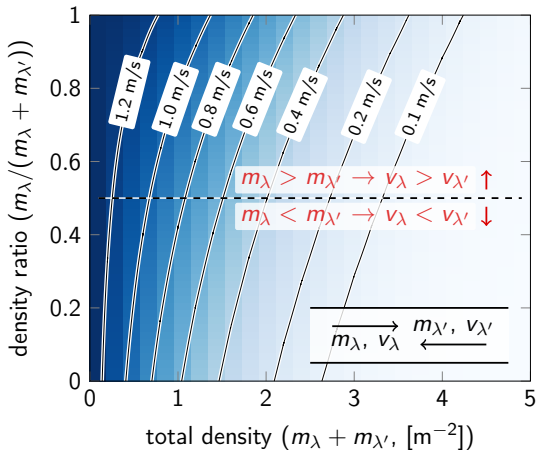


(c) Weidmann ( $L^2$ -error: 47.4 s)



(d) SbFD ( $L^2$ -error: 39.2 s)

# Illustration: Walking speed in counter-flow



$$\lambda \in \Lambda_\xi:$$

$$m_\lambda = M_\lambda / A_\xi$$

$$v_\lambda = V_\lambda / V_f$$

Parameters:

$$V_f = 1.308 \text{ m/s}$$

$$\vartheta = 0.143 \text{ m}^4$$

$$\beta = 0.300 \text{ m}^2$$

(Berlin data set)

## Concluding remarks

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- macroscopic model for congested, multi-directional flow
- explicit consideration of anisotropy
  - stream-based fundamental diagram
- calibration and validation using MLE
  - counter- and cross-flow experiments (Hong Kong and Berlin)
- future work
  - improvement in specification (e.g. fundamental diagram)
  - phenomena of self-organization
  - applications within DTA-framework, demand estimation

# Thank you

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hEART 2015:

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