HKSTS Post-Conference Workshop

A macroscopic loading model for dynamic, multi-directional and congested pedestrian flows

Flurin S. Hänseler, William H.K. Lam, Michel Bierlaire

Hong Kong, December 16, 2014
Modeling of pedestrian behavior

Levels of pedestrian behavior [HB04]

- **strategical**: choice of departure time and activity pattern
- **tactical**: choice of activity scheduling and route
- **operational**: en-route path choice, walking behavior
Modeling of pedestrian behavior

Levels of pedestrian behavior [HB04]

- **strategical**: choice of departure time and activity pattern
- **tactical**: choice of activity scheduling and route
- **operational**: en-route path choice, walking behavior

This work:

- focus on operational level
- interaction across levels kept in mind

→ development of macroscopic network loading model
Unsteady, anisotropic and congested flow

Figure: Passageway in Central Station (MTR), Hong Kong
Aggregate pedestrian flow models

- link transmission models/queuing networks [CS94, Løv94]
  - interaction between links neglected
- cell transmission models [ASKT07, GHW11, HBFM14]
  - inherent assumption of isotropy
- continuum models [Hug02, HWZ⁺09, HvWKDD14]
  - expensive, particularly for multi-class applications
Aggregate pedestrian flow models

- link transmission models/queuing networks \([\text{CS94, Løv94}]\)
  - interaction between links neglected
- cell transmission models \([\text{ASKT07, GHW11, HBFM14}]\)
  - inherent assumption of isotropy
- continuum models \([\text{Hug02, HWZ}^+09, HvWKDD14}]\)
  - expensive, particularly for multi-class applications

Idea: ‘cell-based link-transmission model’
\(\rightarrow\) stream-based pedestrian fundamental diagram \([\text{WLC}^+10, XW14}\]
Decomposition of pedestrian flow into streams

- contiguous area $\xi$ of size $A_\xi$
- each stream $\sigma \in \Sigma_\xi$ characterized by
  - direction (exogenous)
  - area occupation $m_\sigma^\xi$
  - uni-directional velocity $V_\xi^\sigma$

Stream-based fundamental diagram $f(m)$
- bounded velocity: $0 \leq V_\xi^\sigma \leq V_f$, $\forall \sigma \in \Sigma_\xi$
- concave density-speed relation: $\partial V_\xi^\sigma / \partial m_\sigma^\xi' \leq 0$, $\forall \sigma, \sigma' \in \Sigma_\xi$
- speed and density vectors: $v^\Sigma_\xi = [V_\sigma / V_f]$, $m^\Sigma_\xi = [m_\sigma]$

$$v_\xi^\sigma = f_\xi^\sigma (m^\Sigma_\xi, v^\Sigma_\xi; A_\xi)$$

- several specifications available [WLC$^+$10, XW14, FL15]
Time, space and demand

- time interval $\tau \in \mathcal{T}$
  - choice of $\Delta t = |\tau|$ crucial
- cell $\xi \in \mathcal{X}$
  - convex space partitioning
- route $\rho \in \mathcal{R}$
  - origin/destination cell: $\xi^o_\rho, \xi^d_\rho$
  - accessible network: $\mathcal{X}_\rho \subset \mathcal{X}$
- pedestrian group $\ell \in \mathcal{L}$
  - departure interval $\tau_\ell$
  - group size $x_\ell$
  - route $\rho_\ell$
Pedestrian walking network

Pedestrian network $\mathcal{G} = \{\mathcal{N}, \Lambda\}$

- $\mathcal{N}$: set of nodes $\nu \in \mathcal{N}$ connecting adjacent cells
- $\Lambda$: set of links $\lambda \in \Lambda$, $\lambda : \nu^o_\lambda \rightarrow \nu^d_\lambda$
  - $\Lambda_\xi$: set of links associated with cell $\xi$
  - $\Lambda^\sigma_\xi$: set of links associated with stream $\sigma$ in cell $\xi$
  - $\Phi_\lambda^\rho$, $\Theta_\lambda^\rho$: set of up-/downstream adjacent links on route $\rho$
  - $L_\lambda > 0$: length of link $\lambda$, $L_{\text{min}} = \min_{\lambda \in \Lambda} L_\lambda$

• links: uni-directional flow  
• cells: range of interaction  
• nodes: flow valves/splitters
State variables and hydrodynamic flow

• state variables
  – \( m^\ell_{\lambda,\tau} \): size of group \( \ell \) on link \( \lambda \) during interval \( \tau \)
  – aggregation per link: \( m_{\lambda,\tau} = \sum_{\ell \in L} m^\ell_{\lambda,\tau} \)
  – aggregation per stream: \( m^\sigma_{\xi,\tau} = \sum_{\lambda \in \Lambda^\sigma} m_{\lambda,\tau} \)
  – aggregation per cell: \( m_{\xi,\tau} = \sum_{\lambda \in \Lambda^\xi} m_{\lambda,\tau} \)

• ‘hydrodynamic flow’ on link \( \lambda \in \Lambda^\sigma \) during interval \( \tau \)
  – for uni-directional stream: flux = density \( \times \) velocity
  – \( \Delta Q_{\lambda,\tau} = L_{\min}/L_{\lambda} m_{\lambda,\tau} f_\sigma(m^\Sigma_{\xi,\tau}) \) if \( \Delta t = \Delta L/v_f \) (CFL)
  – reaches maximum \( \Delta Q^{\text{opt}}_{\lambda,\tau} \) at \( m^{\text{opt}}_{\lambda,\tau} \)
Hydrodynamic flow capacities

- hydrodynamic inflow capacity

\[ \Delta Q^{\text{in}}_{\lambda,\tau} = \begin{cases} \Delta Q^{\text{opt}}_{\lambda,\tau}, & \text{if } m_{\lambda,\tau} \leq m^{\text{opt}}_{\lambda,\tau} \\ \Delta Q_{\lambda,\tau}, & \text{otherwise} \end{cases} \]

- hydrodynamic outflow capacity

\[ \Delta Q^{\text{out}}_{\lambda,\tau} = \begin{cases} \Delta Q_{\lambda,\tau}, & \text{if } m_{\lambda,\tau} \leq m^{\text{opt}}_{\lambda,\tau} \\ \Delta Q^{\text{opt}}_{\lambda,\tau}, & \text{otherwise} \end{cases} \]
Link model

- receiving capacity on link $\lambda$ during interval $\tau$

$$R_{\lambda,\tau} = \Delta Q_{\lambda,\tau}^{\text{in}}$$

- sending capacity of group $\ell$ on link $\lambda$ during interval $\tau$

$$S_{\lambda \to \lambda',\tau}^{\ell} = \min \left\{ m_{\lambda,\tau}^{\ell}, \frac{m_{\lambda,\tau}^{\ell}}{m_{\lambda,\tau}} \Delta Q_{\lambda,\tau}^{\text{out}} \right\}$$

- $\delta_{\lambda \to \lambda',\tau}$: turning proportions
- free-flow: full local group proceeds
- congestion: demand-proportional supply distribution
Gate model

• candidate inflow to link $\lambda$ during interval $\tau$

$$S_{\lambda,\tau} = \sum_{\lambda' \in \Phi^\rho_\lambda} \sum_{\ell \in \mathcal{L}} S^\ell_{\lambda' \rightarrow \lambda,\tau}$$

• transition flow

$$Y^\ell_{\lambda \rightarrow \lambda',\tau} = \begin{cases} S^\ell_{\lambda \rightarrow \lambda',\tau} & \text{if } S_{\lambda',\tau} \leq R_{\lambda',\tau} \\ \zeta^\ell_{\lambda \rightarrow \lambda',\tau} R_{\lambda',\tau} & \text{otherwise} \end{cases}$$

– congestion: demand-proportional supply

$$\zeta^\ell_{\lambda \rightarrow \lambda',\tau} = \frac{S^\ell_{\lambda \rightarrow \lambda',\tau}}{S_{\lambda',\tau}}$$
Propagation model

- continuity equation \( \forall \tau \in \mathcal{T}, \forall \lambda \in \Lambda, \forall \ell \in \mathcal{L} \)

\[
m_{\lambda, \tau+1}^\ell = m_{\lambda, \tau}^\ell + \sum_{\lambda' \in \Phi_\lambda^\rho_\ell} Y_{\lambda' \rightarrow \lambda, \tau}^\ell - \sum_{\lambda'' \in \Theta_\lambda^\rho_\ell} Y_{\lambda \rightarrow \lambda'', \tau}^\ell + W_{\lambda, \tau}^\ell
\]

- source/sink term
Specification: En-route path choice model

Potential field-based model (see e.g. [GHW11, HBFM14])

- route-specific node potential $P_{\nu, \tau}^\rho$
  - e.g. $P_{\nu, \tau}^\rho \sim$ shortest path distance from node $\nu$ to cell $\xi_{\rho}^d$ along route $\rho$ for traffic conditions prevalent during interval $\tau$

- logit model ($\lambda' \in \Theta_\lambda^\rho$)

$$
\delta_{\lambda \rightarrow \lambda', \tau}^\rho = \frac{\exp\{-P_{\nu, \tau}^\rho\}}{\sum_{\lambda'' \in \Theta_\lambda^\rho} \exp\{-P_{\nu, \tau}^\rho\}}
$$
Specification: Pedestrian fundamental diagram

• model by Xie and Wong, 2014 \([XW14]\)

\[
\nu_\sigma = \exp \left\{ -\vartheta \left( \frac{m_\xi}{A_\xi} \right)^2 \right\} \prod_{\sigma' \in \Sigma_\xi} g(m_\sigma, m_{\sigma'}, \nu_\sigma, \nu_{\sigma'}, \varphi_{\sigma,\sigma'})
\]

- isotropic reduction in speed
- reduction due to pair-wise interaction of streams

• next slide: illustration for counter-flow \((\alpha = 1.0, \beta = 0.132, \vartheta = 0.065 \text{ m}^2; \text{see Xie and Wong, 2014, for details})\)
Specification: Pedestrian fundamental diagram

![Diagram showing the relationship between density ratio and normalized speed of stream 1. Key points include: 
- \( \rho_1 > \rho_2 \rightarrow v_1 > v_2 \) (upward direction)
- \( \rho_1 < \rho_2 \rightarrow v_1 < v_2 \) (downward direction)

Total density is given by \( \rho_1 + \rho_2 \), and is in units of \( \text{m}^{-2} \). Density ratio is \( \frac{\rho_1}{\rho_1 + \rho_2} \). Normalized speed of stream 1 is \( \frac{v_1}{v_f} \).]
Final remarks

Conclusions:

- need for accurate yet affordable network loading model
- pedestrian flow often unsteady, anisotropic and congested
- idea: ‘cell-based link-transmission model’
  - key: stream-based pedestrian fundamental diagram

Next steps:

- implementation (almost complete)
- consideration of test cases/case study
- calibration
HKSTS Post-Conference Workshop:
A macroscopic loading model for dynamic, multi-directional and congested pedestrian flows
Flurin S. Hänseler, William H.K. Lam, Michel Bierlaire

Financial support by SNSF, SBB-CFF-FFS, EPFL and PolyU is gratefully acknowledged.

– flurin.haenseler@epfl.ch

G. Flötteröd and G. Lämmel. 
Bidirectional pedestrian fundamental diagram. 

Collection, spillback, and dissipation in pedestrian evacuation: 
A network-based method. 
Bibliography III


R. L. Hughes.
A continuum theory for the flow of pedestrians.

S. P. Hoogendoorn, F. L. M. van Wageningen-Kessels, W. Daamen, and D. C. Duives.
Continuum modelling of pedestrian flows: From microscopic principles to self-organised macroscopic phenomena.

G. G. Løvås.
Bidirectional pedestrian stream model with oblique intersecting angle.

S. Xie and S. C. Wong.
A Bayesian Inference Approach to the Development of a Multidirectional Pedestrian Stream Model.